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Physics, Chapter 14: Temperature

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Part Two

HEAT

14

Temperature

14-1 Concept of Temperature

Temperature is one of the fundamental concepts of physics. We are all able to recognize that some bodies are hotter than others, but our temperature sense is qualitative rather than quantitative and is capable of only a limited range. The sense of touch can frequently be used to distinguish between hotter and colder objects, provided that these lie in a temperature range consistent with the stability of human tissue. Even within this range the sense of touch is often unreliable as a measure of temperature. The metal bracket holding a wooden rail may feel much colder to the touch than the railing itself, even though both are at the same temperature.

It is a matter of common observation that some of the physical properties of many substances are altered when the temperature is changed. The volume or the pressure of a gas increases when the temperature is raised. The length of a copper rod changes with changing temperature. Some of the electric and magnetic properties of substances vary with changes in temperature. The changes that take place in these physical properties can be used to measure the changes in temperature which produced them.

In order to make a measurement of the temperature of a body, it is important to be able to decide when two bodies are at the same temperature. Suppose that the length of a copper rod is measured while it is in the laboratory and exposed to the air in the room; if the rod is then put into a mixture of ice and water, its length will first decrease and then reach a new value which will remain constant as long as it is in the ice-water mixture. We then say that the temperature of the copper rod is the same as that of the ice-and-water mixture. The two systems, the copper rod and the ice-water mixture, are in thermal contact and have reached *thermal equilibrium.* Thus *two systems are said to be at the same temperature when they are in* *thermal equilibrium.* We may thus consider temperature to be a property of a system which determines whether or not it is in thermal equilibrium with any other system that is placed in thermal contact with it.

14-2 Thermometers

A physical device which measures temperature is called a *thermometer.* In order to construct a thermometer, it is necessary to choose some thermometric property of a system whose value depends upon its temperature. Although the length of a solid rod is seldom used as this thermometric property, the length of a liquid column in a glass tube is used very frequently. In this liquid-in-glass thermometer, the difference in volume expansion between the liquid and the glass container is visible as a change in length of the liquid column when it is allowed to expand into a very fine capillary tube attached to the glass bulb containing most of the liquid, as shown in Figure 14-1.

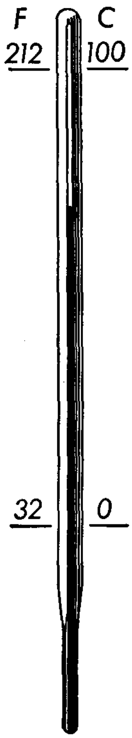

Fig. 14-1 A mercury-in-glass thermometer with the ice point and the steam point marked in both the Fahrenheit and Celsius scales.

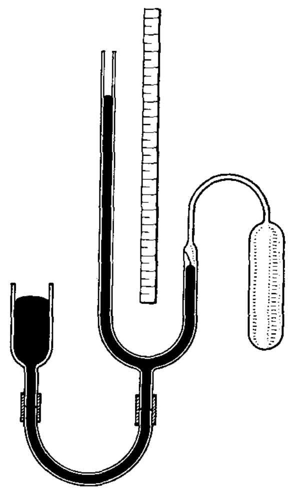

Fig. 14-2 Constant-volume gas thermometer.

The *constant-volume gas thermometer,* in which the pressure of the gas is the thermometric property, provides an extremely accurate measurement of temperature. This thermometer, shown in Figure 14-2, utilizes the change in pressure of a gas with temperature, when the gas is maintained at constant volume. Gases customarily used are air, hydrogen, or helium.

The color and intensity of the light emitted from a furnace vary with the temperature. An *optical pyrometer* is a device in which the temperature of a furnace is measured by comparing the light emitted from the furnace with the light emitted by an electrically heated filament which has been previously calibrated. The character of the light emitted by an incandescent body is a thermometric property suitable for use in the determination of temperature.

The electrical resistance of a wire changes with temperature. As we shall see in a subsequent chapter, the electrical resistance of a metal is an easily measured property. This thermometric property provides the basis for the *resistance thermometer.*

If the ends of two wires made of different metals or alloys are connected together and their junctions maintained at different temperatures, it is found that an electric current flows in the wire loop. The current is produced by an electromotive force whose value depends upon the difference in temperature of the junctions. This thermometric property provides the basis of the *thermocouple* type of thermometer. When a number of small thermocouples are connected together, the effect is enhanced, and the result is called a *thermopile.*

14-3 Temperature Scales

In order to be able to assign a number to the temperature of a body, it is necessary to agree upon a method for setting up a temperature scale. We must first choose some thermometric property of a system, for example, the length of a mercury column in a glass capillary tube, or the electrical resistance of a platinum wire, or the pressure of a gas kept at a constant volume. Let us call the value of the chosen thermometric property X and let T be the temperature of a system that surrounds this thermometer and is in thermal contact with it. When thermal equilibrium is reached, the thermometer and the surrounding system are at the same temperature. If the temperature of the system is changed, the value of the thermometric property of the thermometer is also changed. The temperature, as indicated by the thermometer, will be some function of X; let us call this function $T(X)$. Let us assume that $T(X)$ is a linear function of X, thus

$$T(X) = aX,$$

where a is a constant of proportionality.

