University of Nebraska - Lincoln

DigitalCommons@University of Nebraska - Lincoln

Physics, Chapter 13: Properties of Matter

Henry Semat
City College of New York

Robert Katz
University of Nebraska-Lincoln, rkatz2@unl.edu

13

Properties of Matter

13-1   Internal Forces

When a system is subjected to external forces, it generally undergoes a
change in size or shape or both. We have thus far touched very lightly on
such changes; for example, we have considered the change in length of an
elastic spring and the change in volume of a gas when such systems were
subjected to varying pressures. The changes produced in a system by the
action of external forces depend upon the physical properties of the material
of which the system is composed. A study of the properties of matter leads
to information which is of practical value to both the physicist and the
engineer, and also gives us some information about the internal forces
which act between the constituent parts of the substance. In the final
analysis these physical properties must be explicable in terms of the forces
between the molecules of the substance and, in some cases, between the
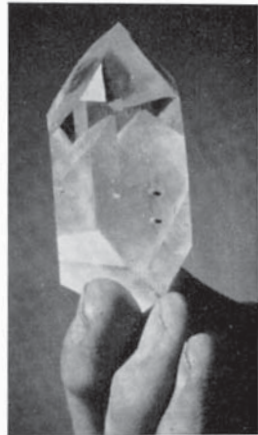atoms of the substance.

   We have so far discussed only one type of force which exists between
particles—the gravitational attraction of two particles because of their
masses. However, gravitational forces are much too small to account for
the observed properties of substances. Furthermore we frequently en-
counter cases in which a force of repulsion is needed to explain the phenom-
ena, whereas gravitational forces are always forces of attraction. One other
fact worth noting here is that the forces which act between molecules pro-
duce their effect only over very short distances, that is, distances of the
order of molecular diameters. These are called *short-range forces*. These
short-range forces are undoubtedly of electrical origin. As we proceed
with our study of physics, we shall study the forces between electrically
charged particles and show how these are thought to be related to the
structure of matter.

   At present, there are 102 different elements known. A chemical
analysis of any substance will show that it is composed of one or more of
these elements. If the substance is a chemical compound, the elements
244

composing it always occur in a definite ratio of their weights. More complex substances consist of several or many compounds. The smallest constituent of a chemical compound is the *molecule;* it is the fundamental structural unit of the compound. A molecule is composed of one or more atoms held together by the short-range forces due to their electrical charges. As we shall show, an atom of an element consists of a very small but massive *nucleus* surrounded by a suitable number of *electrons*. The nucleus consists of two kinds of particles: *neutrons*, which are neutral particles, and *protons*, which are positively charged electrically. Every atom of any one element has exactly the same number of protons in its nucleus. This number is the *atomic number* of the element and can have one of the values from 1 to 102, inclusive. In the normal state of the atom, the atomic

number also represents the number of electrons outside the nucleus. Most of the chemical properties of an element can be explained in terms of the arrangement and behavior of the electrons in the atoms. The electrons exert forces of repulsion on each other and forces of attraction on positively charged nuclei. When a molecule is formed with two or more atoms, there is a change in the electronic arrangement, and a new arrangement of charges is produced. This rearrangement is brought about by the short-range forces which act between the atoms.

When the substance is in the solid phase, the forces which exist between atoms and molecules cause them to form definite geometrical patterns; these show up as the crystalline structure. Sometimes these crystals grow to a fairly large size, as in the case of rock salt or quartz (see Figure 13-1). Sometimes the crystals are very small and can be seen only with the

Fig. 13-1 Photograph of a large quartz crystal grown at the Bell Telephone Laboratories. (Courtesy of Bell Telephone Laboratories.)

aid of a microscope. Most metals consist of such microcrystals. Even in the liquid phase, there is a definite grouping of the atoms and molecules, although individual atoms and molecules often change places. The arrangement of the atoms and molecules of a substance can be determined by means of x-ray analysis (see Chapter 44).

In the gaseous phase, the molecules are comparatively far apart, so that the forces they exert on each other are extremely small. It is this aspect of the structure of gases which makes it possible to analyze the behavior of gases almost without regard to their chemical nature, although it is not possible to neglect the chemical differences in either the liquid or the solid phase.

13-2   Elasticity

The subject of elasticity has already been introduced in connection with our discussion of periodic motion. In order to treat the elastic properties of matter more quantitatively, and in a manner which is not dependent on the configuration or shape of the body being studied, we find it convenient to introduce two new terms: these are (a) *stress* and (b) *strain*.

Stress is defined as *the internal force* F, *brought into play when the substance is distorted in any way, divided by the area* A *over which it acts.*

Thus
$$\text{Stress} = \frac{F}{A}.$$
(13-1)

The stress is therefore the force per unit area. Clearly, the stress indicated in Equation (13-1) is an average stress, for the stress in a member may vary from point to point within that member, as in the case of a bent beam, where one surface of the beam is in tension, while the other surface of the beam is in compression. We shall restrict this discussion to the simplest case, where the stress is uniform, and shall not attempt to treat cases of variable stress for which the stress at a point would be defined through a limiting process.

In the cgs system the stress is expressed in dynes per square centimeter; in the British gravitational system, in pounds per square foot; in the mks system, in newtons per square meter. In most engineering practice the stress is expressed in pounds per square inch.

Strain is defined as *the ratio of the change in size or shape to the original size or shape.* As a ratio, strain has no physical dimensions; that is, it has numerical value only. Methods of expressing the strain will be given in the discussion of the various cases.

The relationship between stress and strain was first given by Robert Hooke (1635–1703) and is known as *Hooke's law.* This states that *for an elastic body the stress divided by the strain is a constant,* or,

$$\frac{\text{Stress}}{\text{Strain}} = K,$$
(13-2)

where K is called the *modulus of elasticity.* The units for K are the same as those for stress, since strain is expressed as a pure number.

13-3   Tensile Stress and Strain

As an example of the stress set up inside a substance, let us consider the increase in the length of a rod produced by the action of two forces, each equal to F, applied at the ends of the rod, as shown in Figure 13-2(a).

These forces are applied by means of clamps C_1 and C_2 attached to the ends of the rod.

