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Physics, Chapter 8: Hydrostatics (Fluids at Rest)

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Hydrostatics (Fluids at Rest)

8-1 Three Phases of Matter

From our everyday experience, we have become familiar with the fact that matter occurs in three different forms—*solid*, *liquid*, and *gas*. Under ordinary conditions stone, iron, copper, and chalk, for example, are solids; water, oil, and mercury are liquids; air, hydrogen, and carbon dioxide are gases. Each one of these forms is called a *phase*. At times it is difficult to distinguish clearly between the solid and the liquid phases, as in a material such as tar which flows under the action of a force at ordinary temperatures. Metals at high temperatures flow or "creep" under the action of a force. Even where the different phases are clearly recognizable, materials undergo a phase change under different conditions of temperature and pressure. For the present we shall confine our discussion to the application of the principles of mechanics to bodies which remain in the same phase.

Liquids and *gases* are sometimes grouped together as *fluids* because they *flow* very readily under the application of an external force, while solids do not. A solid has a definite size and a definite shape, and these change only slightly under the application of external forces. For this reason it is possible to study the statics of *solids* by characterizing them as *rigid* bodies. *Liquids*, on the other hand, possess a definite size or volume but change their shape very readily. Liquids at rest generally take the shape of the containing vessel. If the containing vessel has a volume greater than that of the liquid put into it, there will be a *free surface* at the top of the liquid. A gas differs from a liquid in that a gas has neither size nor shape. A quantity of gas placed in a container will completely fill that container. There is no free surface. The volume of the gas is the volume of the container.

8-2 Pressure

There is a difference in the manner in which a force is applied to a fluid and the way it is applied to a solid. A force can be supported by a single point of a free solid, but it can only be supported by a surface of an enclosed fluid. In a discussion of the results of the application of forces to fluids, it is convenient to introduce a new term called *pressure.* If a force F is applied to the surface of a fluid and acts over an area A perpendicular to it, then the average pressure $\bar{P}$ is defined as

$$\boxed{\bar{P} = \frac{F}{A}.} \tag{8-1}$$

The pressure may be expressed in dynes per square centimeter, in pounds per square foot, in newtons per square meter, or in any other appropriate set of units. Pressure is a scalar quantity.

When a fluid is under pressure, it exerts a force on any surface which contains the fluid. Equation (8-1), which describes the average pressure, is not quite complete, for, while relating the magnitude of the force exerted to the pressure and the area, it must be accompanied by a statement about the direction of the force. We may make the equation more complete by considering *area as a vector quantity.* An element of area $\Delta\mathbf{A}$ may be described as a vector whose magnitude is the numerical value of the area and whose direction is perpendicular to the surface of the area element, as shown in Figure 8-1. If the element of area is part of a closed surface, it is conventional to choose the *outward normal* to the surface as the direction of the area vector. With this convention we may write

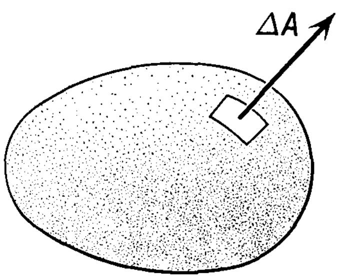

Fig. 8-1 An area vector $\Delta\mathbf{A}$ is directed along the outward normal to the surface.

$$\Delta\mathbf{F} = P\,\Delta\mathbf{A} \tag{8-2}$$

for the force $\Delta\mathbf{F}$ exerted by a fluid of pressure P against any surface element of area $\Delta\mathbf{A}$.

The pressure may vary from point to point within a fluid. We speak of the pressure at a point within a volume of fluid, meaning that we imagine the point to be surrounded by a small container and divide the total force exerted by the fluid against the walls of the container by the area of the container. Following the usual limiting processes of the calculus, we examine this quotient as the volume of the container gets smaller and smaller, and we call its limiting value the pressure at the point.

8-3 Density

In discussing distributions of matter such as solids or fluids, it is convenient to define a quantity called the density ρ (rho) as the mass per unit volume or,

$$\rho = \frac{m}{V}. \tag{8-3}$$

Thus the mass of a homogeneous body of material of volume V is given by $m = \rho V$. The units in which density is expressed are the ones appropriate to the system of units being used. In the cgs system of units the density is expressed in grams per cubic centimeter; in the mks system of units the density is expressed in kilograms per cubic meter; in the British gravitational system of units the density is expressed in slugs per cubic foot.

