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Physics, Chapter 5: Force and Motion

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Force and Motion

5-1 Starting and Stopping Motion

All of us have many times had the experience of setting a body in motion. If we analyze any of these experiences, we readily recall that in each case some *force* was required to start the object moving. In throwing a ball, moving a piece of furniture, or pulling a sled, the force needed to start the object moving is supplied by one's muscular effort as a push or a pull. In more complex cases, such as setting a car or an airplane in motion, the analysis, although more complicated, will also show that a force is required to start the body moving.

There are many cases in which the force that acts on the body to produce the motion is not directly discernible. It was Newton who first showed that the acceleration of a freely falling body is produced by a force which acts between the earth and the body, called the *force of gravitation.* We shall encounter other such action-at-a-distance forces in electricity and magnetism, and in molecular and atomic physics.

Once a body has been set in motion by the action of a force, it will not necessarily stop moving when the force is removed. A sled in motion along a level road will continue to move in a straight line along the road, although with diminishing speed. The reduction in speed is due to the *force of friction* between the runners of the sled and the ground. If there is clean snow on the ground, the force of friction will be very small; if ashes or sand have been dumped on the snow, the force of friction will be greater, and the sled will come to rest much sooner.

The above examples illustrate the fact that a *force is required to change the state of motion of a body.* It was Sir Isaac Newton (1642–1727) who first recognized the relationship between force and the state of motion of the body on which it is acting. He epitomized the entire science of mechanics in the form of three statements which have become known as Newton's laws of motion. Although the first and third laws have been previously stated and discussed, they are sufficiently important to bear repetition.

5-2 Newton's Laws of Motion

Newton's three laws of motion can be stated as follows:

First law: *A body at rest will remain at rest, and a body in motion will continue in motion with constant speed in a straight line, as long as no net force acts on it.*

Second law: *If a net force acts on a body, the body will be accelerated; the magnitude of the acceleration is proportional to the magnitude of the force, and the direction of the acceleration is in the direction of the force.*

Third law: *Whenever one body exerts a force on another, the second body exerts a force equal in magnitude and opposite in direction on the first body.*

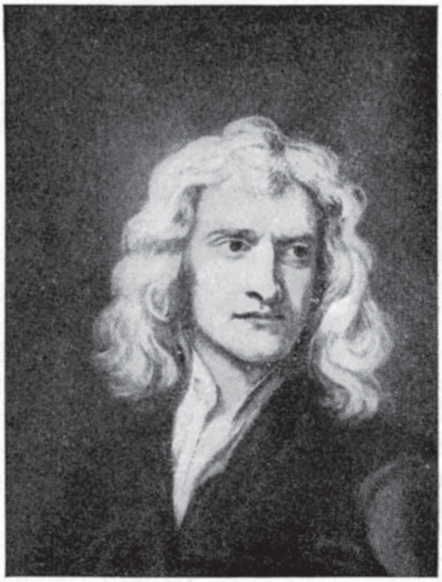


Fig. 5-1 Sir Isaac Newton (1642–1727). One of the greatest physicists of all time, he developed the law of universal gravitation; epitomized the subject of mechanics in the three laws of motion which bear his name; made important contributions to optics. The publication of his *Principia*, the Mathematical Principles of Natural Philosophy, in 1687, was an epoch-making event for science. (Courtesy of *Scripta Mathematica.*)

5-3 Newton's First Law

Newton's first law states that a body at rest will remain at rest, and a body in motion will continue in motion with constant speed in a straight line, as long as no net force acts on it.

An examination of this first law shows that a body at rest and a body moving with constant velocity have one characteristic in common: there is no net external force acting upon either one. This is the case when the resultant of all the external forces acting on the body is zero. As we have already seen, this is the condition for the equilibrium of a particle; this is also the condition for the translational equilibrium of a rigid body.

According to Newton's first law, a train moving at a constant velocity along a level track is in equilibrium. It is acted upon by several external forces whose resultant is zero. Consider the forces acting on a train of cars being pulled by a locomotive (see Figure 5-2). The weights $\mathbf{W}_1$, $\mathbf{W}_2$, $\mathbf{W}_3$, of the cars act vertically downward through the respective centers of gravity. They are opposed by the forces $\mathbf{N}_1$, $\mathbf{N}_2$, $\mathbf{N}_3$, and so on, which the tracks exert upward on the wheels of the train to support the weight. The sum of these upward forces must equal the total weight of the train. There are also frictional forces which oppose the motion of the train. Some of these frictional forces occur between the wheels and the tracks and in the wheel bearings; there is also another type of frictional force owing to the

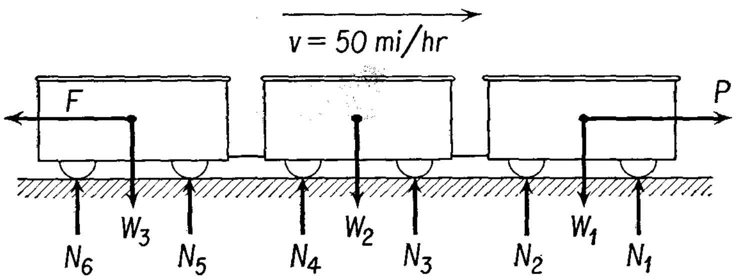


Fig. 5-2 A train moving with constant velocity has no net force acting on it. $W_1 + W_2 + W_3 = N_1 + N_2 + N_3 + N_4 + N_5 + N_6$. $P = F$.

resistance of the air to motion through it. All of these frictional forces are represented in the figure by the single force $\mathbf{F}$. The effect of these frictional forces would be to reduce the speed of the train; to prevent this reduction in speed, the locomotive supplies a force $\mathbf{P}$ equal to $\mathbf{F}$ in magnitude but in the forward direction. There is no net force acting on the train when it is moving with constant velocity.

In this illustration we have taken a very liberal view of the meaning of *body* and of *net force.* We have considered the collection of all the cars of the train as a body, or as a *system* which could be surrounded by an imaginary box. Everything within the walls of the box was considered to be the body, and only the forces acting from outside the box upon objects inside the box were considered as forces acting on the body. In addition to the forces illustrated in the figure, each car exerts a force upon the two cars immediately adjacent to it. Nevertheless, these *internal* forces can be disregarded in our analysis of the over-all motion of the system, and our attention can be focused upon the *external* forces acting upon the system. From Newton's third law the sum of these internal forces must be zero. This procedure is analogous to what we have already done in the study of the equilibrium of a rigid body when we considered only the external forces acting on the rigid body and paid no attention to the internal forces which connected one particle to another particle of the body.