If L is the original length of the rod, and if Δl is the increase in length produced by the application of the forces F, then the strain produced is

$$\text{Strain} = \frac{\text{increase in length}}{\text{original length}} = \frac{\Delta l}{L}. \qquad \textbf{(13-3)}$$

To determine the stress in the rod, let us take any cross-sectional area A through the rod and consider the forces which act on it, as in Figure 13-2(b). The entire rod is in equilibrium under the action of the two external forces, each of magnitude F but acting in opposite directions, so as to extend the rod. These forces produce a *tension* in the rod. Let us consider an area A near the clamp C_2; it is acted on by the external force F downward, and, since it is in equilibrium, it must also be acted upon by a force F upward exerted by that part of the rod which is immediately above this area. If we take any other parallel cross-sectional area A, it will be acted upon by a force downward equal to F exerted by that part of the rod below it, and another force equal to F exerted by that part of the rod above it. The effect of these two forces is to tend to separate the rod across this section; it is opposed by the forces of attraction between the molecules on the two sides of this section. The stress in the rod is the quotient of one of these forces F by the cross-sectional area A, or

$$\text{Stress} = \frac{F}{A}.$$

This type of stress is called a *tensile stress*.

If the material of the rod is elastic, then we know, from Hooke's law, that

$$\frac{\text{Stress}}{\text{Strain}} = K.$$

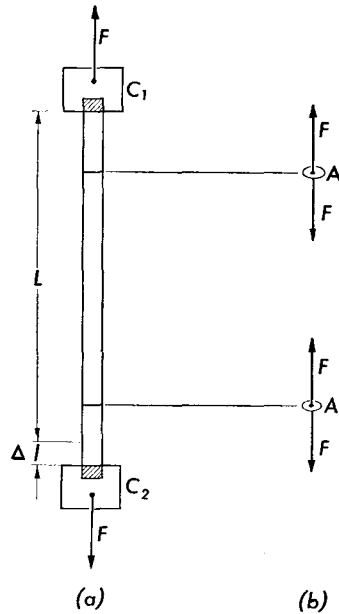Putting in the values of stress and strain found above, and replacing the

Fig. 13-2   Tensile stress in a rod.

letter K by Y, we get

$$\frac{F/A}{\Delta l/L} = Y. \tag{13-4}$$

Y is called *Young's modulus* after Thomas Young (1773–1829), an eminent British physician, physicist, and linguist who made significant contributions to the study of vision and the theory of light, and who deciphered Egyptian hieroglyphics, in addition to his studies in elasticity. The values of Y for several substances are listed in Table 13-1.

TABLE 13-1   ELASTIC CONSTANTS OF SOME SOLIDS

Material	Young's Modulus		Shear Modulus		Bulk Modulus	
	in dynes/cm^2 $\times 10^{11}$	in lb/in.^{2} $\times 10^6$	in dynes/cm^2 $\times 10^{11}$	in lb/in.^{2} $\times 10^6$	in dynes/cm^2 $\times 10^{11}$	in lb/in.^{2} $\times 10^6$
Aluminum, rolled	6.96	10.1	2.37	3.44	7.	10.
Brass	9.02	13.1	3.53	5.12	6.1	8.5
Copper, rolled	12.1 –12.9	17.5 –18.6	4.24	6.14	14	21
Duralumin	6.89	10.0	2.75	3.98		
Iron, cast	8.4 – 9.8	12–14	. . .	. . .	9.6	14
Glass, crown	6.5 – 7.8	9.5 –11.3	2.6 –3.2	3.8 – 4.7		
Lead	1.47– 1.67	2.13– 2.42	0.54	0.78	0.8	1.1
Nickel	20.0 –21.4	29.0 –31.0	7.06–7.55	10.24–10.95		
Platinum	16.67	24.18	6.42	9.32		
Silver, hard drawn	7.75	11.24	2.00	3.77		
Steel, annealed	20.0	29.0	8.11	11.76	16	23
Tin	3.92– 5.39	5.69– 7.82	1.67	2.42		
Tungsten, drawn	35.5	51.5	14.8	21.5		

The extent to which a substance remains elastic as the tensile stress is increased can be determined only by experiment. Figure 13-3 shows the results of a typical experiment on a metallic rod. In this figure the stress is plotted as ordinate and the strain as abscissa. The curve is obtained by exerting a force, measuring the strain, increasing the force, measuring the new strain, and so on. The straight-line portion of this curve, from O to E, represents the values of the stress and the strain for which the rod is elastic. If a stress of value CD is applied to the rod and then removed, the rod will no longer return to its original length. It is said to have a per-

manent set.   The point E is called the *elastic limit* of the material.   At point B the stress was great enough to break the rod.   This value of the stress is known as the *ultimate stress* or the *breaking stress* of the material.

Fig. 13-3   Stress-strain curve for a ductile material.
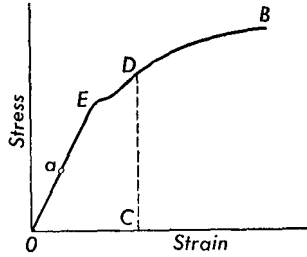
Illustrative Example.   A piece of copper wire 0.0508 in. in diameter and 3 ft long is suspended from a rigid support and supports a load of 8 lb.   Determine (a) the stress in the wire, (b) the increase in length produced by the 8-lb load, and (c) the strain produced.

(a) The wire has a cross-sectional area of 0.00203 in.^{2}.   The stress in this wire is

$$\text{Stress} = \frac{F}{A} = \frac{8 \text{ lb}}{0.00203 \text{ in.}^2} = 3{,}950 \frac{\text{lb}}{\text{in.}^2}.$$

(b) The increase in length can be found from Equation (13-4), provided that Y is known.   From Table 13-1 the value of Young's modulus for copper is given as $17.5 \times 10^6 \dfrac{\text{lb}}{\text{in.}^2}$.   Since both Y and the stress are expressed in the same units, there is no need to convert either quantity into units appropriate to the British gravitational system, and we write

$$Y = \frac{F/A}{\Delta l/L};$$

hence

$$\Delta l = \frac{F}{A} \times \frac{L}{Y},$$

$$\Delta l = 3{,}950 \frac{\text{lb}}{\text{in.}^2} \times \frac{36 \text{ in.}}{17.5 \times 10^6 \dfrac{\text{lb}}{\text{in.}^2}},$$

$$\Delta l = 0.008 \text{ in.}$$

(c) The strain produced in the wire is

$$\text{Strain} = \frac{\Delta l}{L} = \frac{0.008 \text{ in.}}{36 \text{ in.}} = 0.00022,$$

$$\text{Strain} = 2.2 \times 10^{-4}.$$

The strain, or the fractional change in length, is about 2 parts in 10,000.