It is common engineering practice to use the word "density" to express the weight per unit volume in pounds per cubic foot. We distinguish this

TABLE 8-1 DENSITIES OF SOME COMMON SUBSTANCES

Solids	Density in gm/cm^3	Liquids	Density in gm/cm^3
Aluminum	2.70	Alcohol	0.79
Brass	8.44–8.70	Ether	0.74
Copper	8.93	Glycerin	1.26
Cork	0.22–0.26	Mercury	13.596
Glass, common	2.4–2.8	Oil, olive	0.92
Glass, flint	2.9–5.9	Oil, paraffin	0.8
Gold	19.3	Water	1.00
Ice	0.917		
Iron	7.03–7.9	**Gases**	**Density at 0°C, 760 mm Hg in gm/cm^3**
Lead	11.34		
Osmium	22.5		
Nickel	8.9		
Platinum	21.50	Air	0.001293
Silver	10.5	Ammonia	0.000771
Tungsten	18.6–19.1	Carbon dioxide	0.001977
Wood, cedar	0.49–0.57	Helium	0.000179
Wood, ebony	1.11–1.33	Hydrogen	0.000090
Wood, elm	0.54–0.60	Oxygen	0.001429
Wood, white pine	0.35–0.50		
Zinc	7.1		

quantity from the density defined above by referring to the weight per unit volume as the *weight density*.

The density of solids varies only slightly with changes in temperature and pressure and is customarily given in tables as the result of measurement at a standard pressure and temperature. The density of metals ranges over a factor of about 10 from the lightest, magnesium, which has a density of 1.80 gm/cm^3 to the most dense, osmium, which has a density of 22.5 gm/cm^3. The density of commercial metals and alloys depends upon their composition. Some values of the density of various substances are given in Table 8-1.

8-4 Pressure Due to Weight of a Liquid

Let us consider the equilibrium conditions of a small element of liquid volume submerged within the body of the liquid, as shown in Figure 8-2. Since the volume element is at rest, the horizontal forces exerted upon this volume element by the surrounding liquid must have a zero resultant. In the vertical direction we note that there must be a difference in the forces exerted on the top and bottom faces of the volume element sufficient to

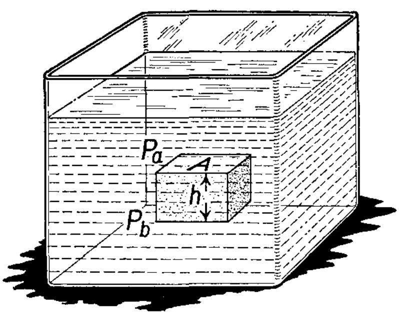

Fig. 8-2

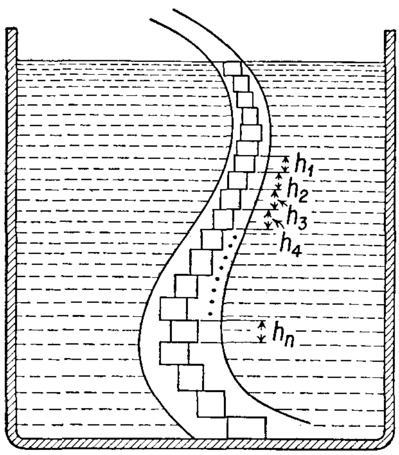

Fig. 8-3

support the weight of the liquid within that volume element. If the pressure at the level of the top face is P_a and the pressure at the level of the bottom face is P_b, the downward force exerted on the top face of the volume element by the surrounding liquid is P_aA, while the upward force exerted against the bottom face is P_bA. The difference between these forces must be equal to the weight of the liquid contained within the element of height h.

Thus we have

$$P_bA - P_aA = Ah\rho g,$$

and, dividing by A,

$$P_b - P_a = h\rho g. \tag{8-4}$$

Thus the pressure difference between two adjacent level surfaces in the liquid is given by $h\rho g$, where h represents the vertical distance between the two surfaces.

In the event that we wish to find the pressure within a liquid confined in a tube of irregular shape, such as the S-shaped tube of Figure 8-3, we may imagine the liquid to be subdivided into a succession of volume elements, one atop the other, each of which contributes a small increment of pressure, depending on its vertical height, to the pressure at the depth h.

Thus the pressure at all points at a given depth beneath the surface of a liquid depends upon the depth but not upon the shape of the container; *all points at the same horizontal level surface within a body of liquid are at the same pressure, as long as the liquid is at rest.* In fact, this statement is often taken as the definition of a level surface. If the pressure on the surface of a liquid is taken as zero, then the pressure P at any point a distance h below this level is given by

$$\boxed{P = h\rho g.} \tag{8-5}$$

Illustrative Example. Determine the pressure at the bottom of a column of mercury 70 cm high.