Implicit in the statement of Newton's first law is a property common to all objects—the property known as *inertia.* The inertia of a body is that property of a body associated with the first law, that a body at rest will remain at rest unless acted on by a net force, and that a body in motion will continue to move with uniform velocity unless acted on by a net force.

The magician who whisks a cloth from under the dinner dishes on a table, the prankster who places a brick under a hat on the sidewalk, have a qualitative understanding of the concept of inertia. We shall attempt to systematize and formalize this concept in the following sections of this chapter.

5-4 Newton's Second Law

Newton's second law states that if a net force acts on a body, the body will be accelerated; the magnitude of the acceleration is proportional to the magnitude of the force, and the direction of the acceleration is in the direction of the force.

Let us examine the meaning of the second law of motion. When a single force acts upon a body, an acceleration results. The body is accelerated in the direction in which the force acts. The magnitude of the acceleration is proportional to the magnitude of the force and to some quality of the body which has not yet been specified. Since both force and acceleration are vector quantities, we may write Newton's second law in the form of an equation:

$$\mathbf{F} = k\mathbf{a}. \tag{5-1}$$

The quantity k is used to represent a constant of proportionality, a scalar quantity, having whatever dimensions are necessary to give the equation formal meaning. The constant k must clearly depend on the properties of the body, for the other quantities in the equation do not. The value of k must also depend upon the choice of units for $\mathbf{F}$ and $\mathbf{a}$. It is desirable to break up k into two parts, one of which depends only upon the properties of the body, and another which depends only upon the choice of units in which to express the magnitudes of these quantities. Thus we rewrite Equation (5-1) as

$$\mathbf{F} = KM\mathbf{a}, \tag{5-2}$$

where both K and M must be scalar quantities. We use the constant K to represent that part of k which is associated with a choice of units, and let the symbol M represent the part of k which is associated with the body being accelerated. We have already referred to the resistance of a body to a change in its state of motion through the qualitative concept of inertia. The quantitative measure of the inertia of a body is its *mass,* represented in Equation (5-2) by the symbol M.

Given a choice of the constant K, to be associated with the choice of units used to represent $\mathbf{F}$ and $\mathbf{a}$, Equation (5-2) represents both a definition of mass and a recipe for its experimental determination. If we had chosen to define a unit of force in terms of the deflection of a spring, we could determine the mass of a body by using the spring to exert a force on the body resting on a frictionless table. The resulting acceleration might be determined through the measurement of the distance traversed in a known time. Such a determination of mass would be called a *dynamic* determination. We have already seen in Chapter 1 that the customary method for the determination of mass is based upon comparing the earth's gravitational force upon the unknown mass and a standard mass in a beam balance.

It is considerably simpler to embody a standard of mass as a preservable physical entity, say in the form of a piece of metal, than it is to embody a standard of force. While the standard of mass may be protected from alteration by wear and corrosion, the properties of a spring which determine the force it exerts vary with the age and condition of the spring. Consequently, mass is often taken to be a fundamental mechanical quantity, along with distance and time, and the force is considered to be a derived quantity, whose definition is based upon Newton's second law of motion.

5-5 Absolute Systems of Units

When numerical values are used with an equation involving physical quantities, such as Equation (5-2), these numerical values must be accompanied by appropriate units. There are many different sets of units in actual use today, each set consistent within itself, each chosen for some special merit which it is supposed to have for the particular group of experiments or investigations under consideration. An *absolute* system of units is one in which the unit of *force* is defined without reference to gravity, as in the two metric systems of units discussed in this section. A *gravitational* system of units is one in which gravity, or weight, is used as the basis of the definition of a unit of force, as in Section 5-6. Quantities expressed in one set of units can be converted more or less readily into any other set of units. Most physicists prefer to base the systems of units upon length, mass, and time as the fundamental concepts. Of these systems one of the most widely used is the cgs system in which the centimeter, gram, and second are the units for the respective fundamental quantities.

For convenience, the constant K of Equation (5-2) is set equal to 1, a pure number without physical dimensions. When $K = 1$ Equation (5-2) becomes

$$\boxed{\mathbf{F} = M\mathbf{a}.} \tag{5-3}$$

Equation (5-3) is the form most commonly used to represent Newton's second law of motion. It must be emphasized that the quantity **F** is the net force, or unbalanced force, or the resultant force acting on the body. If the force is entirely in the x direction, the acceleration must also be in the x direction. Resolving both the force and the acceleration into components parallel to each of the three coordinate axes, we obtain the component form of Equation (5-3).

$$\mathbf{F}_x = M\mathbf{a}_x, \tag{5-4a}$$

$$\mathbf{F}_y = M\mathbf{a}_y, \tag{5-4b}$$

$$\mathbf{F}_z = M\mathbf{a}_z, \tag{5-4c}$$

where the symbols $\mathbf{F}_x$, $\mathbf{F}_y$, and $\mathbf{F}_z$ represent the x, y, and z components of the net or resultant force acting upon the body, and $\mathbf{a}_x$, $\mathbf{a}_y$, and $\mathbf{a}_z$ represent the x, y, and z components of the acceleration.

In the cgs absolute system of units, the mass of a body is expressed in *grams*, and the acceleration is expressed in *centimeters per second per second.* A unit of force must be introduced that will be consistent with Equation (5-3). This unit of force is called a *dyne* and is defined as *that force, which, acting on a one-gram mass, produces an acceleration of one centimeter per second per second.* Thus Equation (5-3), together with legally defined units of mass, length, and time, has been used to generate a unit of force.

Suppose that a force F acts on a body whose mass is 1 gm and that it produces an acceleration of 1 cm/sec^2. Then Equation (5-3) would read

$$F = 1\text{ gm} \times 1\,\frac{\text{cm}}{\text{sec}^2} = 1\,\frac{\text{gm cm}}{\text{sec}^2} = 1\text{ dyne}.$$

Illustrative Example. A loaded car has a mass of 2,800 gm. (a) What horizontal force is required to give this car an acceleration of 80 cm/sec^2? (b) What velocity will this car acquire if it starts from rest and the force acts on it for 8 sec?