13-4 Compressive Stress and Strain

If the ends of a rod of some material are subjected to the action of two
forces, each of magnitude F but directed so as to diminish its length, the
rod is said to be under *compression*, and the stress inside the rod is a *com-
pressive stress*, as illustrated in Figure 13-4(a). If we consider the forces
which act across any cross-sectional area A, that part of the rod to the right
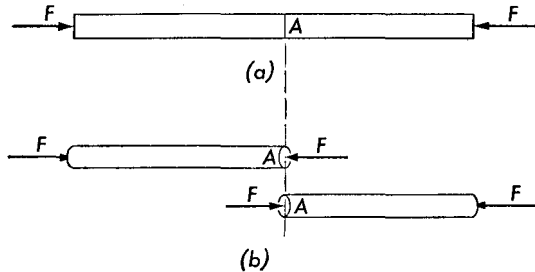
Fig. 13-4   (a) Rod put under compression by action of two external forces each equal
to F.   (b) Any cross section A is acted upon by an internal force F to the left due to
the section of the rod on the right, and by an internal force F to the right due to the
section of the rod on the left.

of this area exerts a force F to the left, while that part to the left of this
area exerts a force to the right, as shown in Figure 13-4(b). The compres-
sive stress in the rod is the quotient of one of these forces divided by the
area over which the force acts, exactly as in the case of the tensile stress.
The molecular forces brought into play by the action of the external forces
must be forces of repulsion. The strain produced by the compressive stress
is the ratio of the decrease in length Δl to the original length, again exactly
as in the case of tensile strain. If the material of which the rod is made is
elastic, then experiment shows that, within the elastic limit, the compressive
stress divided by the compressive strain, or Young's modulus for compres-
sion, is identical with Young's modulus for tension for the same material.
In consequence, no distinction is made in tables between Young's modulus
for tension or for compression, and tabular values of Young's modulus are
intended for use in both types of stress.

13-5 Compressibility of Gases: Boyle's Law

The compressibility of gases was first studied by Robert Boyle (1627–1691).
Suppose we have a mass of gas in a cylinder with a tight-fitting piston,
on which a force F is exerted producing a pressure $P = F/A$, where A is
the area of the piston, as shown in Figure 13-5. The gas will be subject
to this pressure and will occupy a volume V determined by the distance of

the piston from the end of the cylinder. By increasing the force on the piston to a new value F_1, the pressure on the gas will be increased to a new value P_1. If the process takes place slowly, so that the gas remains at constant room temperature, it is found that

$$\frac{V_1}{V} = \frac{P}{P_1},$$

or, stated in words, at constant temperature the volume of a gas varies inversely as the pressure. A more convenient way to express the same result is

$$P_1 V_1 = PV = \text{a constant}; \qquad \text{(13-5)}$$

that is, *the product of the pressure and volume of an enclosed gas remains constant* if the temperature is constant. This statement is known as *Boyle's law*.

Since the mass of gas within an enclosed system remains constant, we can determine the manner in which the density of a gas varies with pressure by dividing both sides of Equation (13-5) by the mass M of the gas within the cylinder. We obtain

$$\frac{P_1}{M/V_1} = \frac{P}{M/V},$$

or

$$\frac{P_1}{P} = \frac{\rho_1}{\rho}, \qquad \text{(13-6)}$$

Fig. 13-5 Gas in a cylinder is compressed by increasing the force on the piston; that is, by increasing the pressure of the gas.
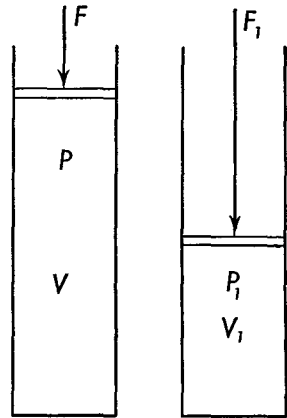
in which ρ is the density of the gas at pressure P, and ρ_1 is the density of the gas at pressure P_1.

Illustrative Example. A steel tank contains 2 ft^3 of oxygen at a gauge pressure of 200 lb/in.^{2}. What volume will this gas occupy at the same temperature at atmospheric pressure?

The gauge pressure is the difference between the pressure of the gas in the cylinder and the pressure of the atmosphere. Hence, the pressure P_1 of the oxygen in the cylinder is 214.7 lb/in.^{2}; the pressure P of the atmosphere is taken as 14.7 lb/in.^{2}. Using Boyle's law in the form of Equation (13-5) and substituting values, we get

$$214.7 \frac{\text{lb}}{\text{in.}^2} \times 2 \text{ ft}^3 = 14.7 \frac{\text{lb}}{\text{in.}^2} \times V,$$

$$V = 29.2 \text{ ft}^3.$$

13-6 Volume Change: Bulk Modulus

In both compressive and tensile stresses, the stress acts along one direction in the body and produces a change in only one dimension. The change produced in the cross-sectional area of a rod under compression or tension is practically negligible. To produce equal strains in all three dimensions of a homogeneous solid, it is necessary to have equal stresses along these three directions. The simplest method of doing this is to subject the solid to a uniform hydrostatic pressure. Since the dimensions of the solid are normally measured at some pressure P_0 which is generally the atmospheric pressure, we are usually interested in the change in the dimensions of the solid when it is subjected to a change in pressure from P_0 to a different pressure $P_0 + \Delta P$. If we call V the volume of the solid and ΔV the change in volume produced by the change in pressure ΔP, then, from Hooke's law,

$$\frac{\text{Stress}}{\text{Strain}} = \frac{\Delta P}{\Delta V/V} = K.$$

Since an increase in pressure always produces a decrease in volume, the modulus K for volume change will always be a negative number. To avoid having a negative number, let us define the *bulk modulus* $B = -K$, so that

$$B = -\frac{\Delta P}{\Delta V/V}. \tag{13-7}$$

Not only solids but liquids and gases undergo volume changes when subjected to changing pressures, and Equation (13-7) can be applied to fluids as well as to solids. Since the denominator of Equation (13-7) is a ratio of the change in volume to the volume, it is a pure number; hence the bulk modulus is expressed in units of pressure, or force per unit area.

