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Decision Making Under Conflicting Criteria In Pension Valuations: An Expected Utility Model

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Abstract

Many of the criteria used by actuaries when selecting assumptions for pension plan valuations often conflict. As a result, actuaries must weigh the various costs and benefits associated with a particular set of assumptions. We use expected utility theory to model the process of choosing actuarial assumptions when faced with potentially conflicting criteria. The three criteria considered are prudence, best estimate, and conservatism.

The actual contribution chosen by the actuary is found to depend on the contribution level that triggers a red flag with respect to tax deductibility. If this level is relatively low, the actuary chooses a high contribution that gives weight to each criterion, incorporating the risk of a penalty by tax authorities. If the tax deductible trigger is of an intermediate level, the actuary chooses this level exactly and insulates the plan from tax scrutiny; if the level is high, the utility maximizing contribution is below that level.

Key words and phrases: *actuarial assumptions, contributions, prudence, best estimate, conservatism*

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1 Introduction

In the United States defined benefit pension plan valuations must be performed periodically by the plan actuary. As noted by Shapiro (1990), however, many of the criteria underlying the choice of assumptions for these valuations often conflict. This observation is not surprising—while the ultimate choice of actuarial assumptions rests with the actuary, the actuary must balance his or her preferences and judgments against those of a number of self-interest groups, including employees, the employer, and tax and labor authorities.

The main purpose of this paper is to describe a methodology that can be used by actuaries to resolve problems associated with conflicting assumptions. The methodology uses expected utility theory[1] to model the process of choosing actuarial assumptions when faced with potentially conflicting criteria. To keep the model simple, only three criteria are considered: prudence, best estimate, and conservatism. This is a first attempt at modeling actuarial decision making.

2 The Criteria

The first criterion, prudence, is satisfied if the contribution that results is in the range of prudent contributions (that is, contributions that would be developed by prudent actuaries in similar circumstances). The context considered is the one where tax authorities are concerned with the possibility of overfunding to escape current taxation and consequently define a deductible contribution as one that is below a certain upper limit.[2] Because excise taxes and other penalties may result if deductions are taken for nondeductible contributions, one limit on the range of prudent assumptions is that such assumptions produce a safe

[1]This is the expected utility theory developed by von Neumann and Morgenstern (1947). See Schoemaker (1982) for a discussion of the pros and cons of expected utility theory.

[2]In the case where, for example, the Pension Benefit Guaranty Corporation (PBGC) is concerned with the adequacy of plan funding, the range defined by a lower limit on the contribution may be equally important. In this model concerns about plan solvency are captured by the conservatism criterion. It is assumed that plan solvency is in the interest of the actuary and the plan sponsor and is imbedded in the utility function. Further pressure by the PBGC is not considered at this stage.

Editor's note: The PBGC is a self-financed public corporation that administered the pension benefits insurance program for qualified plans in the United States. See, for example, McGill (1984, Chapter 24) for more on the PBGC.

harbor contribution.[3] It is assumed that whether the plan contribution satisfies a safe harbor is not of concern, however, if the plan can meet a facts and circumstance test.[4] Moreover, this test is characterized in terms of the relationship between the actual contribution and the contribution that would have funded the plan accurately.

The actual contribution to the plan is denoted as $\hat{C}$; the contribution that would have funded the plan accurately is denoted C;[5] and the contribution that triggers a red flag with respect to deductibility is denoted $\hat{C}^*$. It is assumed the authorities do not investigate the assumptions to determine if they are appropriate unless $\hat{C} > \hat{C}^*$. If $\hat{C} > \hat{C}^*$, then authorities determine whether $\hat{C} - C > D$, where D is an acceptable deviation.