(a) Using Equation (5-3), and noting that **F** and **a** have the same direction, we may write

$$F = Ma,$$

and substituting values for M and a, we get

$$F = 2{,}800\text{ gm} \times 80\,\frac{\text{cm}}{\text{sec}^2},$$

or

$$F = 224{,}000\text{ dynes}.$$

(b) The velocity of the car can be determined with the aid of the equation $v = u + at$ with $u = 0$, $a = 80$ cm/sec^2, and $t = 8$ sec, yielding

$$v = 80\,\frac{\text{cm}}{\text{sec}^2} \times 8\text{ sec},$$

so that

$$v = 640\,\frac{\text{cm}}{\text{sec}}\cdot$$

Another absolute system which is widely used is the mks system of units based upon the meter, kilogram, and second as the respective units of length, mass, and time. The unit of force in the mks system is the *newton,* which is defined as that *force which, acting on a one-kilogram mass, produces an acceleration of one meter per second per second.*

If a force F acts on a body whose mass is 1 kg and produces an acceleration of 1 m/sec^2, then, from Equation (5-3), we have

$$F = 1 \text{ kg} \times 1 \frac{\text{m}}{\text{sec}^2} = 1 \frac{\text{kg m}}{\text{sec}^2} = 1 \text{ nt}.$$

We can obtain the relationship between a newton and a dyne from the above equation thus:

$$1 \text{ nt} = 1 \text{ kg} \times 1 \frac{\text{m}}{\text{sec}^2}$$

$$= 1{,}000 \text{ gm} \times 100 \frac{\text{cm}}{\text{sec}^2} = 100{,}000 \frac{\text{gm cm}}{\text{sec}^2},$$

so that $$1 \text{ nt} = 100{,}000 \text{ dynes} = 10^5 \text{ dynes}.$$

Illustrative Example. A force of 800 nt is applied to a mass of 160 kg. (a) Determine the acceleration produced. (b) If the body starts from rest, determine the distance the body travels if the force acts on it for 12 sec.

(a) Solving Equation (5-3) for the acceleration, we get

$$a = \frac{F}{m},$$

and, substituting numerical values for F and M, we obtain

$$a = \frac{800 \text{ nt}}{160 \text{ kg}} = 5 \frac{\text{m}}{\text{sec}^2}.$$

(b) Since the initial velocity $u = 0$, we can use the equation

$$s = \tfrac{1}{2}at^2$$

for determining the distance s traveled at the constant acceleration of 5 m/sec^2 for 12 sec, obtaining

$$s = \tfrac{1}{2} \times 5 \frac{\text{m}}{\text{sec}^2} \times 144 \text{ sec}^2,$$

from which $$s = 360 \text{ m}.$$

The above two systems of absolute units are based on the metric system and will be used throughout this book.

5-6 British Gravitational System of Units

While the legally defined unit of mass is the pound mass, defined as 1/2.20462 kilogram, it is inconvenient to use the pound mass as the basis of a system of units, for in everyday terminology, and in many engineering applications, the word "pound" commonly refers to weight rather than to mass. To conform to this common usage, the British gravitational system of units has chosen to define the pound of force as the weight of the standard one-pound body at sea level and at 45° latitude.

Modern engineering practice tends to avoid the use of the pound mass by introducing a new unit of mass called a *slug*. The *slug* is defined as *that unit of mass, which, when acted on by a force of one pound, will acquire an acceleration of one foot per second per second.* In the British gravitational system of units, the unit of force is the *pound,* the unit of mass is the *slug,* the unit of distance is the *foot,* and the unit of time is the *second.*

In everyday language the word "pound" is commonly used as a unit of mass and as a unit of force. Usually one can infer from the context of a statement whether the pound is used as a unit of force or as a unit of mass. For example, in Chapter 3 the pound was consistently used as a unit of force. In statements where it is possible to interpret the word "pound" as either force or mass, the terms "pound force" or "pound mass" should be used to avoid ambiguity.

Let us examine the relationship between the pound mass and the slug. From Table 2-1 we see that at latitude 45° the acceleration of gravity is 32.17 ft/sec^2. This is the acceleration acquired by any freely falling body at sea level, and in particular it is the acceleration which would be acquired by a pound mass falling freely at sea level. The weight of a pound mass at this latitude and elevation has been defined as the pound of force. But the weight of a body is the force of the earth's gravitational attraction. Writing M as the mass in appropriate units, and substituting in Equation (5-3), with $F = 1$ lb and $a = 32.17$ ft/sec^2, we find

$$1 \text{ lb} = M \times 32.17 \text{ ft/sec}^2;$$

hence

$$M = \frac{1}{32.17} \text{ slug.}$$

A pound mass has a mass in slugs given by 1/32.17 slug; thus the mass of one slug is 32.17 lb mass.

In general, if the weight of a body W is the only force which acts upon it, it is a freely falling body and has an acceleration g. We can apply Newton's second law to a freely falling body by setting $F = W$ and $a = g$ in the equation $F = Ma$, to obtain

$$W = Mg. \tag{5-5}$$

The appropriate units for mass, length, time, and force for use in Equation (5-3) are shown in Table 5-1. Only when units appropriate to a

TABLE 5-1 SYSTEMS OF UNITS

System	Mass	Length	Time	Force
Metric absolute—cgs	Gram	Centimeter	Second	Dyne
Metric absolute—mks	Kilogram	Meter	Second	Newton
British gravitational	Slug	Foot	Second	Pound

particular system are used is the constant K of Equation (5-2) equal to 1, and, in fact, it is this consideration which converts a collection of apparently unrelated quantities into a system of units.

Illustrative Example. An automobile weighing 3,200 lb starts from rest and acquires a speed of 30 mi/hr in 5 sec. Determine the resultant force on the automobile.

The acceleration of the automobile is

$$a = \frac{30 \text{ mi/hr}}{5 \text{ sec}} = \frac{44}{5} \frac{\text{ft}}{\text{sec}^2}.$$

The mass of the automobile is found by substitution in Equation (5-5):

$$3{,}200 \text{ lb} = M \times 32 \frac{\text{ft}}{\text{sec}^2},$$

yielding

$$M = 100 \text{ slugs}.$$

To find the force we substitute the now known values of M and a in appropriate units in Equation (5-3):

$$F = 100 \text{ slugs} \times \frac{44}{5} \frac{\text{ft}}{\text{sec}^2} = 880 \text{ lb}.$$

A general class of problems of great value in developing understanding of Newton's second law deals with two or more bodies connected by a rope which is passed over a pulley. The bodies may be hanging freely or may be supported on inclined planes, and to add additional complications the planes may be made rough. In such problems we analyze the forces acting on each body separately, and then tie the system together through an algebraic statement about the way the various parts are connected. We call these connections the *constraints* under which the system is required to move. Thus a body sliding on a horizontal table is constrained to move in the horizontal plane of the table. A bead sliding along a wire is constrained to move along that wire. Two bodies connected by an inextensible rope must always be a fixed distance apart.