TABLE 13-2 BULK MODULUS OF LIQUIDS

Material	in dynes/cm^2 $\times$ 10^{11}
Carbon disulphide	0.15
Ethyl alcohol	0.09
Glycerin	0.45
Mercury	2.6
Nitric acid	0.03
Water	0.23

The bulk modulus of a gas which obeys Boyle's law at constant temperature is given by the pressure of the gas, for we may state Boyle's law as

$$PV = \text{constant},$$

and, taking differentials of this equation, we find

$$P \, dV + V \, dP = 0,$$

and, solving for P, we find

$$P = -\frac{dP}{dV/V}.$$

If we replace the differentials by small increments so that the differential dP is replaced by ΔP and the differential dV is replaced by ΔV, we have

$$P = -\frac{\Delta P}{\Delta V/V} = B.$$

From the above analysis it can be seen that, if the pressure of a gas is changed by any amount while the temperature is kept constant, the bulk modulus will vary and, at any stage of the process, will be equal to the pressure of the gas at that stage. This is the reason that tires or basketballs inflated to high pressures seem hard, or difficult to deform, while the same object inflated to low pressure is easy to deform, or soft.

13-7　Shearing Stress and Strain

It is possible to produce a change in the shape of a solid without changing its volume. Such a distortion is called a *shear*. A simple method of producing a shear is illustrated in Figure 13-6(a). If we take a rectangular solid and apply a force F along its top surface, and an equal force F acting in the opposite direction along its bottom surface, the rectangular surfaces such as $BCDE$ at right angles to the top and bottom surfaces will be distorted into parallelograms such as $B'C'DE$, whose angles are not right angles. If we imagine the solid as made up of a series of layers parallel to the top and bottom surfaces, each of area A, then the effect of the shear is to cause one layer to slide with respect to another layer, much as in a deck of cards.
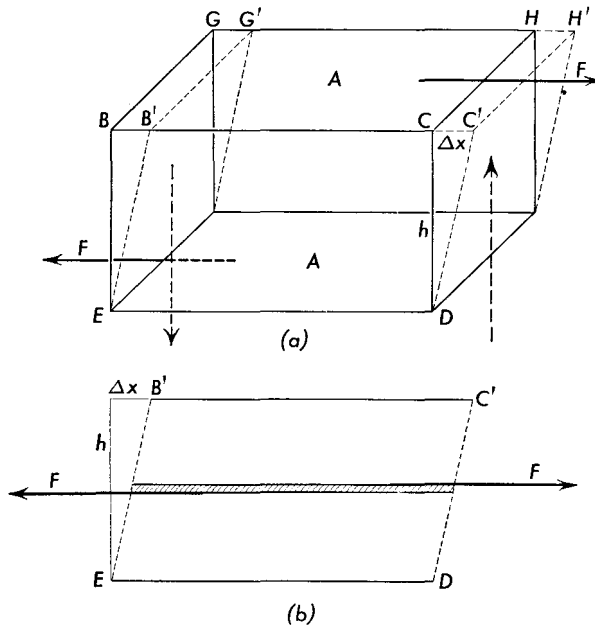
The shearing stress set up on the solid is the force F divided by the area A of the surface over which it acts. Each layer parallel to the top and bottom surfaces is acted upon by two forces, as shown in Figure 13-6(b), the layer above it exerting a force to the right, and the layer below it exerting a force to the left. These forces between the molecules of the layers oppose the sliding of one layer with respect to another. If the applied force becomes too great, the solid will be sheared; that is, it will be separated into two parts, with the surfaces of separation parallel to the direction of the applied force.

The shearing strain produced is measured by the ratio of the distance Δx through which the top surface has been moved relative to the bottom

surface, to the height h, that is, the distance between the two surfaces.
Thus

$$\text{Shearing strain} = \frac{\Delta x}{h}.$$

It will be noted that the tangent of the angle $B'EB$ is $\Delta x/h$.

Fig. 13-6 (a) Shearing stress set up in a solid by parallel forces F acting on top and bottom surfaces each of area A. Rectangle $BCDE$ is distorted into parallelogram $B'C'DE$. There is no change in the volume of the solid. (b) Shaded area is section of layer in the solid parallel to top and bottom surfaces. Shearing strain is $\Delta x/h$.

Applying Hooke's law, we get for the *shear modulus M*,

$$M = \frac{F/A}{\Delta x/h}, \tag{13-8}$$

and once again we note that the units of the shear modulus are the units of force per unit area.

When a thin-walled tube or a rod is twisted, its deformation depends on the shear modulus. The force constant of a helical spring may be computed from the dimensions of the spring and the shear modulus of the wire from which it is made. When a spring is elongated or compressed, the wire from which it is made does not stretch, but rather, any short length of the wire acts like a rod which is being twisted.

While we have defined three constants of elasticity as though they were completely independent, it may be shown that, for a homogeneous material in which the elastic properties are the same in every direction, only two of these constants are independent, and the third may be expressed in terms of the other two.

While all three of the elastic constants are meaningful for solids, only the bulk modulus has meaning for fluids. This is because neither a liquid nor a gas is capable of supporting a tensile or a compressive stress, nor can either of these fluids support a shearing stress under ordinary conditions. When a fluid is subjected to a shearing stress, the upper surface acquires a uniform velocity with respect to the lower surface of Figure 13-6, as we have seen in the discussion of viscosity. Only the bulk modulus is appropriate to a liquid or gas, for these are capable of withstanding only hydrostatic pressures.

Illustrative Example. Two clamps are fastened near the ends of a rectangular steel rod 5 in. long. The rectangular cross section of the rod has an area of 2.5 in.^{2}. A force of 800 lb is exerted on each of these clamps parallel to this area but in opposite directions. Determine (a) the shearing stress, (b) the shearing strain, and (c) the relative displacement of the top surface with respect to the bottom surface.

(a) The shearing stress is

$$\frac{F}{A} = \frac{800 \text{ lb}}{2.5 \text{ in.}^2} = 320 \frac{\text{lb}}{\text{in.}^2}.$$

(b) The shearing strain can be found by solving Equation (13-8) for $\Delta x/h$, obtaining

$$\frac{\Delta x}{h} = \frac{320}{11.8 \times 10^6} = 27.2 \times 10^{-6}.$$

(c) Since $h = 5$ in.,

$$\Delta x = 5 \text{ in.} \times 27.2 \times 10^{-6}.$$

so that $$\Delta x = 1.36 \times 10^{-4} \text{ in.}$$

for the relative displacement of the two surfaces.

