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Skyrmionic spin Seebeck effect via dissipative thermomagnonic torques

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We derive thermomagnonic torque and its “ β -type” dissipative correction from the stochastic Landau-Lifshitz-Gilbert equation. The β -type dissipative correction describes viscous coupling between magnetic dynamics and magnonic current and it stems from spin mistracking of the magnetic order. We show that thermomagnonic torque is important for describing temperature gradient induced motion of skyrmions in helical magnets while dissipative correction plays an essential role in generating transverse Magnus force. We propose to detect such skyrmionic motion by employing the transverse spin Seebeck effect geometry.

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Introduction. Out-of-equilibrium effects involving spin are essential to understanding many spintronic discoveries, such as spin-transfer torque [1–4], spin pumping [5,6], and the spin Seebeck effect [7–11]. These discoveries enabled unprecedented degree of control in magnetic information-storage devices in which the magnetization can be flipped at will [12] or the domain wall can be moved in order to change the magnetization configuration [13]. Sizable coupling of spin to thermal flows [14] leads to yet another knob by which we can control magnetization and magnetic textures such as domain walls [15–18] as confirmed in recent experiments [19,20]. In addition, such coupling can enable energy harvesting from temperature gradients by employing the spin Seebeck effect [21] or magnetic texture dynamics [22].

Particularly strong coupling between heat flows carried by magnons and magnetic textures is expected in magnetic insulators such as Cu_2OSeO_3 [23], $\text{BaFe}_{1-x-0.05}\text{Sc}_x\text{Mg}_{0.05}\text{O}_{19}$ [24], and $\text{Y}_3\text{Fe}_5\text{O}_{12}$ [25], where the magnetization dynamics have low dissipation as coupling to electron continuum is absent [18,26]. At the same time, even at relatively low temperatures, thermal magnons have very small wavelength and thus can be treated as particles on the scale of magnetic texture [18,27]. This brings a lot of analogies to magnetization dynamics in metallic systems where the dynamics can be controlled by flows of electrons [28–31]. In conducting materials, on the other hand, thermoelectric spin torque can also couple magnetization to heat flows even in the absence of charge flows [15,22,32].

A skyrmion is an example of a magnetic texture that can arise in helical magnets due to inversion asymmetry induced Dzyaloshinsky-Moriya (DM) interaction [33] as observed in bulk samples of MnSi by neutron scattering techniques [34]. Skyrmion crystal (SkX) is particularly stable in two-dimensional (2D) systems or thin films as was predicted theoretically [35,36] and later confirmed experimentally [37,38]. Just like other textures, such as domain walls, skyrmions can be manipulated by temperature gradients [39,40]. However, many aspects related to interaction of magnon spin currents to magnetic textures are still unclear [41,42].

In this Rapid Communication we address the dissipative β -type corrections to magnonic spin torques. Such corrections play an important role in domain wall dynamics [28–31]; however, they were introduced only phenomenologically in relation to magnonic torques [18,42]. Here we derive

thermomagnonic torque and its β -type correction from the stochastic Landau-Lifshitz-Gilbert (LLG) equation. By applying this theory to skyrmionic textures, we conclude that β -type correction influences the sign of the transverse Magnus force which can be tested in the spin Seebeck effect geometry termed as skyrmionic spin Seebeck effect (see Fig. 1). The sign of the Magnus force is indicative of two regimes (i) $\beta > \alpha$ and (ii) $\beta < \alpha$, where according to our calculations the stochastic LLG equation results in the regime (i), here α is the Gilbert damping. We also analyze thermoelectric spin torque contributions that can arise in conducting ferromagnets even in the absence of the charge transport. We compare thermoelectric contributions to their thermomagnonic counterparts.

Thermal magnons and magnetic texture. We consider a ferromagnet well below the Curie temperature in which the magnetization dynamics is described by the stochastic LLG equation:

$$s(1 + \alpha \mathbf{m} \times) \dot{\mathbf{m}} + \mathbf{m} \times (\mathbf{H}_{\text{eff}} + \mathbf{h}) = 0, \quad (1)$$

where s is the saturation spin density, $\mathbf{m}(\mathbf{r}, t)$ is a unit vector in the direction of the spin density, and $\mathbf{H}_{\text{eff}} = -\delta_{\mathbf{m}} F$ describes the effective magnetic field. For the purposes of this paper, we consider the free energy density $F = (A/2)(\partial_{\alpha} \mathbf{m})^2 + D \mathbf{m} \cdot (\nabla \times \mathbf{m}) - \mathbf{m} \cdot \mathbf{H}$, where $A_x = A/M_s$ is the exchange stiffness, D_{dm} describes DM interaction with $D \equiv D_{\text{dm}} M_s$, M_s is the saturation magnetization, and \mathbf{H}_e is the external magnetic field with $\mathbf{H} \equiv \mathbf{H}_e M_s$. Note, however, that the discussion in this section is general and should also apply to the most general form of the free energy density as long as other energy contributions can be ignored in comparison to exchange energy of thermal magnons which is true at sufficiently high temperatures and for sufficiently small strength of DM interactions. In Eq. (1) \mathbf{h} is the random Langevin field corresponding to thermal fluctuations at temperature T with correlator [43]