If $\hat{C} > \hat{C}^*$ and $\hat{C} - C > D$, the actuary is penalized.[6] The penalty is modeled here as a monetary penalty of P dollars. This can represent anything ranging from a fine to damage to one's reputation that would reduce earning power. An excise tax may be levied upon the plan sponsor that may have repercussions for the actuary in terms of compensation, job security, or future job prospects. The actuary may face a lawsuit and possible loss of accreditation.[7] Furthermore, it is assumed that the damage to the actuary's reputation also leaves the actuary with a lower level of utility for any given wealth level in the event that the actuary is penalized. The prudent actuary's rule is characterized by a variable, ρ (the penalty), which takes the following values:

$$\rho = \begin{cases} 0 & \text{if } \hat{C} \leq \hat{C}^* \text{ or if } \hat{C} > \hat{C}^* \text{ and } \hat{C} - C \leq D; \\ P & \text{if } \hat{C} > \hat{C}^* \text{ and } \hat{C} - C > D. \end{cases}$$

Deductibility raises a perplexing problem. Solvency is one of the primary considerations underlying the funding of a pension plan, but the taxing authority may not explicitly allow a contingency reserve to protect this solvency. Additionally, as modeled above, there may be an arbitrary limit to the maximum deductible contribution to a plan.

[3] This would be the case, for example, if the contribution were no larger than if all the assumptions used were the most generous allowed under IRS standards.

[4] This test is satisfied if the facts and circumstances surrounding the plan justify the assumptions.

[5] The quantity C is known after the actual plan experience has unfolded.

[6] In this model, we assume that the authorities always will discover the fact that $\hat{C} > \hat{C}^*$ and $\hat{C} - C > D$. The authors currently are investigating a model where the occurrence of this event is a random variable.

[7] The simplifying assumption is made that the size of the monetary penalty is not a function of the size of the contribution or the experience of the plan. It is possible that these and other factors may have an impact on the size of the penalty, in which case the penalty would not be a constant.

Because of adverse experience, however, the deductible contribution may not be sufficient to keep the plan solvent.

The second criterion, conservatism, follows from the concern about plan solvency. The actuary (as well as the plan sponsor) prefers to keep the probability that the plan has actuarial losses to a minimum. Therefore, the contribution is conservative if $\Pr[\hat{C} < C] < \epsilon$ or, equivalently, $\Pr[C > \hat{C}] < \epsilon$, where ϵ is the tolerance level for conservatism (that is, if the probability of actuarial losses is below ϵ). The actuary uses only his or her beliefs about the distribution of C to determine $\Pr[C > \hat{C}]$. It is assumed that the actuary believes that C has a cumulative distribution function $F(C)$. This is the actuary's subjective belief about the distribution of C and is necessary if expected utility theory is to be used. Therefore, the actuary concerned about conservatism prefers a contribution, $\hat{C}$, for which $1 - F(\hat{C}) < \epsilon$.

The final criterion incorporated in the model is the best estimate. For this analysis, best estimate is interpreted to mean the estimate for which the expected value of the absolute deviation of the actual value from the estimate is minimized, as suggested by Anderson (1985, p. 110). Again, the plan experience is characterized in terms of C, the contribution that would have funded the plan accurately. The actuary's best estimate of C is μ, i.e., the actuary believes that if C^E is defined as any estimate of C, then $E[|C^E - C|]$ is minimized when the estimate $C^E = \mu$.

3 The Expected Utility Model

At this point the actuary's decision process is modeled explicitly using the theory of expected utility. We assume that the actuary obtains utility from three sources: (i) wealth, (ii) the appropriateness of his or her assumptions, and (iii) plan solvency. The appropriateness of the assumptions may affect wealth through a potential penalty. Apart from that, the actuary simply feels good about making an appropriate estimate and enjoys positive recognition from his or her employer.[8] The two aspects of the appropriateness of the assumptions that are modeled here are accuracy and conservatism. The accuracy (or inaccuracy) of the assumptions is measured by $|\hat{C} - C|$, with smaller values representing greater accuracy. The larger the value of $|\hat{C} - C|$, the lower the actuary's utility, and this inaccuracy is weighted by a constant λ in