13-8   Cohesion and Adhesion

The fact that molecular forces have a short range would lead us to expect some distinctive types of phenomena to be observable at the surfaces of substances. Conversely, the appearance of these surface phenomena should lead to information about these molecular forces. For example, if we take two pieces of metal, each with an accurately plane surface, and bring them together, there will be no observable force between them until the two surfaces are placed in contact. Once they are placed in contact,

a very great force will be required to pull them apart. This experiment, which can readily be performed with two pieces of steel or two pieces of lead, shows that the forces between the molecules in the two surfaces have a very short range of effectiveness. It is for this reason that a bearing in which a shaft rotates is never made of the same material as the shaft itself. The force of attraction between molecules of the same substance is sometimes called *cohesion*.
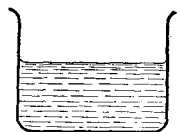

Fig. 13-7 Free surface of water in a glass jar is level (horizontal) except near the glass.
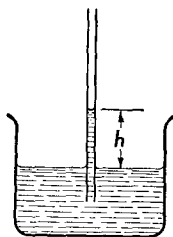

Fig. 13-8 Level of water in a capillary tube is at a height h above level in the large vessel.

If some water is poured into a glass vessel, the free surface of the water will be a level surface, except at the region of contact with the glass; at this region, the water will be seen to cling to the glass for a short distance above the level surface, as shown in Figure 13-7. This phenomenon can be accentuated by immersing a glass tube with a narrow bore, a *capillary tube*,
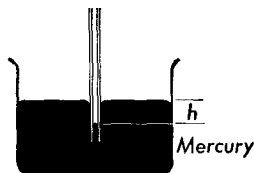
Fig. 13-9 Level of mercury in a glass capillary tube is at a level h below that in a large vessel.
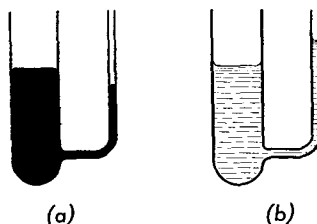

Fig. 13-10 (a) **U** tube containing mercury. (b) **U** tube containing water.

into the water, as in Figure 13-8. The level of the water inside the capillary tube will be found to be considerably higher than the level inside the larger jar, and an examination of the surface of the water in the capillary tube shows that it is not plane but is concave, and spherical in shape. Not all liquids behave like water. For example, if a glass capillary tube is immersed in mercury contained in a larger dish, as in Figure 13-9, the level

of the mercury will be lower in the capillary tube than in the dish. If a glass **U** tube is constructed with one arm about 1 cm in diameter and the other about 0.2 cm in diameter, and mercury is poured into the tube, the level in the narrower tube will be lower than in the wider tube, while if water is poured into such a **U** tube, the level of the water will be higher in the narrower tube, as shown in Figure 13-10.

One method of accounting for the behavior of liquids in capillary tubes is to assume that there are forces of attraction, also of short range, between the molecules of the liquid and the molecules of the solid at the surfaces
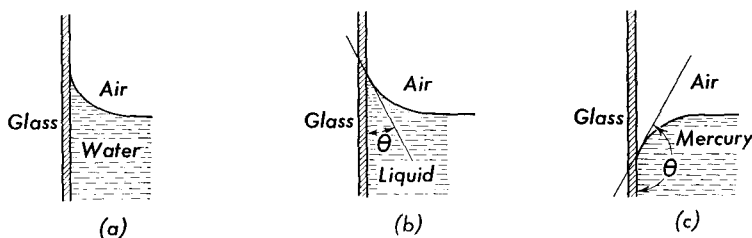
Fig. 13-11   Angles of contact. (a) $0°$ between water and glass; (b) angle of contact $\theta < 90°$ between a liquid and glass; (c) angle of contact $\theta > 90°$ between mercury and glass.

of contact. This type of attractive force between molecules in the surface of one substance for those in the surface of another substance is sometimes called *adhesion,* to distinguish it from the force of cohesion between like molecules. If the force of adhesion is greater than the force of cohesion, the liquid will cling to the solid surface; that is, it will wet the solid. If the force of cohesion is greater than the force of adhesion, the liquid will cling to itself; that is, it will tend to form droplets when placed on a smooth surface, rather than spreading out to wet the surface.

The angle between the liquid surface and the solid surface at the region of contact is an indication of the relative values of the forces of adhesion and cohesion. This angle is known as the *angle of contact.* For water and glass the angle of contact is practically $0°$, while for some other liquid, the angle of contact will have some value θ, as shown in Figure 13-11. If the force of cohesion is much greater than the force of adhesion, as in the case of mercury and glass, the angle of contact θ is greater than $90°$. For mercury and glass, $\theta = 139°$. Values of some contact angles are given in Table 13-3.
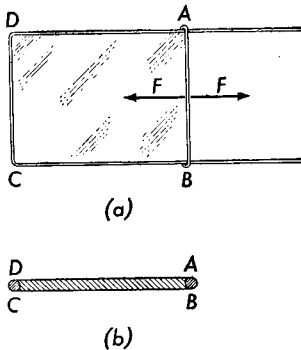
13-9   Surface Tension

We have seen that the liquid inside a capillary tube has a curved surface, and, if the tube is cylindrical, the surface of the liquid may be nearly

TABLE 13-3 CONTACT ANGLES

Liquid	Tube	Angle, Degrees
Alcohol	Glass	0
Ether	Glass	0
Glycerin	Glass	0
Mercury	Glass	139
Water	Glass	0
Water	Paraffin	107

spherical. The interesting phenomena associated with liquid surfaces can be most easily explained by introducing the concept of *surface tension*.

That a liquid surface behaves as though it is under tension can be demonstrated in a variety of experiments. Let us construct a rectangular wire frame having one side movable; this can be done by curving the ends of a wire AB so that it slides easily on two legs of the frame, as in Figure

Fig. 13-12 (a) Wire frame with movable slide AB used to measure the surface tension of a film in the frame $ABCD$. (b) Shows the thickness of the film.

13-12. We can pick up a film on this frame by dipping it in a soap solution. This film will have two rectangular surfaces. The film will tend to contract, and, since AB is movable, the film will pull this wire toward CD with some force F.