$$\langle h_i(\mathbf{r}, t) h_j(\mathbf{r}', t') \rangle = 2\alpha s k_B T(\mathbf{r}) \delta_{ij} \delta(\mathbf{r} - \mathbf{r}') \delta(t - t'), \quad (2)$$

which is consistent with the fluctuation dissipation theorem. The fluctuating field results in fast magnetization dynamics that happens on top of the slow magnetic texture dynamics with long characteristic length scale. The purpose of this paper is to describe how the fast magnetization dynamics influence the slow dynamics. In Eq. (2) we assume a uniform temperature gradient along the x axis, i.e., $\partial T(x)/\partial x = \text{const}$.

We consider small fast oscillations of the magnetization with time scale $1/\omega_q$ defined by the magnon frequency ω_q on top of the slow magnetization dynamics with time scale defined by external magnetic field and current. The vectors for the fast $\mathbf{m}_f(\mathbf{r}, t)$ and slow $\mathbf{m}_s(\mathbf{r}, t)$ magnetization dynamics are related by $\mathbf{m} = (1 - \mathbf{m}_f^2)^{1/2} \mathbf{m}_s + \mathbf{m}_f$, where $\mathbf{m}_s \cdot \mathbf{m}_f = 0$. We apply a coordinate transformation after which the z axis points along the spin density of the slow dynamics. In the new coordinate system, small excitations will only have m_x and m_y components. In order to describe our system in the new coordinates, we introduce 3×3 rotation matrix $\hat{R} = \exp \theta \hat{J}_y \exp \phi \hat{J}_z$, with \hat{J}_α being the 3×3 matrix describing infinitesimal rotation along the axis with index α . In the new coordinates, we have $\mathbf{m} \rightarrow \mathbf{m}' = \hat{R} \mathbf{m}$ and the covariant derivative $\partial_\mu \rightarrow (\partial_\mu - \hat{A}_\mu)$, with $\hat{A}_\mu = (\partial_\nu \hat{R}) \hat{R}^{-1}$ (the index $\mu = 0, \dots, 3$ denotes the time and space coordinates). Since the matrix \hat{A}_μ is skew symmetric, we can introduce a vector \mathcal{A}_μ so that $\hat{A}_\mu \mathbf{m} = \mathcal{A}_\mu \times \mathbf{m}$. In some specific gauge, the elements of \mathcal{A}_μ become $\mathcal{A}_\mu = (-\sin \theta \partial_\mu \phi, \partial_\mu \theta, \cos \theta \partial_\mu \phi)$. The equation describing fast dynamics follows from the LLG equation subject to the coordinate transformation:

$$is[\partial_t(1 - i\alpha) - i\mathcal{A}_0^z]m_+ = A(\partial_\alpha/i - \mathcal{A}_\alpha^z)^2 m_+ + Hm_+, \quad (3)$$

where we disregarded various anisotropy terms assuming that the exchange interactions are dominant. As a result, the coupling between the circular components [27] of spin wave $m_\pm = m'_x(\mathbf{r}, t) \pm im'_y(\mathbf{r}, t)$ can be disregarded where $m'_{x(y)}$ are the transverse excitations in the transformed coordinates with the z axis pointing along the direction of \mathbf{m}_s . Equation (3) describes thermal magnons with spectrum $\omega_q = (H + Aq^2)/s$, where the magnon current can be written as $j_\alpha = \frac{A}{2i}(m_- \partial_\alpha m_+ - m_+ \partial_\alpha m_-)$. Note that formally Eq. (3) describes charged particles moving in the fictitious electric $\mathcal{E}_\alpha = -\partial_t \mathcal{A}_\alpha^z - \partial_\alpha \mathcal{A}_0^z = \hbar \mathbf{m}_s \cdot (\partial_t \mathbf{m}_s \times \partial_\alpha \mathbf{m}_s)$ and magnetic $\mathcal{B}_i = (\hbar/2)\epsilon^{ijk} \mathbf{m}_s \cdot (\partial_k \mathbf{m}_s \times \partial_j \mathbf{m}_s)$ fields produced by the magnetic texture.