From Equation (8-5) we have

$$P = h\rho g.$$

We substitute the value of $h = 70$ cm, the density ρ of mercury $= 13.6$ gm/cm^3, and $g = 980$ cm/sec^2, and obtain

$$P = 70\ \text{cm} \times 13.6\,\frac{\text{gm}}{\text{cm}^3} \times 980\,\frac{\text{cm}}{\text{sec}^2},$$

$$P = 9.33 \times 10^5\,\frac{\text{dynes}}{\text{cm}^2}.$$

8-5 Pressure in a Confined Liquid

In addition to the pressure due to its weight, a confined liquid may be subjected to an additional pressure by the application of an external force. Suppose the liquid is in a cylinder, as shown in Figure 8-4, and that a tight-fitting piston is placed on the surface of the liquid. If a force F is applied to the piston, it will remain in practically the same position, since the compressibility of liquids is very small. If A is the area of the piston, this external force produces a pressure $P = F/A$ at the surface of the liquid. As we have seen from the preceding discussion, as we examine the pressure at increasing depth beneath the surface of the liquid, each element of volume adds a contribution to the pressure which is due to its own weight

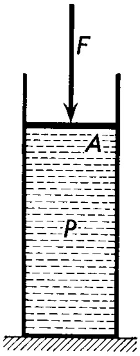

Fig. 8-4 A pressure P is produced in a confined liquid by the application of a force F on a piston of area A in contact with the liquid.

to the pressure at the top surface of that volume, so that the pressure P due to the external force is transmitted throughout every part of the liquid and acts on all surfaces in contact with the liquid. This is sometimes known as *Pascal's principle* and may be stated as follows:

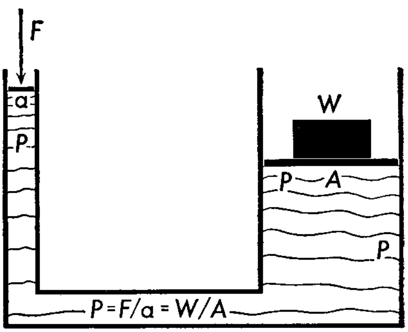

Fig. 8-5 Hydraulic press. The pressure P is the same everywhere throughout the liquid, if the pressure due to the weight of the liquid is negligible.

Whenever the pressure in a confined liquid is increased or diminished at any point, this change in pressure is transmitted equally throughout the entire liquid.

The operation of the hydraulic press, the hydraulic brakes of a car, and the hydraulic lift is based upon Pascal's principle. The hydraulic press, sketched in Figure 8-5, consists essentially of two connected cylinders, one of small cross-sectional area a, the other of large cross-sectional area A, each fitted with a piston. A liquid, usually oil or water, is supplied to it from a reservoir. By exerting a force F on the small piston, an additional pressure $P = F/a$ is produced. This pressure is transmitted throughout the liquid and hence acts on the larger piston of area A. The force that can be exerted by the larger piston is then PA. If the hydraulic press is designed to lift a weight W, then

$$W = PA = F\frac{A}{a}.$$

The hydraulic press may be considered as a simple machine in which the force exerted by the machine divided by the force exerted on the machine, is equal to the ratio of the areas of its pistons; thus

$$\frac{W}{F} = \frac{A}{a}.$$

A force-distributing system such as that used to operate the brakes of a car offers the very great advantage that the force applied on each of the brakes is automatically equal. Furthermore, the force is transmitted with very little mechanical movement of the hydraulic link.

8-6 Atmospheric Pressure

The atmosphere is a layer of air surrounding the earth; its thickness has been estimated as about 500 to 600 mi. The density of the air decreases with increasing altitude. Since air has weight, this layer of air produces a pressure, called the atmospheric pressure, at the surface of the earth. The atmospheric pressure varies from day to day by about 5 per cent, the variations often accompanying changes in the weather. The pressure of the air is measured by a *barometer*, which often consists of an evacuated tube inverted in a dish of mercury, as shown in Figure 8-6.

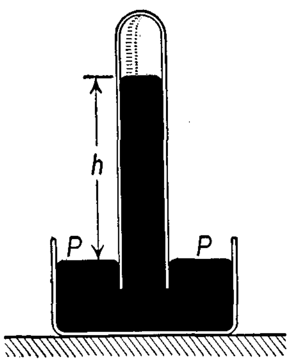

Fig. 8-6 Mercury barometer.