[8] We assume that neither the actuary nor the employer is motivated to overfund the plan for the specific purpose of deferring taxation.

the utility function. But due to concerns about solvency and conservatism, the actuary's utility is reduced further when there are actuarial losses. Therefore, whenever $\hat{C} < C$, the actuary has additional disutility equal to a constant γ. The actuary's disutility due to actuarial losses is characterized by a variable Γ that takes the following values:

$$\Gamma = \begin{cases} 0 & \text{if } \hat{C} \geq C; \\ \gamma & \text{if } \hat{C} < C. \end{cases}$$

The actuary's utility function U is given by $U(W-\rho, |\hat{C}-C|, \Gamma)$, where W represents his or her wealth before the penalty is determined.[9] Assuming additivity,[10] this utility function can be written more explicitly as:

$$U(W-\rho, |\hat{C}-C|, \Gamma) = u_s(W-\rho) - \lambda|\hat{C}-C| - \Gamma \tag{1}$$

where $u_s(W-\rho)$ is the utility of wealth in state of nature s. We assume there are two states of nature: $s = 0$ represent the state where the actuary is not penalized, and $s = 1$ represent the state where the actuary is penalized. For each state s, utility increases with wealth (so $u_s' > 0$), and risk aversion with respect to wealth implies that $u_s'' < 0$. We assume that $u_0(w) > u_1(w)$ for any given wealth level w. This implies that the actuary suffers more than just a monetary fine when penalized by the authorities. Once sanctioned by the authorities, the actuary is worse off, in utility terms, at any given wealth level.

In choosing the contribution, $\hat{C}$, the actuary maximizes his or her expected utility where the expectation is taken over the distribution $F(C)$. So, the actuary solves for the $\hat{C}$ that maximizes the right hand side of the equation:

$$\begin{aligned} M_1 &= \max_{\hat{C}} E[u_s(W-\rho) - \lambda(|\hat{C}-C|) - \Gamma] \\ &= \max_{\hat{C}} E[u_s(W-\rho)] - \lambda E[|\hat{C}-C|] - \gamma(1-F(\hat{C})) \end{aligned} \tag{2}$$

where $0 \leq \rho \leq P$.

Each of the three terms on the right hand side of equation (2) represents one of the criteria that the actuary uses in making the funding decision. The second term represents the best estimate criterion. The

[9] A more general representation of the actuary's utility function is $U(Y(W,\rho), |\hat{C}-C|, \Gamma)$, where Y and W represent wealth after and before the penalty, respectively, and ρ represents an arbitrary penalty function.

[10] The authors are currently investigating a more general formulation of this utility function. The simplified version in the text, however, is sufficient to convey the essence of the model.

contribution that minimizes this term (and hence maximizes its contribution to expected utility) is the best estimate, μ. But the best estimate may not be the optimal contribution for the plan due to the offsetting effects of the two other criteria. The third term, representing conservatism, is the probability of an actuarial loss weighted by the disutility such a loss brings. This term is subtracted from expected utility. To maximize this term's contribution to expected utility, the probability of an actuarial loss $1 - F(\hat{C})$ must be minimized. This provides an incentive for the actuary to choose a contribution that is above the best estimate, i.e., to play it safe. On the other hand, the first term is the expected utility of wealth which is dependent upon whether a penalty is received from the authorities for choosing a contribution that may not be deductible.

The actuary has an incentive to choose a contribution that is higher than the best estimate because of concerns about solvency. But government officials may choose to interpret this behavior as an attempt to avoid current taxation. This exerts pressure on the actuary to choose a lower contribution. This first term is maximized when the chance of receiving a penalty and the subsequent damage to the actuary's reputation is eliminated (that is, when the contribution is below the authorities' upper bound). The relative weight with which each of the three criteria enters expected utility determines the trade-off that must be made. Other factors that determine this trade-off are initial wealth, W, the size of the acceptable deviation, D, and the size of the penalty, P. A final factor is the actuary's perception of the distribution of C, in particular, how probable it is that the deviation will be greater than zero and/or greater than D. We now analyze this more formally.