Illustrative Example. A useful experimental device, called *Atwood's machine*, consists of two bodies suspended by a rope which is passed over a frictionless fixed pulley, as illustrated in Figure 5-3. Let us call the masses of the suspended objects M_1 and M_2, the tension in the ropes S_1 and S_2, as illustrated in the figure, and their respective accelerations a_1 and a_2. Suppose that M_1 and M_2 are known and we wish to find the accelerations of the two bodies and the tension in the rope.

The force of gravity acting upon these bodies is equal to the weights of the bodies M_1g and M_2g, respectively. Each body experiences an upward force

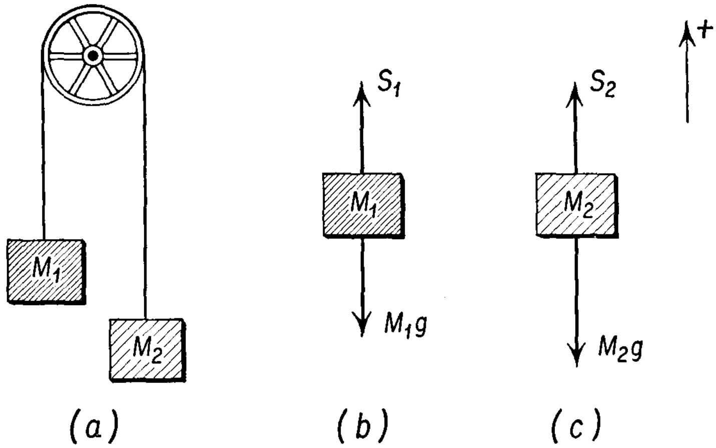


Fig. 5-3 Atwood's machine.

produced by the pull of the rope on it. Let us call the tension in the rope acting on the first body S_1, and the tension in the rope acting on the second body S_2. Following our customary sign conventions, we call the direction vertically upward positive.

First we imagine the bodies M_1 and M_2 to be completely isolated in space, as shown in Figures 5-3(b) and 5-3(c), and apply Equation (5-3) to determine their motion. Applying Newton's second law, we get

for M_1: $$S_1 - M_1g = M_1a_1; \qquad \textbf{(a)}$$

for M_2: $$S_2 - M_2g = M_2a_2. \qquad \textbf{(b)}$$

Now we examine the connection between the two bodies. First, because the rope which connects them passes over a frictionless pulley, the tension in the rope is everywhere the same. By the very nature of a tensile force, when the rope pulls M_1 upward, it must also pull M_2 upward. Thus the rope acts simultaneously on M_1 and M_2 in the directions indicated in Figures 5-3(b) and 5-3(c), and we may write

$$S_1 = S_2 = S. \qquad \textbf{(c)}$$

Now we consider a second aspect of the connection between the two bodies. The distance between them, measured along the rope, is always the length of the rope. Thus if the body M_1 moves 1 ft upward, the body M_2 must move 1 ft downward. A positive displacement of M_1 generates an equal negative displacement of M_2. If the body M_1 is given a positive acceleration, the body M_2 must be given an equal negative acceleration. We may write

$$a_1 = -a_2 = a. \tag{d}$$

Substituting equations (c) and (d) into (a) and (b), we have

$$S - M_1 g = M_1 a,$$

$$S - M_2 g = -M_2 a.$$

Subtracting the second equation from the first, we obtain

$$M_2 g - M_1 g = M_2 a + M_1 a,$$

from which

$$a = \frac{M_2 - M_1}{M_2 + M_1} g.$$

Thus, if M_2 is greater than M_1, the body M_2 receives a positive acceleration of magnitude a, while the body M_2 experiences a negative acceleration of equal magnitude. If we multiply the first equation by M_2, the second by M_1, and add, we find

$$(M_1 + M_2)S = M_1 M_2 g,$$

from which

$$S = \frac{M_1 M_2}{M_1 + M_2} g.$$

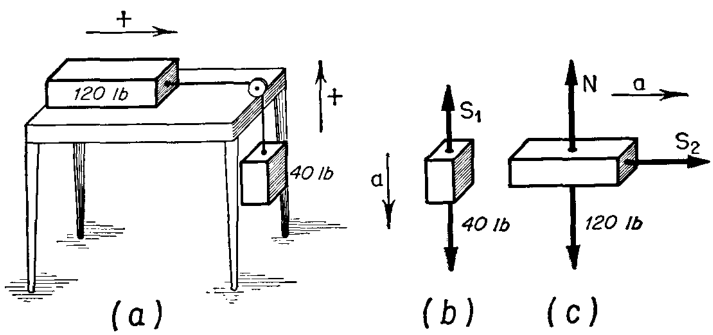


Fig. 5-4

Illustrative Example. A box weighing 120 lb is placed on a smooth table. A cord tied to this box passes over a smooth pulley fixed to the edge of the table. Another box weighing 40 lb is fastened to the other end of the cord, as shown in Figure 5-4(a). Determine the acceleration of the two bodies and the tension in the cord.