The atmosphere exerts a pressure P on the open surface of the mercury in the dish, and this is transmitted to the liquid in the tube. This pressure is balanced by the pressure due to the mercury in the tube at a height h above the open surface of the dish. To look at the barometer another way, we recall that the pressure is always the same at any level surface in a liquid. Consider the level surface defined by the surface of the mercury in the dish. Outside the barometer tube the pressure at this surface is entirely due to the atmosphere, so that the pressure of the mercury here is atmospheric pressure. Hence the pressure of the mercury within the tube at the level of the surface of the mercury in the dish is also atmospheric pressure. Knowing the density of mercury and the height to which the column of mercury rises within the evacuated barometer tube, we can calculate the atmospheric pressure.

Since the atmospheric pressure varies from day to day and from place to place, scientific data are often corrected and reported for a standard atmospheric pressure, the pressure of the atmosphere when the mercury barometer stands 76.0 cm above the free surface of mercury in the dish. The density of mercury at 0°C is 13.60 gm/cm^3, so that the pressure of a standard atmosphere at 0°C is

$$P = 76\ \text{cm} \times 13.60\ \frac{\text{gm}}{\text{cm}^3} \times 980\ \frac{\text{cm}}{\text{sec}^2},$$

$$P = 1{,}013{,}000\ \frac{\text{dynes}}{\text{cm}^2}.$$

This pressure can also be expressed as

$$P = 14.70\ \frac{\text{lb}}{\text{in.}^2}.$$

A pressure of 1.013×10^6 dynes/cm^2, or 14.70 lb/in.^{2}, or the pressure at the base of a column of mercury 76 cm high at 0°C is called a pressure of one *atmosphere* (abbreviated atm). In meteorology other units of pressure are used; they are mentioned here only for the sake of completeness. *A pressure of one bar is defined as one million dynes per square centimeter. A millibar is one thousandth of a bar.* Standard atmospheric pressure can thus be expressed as 1,013 millibars.

8-7 Pressure Gauges: Manometers

We have already encountered one type of pressure gauge, the mercury barometer. A second type of pressure gauge is the *aneroid* barometer, which consists of a partially evacuated cylindrical box made of corrugated metal, as shown in Figure 8-7. The difference in pressure between the inside of the chamber and the atmosphere causes the faces of the box to deflect, and this is balanced by a steel spring. The motion of the upper surface of the box is coupled to a pointer whose indication is read on a scale. Such a gauge is often used as a pressure altimeter on board aircraft, for atmospheric pressure varies with altitude, and so can be used as a measure of altitude. Another type of gauge, a *Bourdon* gauge, consists essentially of a flattened brass tube, closed at one end and bent into circular form. When a fluid under pressure is admitted to the open end of the tube, the tube straightens slightly, and this motion is coupled to a pointer whose deflection is calibrated.

The simplest type of pressure gauge is the *open-tube manometer*, illustrated in Figure 8-8, which consists essentially of a bent tube of transparent material with both arms vertical. One end A is open to the atmosphere, and the other end B is connected to the vessel in which the pressure

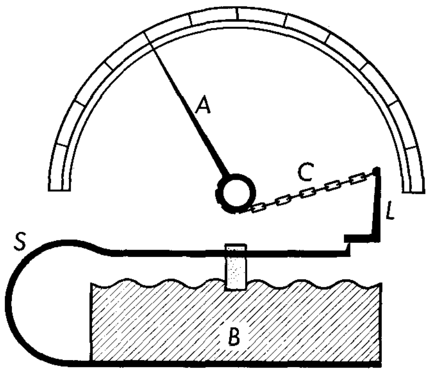

Fig. 8-7 Aneroid barometer. B is a box containing air at reduced pressure, S is a spring, L a lever, C a chain, and A a pointer.

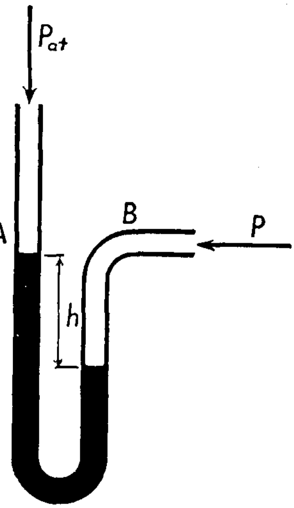

Fig. 8-8 Open-tube manometer.