To keep the wire AB in equilibrium, a force F to the right has to be applied to it. This force can be used to measure the surface tension. The force exerted on the wire depends on the length of the wire. *We define the surface tension* S *as the force exerted by a single surface on a section of unit length.* In this example the force exerted by each surface is $F/2$. If l is the length of the wire, the surface tension is therefore

$$S = \frac{F}{2l}.$$

If the surface area is increased by moving the wire AB through a

distance x, the work done is

$$W = Fx,$$

and, since $$F = 2lS,$$

therefore $$W = S \times 2lx.$$

Now $2lx$ is the increase in surface area of the film; setting $2lx = A$, we find

$$S = \frac{W}{A}.$$ (13-9)

The surface tension thus represents the work done per unit area in increasing the area of a film. From mechanics we know that the most stable configuration of a mechanical system is the position of lowest potential energy. Thus a soap bubble or a water droplet assumes a spherical shape, for the spherical surface contains the greatest volume per unit of surface area of any three-dimensional figure and is therefore the surface of lowest energy.

The surface tension of a liquid depends on the nature of the liquid and the nature of the substance outside the liquid surface, that is, whether it is air or the vapor of the liquid itself. The values of the surface tensions of liquids are given in Table 13-4. The surface tension also depends on the temperature of the system, decreasing as the temperature rises.

TABLE 13-4   SURFACE TENSION

Liquid in Contact with Air	Temperature in °C	Surface Tension in dynes/cm
Ethyl alcohol	20	22.3
Water	0	75.6
	20	72.8
	60	66.2
	100	58.9
Mercury	25	473
Olive oil	20	32
Glycerin	20	63.1
Soap solution		26

Soap films provide an analogue solution to many engineering problems. The soap film has the property that the surface tension at every point in the surface is a constant. In an ideal engineering structure every element might well be stressed to the same tensile or compressive stress, to gain the maximum strength per unit weight of the total structure. Suppose one wished to design a Plexiglas dome for a pressurized vessel in which the

opening was to be of irregular shape. One way to find the best shape for the dome is to build a model vessel with the desired opening, cover the opening with a soap film, and blow a bubble. The soap bubble automatically will come to the best design shape for the desired pressure differential. If proper scaling factors are known, the design of the desired dome may be determined from this experiment.

13-10   Pressure and Curved Surfaces

Let us consider the forces holding a spherical surface together. Suppose a sphere of radius R is imagined to be parted by a plane diaphragm, as shown in Figure 13-13. Let us imagine that the upper (dotted) part of the sphere
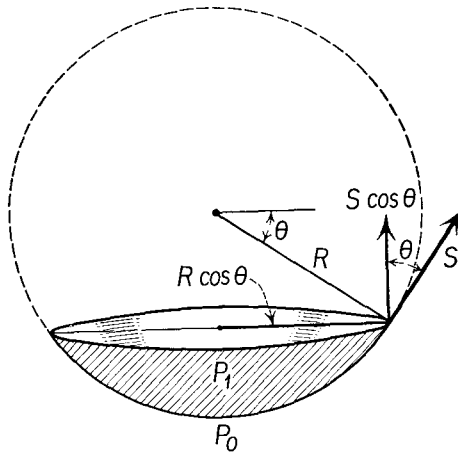

Fig. 13-13

has been removed. The interior of the cap contains fluid at pressure P_1 which is greater than the pressure P_0 outside the volume bounded by the cap and diaphragm. The force which tends to blow the diaphragm off the spherical cap is given by the difference in pressure $P_1 - P_0$ multiplied by the area of the diaphragm $\pi(R \cos \theta)^2$. The only force which tends to hold the spherical cap to the diaphragm is the surface tension S, whose component perpendicular to the diaphragm is $S \cos \theta$. The total force exerted by the surface tension is the product of the perpendicular component by the perimeter of the circle which bounds the diaphragm and the spherical cap, for, by definition, the surface tension is a force per unit length of surface. The spherical cap will be in equilibrium on the diaphragm when these two forces are equal. Thus

$$(P_1 - P_0) \times (\pi R^2 \cos^2 \theta) = (S \cos \theta) \times (2\pi R \cos \theta),$$

or
$$P_1 - P_0 = \frac{2S}{R}. \tag{13-10}$$

Equation (13-10) has been deduced by replacing the upper (dotted) portion of the sphere by an imaginary diaphragm. We may now imagine the dotted portion of the sphere to be replaced over the lower spherical cap, and the diaphragm removed. The equation now describes the relationship between the gauge pressure within the sphere $P_1 - P_0$, the surface tension S, and the radius R of the sphere.

Thus the difference in pressure on the two sides of a spherical surface due to the surface tension depends inversely on the radius of the sphere. P_0 is the pressure on the convex side of the surface and P_1 is the pressure on the concave side. The above result is not dependent on the angle θ and is therefore independent of the size of the spherical cap analyzed. It applies

Fig. 13-14 A soap bubble has two spherical surfaces whose radii R_1 and R_2 differ very slightly.
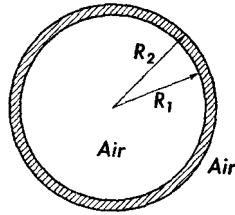
to a whole sphere or any part of it. Thus the pressure inside a water droplet is greater than the atmospheric pressure, and the pressure inside a small droplet is greater than the pressure inside a large droplet. When two water droplets are placed in contact, the water tends to move from a region of high pressure into a region of low pressure, and the large droplet swallows the small one. From yet another point of view, Equation (13-10) shows why water droplets at rest are spherical rather than, say, pear-shaped, for the water within the smaller end would be at higher pressure than the water at the large end of a pear-shaped droplet. Equation (13-10) is applicable to the determination of the stress in any spherical shell. In a metallic shell of a given thickness, the stress must be multiplied by the thickness of the shell to give a quantity whose dimensions are force per unit length. In this way we can determine the pressure differential a spherical shell can withstand, or the breaking strength of a spherical diaphragm.

In the case of a soap bubble blown in air, the pressure difference is practically twice as great as given by Equation (13-10), that is, $4S/R$, since a soap bubble has two spherical surfaces of radius R_1 and R_2, as in Figure 13-14. Since the thickness of the soap film is very small, the difference between the two radii may be neglected, and in the equation for the pressure differential between the inside and the outside of the bubble we simply write the average radius R for the radius of the bubble.

Illustrative Example. Determine the gauge pressure inside a small raindrop 3 mm in diameter.