Let us assume F is such that $\Pr[C_L \leq C \leq C_U) = 1$. There are two possible ranges within which the chosen contribution, $\hat{C}$, can fall, the prudent range where $\hat{C} \in [C_L, \hat{C}^*]$ or the other range where $\hat{C} \in [\hat{C}^*, C_U]$.[11] A penalty is imposed when $\hat{C} \in (\hat{C}^*, C_U]$ and $\hat{C} - C > D$. On the other hand, when $\hat{C} \in [C_L, \hat{C}^*]$ there is no possibility of receiving a penalty. Therefore, analysis of the decision requires that the maximization problem given by equation (2) be separated into two steps because the expected utility function is discontinuous at the point $\hat{C} = \hat{C}^*$.

There are two expected utility functions that must be considered, one that applies for values of $\hat{C} \leq \hat{C}^*$ and one that applies for values of $\hat{C} > \hat{C}^*$. We will graph both of these expected utility functions over the entire range of potential contributions, $\hat{C}$, and illustrate how the

[11] We assume that $[C_L, \hat{C}^*]$ is not empty.

actuary's choice is affected by the tax authorities' choice of an upper limit on the prudent range, $\hat{C}^*$.

The expected utility function that pertains to the range of contributions $\hat{C} \leq \hat{C}^*$ is:

$$\begin{aligned} E[U_{\hat{C}\leq\hat{C}^*}] &= u_0(W) - \lambda E[|\hat{C} - C|] - \gamma(1 - F(\hat{C})) \\ &= u_0(W) - \lambda\left[\int_{C_L}^{\hat{C}} (\hat{C} - C)dF(C) + \int_{\hat{C}}^{C_U} (C - \hat{C})dF(C)\right] \\ &\quad - \gamma(1 - F(\hat{C})), \end{aligned} \tag{3}$$

while for the range of contributions $\hat{C} > \hat{C}^*$:

$$\begin{aligned} E[U_{\hat{C}>\hat{C}^*}] &= u_1(W - P)F(\hat{C} - D) + u_0(W)(1 - F(\hat{C} - D)) \\ &\quad - \lambda E[|\hat{C} - C|] - \gamma(1 - F(\hat{C})) \\ &= u_1(W - P)F(\hat{C} - D) + u_0(W)(1 - F(\hat{C} - D)) \\ &\quad - \lambda\left[\int_{C_L}^{\hat{C}} (\hat{C} - C)dF(C) + \int_{\hat{C}}^{C_U} (C - \hat{C})dF(C)\right] \\ &\quad - \gamma(1 - F(\hat{C})). \end{aligned} \tag{4}$$

Comparison of equations (3) and (4) indicates that for any given value of $\hat{C}$ greater than $C_L + D$, the expression on the left hand side of equation (3) is greater than that of equation (4) because $u_0(W) > u_1(W - P)$.[12] Furthermore, the gap between these two functions increases as $\hat{C}$ increases because more weight is given to $u_1(W - P)$ as $F(\hat{C} - D)$ increases.

Next, the contribution that provides the maximum level of expected utility must be determined. Differentiating[13] equations (3) and (4) with respect to $\hat{C}$ yields equations (5) and (6), respectively. Setting equations (5) and (6) each equal to zero gives the conditions for the maximum values of $E[U_{\hat{C}\leq\hat{C}^*}]$ and $E[U_{\hat{C}>\hat{C}^*}]$, respectively:

$$\begin{aligned} \frac{d}{d\hat{C}}E[U_{\hat{C}\leq\hat{C}^*}] &= -\lambda(2F(\hat{C}) - 1) + \gamma\frac{d}{d\hat{C}}F(\hat{C}) \\ &= 0 \end{aligned} \tag{5}$$