is to be measured. The tube is partially filled with a liquid of density ρ. Suppose that the pressure within the vessel is greater than the atmospheric pressure, causing the liquid to rise in the column exposed to the atmosphere. Again we note that the pressure within a body of liquid is the same everywhere along a level surface. Choosing as the level surface of reference that of the lower liquid surface of the manometer, we observe that the pressure within the vessel P must be equal to the pressure of the atmosphere plus the pressure due to a column of liquid of height h, the difference in level between the liquid surfaces in the two arms of the manometer tube. Thus we have

$$P = P_{\text{atm}} + h\rho g. \tag{8-6}$$

In many technical applications it is the difference in pressure between the inside of a container and the atmosphere which is of importance, rather than the pressure itself. This is the case in a steam boiler, or in a gas line, and in many other applications of the manometer. The difference $P - P_{\text{atm}}$ is called the *gauge pressure.* The pressure P is called the *absolute pressure.* A barometer thus reads the absolute pressure of the atmosphere, while the height h of an open-tube manometer is a measure of the gauge pressure. The gauge pressure of the atmosphere itself is, of course, zero.

8-8 Archimedes' Principle

The fact that some objects float in water while others sink to the bottom has been known for centuries; Archimedes (287–212 B.C.) was the first to discover the principle underlying these phenomena. To understand Archimedes' principle it is necessary to consider the forces acting on a body totally immersed in a liquid, as shown in Figure 8-9. There is a downward force on the body equal to its own weight W, and, in addition, there is a buoyant force B on the body, which acts in the upward direction. This can be understood by imagining the volume now occupied by the body to be occupied instead by an equal volume of liquid. This volume of liquid would have been in equilibrium, which means that its weight would have been supported by the action of the rest of the liquid. This support comes from the difference in pressure between the top and the bottom of this volume. Hence, no matter what material occupies this volume, there will be a force upward on it equal to the weight of the liquid displaced.

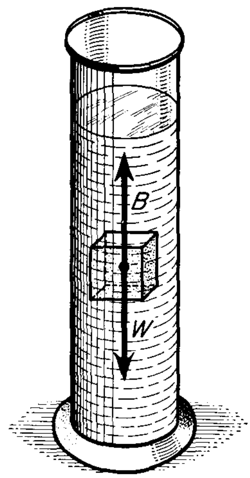

Fig. 8-9 Buoyant force on a block immersed in a liquid.

Archimedes' principle is a generalization of the result obtained above; it states that *any object partly or completely immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced.* The principle is applicable to both liquids and gases.

A body completely immersed in a fluid will sink if its weight is greater than the buoyant force and will rise if the buoyant force is greater than its weight. Equilibrium will be established when the weight of the liquid displaced is equal to the weight of the body. Thus a solid body which is less dense than water floats on the surface of the water, part below the water and part above the water. A ship afloat, for example, displaces its own weight of water. The weight of a ship is frequently expressed in terms of the weight of the water it displaces, and one speaks of ships with 10,000 tons displacement, and so on. There is usually a definite water line painted on a ship, indicating the limit to which a ship may be submerged and still be safe.

A submarine is so designed that it can take water into specially built tanks so as to make its weight greater than the weight of the water it displaces when fully submerged. Because the density of water is essentially independent of pressure, water being an almost incompressible fluid, a submarine which has no forward motion when submerged will sink to the bottom of the sea. To enable the submarine to rise to the surface, water is forced out of the tanks by pumps.

Illustrative Example. A cylinder of brass 6 cm high and 4 cm^2 in cross-sectional area is suspended in water by means of a string so that its upper surface is 7 cm below the surface of the water, as shown in Figure 8-10. Determine (a)

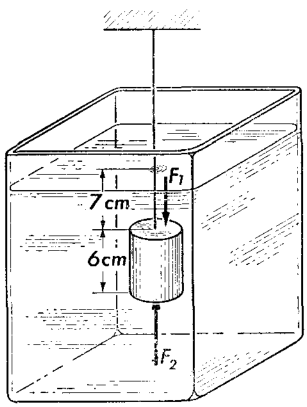

Fig. 8-10

the force acting on the top of the cylinder, (b) the force acting on the bottom of the cylinder, and (c) the buoyant force acting on this cylinder.

(a) The force F_1 acting on the top of the cylinder is that due to the pressure of the water above it and is

$$F_1 = P_1A = h_1\rho gA$$

$$= 7 \text{ cm} \times 1 \frac{\text{gm}}{\text{cm}^3} \times 980 \frac{\text{cm}}{\text{sec}^2} \times 4 \text{ cm}^2,$$

$$F_1 = 27{,}440 \text{ dynes}.$$

This force pushes down on the cylinder.

(b) The force F_2 acting on the bottom of the cylinder is due to the pressure of the water above it. Since the depth of the water at the bottom of the cylinder is 13 cm, this force is

$$F_2 = 13 \text{ cm} \times 1 \frac{\text{gm}}{\text{cm}^3} \times 980 \frac{\text{cm}}{\text{sec}^2} \times 4 \text{ cm}^2,$$

$$F_2 = 50{,}960 \text{ dynes}$$

acting upward on the cylinder.