The gauge pressure is $P_1 - P_0$, for the pressure P_1 is the pressure inside the spherical surface, and the outside pressure P_0 is the atmospheric pressure. Substituting $S = 73$ dynes/cm for the surface tension of water, and $R = 0.15$ cm in Equation (13-10), we find

$$P_1 - P_0 = \frac{2 \times 73 \text{ dynes/cm}}{0.15 \text{ cm}},$$

$$P_1 - P_0 = 973 \text{ dynes/cm}^2.$$

13-11 Capillarity

We have already seen that, if a capillary tube is inserted into a liquid, the levels inside and outside the tube will differ by an amount h. In some cases the liquid will be higher in the capillary tube; in other cases it will be lower, depending upon the relative values of the forces of adhesion and cohesion. Furthermore, the surface of the liquid in the capillary tube, sometimes called the *meniscus*, may be approximated as spherical, if the bore is cylindrical and of sufficiently small diameter.

The concept of surface tension, and the pressure differences associated with curved surfaces discussed in the preceding paragraphs, enables us to obtain a simple relationship between the difference in levels h inside and outside a capillary tube and the radius r of this tube. Let us suppose that the angle of contact between the liquid and the material of the capillary tube is θ. The meniscus is of spherical shape of radius R such that

$$R \cos \theta = r.$$

If, as in Figure 13-15(a), we complete the spherical surface generated by the meniscus in dotted lines, we see that the atmosphere is inside the sphere and that the liquid in the capillary tube is outside the sphere. Thus the pressure in the liquid just outside the meniscus must be below atmospheric pressure by an amount given by Equation (13-10).

$$P_1 - P_0 = \frac{2S}{R},$$

$$P_{\text{atm}} - P_{\text{liquid}} = \frac{2S}{r/\cos \theta},$$

$$P_{\text{atm}} - P_{\text{liquid}} = \frac{2S \cos \theta}{r}.$$

The pressure at the free surface of the liquid outside the capillary tube is the atmospheric pressure, and at the same level inside the capillary tube the pressure must also be atmospheric. But the pressure beneath the meniscus has been reduced by an amount given by the formula above, so that the

meniscus must rise until the hydrostatic pressure generated by the column of liquid is equal to the reduction in pressure generated by the curvature of the meniscus, and we have

$$h\rho g = \frac{2S \cos \theta}{r},$$

$$h = \frac{2S \cos \theta}{r\rho g},$$ **(13-11)**

where ρ is the density of the liquid.
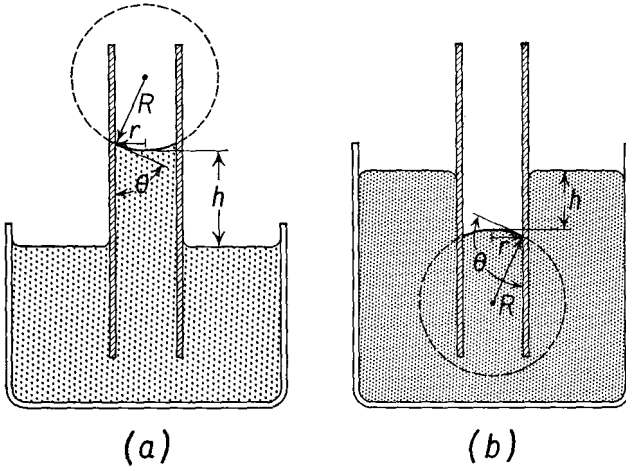
Fig. 13-15 Surface-tension forces in capillary tubes (a) when contact angle θ is less than 90°; (b) when contact angle θ is greater than 90°.

The same analysis will hold if the surface in the capillary tube is depressed by an amount h, as in Figure 13-15(b). Here the liquid may be thought of as being inside the sphere and at a higher pressure than the atmospheric pressure. In Equation (13-11), when the angle of contact is greater than 90°, the value of its cosine is a negative number, hence h will be negative, indicating that the level is depressed in the capillary tube.

From Equation (13-11) we see that a liquid will rise higher in a capillary tube of small bore than in a tube of large diameter. Capillary action is the basis of operation of mops, of sponges, of lampwicks, and of many other devices, where the fine spaces between the threads act as capillary tubes. In reading a mercury barometer, correction must be made for the effect of surface tension upon the height of the mercury column in order to obtain accurate measurements of the atmospheric pressure, for the height of the column is depressed from its true height by capillary action, which depends upon the diameter of the tube.

Illustrative Example. Two glass capillary tubes, each 1 mm in radius, are put into two different liquids, one in water and the other in mercury. Compare the liquid levels in the two tubes.

Let us take the level of the liquid outside each capillary tube as the zero reference level. From Equation (13-11) the level of the liquid inside the capillary will differ from that outside by an amount

$$h = \frac{2S \cos \theta}{r \rho g}. \tag{13-11}$$

For the case of water, $\theta = 0°$, hence $\cos \theta = 1$; $S = 73$ dynes/cm, $r = 0.1$ cm, $\rho = 1$ gm/cm^3, and $g = 980$ cm/sec^2. Letting $h = h_1$ for water, we get

$$h_1 = \frac{2 \times 73 \text{ dynes/cm}}{0.1 \text{ cm} \times 1 \text{ gm/cm}^3 \times 980 \text{ cm/sec}^2},$$

from which $h_1 = \dfrac{146}{98}$ cm $= 1.5$ cm.

For the case of mercury, $\theta = 139°$, hence $\cos \theta = \cos 139° = -\cos 41° = -0.755$; $S = 473$ dynes/cm, $r = 0.1$ cm, $\rho = 13.6$ gm/cm^3, and $g = 980$ cm/sec^2. Letting $h = h_2$ for mercury, we get

$$h_2 = -\frac{2 \times 473 \text{ dynes/cm} \times 0.755}{0.1 \text{ cm} \times 13.6 \text{ gm/cm}^3 \times 980 \text{ cm/sec}^2},$$

from which $h_2 = -0.536$ cm.

13-12   Phenomena Associated with Surface Tension

There are many phenomena associated with surface tension, a few of which will be described here. For example, if a drop of oil is allowed to fall on a large clean surface of water, the oil will spread out over this surface until it is 1 molecule thick; it forms a monomolecular layer. If the volume of the original oil drop is known, and if the area of the monomolecular layer is measured, its thickness can be computed. This thickness will then give us one dimension of the oil molecule. Studies of the structure of oil molecules by means of x-rays show that they are not spherical but have one long dimension and two shorter dimensions. In a monomolecular layer, the molecules stand on end, so that a measure of the thickness of the oil film will yield the longest dimension.