[12]Only contributions that are greater than $C_L + D$ are potential choices as the maximization problem is established. Intuitively, as long as the best estimate, μ, is greater than $C_L + D$ (as will be assumed), contributions that are less than $C_L + D$ will not be chosen because $\hat{C} - C$ cannot be greater than D, implying that there are no potential penalties in this range and, therefore, no benefits to be gained from reducing the contribution below $C_L + D$. This further implies that $\hat{C} - D > C_L$ for any possible solution, so $F(\hat{C} - D) > 0$.

[13]We now assume that $F(C)$ is differentiable in the relevant regions.

$$\begin{aligned}\frac{d}{d\hat{C}}E[U_{\hat{C}>\hat{C}*}] &= (u_1(W-P)-u_0(W))\frac{d}{d\hat{C}}F(\hat{C}-D) \\ &\quad -\lambda(2F(\hat{C})-1)+\gamma\frac{d}{d\hat{C}}F(\hat{C}) \qquad (6) \\ &= 0.\end{aligned}$$

A useful point of comparison is the best estimate μ, which is obtained by minimizing $E[|\hat{C}-C|]$; that is, by solving:

$$M_2 = \min_{\hat{C}}\left[\int_{C_L}^{\hat{C}}(\hat{C}-C)dF(C)+\int_{\hat{C}}^{C_U}(C-\hat{C})dF(C)\right]. \qquad (7)$$

The first order condition for the problem of equation (7) is:

$$2F(\hat{C})-1=0. \qquad (8)$$

The value of $\hat{C}$ that solves equation (8) is the best estimate and is denoted μ, where μ is the median because $F(\mu)=1/2$.

4 An Example

For the purpose of example, assume that C is uniformly distributed on the interval $[C_L, C_U]$.[14] The best estimate is:

$$\mu = \frac{C_L+C_U}{2}. \qquad (9)$$

Furthermore, the contributions that maximize $E[U_{\hat{C}\le\hat{C}*}]$ and $E[U_{\hat{C}>\hat{C}*}]$, respectively, and the shapes of these expected utility functions can be obtained by substituting for $F(C)$ into equations (5) and (6).[15] Using equation (9), we have:

$$\begin{aligned}\frac{d}{d\hat{C}}E[U_{\hat{C}\le\hat{C}*}] &= \frac{\gamma-2-\lambda(\hat{C}-\mu)}{(C_U-C_L)} \qquad (10) \\ &= 0 \\ \frac{d}{d\hat{C}}E[U_{\hat{C}>\hat{C}*}] &= \frac{u_1(W-P)-u_0(W)}{(C_U-C_L)}+\frac{\gamma-2-\lambda(\hat{C}-\mu)}{(C_U-C_L)} \qquad (11) \\ &= 0.\end{aligned}$$

[14]This assumption is not meant to imply that this is the appropriate distribution for C, but is used to allow a clear characterization of the solution that may be obtained using this model.

[15]We assume that the contribution that maximizes $E[U_{\hat{C}\le\hat{C}*}]$ and the contribution that maximizes $E[U_{\hat{C}>\hat{C}*}]$ are elements of the interval $[C_L, C_U]$.

Both $E[U_{\hat{C}\leq\hat{C}^*}]$ and $E[U_{\hat{C}>\hat{C}^*}]$ are strictly concave, as the second derivative with respect to $\hat{C}$ of each is negative.