We are now in the position to calculate the force that fast oscillations exert on the slow magnetization dynamics $\mathbf{m}_s(\mathbf{r}, t)$. For simplicity, we assume that the texture is static as we can add the time dependent contribution, e.g., corresponding to \mathcal{A}_0^z , later by involving the Onsager reciprocity principle. The force due to rapid oscillations of the fast component averages out in the first order terms with respect to $\mathbf{m}_f(\mathbf{r}, t)$, hence such force can only come from the second order terms. In the effective field $\mathbf{H}_{\text{eff}} = \mathbf{H} + A\nabla^2 \mathbf{m} - 2D\nabla \times \mathbf{m}$ relevant terms can be immediately identified leading to the following contribution:

$$\mathcal{T} = -(\mathbf{m}_f \times \mathbf{H}_{\text{eff}}) = A\mathbf{m}_s \times \mathcal{S} \approx A(\mathbf{m}_f \times \nabla^2 \mathbf{m}_f), \quad (4)$$

where we formally defined \mathcal{S} as the nonequilibrium transverse accumulation of magnon spins, $\langle \dots \rangle$ stands for averaging over the fast oscillations induced by random fields, and the terms corresponding to DM interactions have been dropped since they contain one spacial derivative which for thermal fluctuations with small wavelength leads to a small parameter $D/(Aq)$. We now calculate the torque in Eq. (4) in the reference frame associated with the slow dynamics by employing the covariant derivative. By dropping the terms that average out due to fast oscillations and concentrating only on the torque components that are in the x' - y' plane we obtain the following

result for the transverse accumulation of magnon spins:

$$\mathcal{S} = 2\langle \mathbf{m}_f(\partial_\alpha \mathbf{m}_s \cdot \partial_\alpha \mathbf{m}_f) \rangle. \quad (5)$$

In the transformed reference frame vectors \mathcal{S} , \mathbf{m}_f , $\partial_\alpha \mathbf{m}_s$, and $\partial_\alpha \mathbf{m}_f$ are in the x' - y' plane, thus it is convenient to switch to complex notations $a \equiv a'_x + ia'_y$, where \mathbf{a} is an arbitrary vector in the x' - y' plane which leads to $S = \langle m_f(\partial_\alpha m_s \partial_\alpha m_f^* + \partial_\alpha m_s^* \partial_\alpha m_f) \rangle = \partial_\alpha m_s \langle m_+ \partial_\alpha m_- \rangle$. For the steady state solution we find

$$S = \partial_x m_s \int \frac{d^{d-1} \mathbf{q} d\omega}{(2\pi)^d} \frac{\langle m_+(\mathbf{q}, \omega, x) \partial_x m_-(\mathbf{q}', \omega', x) \rangle}{(2\pi)^d \delta(\mathbf{q} - \mathbf{q}') \delta(\omega - \omega')}, \quad (6)$$

where $d = 2$ or 3 depending on the dimensionality of the magnet and $S = S_x + iS_y$ describes two components of spin accumulation leading to the dissipative and nondissipative torques. Here we used $m_\pm(\mathbf{r}, t)$ Fourier transformed with respect to time and transverse coordinate:

$$m_\mp(\mathbf{q}, \omega, x) = \int \frac{d^{d-1} \rho d\omega}{(2\pi)^d} e^{\pm i(\omega t - \mathbf{q}\rho)} m_\mp(\mathbf{r}, t). \quad (7)$$

The δ functions in the denominator of Eq. (6) cancel out after the averages over stochastic fields are evaluated. For the corresponding stochastic fields we obtain [26]

$$\frac{\langle h(x, \mathbf{q}, \omega)^* h(x', \mathbf{q}', \omega') \rangle}{4(2\pi)^d \alpha s k_B} = T(x) \delta(x - x') \delta(\mathbf{q} - \mathbf{q}') \delta(\omega - \omega'). \quad (8)$$

Since we are after the first order terms in magnetic texture gradients we can use Eq. (3) in the absence of vector potentials. The stochastic LLG Eq. (1) takes the form of the homogeneous Helmholtz equation:

$$A(\partial_x^2 + k^2)m_-(x, \mathbf{q}, \omega) = h(x, \mathbf{q}, \omega), \quad (9)$$

where this equation corresponds to Eq. (3) with an added stochastic term and $k^2 = [(1 + i\alpha)s\omega - H]/A - q^2$. Equation (9) can be easily solved by employing Green's function $G(x - x_0) = ie^{ik|x-x_0|}/(2k)$. We substitute this solution in Eq. (6) and carry through integrations over variables x and x' in Eq. (8). The final expression for the magnon spin torque becomes

$$\mathcal{T} = -\frac{s\alpha}{4A} \partial_x m_s \int \frac{d^{d-1} \mathbf{q}}{(2\pi)^d} \int_{\omega_0}^{\infty} d\omega \frac{\partial_x (\hbar\omega \coth \frac{\hbar\omega}{2k_B T})}{k^* (\text{Im}k)^2}. \quad (10)$$