If two small wooden matchsticks are floated near each other on the surface of water, they will be pulled toward each other. The liquid between the matchsticks is raised to a level higher than the rest of the surface by the surface tension; the pressure in the liquid between the sticks is thus decreased to a value less than atmospheric pressure. If a drop of alcohol is placed between them, the two matchsticks will be pulled apart. The effect of the alcohol is to decrease the surface tension; that is, the surface tension

of the solution of alcohol in water is less than that of pure water. The difference in the surface tensions of the two sides of each stick supplies the forces which pull them apart.

When small pieces of camphor are dropped onto a clean surface of water, these pieces will perform very erratic motions. Although camphor is soluble in water only to a very slight extent, wherever it is dissolved, the surface tension is reduced. Each little piece of camphor will experience forces caused by the different surface tensions around it, and these forces set the particle in motion. The motion will cease when the surface tension becomes uniform and equal to that of the solution of camphor in water.

Problems

13-1. A copper wire 80 cm long and 0.25 cm in diameter is suspended from a rigid framework. A body whose mass is 5 kg is hung at the end of the wire. Determine (a) the stress in the wire and (b) the strain produced.

13-2. A steel wire 1.5 m long and 0.04 cm in diameter supports a cylinder whose mass is 4.0 kg. Determine (a) the stress in the wire, (b) the strain produced, and (c) the elongation of the wire.

13-3. A brass wire 3.0 ft long and 0.04 in. in diameter supports a body whose weight is 3 lb. Determine (a) the stress in the wire, (b) the strain produced, and (c) the increase in length of the wire.

13-4. A brass wire 4.0 m long and 2.0 mm in diameter is suspended from a hook in a beam in the ceiling. A cylinder whose mass is 6.0 kg is hung from the other end. Determine the increase in length of the brass wire.

13-5. An aluminum wire 200 cm long and 0.5 mm in diameter has a series of cylinders hung from it in succession. Each cylinder has a mass of 10 gm. The measured changes in length expressed in centimeters, as determined by a telescope and scale method, are: 0.0014, 0.0029, 0.0042, 0.0056, and 0.0070. Plot a graph with the stress as ordinate and the strain as abscissa; from the slope of this graph, determine Young's modulus for this aluminum wire.

13-6. A steel rod 6.0 in. long and 0.5 in. in diameter is to be used as a piston in a cylinder to produce a pressure of 2,000 lb/in.^{2}. Determine the decrease in length of the rod produced by this stress.

13-7. Glycerin is subjected to a pressure of 850 atm. Determine the percentage change in its volume.

13-8. Determine the bulk modulus of an oil if a volume of 1,000 cm^3 shows a decrease in volume of 0.3 cm^3 when subjected to a pressure of 12 atm.

13-9. A cube of copper 5 cm on an edge is subjected to two oppositely directed shearing forces along two of its faces. Each force is 900 nt. Determine (a) the shearing stress, (b) the shearing strain, and (c) the angle, in degrees, through which the cube has been sheared.

13-10. The maximum permissible design stress for an elevator cable is 10,000 lb/in.^{2}. What diameter cable should be used for an elevator weighing 2 tons when fully loaded, if the acceleration is to be 10 ft/sec^2?

13-11. A brass rod and a copper rod, each 2 ft long and 1 in. in diameter, are

joined at one end, and the combined rod is subjected to a compressive force of 10,000 lb. Find (a) the stress in each rod and (b) the strain in each rod.

13-12. What is the density of water at a point where the pressure is 100 atm if the density at sea level is 1 gm/cm³?

13-13. Suppose that a square specimen of cross-sectional area A is placed in tension by equal and opposite forces F applied to its ends. Calculate the tensile stress (stress component perpendicular to the face) and the shear stress (stress component parallel to the face) across a plane face inclined at an angle θ to the normal cross section of the specimen. (a) At what angle is the tensile stress a maximum? (b) At what angle is the shear stress a maximum?

13-14. Three capillary tubes of diameters 0.5 mm, 1.0 mm, and 1.5 mm, respectively, are supported in a jar of water. Determine the height to which the water will rise in each of these tubes.

13-15. Three holes of diameters 1.0 mm, 1.5 mm, and 2.0 mm, respectively, are bored in a block of paraffin. The paraffin is partly immersed in water. Determine the level of the water in each hole.

13-16. A capillary tube 1.0 mm in diameter is placed in a soap solution of density 1 gm/cm³. The liquid in the tube rises to a height of 0.45 cm above the level of the rest of the surface. Determine the surface tension of this solution, assuming the contact angle to be zero.

13-17. A soap film is formed on a rectangular frame 2 cm by 8 cm, as in Figure 13-12. (a) Determine the force that the film exerts on the shorter wire. (b) If this wire is moved through a distance of 5 cm, determine the amount of work done. Assume that the temperature remains constant in this process.

13-18. Calculate the gauge pressure inside a raindrop which is 4 mm in diameter. Assume the temperature to be 20°C.

13-19. Calculate the gauge pressure inside a drop of mercury whose temperature is 25°C and whose diameter is 4 mm.

13-20. Determine the gauge pressure inside a soap bubble which is 5 cm in diameter. Assume the temperature to be 20°C.

13-21. Two rectangular glass plates are spaced 1 mm apart. They are partly immersed in a dish of water at 20°C, with the plates placed so that the air space between them is in a vertical plane. Determine how high the water will rise in this air space above the level of the water in the dish. [NOTE: Consider the forces acting on a surface film 1 cm wide in contact with each plate. Balance these forces against the weight of water lifted through a height h.]

13-22. A hollow glass tube has a soap bubble of 5 cm diameter formed on one end and another soap bubble of 2 cm diameter formed on the other end. Determine the pressure difference at the ends of the tube. Explain what will happen as a result of this pressure difference.

13-23. Two glass plates, each having a large surface, are clamped together along one edge and separated by spacers a few millimeters thick along the opposite edge to form a wedge-shaped air film. These plates are then placed vertically in a dish of colored liquid. Calling x the horizontal distance measured from the edge where the thickness of the air film is zero, show that the vertical distance y through which the liquid rises in the air space varies inversely as x. [NOTE: The thickness of the air film increases as the distance x increases.]