First we imagine the two bodies to be isolated in space and examine the forces acting on them, as in Figures 5-4(b) and 5-4(c). The only forces acting on the 40-lb body are its weight and the tension in the rope supporting it, which we shall call S_1. Three forces act on the 120-lb body. These are the weight of the body of 120 lb, acting vertically downward, the force of the smooth table on the body acting vertically upward, which we call N, and the tension in the rope acting on it, which we call S_2. Since the 120-lb body is constrained to move in a horizontal plane, it can have no vertical acceleration, and

$$N = 120 \text{ lb}.$$

The resultant of the forces acting on the 120-lb body is S_2 acting to the right. Writing a_1 for the acceleration of the 40-lb body and a_2 for the acceleration of the 120-lb body, we have, from Equations (5-4a) and (5-4b),

$$S_1 - 40 \text{ lb} = \tfrac{40}{32} \text{ slug} \times a_1, \tag{a}$$

and

$$S_2 = \tfrac{120}{32} \text{ slug} \times a_2. \tag{b}$$

Examining the nature of the constraint imposed by the rope, we note first that the magnitude of the tension in the rope must be the same at both ends of the rope, and that the directions chosen for S_1 and S_2 are appropriate, for a positive value of S_1 implies a positive value of S_2. We write

$$S_1 = S_2 = S. \tag{c}$$

Next we find that a displacement of the 120-lb weight to the right implies an equal displacement of the 40-lb weight downward. Thus a positive displacement of the 120-lb weight implies an equal negative displacement of the 40-lb weight, and a positive acceleration of the 40-lb weight implies an equal negative acceleration of the 120-lb weight, and we write

$$+a_2 = -a_1 = a. \tag{d}$$

Substituting equations (c) and (d) into (a) and (b), we have

$$S - 40 \text{ lb} = \tfrac{40}{32} \text{ slug} \times (-a), \tag{e}$$

and

$$S = \tfrac{120}{32} \text{ slug} \times a. \tag{f}$$

Subtracting the second from the first of these equations, we have

$$-40 \text{ lb} = -\tfrac{160}{32} \text{ slug} \times a, \tag{g}$$

from which

$$a = 8 \frac{\text{ft}}{\text{sec}^2}; \tag{h}$$

substituting this value of a in equation (f) yields

$$S = 30 \text{ lb}.$$

We note that the 120-lb weight receives a positive acceleration of 8 ft/sec^2, while the 40-lb weight receives an equal negative, or downward, acceleration.

It is particularly important to emphasize the procedure used in solving these pulley problems. In each case we have systematically isolated the bodies involved in the problem and have examined the forces acting on each of them. To each body we then applied Newton's second law. The constraints which related the several motions to give a sufficient number of relationships to solve the problem were then introduced. In more advanced courses in mechanics, more sophisticated methods are developed for the solutions of such problems, but in all cases the methods depend on Newton's equation, and if a problem is soluble by any method it is soluble by the persistent and systematic application of Newton's second law of motion.

5-7 Weight and Mass

Although considerable space has already been devoted to a discussion of the distinction between weight and mass, the subject is of sufficient importance to warrant further emphasis. If we consider two freely falling bodies at the same place on the earth's surface, one of which has a mass M and weight W, while the other has a mass m and weight w, we find, by application of Equation (5-5) to each of the bodies, that

$$W = Mg,$$

and

$$w = mg,$$

from which

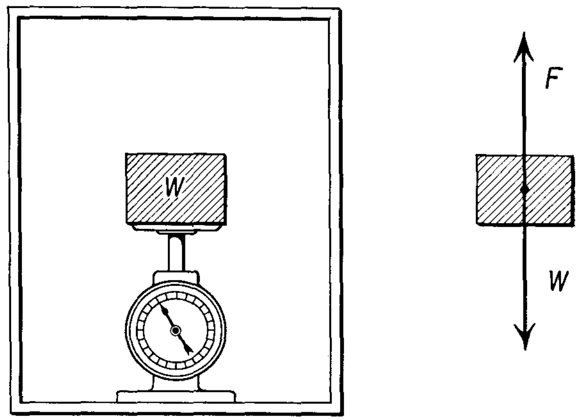
$$\frac{W}{w} = \frac{M}{m}. \tag{5-6}$$

Thus the magnitudes of the weights of two bodies at the same place are in the same ratio as their masses. This is the reason the beam balance can be used to determine the mass of an unknown object in terms of a standard mass.

As we go from place to place, the value of g changes with latitude and with altitude. The mass of a body, however, remains constant unless the body is traveling with a speed comparable to the speed of light, which is about 186,000 mi/sec, in which case the mass of the body increases over its mass at rest. We shall restrict this discussion to bodies moving with speeds which are small in comparison with the speed of light. The weight W of a body of constant mass M depends upon the particular place where the weight is measured. In the systems of units used in this book, the weight of a body is properly referred to in units of force. Thus the weight of a body is properly expressed in dynes, in newtons, or in pounds. In these terms the operations customarily undertaken in a chemical laboratory, called "weighings," are more properly "massings," for the analytical chemist is not interested in the force of the earth's attraction but in the quantity of matter present in a sample.

The weight of an object is measured by the deflection of a calibrated spring. The reading of such a spring scale varies with the state of motion of the scale. When the scale is at rest, or moving with uniform motion in a straight line, the reading of the scale is the same, but when the motion is accelerated, the reading of the scale depends on both the mass of the object and the amount of acceleration. This is the source of the sensation of heaviness which is experienced when standing in an elevator being accelerated upward, or of lightness in the same elevator when its acceleration is downward.

Fig. 5-5



Illustrative Example. A weight W rests on a spring scale which is placed on the floor of an elevator. The scale reads 50 lb when the elevator is at rest. The elevator is started, accelerating upward at a rate of 16 ft/sec² for 1 sec, then continues at constant speed for 5 sec, and finally is decelerated at the rate of 16 ft/sec² for 1 sec. What is the reading of the scale during the first second? During the next 5 sec? During the last second?

The forces acting on the body are its weight W, acting downward, and the force of the scale F, acting upward, as shown in Figure 5-5.

During the first second the acceleration is 16 ft/sec² in the upward direction. Substituting in Newton's equation, we find that

$$F - 50\text{ lb} = \tfrac{50}{32}\text{ slug} \times 16\,\frac{\text{ft}}{\text{sec}^2},$$

so that

$$F = 75\text{ lb}.$$

During the next 5 sec the acceleration is zero. Hence the scale reads 50 lb.

For the last second the acceleration is 16 ft/sec² in the downward direction. Thus $a = -16$ ft/sec², and we find that

$$F - 50\text{ lb} = \tfrac{50}{32}\text{ slug} \times \left(-16\,\frac{\text{ft}}{\text{sec}^2}\right),$$

or

$$F = 25\text{ lb}.$$

5-8 Motion on an Inclined Plane

When a block of mass M is placed on an inclined plane, the forces acting on the block are due to the force of gravity and to the forces exerted on the block by the plane itself.