(c) The buoyant force B is the net force upward caused by the difference in pressures in the liquid. The forces which act on the walls of the cylinder are all directed horizontally, and their resultant is zero, as can be seen from the symmetry of the figure. Therefore, the buoyant force is simply the difference between the two vertical forces F_1 and F_2, thus

$$B = F_2 - F_1$$

$$= 50{,}960 \text{ dynes} - 27{,}440 \text{ dynes},$$

so that

$$B = 23{,}520 \text{ dynes}$$

and acts upward.

It is interesting to compare this buoyant force with the weight of water displaced. The volume of the cylinder is $h \times A = 24 \text{ cm}^3$. This is also the volume of water displaced. The weight of this displaced water is

$$W = mg = \rho V g$$

$$= 1 \frac{\text{gm}}{\text{cm}^3} \times 24 \text{ cm}^3 \times 980 \frac{\text{cm}}{\text{sec}^2},$$

$$W = 23{,}520 \text{ dynes},$$

which is in agreement with the earlier calculation.

Illustrative Example. A block of aluminum is attached to a balance. When suspended in air, the balance reads 250 gm. When the aluminum block is lowered so that it is completely immersed in water, the balance reads 160 gm. When the aluminum block is lowered so that it is completely immersed in alcohol, the balance reads 180 gm. The density of water is 1 gm/cm^3. Determine (a) the density of aluminum, and (b) the density of alcohol.

(a) The buoyant force of the water is the difference between the weight of the aluminum block in air and its weight when immersed in water; that is,

$$B = 250 \times 980 \text{ dynes} - 160 \times 980 \text{ dynes} = 90 \times 980 \text{ dynes}.$$

Thus the aluminum block displaces a mass of 90 gm of water. The volume of water displaced is

$$V = \frac{m}{\rho} = \frac{90 \text{ gm}}{1 \text{ gm/cm}^3}$$

$$= 90 \text{ cm}^3,$$

and this is equal to the volume of aluminum. Thus the density of aluminum is

$$\rho = \frac{m}{V} = \frac{250 \text{ gm}}{90 \text{ cm}^3},$$

$$\rho = 2.78 \frac{\text{gm}}{\text{cm}^3}.$$

(b) The amount of alcohol displaced by the aluminum block may be found from the buoyant force exerted by the alcohol, which is given by

$$B = 250 \times 980 \text{ dynes} - 180 \times 980 \text{ dynes} = 70 \times 980 \text{ dynes},$$

so that the mass of alcohol displaced by the aluminum block is 70 gm. The volume of alcohol displaced is the volume of the aluminum block, which we have found to be equal to 90 cm^3. Thus the density of alcohol is

$$\rho = \frac{70}{90} \frac{\text{gm}}{\text{cm}^3},$$

$$\rho = 0.78 \frac{\text{gm}}{\text{cm}^3}.$$

For many technical purposes it is not necessary to make precise measurements of density; it is only important to know whether the density is greater than some minimum value or whether it lies between certain limits. For example, in testing die castings for internal porosity, one technique for rapid inspection of large quantities of castings is a flotation test where the die castings are immersed in a suitable liquid. If the castings are sound, their average density is greater than the density of the liquid, and the castings sink. If the castings contain excessive porosity, their average density is too low, and the defective castings may be skimmed off the surface of the flotation liquid.

8-9 Specific Gravity

It is sometimes convenient to refer the mass of a given object to the mass of an equal volume of water. The term *specific gravity* is used to denote the magnitude of this quotient, and since the specific gravity is a property of the material of which the object is made rather than of the size or shape of the object, we may conveniently define the specific gravity as the *ratio of the density of a body to the density of water.* Thus water has a specific gravity of 1. The specific gravity is a pure number and is independent of the system of units used to measure the density. Since the density of water in the cgs system of units is 1 gm/cm^3, the specific gravity of a substance has the same numerical value as its density in the cgs system.

From the definition of specific gravity, we see that a body of specific gravity less than 1 will float in water, while a body of specific gravity greater than 1 will sink.