Here we limited frequency integration by $\omega_0 = (H + Aq^2)/s$ and replaced $2k_B T \rightarrow \hbar\omega \coth(\hbar\omega/2k_B T)$ by employing the quantum fluctuation dissipation theorem which introduces high frequency cut off at $\hbar\omega \gg k_B T$. The former allows us to limit our consideration to magnonic excitations with energies above the magnonic gap and the latter allows us to relate our expression to magnon currents obtained by the Boltzmann approach. By replacing integration over ω with integration over k and keeping only the first two orders in α we obtain

$$\mathcal{T} = -\hbar \partial_x m_s j_x (1 + i\beta), \quad (11)$$

where $j_x = (\partial_x T/T) \int d^d \mathbf{k} / (2\pi)^d \tau(\epsilon) \epsilon v_x^2 \partial f_0 / \partial \epsilon$, with $\tau(\epsilon) = (2\alpha\omega)^{-1}$ can be interpreted as the magnon current calculated within the relaxation time approximation from nonequilibrium distribution correction $\delta f = \tau \epsilon (\partial f / \partial \epsilon) v_\alpha (\partial_\alpha T/T)$ [44]. Here $\epsilon(\mathbf{k}) = \hbar(Ak^2 + H)/s$,

$v_x = \partial\omega_k/\partial k_x$, and $f_0 = \{\exp[\varepsilon/k_B T] - 1\}^{-1}$ is the Bose-Einstein equilibrium distribution. This result is expected given that each magnon carries angular momentum $-\hbar$. The second term in Eq. (11) corresponds to the dissipative correction with $\beta/\alpha = (d/2)F_1(x)/F_0(x) \sim d/2$, with $F_0(x) = \int_0^\infty d\epsilon \epsilon^{d/2-1} e^{\epsilon+x}/(e^{\epsilon+x} - 1)^2$ and $F_1(x) = \int_0^\infty d\epsilon (\epsilon+x) \epsilon^{d/2-1} e^{\epsilon+x}/(e^{\epsilon+x} - 1)^2$ evaluated at the magnon gap $x = \hbar\omega_0/k_B T$, where $d = 2$ or 3 . The magnon current density is given by [18]

$$j_\alpha = k_B \partial_\alpha T F_0 / (6\pi^2 \lambda \hbar \alpha), \quad (12)$$

where $d = 3$ and $\lambda = \sqrt{\hbar A / (s k_B T)}$ is the thermal magnon wavelength [for $d = 2$ we obtain $j_\alpha = k_B \partial_\alpha T F_0 / (4\pi \hbar)$]. We can express result in Eq. (11) in the form of LLG equation:

$$\begin{aligned} \mathfrak{s}(1 + \alpha^s \mathbf{m}_s \times) \dot{\mathbf{m}}_s = & \mathbf{H}_{\text{eff}}^s \times \mathbf{m}_s - [1 + \beta \mathbf{m}_s \times] (\mathbf{j}_m^s \boldsymbol{\partial}) \mathbf{m}_s \\ & - [1 + \beta_e \mathbf{m}_s \times] (\mathbf{j}_e^s \boldsymbol{\partial}) \mathbf{m}_s, \end{aligned} \quad (13)$$

where $\mathfrak{s} = \langle \mathbf{m} \rangle_s$ is the renormalized spin density, $\mathbf{H}_{\text{eff}}^s = -\langle \mathbf{m} \rangle \langle \delta_{\mathbf{m}} F \rangle$ is the effective field, $\alpha^s = \langle \mathbf{m} \rangle \alpha$ is the renormalized Gilbert damping, $\mathbf{j}_m^s = -\hbar \mathbf{j}$ is the spin current with polarization along \mathbf{m}_s carried by magnons, and we added the terms proportional to thermoelectric spin current $\mathbf{j}_e^s = \partial T \wp_S S \sigma_F (1 - \wp^2) \hbar / 2e$ arising under the open circuit conditions in conducting materials (i.e., in the absence of charge currents) [15] with S being the Seebeck coefficient, \wp_S being the polarization of the Seebeck coefficient, σ_F being the conductivity of the ferromagnet with polarization \wp , and β_e being the β -type correction corresponding to thermoelectric spin currents.