When there is no friction, the force exerted by the plane must be perpendicular to the surface of the plane, as illustrated in Figure 5-6(a). The

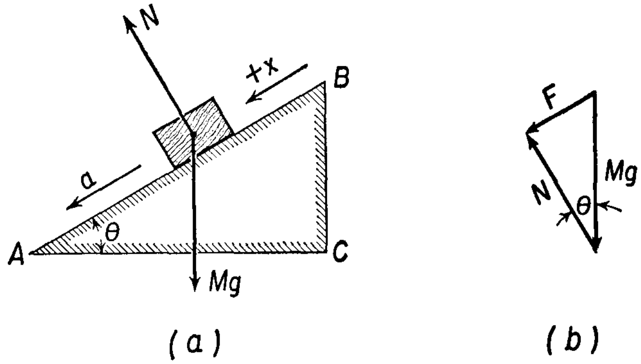


Fig. 5-6 Forces acting on a block placed on a frictionless inclined plane.

resultant of the force of gravity Mg and the normal force N of the plane must be a force F parallel to the plane, as shown in Figure 5-6(b) whose magnitude is given by

$$F = Mg \sin \theta,$$

where the angle θ is the angle the plane makes with the horizontal direction, and the direction of F is down the plane, as shown in the figure. Choosing the direction of the x axis as parallel to the plane, positive downward, we may find the acceleration of the block, from Equation (5-4a), as

$$Mg \sin \theta = Ma,$$

yielding

$$a = g \sin \theta. \tag{5-7}$$

The acceleration of a body on a frictionless inclined plane is down the plane and depends on the angle of inclination but is independent of the mass of the body. Looking at the same problem another way, we see that the acceleration g due to gravity is a vector quantity, directed vertically downward. The constraint of the plane prohibits such motion and only permits the body to move along the plane itself. The component of the acceleration of gravity along the plane is of magnitude $a = g \sin \theta$ directed down the plane.

When Galileo was studying the laws of motion, clocks of sufficient accuracy to time the motion of freely falling bodies were not yet available.

To slow down the motion so that it could be studied with available timing devices, he made use of the properties of a smooth inclined plane.

If a body is in motion on a rough inclined plane, the force of friction between the body and the plane affects the motion. The frictional force F_r acts so as to oppose the motion of the body along the plane. To determine the magnitude of the frictional force, we resolve the force of gravity W into components parallel and perpendicular to the plane, as shown in Figure 5-7, and find the parallel component to be of magnitude $W \sin \theta$ and the perpendicular component to be $W \cos \theta$.

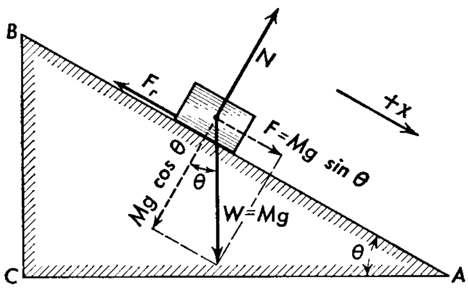


Fig. 5-7 Analysis of forces which act on a body that is sliding down a rough inclined plane.

The relationship between the frictional force and the normal force has been given as

$$F_r = fN,$$

so that

$$F_r = fMg \cos \theta,$$

where f is the coefficient of sliding friction between the body and the plane. If no other forces act on the body, its motion will be down the plane, and the frictional force will be directed up the plane, as shown in the figure. If we consider the x direction as parallel to the plane, positive downward, we find, on substituting in Newton's equation,

$$Mg \sin \theta - fMg \cos \theta = Ma,$$

$$a = g \sin \theta - fg \cos \theta. \tag{5-8}$$

Equation (5-8) becomes equivalent to Equation (5-7) when there is no friction.

If the angle of the plane is reduced to some critical value θ_c, the object will just slide down the plane with no acceleration. Equation (5-8) then gives

$$0 = g \sin \theta_c - fg \cos \theta_c,$$

from which

$$f = \tan \theta_c, \tag{5-9}$$

which is identical with a result obtained in Section 3-6.

Illustrative Example. A heavy wooden crate weighing 200 lb is pulled up a wooden plane, inclined at an angle of 20°, by a force of 150 lb. The coefficient of kinetic friction between the two surfaces is 0.30. (a) Determine the acceleration of the crate. (b) If the rope hauling it breaks, discuss the subsequent motion of the crate.

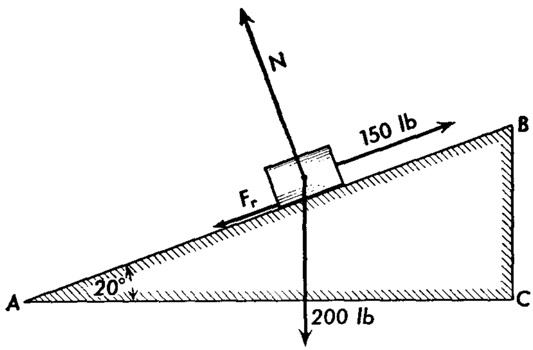


Fig. 5-8

(a) Figure 5-8 shows the forces which act on the crate as it moves up the plane. The force N which the plane exerts on the crate is perpendicular to its surface and is equal to the normal component of the weight, since there is no acceleration normal to the plane. Hence

$$N = W\cos\theta = 200\text{ lb} \times 0.9397 = 187.9\text{ lb.}$$

The force of friction is

$$F_r = fN = 0.3 \times 187.9\text{ lb} = 56.4\text{ lb.}$$

The component of the weight parallel to the plane is

$$F = W\sin\theta = 200\text{ lb} \times 0.3420 = 68.4\text{ lb.}$$

Choosing the direction of the x axis as parallel to the plane, positive downward, we find, from Equation (5-4a),

$$-150\text{ lb} + 56.4\text{ lb} + 68.4\text{ lb} = \tfrac{200}{32}\text{ slug} \times a,$$

so that

$$a = -4.03\,\frac{\text{ft}}{\text{sec}^2}\cdot$$

(b) When the rope breaks, the forces parallel to the plane which act on the crate are the component of the weight parallel to the plane and the frictional force. The direction of the frictional force is now reversed, since it always acts in a direction to oppose the motion. We find, substituting in Equation (5-4a),

$$-56.4\text{ lb} + 68.4\text{ lb} = \tfrac{200}{32}\text{ slug} \times a,$$

so that

$$a = +1.92\,\frac{\text{ft}}{\text{sec}^2}\cdot$$

5-9 Motion through the Air

Objects falling through the air are acted upon by the resistance of the air as well as by the force of gravity. Some of this resistance may be due to the viscosity of the air and the rest to turbulence. This resistive force is often called *drag*, in connection with the flight of aircraft, and is the only reason an airplane does not cut off its engines after once assuming flying speed at a desired altitude. The effects of drag are also evident in the dust raised by the wind and in the transport of gravel and sand by flowing water. Drag is often put to use in engineering in the pneumatic conveying of grain and similar materials.