The specific gravity of liquids is commonly determined by use of an instrument called the *hydrometer,* which is usually made in the form of a cylinder with a weighted bulb at one end, as shown in Figure 8-11. The depth to which the hydrometer will sink in a liquid depends on its specific gravity, so the hydrometer may be provided with a calibrated scale to read specific gravity directly, or to read some property associated with specific gravity which is of immediate interest. Since the freezing point of a mixture of antifreeze and water is determined by the fraction of antifreeze in the solution, the hydrometer may be directly calibrated to read the freezing temperature of the mixture. Similarly, the specific gravity of a mixture of alcohol and water may be used to determine the percentage of alcohol in the mixture. A hydrometer is customarily used as a proof tester in alcoholic beverages. On such a scale, "100 proof" means 50 per cent alcohol content.

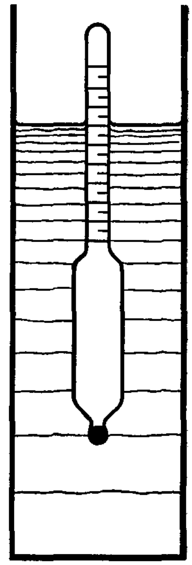

Fig. 8-11 Hydrometer.

8-10 The Centrifuge

In our discussion of rotational motion, we emphasized that a particle could move in uniform circular motion only if there was a centripetal force acting on the particle. Let us consider a particle of mass m placed on a horizontal turntable which is rotating with uniform angular speed ω about a vertical axis through its center. If the particle is at a distance r from the axis, a centripetal force equal to $m\omega^2 r$ must act on this particle to keep it in place on the turntable. This force may be supplied by friction between the particle and the table or, if there is no friction, by a string tied to a shaft at this axis and to the particle. If the motion of the particle is now considered from the turntable as the reference system, the particle seems to be experiencing a force $m\omega^2 r$ away from the center of the table, and it is kept in place by the opposing force produced by the pull of the string on it. The particle appears to experience a new kind of force field whose intensity is now given by $\omega^2 r$ instead of g. We may thus say that an observer located on a rotating coordinate system is led to believe that he is located in a force field directed radially outward from the center of the circle.

If a liquid contains particles of greater density than that of the liquid itself, these particles will separate out at the bottom of the liquid in the earth's gravitational field, given sufficient time. If the volume of the particle is V and its density is ρ, its weight will be given by $\rho V g$. The buoyant force on such a particle in a fluid of density ρ_0 is the weight of the displaced fluid $\rho_0 V g$, and the resultant downward force on the particle is $(\rho - \rho_0) V g$. The particles of sediment suspended in the fluid are acted upon by a net force in the direction of the gravitational field and, in time, will settle out on the bottom of the container. Because of internal friction in the liquid, the settling-out process may be quite slow. The rate of separation of the solid particles from the liquid might be greatly expedited by placing the suspension in an intense gravitational field, if such were available. A centrifuge, which is a machine designed to rotate a liquid at high speeds, simulates such a gravitational field, and the apparent force on the particle becomes $(\rho - \rho_0)V\omega^2 r$. Thus if a liquid is placed in a tube,

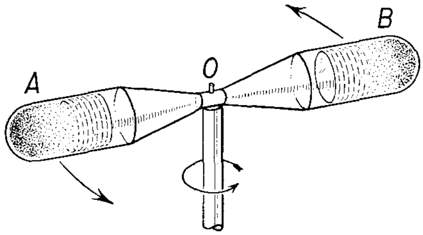

Fig. 8-12 The centrifuge. Tubes A and B are whirled around in a circular path about an axis through O. Particles of greatest density go to the ends of tubes farthest removed from O.

such as that shown in Figure 8-12, and the tube is rotated at high speed, the particles in the liquid will settle out very rapidly. Those particles which are denser than the liquid will be found at the bottom, that is, farthest from the axis of rotation, while those particles which are less dense than the liquid will be found near the top.

Illustrative Example. A liquid containing some solid particles is poured into the cup of a centrifuge which is then rotated at a speed of 6,000 rpm. Determine the apparent gravitational field intensity acting on a particle at a distance of 12 cm from the axis of rotation. Express this in terms of the earth's gravitational field intensity g.

The apparent gravitational field intensity acting on the particle is given by the formula $\omega^2 r$. We have

$$\omega = \frac{6{,}000 \times 2\pi}{60} \frac{\text{radians}}{\text{sec}} = 200\pi \frac{\text{radians}}{\text{sec}}.$$

Thus

$$\omega^2 r = (200\pi)^2 \times 12 \frac{\text{cm}}{\text{sec}^2},$$

so that

$$\omega^2 r = 474 \times 10^4 \frac{\text{cm}}{\text{sec}^2}.$$

Expressing this in terms of g, we have

$$\omega^2 r = \frac{474 \times 10^4}{980} g,$$

or

$$\omega^2 r = 4{,}840 g.$$

Thus the particle in the centrifuge experiences an apparent gravitational field almost 5,000 times the magnitude of the earth's gravitational field.