Application to skyrmion dynamics. Equation (13) can be used in order to describe the magnetization dynamics in response to temperature gradients. Here we analyze motion of skyrmions in response to temperature gradients in ferromagnetic insulators. For describing motion of magnetic textures such as magnetic domain walls and vortices one can use the approach proposed by Thiele [45]. Within such an approach, the dynamics are constrained to a subspace described by the generalized coordinates q , $\dot{\mathbf{m}} = \sum_i \dot{q}_i \partial_{q_i} \mathbf{m}$ where the equation of motion for q can be found by integrating $\int d^3 \mathbf{r} \partial_{q_i} \mathbf{m} \cdot (\mathbf{m} \times \dots)$, where instead of dots one needs to substitute Eq. (13). One can apply this approach in order to describe motion of a skyrmion in response to magnon currents in Eq. (13) as long as the thermal magnon wavelength is smaller than the typical size of a skyrmion. The following equation describing skyrmion dynamics in a thin film results from Eq. (13):

$$\hat{z} \times (\mathbf{j}_m^s + \mathbf{j}_e^s - \mathfrak{s} \mathbf{v}) + \eta (\beta \mathbf{j}_m^s + \beta_e \mathbf{j}_e^s - \alpha \mathfrak{s} \mathbf{v}) = 0, \quad (14)$$

where $\eta = \eta_\alpha = \int d^2 \mathbf{r} (\partial_\alpha \mathbf{m}_s)^2 / (4\pi)$ is the form factor of skyrmion which is of the order of 1 and \mathbf{v} is the skyrmion velocity. The skyrmion will move along the temperature gradient with velocity:

$$v_x = \frac{j_m^s + j_e^s + \alpha \eta^2 (\beta j_m^s + \beta_e j_e^s)}{\mathfrak{s}(1 + \alpha^2 \eta^2)}, \quad (15)$$

in addition acquiring extra transverse Hall-like motion:

$$v_y = \eta \frac{\alpha (j_m^s + j_e^s) - (\beta j_m^s + \beta_e j_e^s)}{\mathfrak{s}(1 + \alpha^2 \eta^2)}. \quad (16)$$

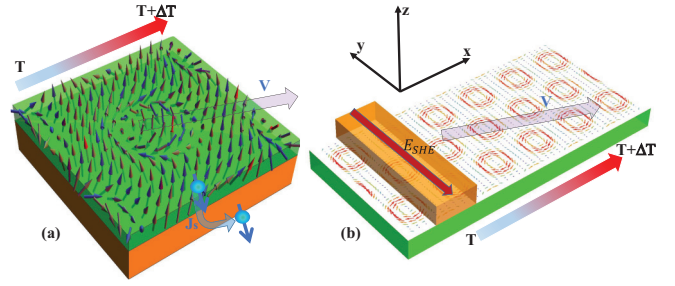


FIG. 1. (Color online) (a) The magnon current induced by temperature gradient exerts spin torque on magnetization which leads to skyrmion motion in the direction of the hot region with an additional Hall-like side motion. An additional nonmagnetic layer, such as Pt, can be used in order to detect the spin pumping resulting from skyrmionic motion, i.e., via the inverse spin Hall effect. (b) Spin Seebeck effect geometry can be used for detection of the Hall-like motion of skyrmions. Due to mostly out-of-plane magnetization configuration the ordinary spin Seebeck effect should be suppressed.

Here we define the Hall angle as $\tan \theta_H = v_y / v_x$. For a ferromagnetic insulator ($j_e^s = 0$) it is easy to see that the skyrmion will move towards the hot region as follows from Eq. (14). Earlier approaches either did not consider the β -type corrections [41] or introduced them heuristically [42]. Note that Eq. (10) results in $\beta \approx 1.5\alpha > 0$.

Transverse spin Seebeck effect. In ferromagnetic insulators the coupling to the electron continuum is absent which may complicate the detection of skyrmion dynamics. Here we propose to detect temperature induced skyrmion dynamics by employing spin pumping into the neighboring nonmagnetic metallic layer, such as Pt. The whole setup in Fig. 1 then corresponds to the geometry of the transverse spin Seebeck effect [7–11] in which the ordinary spin Seebeck response should be weak since we are not considering the in-plane magnetization configuration. We assume that the spin injection is locally homogeneous which is justified when $\lambda_{sd} \ll \xi$, where $\xi = 2\pi A/D$ is the characteristic length of a skyrmion estimated by the spiral period. To calculate the spin accumulation and the spin current in normal metal we employ the spin diffusion equation for the spin accumulation $\nabla^2 \mu_s = \mu_s / \lambda_{sd}^2$, where λ_{sd} is the spin-diffusion length in the normal metal. The boundary conditions at the interface between the normal metal and ferromagnetic insulator enforce continuity of the spin current $\partial_z \mu_s |_{z=0} = -(G_0 / \sigma_N) \mathbf{j}_{s,z}$, where σ_N is the conductivity of the normal metal and $G_0 = 2e^2 / h$ is the quantum conductance. The spin current $\mathbf{j}_{s,z} = \mathbf{j}_s^{\text{pmp}} + \mathbf{j}_s^{\text{bf}}$ is the sum of the spin-pumped and backflow contributions:

$$\mathbf{j}_{s,z} = \frac{\hbar g^{\uparrow\downarrow}}{4\pi} \mathbf{m}_s \times \frac{\partial \mathbf{m}_s}{\partial t} + \frac{g^{\uparrow\downarrow}}{4\pi} \mu_s |_{z=0}, \quad (17)$$

where $g^{\uparrow\downarrow}$ is the spin mixing conductance per unit of interface area and the conductance quantum in which the imaginary part is disregarded which is usually justified for realistic interfaces with metals [46].