Experience shows that the resistance of air to motion through it increases as the velocity of the body increases. A body falling through the air for a sufficient time will ultimately reach a *terminal velocity*, at which time the force due to the resistance of the air is equal to the weight of the body. The body then continues to move downward with this limiting velocity.

For simplicity, let us assume that, for the case of a spherical body moving slowly through the air the resistance varies directly with the velocity. We may write

$$\mathbf{R} = K\mathbf{v}, \tag{5-10}$$

where K is a constant of proportionality depending on the cross-sectional area of the body and the viscosity of the air. As the velocity of fall increases, the magnitude of the force $\mathbf{R}$ increases until it becomes equal to the weight of the body. Thus

$$R = Kv_l = W,$$

from which

$$v_l = \frac{W}{K}, \tag{5-11}$$

where v_l is the limiting or the terminal velocity of the body.

Thus the terminal velocity of fall v_l of raindrops depends upon their weight. When raindrops reach the surface of the earth, the larger and heavier drops are moving faster than the light ones. The effective cross-sectional area of a man wearing an opened parachute is considerably greater than that of a man wearing a closed parachute, and the associated increase in K makes a very important difference to the man who is forced to leave an airplane in flight. The variation in K with cross-sectional area has been applied for centuries in the winnowing of grain to remove chaff and is today widely used in the cleaning of seed.

While the path of a freely falling projectile is parabolic, the path of a projectile in air is not. The speed of the projectile is steadily diminished by the resistance of the air. A baseball caught in the outfield is much easier to catch than the same ball would have been if it could have been caught in the infield.

5-10 Pairs of Forces. Newton's Third Law

In our discussion of Newton's first and second laws, our attention was focused on one body on which a set of external forces acted. If we now analyze the origin of each of these forces, we find that each force is produced by the action of some other body on the one under discussion. If we push a trunk along the floor, the trunk exerts a force against our hands. If a ball is hit with a bat, not only is there a force exerted by the bat on the ball, but the ball also exerts a force on the bat. An automobile which is standing still pushes down on the ground at each of the surfaces of contact between its tires and the ground. At each region of contact, the ground exerts a force upward equal to that exerted by the car. *Newton's third law states that whenever one body exerts a force on another, the second body exerts a force equal in magnitude and opposite in direction on the first body.* This law is sometimes called the law of *action* and *reaction.*

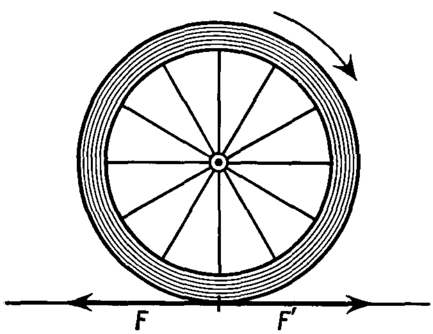


Fig. 5-9 Force F' exerted by the ground on the wheel is equal in magnitude but opposite in direction to the force F exerted by the wheel on the ground.

As an illustration of Newton's third law, consider the manner in which a car is set in motion. To start the car moving forward, there must be a net or unbalanced horizontal force acting on the car. To produce this horizontal force, the engine is started and then connected by means of gears and shafts to the rear wheels, causing them to turn in a clockwise direction, as shown in Figure 5-9. Because of the friction between the tires and the ground, the wheels exert a force F to the left (backward) on the ground; the ground exerts an equal and opposite force F' forward on the rear wheels. It is this horizontal force F' which makes the car go forward. To understand that it is the push of the ground on the driving wheels which makes the car go forward, just think of driving experiences on a winter day with ice on the ground, when the friction between the tires and the ground is very small. What usually happens is that the wheels spin in a clockwise direction, but since there is little frictional force available, the wheels merely spin around, and the car does not move.

Problems

5-1. How big a force is required to give a 40-gm mass an acceleration of 150 cm/sec^2?

5-2. What constant force is required to give a body weighing 120 lb an acceleration of 4 ft/sec^2?

5-3. An automobile weighing 2,800 lb starting from rest acquires a speed of 40 mi/hr in 12 sec. Assuming that the acceleration is uniform, determine the unbalanced force which is acting on the automobile during this time.

5-4. A box whose mass is 350 gm rests on a table. A steady horizontal force is applied to this box. After 5 sec the box has acquired a speed of 40 cm/sec. Determine the force acting on the box.

5-5. A box of 800 gm mass is projected across a horizontal table with an initial speed of 150 cm/sec. It comes to rest on the table after having traversed a distance of 180 cm. Determine the frictional force opposing the motion.

5-6. A box whose mass is 12 kg is given an acceleration of 25 m/sec^2 on a horizontal surface. (a) Determine the resultant force acting on the box. (b) If the box starts from rest, determine the speed it will acquire in 8 sec.

5-7. A train weighing 450 tons has its speed increased from 20 mi/hr to 50 mi/hr in 15 sec. What force is supplied by the locomotive to produce this acceleration?

5-8. A steel cable supports an elevator weighing 2,500 lb. What is the tension in the cable when the elevator is moving (a) upward with a uniform velocity of 600 ft/min and (b) downward with a uniform velocity of 500 ft/min?

5-9. A steel cable supports an elevator weighing 1,800 lb. Starting from rest, the elevator acquires a velocity upward of 600 ft/min in 2 sec. (a) What is the resultant force acting on the elevator? (b) What is the tension in the cable?

5-10. The elevator of Problem 5-9, when going down, acquires a velocity of 500 ft/min in 2 sec. (a) What is the resultant force acting on the elevator? (b) What is the tension in the cable?

5-11. A steel ball whose mass is 250 gm is attached to the end of a cord. The ball is pulled upward with an acceleration of 120 cm/sec^2. Determine (a) the unbalanced force acting on the ball and (b) the tension in the cord.