Problems

8-1. A block of metal weighs 120 lb in air, 105 lb when immersed in water, and 108 lb when immersed in a certain liquid. Determine (a) the density of the metal, (b) the density of the liquid, and (c) the specific gravity of the liquid.

8-2. A metal sphere whose mass is 36 gm is attached by means of a string to one arm of an equal-arm balance. When the sphere is completely immersed in water, a mass of 23 gm is sufficient to balance it. Determine (a) the volume of the sphere and (b) the density of the metal.

8-3. A raft is made in the form of a rectangular box 8 ft by 10 ft by 4 ft deep. The raft weighs 2,500 lb. (a) How deep will this raft go when placed in fresh water which weighs 62.4 lb/ft^3? (b) What load can this raft carry without sinking? (c) What is the total force exerted by the water on the bottom of the raft when so loaded? (d) Assuming the load is uniformly distributed, will the bottom of the raft tend to bulge in at the center, bulge out, or remain flat?

8-4. A beaker partially filled with water is placed on a scale pan and found to have a mass of 500 gm. A string is attached to a stone and held so that the stone is completely submerged in the water but does not touch the beaker at any point. The scale now reads 550 gm. When the string is released and the stone rests on the bottom of the beaker, the scale reads 620 gm. Determine (a) the mass of the stone, (b) the density of the stone, and (c) the tension in the string.

8-5. A cube of iron 3 cm on an edge is placed in a dish of mercury. (a) How much of the cube is immersed in the mercury? (b) If water is poured over the mercury to a depth of 4 cm, what will be the depth of the iron in the mercury?

8-6. A piece of concrete whose mass is 150 kg has a density of 2,500 kg/m^3. A block of wood of density 500 kg/m^3 is to be fastened to the concrete block and placed in water so that they will both float almost completely submerged. What is the minimum mass of wood which can be used?

8-7. A **U** tube contains mercury at the bottom. Glycerin is poured into one arm so that the height of the glycerin column is 30 cm. How high a column of water must be poured into the other arm to bring the mercury to the same level in both arms?

8-8. What is the atmospheric pressure in dynes per square centimeter when the reading of a mercury barometer is 77 cm?

8-9. Hoover Dam is 1,180 ft long and 726 ft high. (a) What is the pressure at the bottom of the dam when the reservoir is full? (b) Assuming the face of the dam to be a plane rectangle, determine the total force pushing against the face of the dam.

8-10. From Problem 8-9, determine the torque exerted by the force of the water on the face of the dam (a) about a horizontal axis through the bottom of the dam and (b) through the top of the dam. [HINT: Use the methods of the calculus.]

8-11. An open-tube manometer containing water has one end connected to a city gas supply outlet. The difference in level between the two arms is 2.0 in. (a) Determine the gauge pressure of the gas. (b) If the height of the barometer is 76 cm of mercury, determine the absolute pressure of the gas.

8-12. Express the density of lead in slugs per cubic foot and in kilograms per cubic meter.

8-13. If every gram of air in the atmosphere were replaced by mercury, how deep would be the layer of mercury on the surface of the earth?

8-14. If it is desired to determine the mass of 1 cm^3 of aluminum to an accuracy of 1 per cent using a beam balance and calibrated brass weights, should the result of the measurement be corrected for the buoyant effect of the air? At what accuracy does the measurement need to be corrected for buoyancy?

8-15. A submarine having a volume of 165,000 cubic feet and weighing 4830 tons is floating at the surface of sea water of weight density 64 lb/ft^3. (a) What is the buoyant force on the submarine? (b) What volume of sea water must be admitted into the submarine so that it will just submerge?

8-16. A uniform wooden rod which weighs 5 lbs has a length of 8 ft and a volume of 0.1 ft^3. The rod is pinned at one end and is submerged in water, and is held in a horizontal position with the aid of a vertical force F at the opposite end. Assume the pinned end to be fixed. (a) What is the magnitude and direction of the total force on the rod due to the water? (b) What is the magnitude of the force F needed to keep the rod in equilibrium?

8-17. A 2.7 gm block of aluminum is suspended from a spring scale which hangs from the ceiling of an elevator. The block is then immersed in water. (a) What is the reading of the scale when the elevator is at rest? (b) The elevator is accelerated upwards at 490 cm/sec^2. What is the reading of the scale during this time? (c) What is the reading of the scale when the elevator is moving upward at a steady speed of 500 cm/sec? (d) Finally the elevator is brought to rest with an acceleration of 245 cm/sec^2. What is the reading of the scale during this time?