After solving the diffusion equation, we obtain the full spin flow as well as the average voltage difference due to the inverse spin Hall effect (ISHE) [47] by averaging the result over the magnetic texture of a skyrmion. The voltage due to the ISHE

becomes

$$\frac{V_{\text{ISH}}}{W} = \frac{|\langle \mathbf{j}_s^{\text{pmp}} \rangle_s| \theta_{\text{SH}}}{d_N |e|} \frac{\cosh \frac{d_N}{\lambda_{\text{sd}}} - 1}{\frac{\sigma_N}{G_0 \lambda_{\text{sd}}} \sinh \frac{d_N}{\lambda_{\text{sd}}} + \frac{g^{\uparrow\downarrow}}{4\pi} \cosh \frac{d_N}{\lambda_{\text{sd}}}}, \quad (18)$$

where θ_{SH} is the spin Hall angle of the normal metal, $\langle \mathbf{j}_s^{\text{pmp}} \rangle_s$ is the pumped spin current density averaged over one skyrmion, and W is the width over which the voltage is measured. The averaged spin current becomes

$$\langle \mathbf{j}_s^{\text{pmp}} \rangle_s = \frac{\hbar g^{\uparrow\downarrow}}{4\pi \xi} \chi_\alpha v_\alpha, \quad (19)$$

where we introduced another form factor of a skyrmion $\chi_\alpha = \int d^2 \mathbf{r} (\mathbf{m}_s \times \partial_\alpha \mathbf{m}_s) / \xi$ which defines the polarization of the averaged spin current density in Fig. 1 (in-plane and orthogonal to skyrmion velocity). Thus the voltage in Eq. (18) is generated along the direction of skyrmion flow which includes a conventional Seebeck-like contribution along the applied temperature gradient as well as transverse component corresponding to Nernst effect.

Estimates. For estimates we consider Cu_2OSeO_3 thin insulating layer of thickness $d_F = 15$ nm in skyrmionic phase in contact with a thin Pt layer, $d_N = 1$ nm (see Fig. 1). By taking the lattice spacing $a = 0.5$ nm, $\varsigma = 0.5\hbar/a^3$, $A/(a^2 k_B) = 50$ K, and $D/(ak_B) = 3$ K we obtain the spiral period $\xi \approx 50$ nm [23]. Even at low temperatures comparable to 1 K the magnon wavelength satisfies the condition $\lambda \ll d_F$ which should justify three-dimensional treatment in Eq. (11). We further take the temperature $T = 50$ K, $\alpha = 0.01$, and a gradient $\partial_\alpha T = 1$ K/ μm arriving at the skyrmion longitudinal velocity towards the hot region $v_x \approx 0.1$ m/s from Eq. (14). From Eq. (19) the skyrmion motion leads to the ISHE voltage $V_{\text{ISH}}/W = 2 \times 10^{-2}$ V/m for $\theta_{\text{SH}} = 0.11$, $\lambda_{\text{sd}} = 1.5$ nm, $\sigma_N = 1 \mu\Omega^{-1} \text{m}^{-1}$, and $g^{\uparrow\downarrow} = 1.5 \times 10^{19} \text{m}^{-2}$ [48]. This signal is much smaller than the conventional Seebeck signal of a Pt layer but should be detectable with sensitive techniques. In addition, the skyrmion will acquire transverse Hall-like motion $v_y \approx -0.001$ m/s resulting in the Nernst response given by $V_{\text{ISH}}^\perp/W = V_{\text{ISH}} \sin \theta_H/W = 2 \times 10^{-4}$ V/m which can be measured in the setup in Fig. 1(b).