Next, let the solution to (10) be denoted $\hat{C}_3$ and the solution to (11) be denoted $\hat{C}_4$.[16] Then $E[U_{\hat{C}\leq\hat{C}^*}]$ is maximized at:

$$\hat{C}_3 = \mu + \frac{\gamma}{2\lambda} \tag{12}$$

and $E[U_{\hat{C}>\hat{C}^*}]$ is maximized at:

$$\hat{C}_4 = \mu + \frac{\gamma}{2\lambda} - \frac{u_0(W) - u_1(W-P)}{2\lambda}. \tag{13}$$

When there is no concern about a penalty by the authorities, as is the case when $\hat{C} \leq \hat{C}^*$, the optimal contribution, $\hat{C}_3$, is the best estimate, μ, plus a contingency reserve equal to one half of the relative disutility of insolvency (that is, disutility, γ, relative to the weight given to accuracy, λ). When there is the possibility of a penalty (because the contribution is not in the prudent range), then the optimal contribution, $\hat{C}_4$, is $\hat{C}_3$ reduced by one half of the relative disutility of being penalized (that is, the change in utility caused by a penalty, $u_0(W) - u_1(W-P)$, relative to the weight given to accuracy, λ).

The two expected utility functions, $E[U_{\hat{C}\leq\hat{C}^*}]$ and $E[U_{\hat{C}>\hat{C}^*}]$ now can be graphed over the range of potential contributions to illustrate how the value of $\hat{C}^*$ impacts the actuary's funding choice. The following characteristics of the expected utility functions have been determined from the above analysis. $E[U_{\hat{C}\leq\hat{C}^*}]$ is greater than $E[U_{\hat{C}>\hat{C}^*}]$ for any given value of $\hat{C}$, and the difference between these two functions increases as $\hat{C}$ increases. Both $E[U_{\hat{C}\leq\hat{C}^*}]$ and $E[U_{\hat{C}>\hat{C}^*}]$ are strictly concave, and $E[U_{\hat{C}\leq\hat{C}^*}]$ reaches its maximum value at a higher contribution level than $E[U_{\hat{C}>\hat{C}^*}]$ does (because $\hat{C}_3 > \hat{C}_4$).

Figure 1 is based on the foregoing observations. An important point on this graph is the lowest contribution level at which $E[U_{\hat{C}\leq\hat{C}^*}]$ is exactly equal to the maximum value of $E[U_{\hat{C}>\hat{C}^*}]$. This contribution level is denoted $\hat{C}_5$ (that is, $E[U_{\hat{C}\leq\hat{C}^*}(\hat{C}_5)] = E[U_{\hat{C}>\hat{C}^*}(\hat{C}_4)]$).[17] It is necessary

[16] Both of these points exist and are unique because the functions that are being maximized have been shown to be concave.

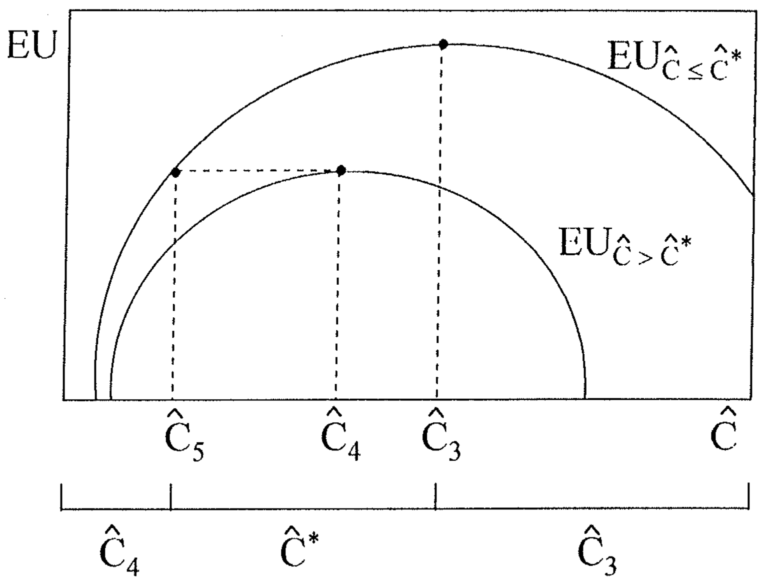
[17] Solving for $\hat{C}_5$ explicitly gives:

$$\hat{C}_5 = \hat{C}_4 - \frac{u_0(W) - u_1(W-P)}{2\lambda\mu + \gamma}(\hat{C}_4 - D - C_L).$$