5-12. A cube whose mass is 1,600 gm rests on a smooth table. A cord which is attached to the center of one face of the cube passes over a frictionless pulley at the edge of the table. A steel ball whose mass is 800 gm is fastened to the free end of the cord. Determine (a) the acceleration of each body and (b) the tension in the cord.

5-13. A box weighing 72 lb is placed on a smooth horizontal table. A cord which is connected to the center of one face of the box passes over a smooth pulley at the edge of the table. A steel ball weighing 24 lb is then fastened to the other end of the cord. Determine (a) the acceleration of each body and (b) the tension in the cord.

5-14. Two boxes, one weighing 16 lb and the other weighing 4 lb, are attached to the ends of a cord. The cord is placed over a frictionless pulley which is free to rotate about a horizontal axis. Determine the acceleration of each box.

5-15. A cord passes over a fixed frictionless pulley. A cylinder whose mass is 3 kg is suspended from one end of the cord, and another cylinder whose mass is 2 kg is suspended from the other end. Determine (a) the acceleration of the system and (b) the tension in the cord.

5-16. A series of frictionless inclined planes all have the same heights but have different lengths. Show that the time required for an object to slide down any of these planes is directly proportional to the length of the plane.

5-17. A car weighing 3,000 lb and moving with a speed of 20 mi/hr reaches a hill having a 5 per cent grade and starts coasting downhill. Determine (a) the component of the weight acting downhill and (b) the speed the car will acquire if it coasts for 400 ft, assuming friction is negligible. [NOTE: A hill having a 5 per cent grade is one which rises 5 ft for every 100 ft of length.]

5-18. A body whose mass is 3 kg is projected up an inclined plane with an initial velocity of 5 m/sec. The plane is inclined at an angle of 30° to the horizontal, and the coefficient of kinetic friction between the plane and the body is 0.2. Determine (a) how far up the plane the body will go before coming to rest, (b) its acceleration down the plane, and (c) the speed it will have when it reaches its starting point.

5-19. A box slides down a 30° inclined plane with an acceleration of 4 ft/sec^2. Determine the coefficient of friction between the box and the plane.

5-20. A box whose mass is 18 kg rests on a table. A cord tied to this box passes over a frictionless pulley at the edge of the table. A cylinder whose mass is 6 kg is hung from the free end of the cord. The coefficient of friction between the box and the table is 0.25. Determine (a) the acceleration of the box, (b) the tension in the cord, and (c) the distance the cylinder will move in 3 sec.

5-21. A boy takes a running start with a sled and acquires a speed of 8 ft/sec. If the coefficient of friction between sled and snow is 0.10, how far will the sled move on a level road before coming to rest?

5-22. A boy coasts down a hill on a sled, reaching level ground with a speed of 30 ft/sec. If the coefficient of friction between the steel runners and the snow is 0.05 and the boy and sled weigh 150 lb, find how far the sled will travel before coming to rest.

5-23. Show that if the force due to the resistance of the air varies with the square of the velocity of a falling body, the limiting velocity of fall is proportional to the square root of the weight of the body.

5-24. Two men, one weighing 180 lb and the other weighing 120 lb, are on ice skates. Each holds one end of a taut rope. The heavier man exerts a force of 20 lb on the rope. (a) How big a force does the lighter man exert? (b) What is the acceleration of each man? Neglect friction.

5-25. A 5-gm bullet is fired from a gun whose barrel is 60 cm long. The bullet leaves the gun with a muzzle velocity of 2,500 cm/sec. What was the average force acting on the bullet?

5-26. A man weighing 150 lb stands on a platform weighing 42 lb. The platform is suspended by a rope which passes over a frictionless pulley. The man pulls down on the free end of the rope to lift himself and the platform. (a) With what force must he pull on the rope if the system consisting of the man and the platform is to receive an upward acceleration of 3 ft/sec^2? (b) What is the maximum acceleration with which the man can raise the platform and still stay on the platform?

5-27. A rope inclined at an angle of 37° with the horizontal is used to drag a 50-kg block along a level floor with an acceleration of 1 m/sec^2. The coefficient of friction between the block and the floor is 0.2. What is the tension in the rope?

5-28. A projectile of mass 5 gm is fired from a gun with a muzzle velocity of 2,500 cm/sec directed due east at an angle of 45° with the horizontal. A wind is blowing from the north, exerting a steady force of 1,000 dynes against the projectile. Find the position of the projectile when it strikes the ground.

5-29. A body of mass 100 kg is hung from a rope which is passed over a frictionless pulley to a man on the ground who is interested in raising the body a distance of 25 m in the shortest possible time. The pulley is hung from the ceiling by a chain whose breaking strength is 2,000 nt. What is the shortest time in which the body can be raised?

5-30. Two bodies, each weighing 10 lb, are connected by a cord which passes over a light frictionless pulley. What vertical force must be applied to the pulley to raise the system with an acceleration of 5 ft/sec^2?

5-31. Two bodies, weighing 10 lb and 20 lb, rest upon a table. The two bodies are connected by a cord which passes over a light frictionless pulley. What is the least vertical force which can be applied to the pulley (a) to raise the 10-lb weight? (b) To raise the 20-lb weight?

5-32. If in Problem 5-31 a force of 50 lb is applied to raise the pulley, what will be the acceleration of (a) the 10-lb weight and (b) the 20-lb weight?

5-33. A block weighing 5 lb rests on a horizontal surface. The coefficient of friction between the block and the surface is 0.2. A horizontal force of 2 lb is applied to the block. (a) What is the acceleration of the block? (b) The system consisting of block, table, and applied force is placed on an elevator which rises at a constant speed of 5 ft/sec. What is now the acceleration of the block? (c) The elevator is then brought to a stop with a uniform acceleration of 4 ft/sec^2. During this period of vertical acceleration what is the horizontal acceleration of the block?

5-34. A pendulum bob weighing 1 lb is hung from the roof of a railroad car. The train is started with a constant acceleration of 3.2 ft/sec^2. (a) At what angle with the vertical does the pendulum bob hang? (b) What is the tension in the string?

5-35. In a train moving with constant acceleration it is observed that the chandeliers hang at an angle of 0.57° with the vertical. The train starts from rest. With what velocity is the train moving at the end of 2 minutes?

5-36. A mass of 10 kg and a second mass of 5 kg are connected by a string and rest on a horizontal frictionless table. A constant pull of 60 nt is applied to the 5 kg mass. Find (a) the acceleration of the system and (b) the tension in the string.