We repeat the same estimate for MnSi for which we take $a = 0.5$ nm, $\varsigma = 0.4\hbar/a^3$, $A/(a^2 k_B) = 12$ K, $Da = 0.18A$, $\xi \approx 18$ nm [34], $T = 30$ K, and a gradient $\partial_\alpha T = 1$ K/ μm arriving at the skyrmion longitudinal velocity $v_x = 0.02$ m/s in the absence of thermoelectric spin currents. The thermoelectric spin current \mathbf{j}_e^s arising under the open circuit conditions in conducting materials (i.e., in the absence of charge currents) [15] leads to additional contribution to velocity in Eqs. (15) and (16). Taking the Seebeck coefficient $S = 10 \mu\text{V/K}$, its polarization $\wp_S = 0.3$, and the conductivity $\sigma_{\text{MnSi}} = 3 \mu\Omega^{-1} \text{m}^{-1}$ we arrive at $v_x = -0.01$ m/s which shows that this contribution pushes the domain wall away from the hot region when $\wp_S S > 0$. Since thermomagnonic forces strongly depend on the temperature it seems to be feasible to reverse the direction of longitudinal skyrmion motion at lower temperatures. For estimating the transverse motion we use $\beta_e \approx \alpha$ for which the magnonic contribution dominates in Eq. (16) arriving at $v_y \approx -0.002$ m/s and the Nernst response $V_{\text{ISH}}^\perp/W = V_{\text{ISH}} \sin \theta_H/W = 10^{-3}$ V/m.

Conclusions. We derived thermomagnonic torque and its β -type dissipative correction from the stochastic LLG equation. Such corrections are important for magnetic texture dynamics and here we show that they influence the sign of the magnus force acting on skyrmion under temperature gradient. For experimental detection we propose to use the transverse spin Seebeck effect geometry (see Fig. 1). Parametric excitation of spin waves of sufficiently short wavelength can also be described by our theory while effects related to linear momentum transfer should be included for longer wavelength [49]. Our theory provides the minimalistic phenomenological description while further studies could incorporate magnon-magnon, magnon-phonon, and magnon scattering on impurities by constructing a microscopic kinetic theory.

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- [1] J. Slonczewski, *J. Magn. Magn. Mater.* **159**, L1 (1996).
 [2] L. Berger, *Phys. Rev. B* **54**, 9353 (1996).
 [3] M. Tsoi, A. G. M. Jansen, J. Bass, W.-C. Chiang, M. Seck, V. Tsoi, and P. Wyder, *Phys. Rev. Lett.* **80**, 4281 (1998).
 [4] E. B. Myers, *Science* **285**, 867 (1999).
 [5] Y. Tserkovnyak, A. Brataas, and G. E. W. Bauer, *Phys. Rev. Lett.* **88**, 117601 (2002).
 [6] B. Heinrich, Y. Tserkovnyak, G. Woltersdorf, A. Brataas, R. Urban, and G. E. W. Bauer, *Phys. Rev. Lett.* **90**, 187601 (2003).
 [7] K. Uchida, S. Takahashi, K. Harii, J. Ieda, W. Koshibae, K. Ando, S. Maekawa, and E. Saitoh, *Nature (London)* **455**, 778 (2008).
 [8] K. Uchida, T. Ota, K. Harii, K. Ando, H. Nakayama, and E. Saitoh, *J. Appl. Phys.* **107**, 090000 (2010).
 [9] C. M. Jaworski, J. Yang, S. Mack, D. D. Awschalom, J. P. Heremans, and R. C. Myers, *Nat. Mater.* **9**, 898 (2010).
 [10] K. Uchida, J. Xiao, H. Adachi, J. Ohe, S. Takahashi, J. Ieda, T. Ota, Y. Kajiwara, H. Umezawa, H. Kawai *et al.*, *Nat. Mater.* **9**, 894 (2010).
 [11] S. Bosu, Y. Sakuraba, K. Uchida, K. Saito, T. Ota, E. Saitoh, and K. Takanashi, *Phys. Rev. B* **83**, 224401 (2011).
 [12] S. Maekawa, H. Adachi, K.-I. Uchida, J. Ieda, and E. Saitoh, *J. Phys. Soc. Jpn.* **82**, 102002 (2013).
 [13] S. S. P. Parkin, M. Hayashi, and L. Thomas, *Science* **320**, 190 (2008).
 [14] G. E. W. Bauer, E. Saitoh, and B. J. van Wees, *Nat. Mater.* **11**, 391 (2012).
 [15] A. A. Kovalev and Y. Tserkovnyak, *Phys. Rev. B* **80**, 100408 (2009).