It is possible that the value $\hat{C}_5$ is less than C_L, in which case it does not appear on the graph. The following analysis will make clear that if this is the case, the optimal contribution must be in the prudent range.

to know the value of $\hat{C}^*$ to determine which expected utility function is relevant over which range of contributions. $E[U_{\hat{C}\leq\hat{C}^*}]$ is the applicable expected utility function for all contributions below $\hat{C}^*$, and $E[U_{\hat{C}>\hat{C}^*}]$ is applicable for contributions above $\hat{C}^*$. If $\hat{C}^*$ is greater than $\hat{C}_3$, then $\hat{C}_3$ is the contribution that is chosen by the actuary because expected utility is maximized at that point. If $\hat{C}_5 < \hat{C}^* \leq \hat{C}_3$, then the optimal contribution is $\hat{C}^*$. If $\hat{C}^* = \hat{C}_5$, then the actuary is indifferent between $\hat{C}^*$ and $\hat{C}_4$. Finally, if $\hat{C}^* < \hat{C}_5$, then the optimal contribution is $\hat{C}_4$.

Figure 1
Optimum Expected Utilities



Further, Figure 1 indicates that when the upper bound on the prudent range is relatively high (that is, higher than the contribution that maximizes the expected utility in the absence of a penalty), then the constraint provided by the authorities is not binding and the actuary's choice ($\hat{C}_3$) is a trade-off between the criteria of best estimate and conservatism. When the upper bound on the prudent range is in some middle range (that is, $\hat{C}_5 < \hat{C}^* \leq \hat{C}_3$), then the actuary chooses the upper bound as the optimal contribution because it is preferable to avoid the possibility of a penalty. When the upper bound on the prudent range is relatively low (that is, below $\hat{C}_5$), then it is in the actuary's best interest to choose a contribution above the prudent range that makes a trade-off between the criteria of conservatism, prudence, and best estimate. In this case, the chosen contribution is $\hat{C}_4$. If $\hat{C}_5$ is less than C_L, then it is necessarily less than $\hat{C}^*$, implying that for any $\hat{C}^* \in [C_L, C_U]$ the optimal contribution, $\hat{C}$, is in the prudent range.

5 Summary

The purpose of this paper has been to explore the use of expected utility theory to model the process by which an actuary chooses the appropriate contribution for a pension plan. Because this is just a first attempt, however, only a simple expected utility model is used and only three criteria are considered: prudence, best estimate, and conservatism. Nonetheless, we are able to conceptualize the essence of some of the relationships.

Based on our model, the actual contribution chosen by the actuary depends on the contribution level that triggers a red flag with respect to tax deductibility. If this level is relatively low, the actuary chooses a high contribution that gives weight to each criterion, incorporating the risk of a penalty by tax authorities. If the tax deductible trigger is of an intermediate level, the actuary chooses this level exactly and insulates the plan from tax scrutiny; if the level is high, the utility maximizing contribution is below that level.

References

Anderson, A.W. *Mathematics of Pension Funding.* Brightwaters, NY: Windsor Press, Inc., 1985.

McGill, D.M. *Fundamentals of Private Pensions.* Homewood, Ill.: Irwin, 1984.

Schoemaker, P.J. "The Expected Utility Model: Its Variants, Evidence and Limitations." *Journal of Economic Literature* 20 (1982): 529-563.

Shapiro, A.F. "The Parameters of a Multiple Criteria Model of Actuarial Assumptions for Pension Plan Valuations." *Insurance: Mathematics and Economics* 9 (1990): 197-206

von Neumann, J. and Morgenstern, O. *Theory of Games and Economic Behavior.* Princeton University Press, 1947.