- [16] G. E. W. Bauer, S. Bretzel, A. Brataas, and Y. Tserkovnyak, *Phys. Rev. B* **81**, 024427 (2010).
- [17] D. Hinzke and U. Nowak, *Phys. Rev. Lett.* **107**, 027205 (2011).
- [18] A. A. Kovalev and Y. Tserkovnyak, *Europhys. Lett.* **97**, 67002 (2012).
- [19] J. Torrejon, G. Malinowski, M. Pelloux, R. Weil, A. Thiaville, J. Curiale, D. Lacour, F. Montaigne, and M. Hehn, *Phys. Rev. Lett.* **109**, 106601 (2012).
- [20] W. Jiang, P. Upadhyaya, Y. Fan, J. Zhao, M. Wang, L.-T. Chang, M. Lang, K. L. Wong, M. Lewis, Y.-T. Lin *et al.*, *Phys. Rev. Lett.* **110**, 177202 (2013).
- [21] A. B. Cahaya, O. A. Tretiakov, and G. E. W. Bauer, *Appl. Phys. Lett.* **104**, 042402 (2014).
- [22] A. A. Kovalev and Y. Tserkovnyak, *Solid State Commun.* **150**, 500 (2010).
- [23] S. Seki, X. Z. Yu, S. Ishiwata, and Y. Tokura, *Science* **336**, 198 (2012).
- [24] X. Yu, M. Mostovoy, Y. Tokunaga, W. Zhang, K. Kimoto, Y. Matsui, Y. Kaneko, N. Nagaosa, and Y. Tokura, *Proc. Natl. Acad. Sci. U.S.A.* **109**, 8856 (2012).
- [25] S. Bhagat, H. Lessoff, C. Vittoria, and C. Guenzer, *Phys. Status Solidi (a)* **20**, 731 (1973).
- [26] S. Hoffman, K. Sato, and Y. Tserkovnyak, *Phys. Rev. B* **88**, 064408 (2013).
- [27] V. K. Dugaev, P. Bruno, B. Canals, and C. Lacroix, *Phys. Rev. B* **72**, 024456 (2005).
- [28] S. Zhang and Z. Li, *Phys. Rev. Lett.* **93**, 127204 (2004).
- [29] H. Kohno, G. Tatara, and J. Shibata, *J. Phys. Soc. Jpn.* **75**, 113706 (2006).
- [30] Y. Tserkovnyak, H. J. Skadsem, A. Brataas, and G. E. W. Bauer, *Phys. Rev. B* **74**, 144405 (2006).
- [31] Y. Tserkovnyak, A. Brataas, and G. E. Bauer, *J. Magn. Magn. Mater.* **320**, 1282 (2008).
- [32] K. M. Hals, A. Brataas, and G. E. Bauer, *Solid State Commun.* **150**, 461 (2010).
- [33] U. K. Röbler, A. N. Bogdanov, and C. Pfleiderer, *Nature (London)* **442**, 797 (2006).
- [34] S. Mühlbauer, B. Binz, F. Jonietz, C. Pfleiderer, A. Rosch, A. Neubauer, R. Georgii, and P. Böni, *Science* **323**, 915 (2009).
- [35] S. D. Yi, S. Onoda, N. Nagaosa, and J. H. Han, *Phys. Rev. B* **80**, 054416 (2009).
- [36] A. B. Butenko, A. A. Leonov, U. K. Röbler, and A. N. Bogdanov, *Phys. Rev. B* **82**, 052403 (2010).
- [37] X. Z. Yu, Y. Onose, N. Kanazawa, J. H. Park, J. H. Han, Y. Matsui, N. Nagaosa, and Y. Tokura, *Nature (London)* **465**, 901 (2010).
- [38] X. Z. Yu, N. Kanazawa, Y. Onose, K. Kimoto, W. Z. Zhang, S. Ishiwata, Y. Matsui, and Y. Tokura, *Nat. Mater.* **10**, 106 (2011).
- [39] M. Mochizuki, X. Z. Yu, S. Seki, N. Kanazawa, W. Koshibae, J. Zang, M. Mostovoy, Y. Tokura, and N. Nagaosa, *Nat. Mater.* **13**, 241 (2014).
- [40] K. Everschor, M. Garst, B. Binz, F. Jonietz, S. Mühlbauer, C. Pfleiderer, and A. Rosch, *Phys. Rev. B* **86**, 054432 (2012).
- [41] L. Kong and J. Zang, *Phys. Rev. Lett.* **111**, 067203 (2013).
- [42] S.-Z. Lin, C. D. Batista, C. Reichhardt, and A. Saxena, *Phys. Rev. Lett.* **112**, 187203 (2014).
- [43] W. F. Brown, *Phys. Rev.* **130**, 1677 (1963).
- [44] N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Saunders College, Philadelphia, 1976).
- [45] A. A. Thiele, *Phys. Rev. Lett.* **30**, 230 (1973).
- [46] K. Xia, P. J. Kelly, G. E. W. Bauer, A. Brataas, and I. Turek, *Phys. Rev. B* **65**, 220401 (2002).
- [47] E. Saitoh, M. Ueda, H. Miyajima, and G. Tatara, *Appl. Phys. Lett.* **88**, 182509 (2006).
- [48] M. Weiler, M. Althammer, M. Schreier, J. Lotze, M. Pernpeintner, S. Meyer, H. Huebl, R. Gross, A. Kamra, J. Xiao *et al.*, *Phys. Rev. Lett.* **111**, 176601 (2013).
- [49] P. Yan, A. Kamra, Y. Cao, and G. E. W. Bauer, *Phys. Rev. B* **88**, 144413 (2013).