Thus, if at one temperature the value of the thermometric property is X_1, and if at a higher temperature the value of this thermometric property is X_2, then we can write

$$\frac{T(X_1)}{T(X_2)} = \frac{X_1}{X_2}. \tag{14-1}$$

Before 1954 two *fixed points* were used to assign numbers in calibrating a thermometer. Since 1954, by international agreement, only one fixed point is being used. Since the older scales are still in common use, we shall describe the earlier method first.

A *fixed point* on a temperature scale is a number that is assigned to the temperature of an easily reproducible state of a system. Before 1954 the two fixed points were (a) the temperature of a system consisting of a mixture of ice in equilibrium with water open to the air at standard atmospheric pressure with the water saturated with air (called the *ice point*) and (b) the temperature of steam in equilibrium with pure water at standard atmospheric pressure (called the *steam point*).

To measure the temperature of a system, the thermometer is put in thermal contact with it; the value X of the thermometric property at this temperature $T(X)$ is measured. Calling X_i the value of this property at the ice point, we have

$$\frac{T(X_i)}{T(X)} = \frac{X_i}{X}. \tag{14-2}$$

Similarly, if X_s is the value of the thermometric property at the steam point, then

$$\frac{T(X_s)}{T(X)} = \frac{X_s}{X}. \tag{14-3}$$

From the above equations we get

$$\frac{T(X_s) - T(X_i)}{T(X)} = \frac{X_s - X_i}{X}, \tag{14-4}$$

so that

$$T(X) = \frac{T(X_s) - T(X_i)}{X_s - X_i} X. \tag{14-5}$$

On the Celsius scale of temperature, also called the centigrade scale of temperature, first devised by A. Celsius (1701–1744), the temperature interval $T(X_s) - T(X_i)$ is assigned the value of 100°C, the ice point is called 0°C, and the steam point 100°C, the interval being divided into 100 equal divisions called degrees Celsius.

On the Fahrenheit scale, first devised by G. Fahrenheit (1686–1736), the interval between the ice point and the steam point is assigned the value 180°F, the ice point is called 32°F, and the steam point 212°F, the interval being divided into 180 equal divisions called degrees Fahrenheit.

As the accuracy of temperature measurements increased, discrepancies arose principally because of the difficulty of reproducing the ice point. It will be recalled that the ice point is the temperature of an equilibrium mixture of ice and air-saturated water at atmospheric pressure. The discrepancies between measurements made at standardizing laboratories amounted to as much as 0.04°C, whereas the accuracy of measurement in this temperature region was good to about 0.001°C. In 1954 it was decided to change the method of calibrating thermometers and to use only *one fixed point;* this fixed point is the temperature of an equilibrium mixture of ice, water, and water vapor; this state is called the *triple point of water* and exists at only one definite temperature and pressure (see Chapter 17). The number chosen for this fixed point is 273.16 degrees Kelvin, written as 273.16°K. (This is in honor of Lord Kelvin who developed the thermodynamic scale of temperature. See Chapter 19.) Thus if X_3 is the value of the thermometric property of the thermometer at the triple point, then from Equation (14-1)

$$\frac{T(X)}{T(X_3)} = \frac{X}{X_3}, \tag{14-6}$$

and since

$$T(X_3) = 273.16°\text{K}, \tag{14-7}$$

we have

$$T(X) = 273.16°\text{K}\,\frac{X}{X_3}\cdot \tag{14-8}$$

The triple-point cell used by the National Bureau of Standards is illustrated in Figure 14-3. Very pure, air-free water is introduced into the cell which is then sealed. The cell is then cooled in a thermos flask by an ice bath until ice, water, and water vapor are present simultaneously in the cell, indicating that the triple point of water has been attained.

If the constant-volume gas thermometer is used for measuring temperature, the thermometric property that is measured is the pressure P of the gas. For this case Equation (14-8) becomes

$$T(P) = 273.16°\text{K}\,\frac{P}{P_3}, \tag{14-9}$$

where P_3 is the pressure of the gas at 273.16°K or the triple-point temperature. As the result of many careful experiments, we find that the value of the temperature of a particular system depends upon the nature of the gas that is used in the gas thermometer. However, as smaller and smaller amounts of gas are used in the thermometers, so that the pressure of the gas gets smaller and the density gets smaller, all gas thermometers give the same result for the temperature T of the system. Thus

$$T = \underset{(\text{density}\to 0)}{273.16°\text{K}} \left(\frac{P}{P_s}\right)\cdot \tag{14-10}$$

We shall consider some of the interesting properties of gases in this and later chapters.

The relationship between the Celsius scale and the Kelvin scale can now be stated in terms of the single fixed point. On the Celsius scale the triple point is defined as 0.01°C; any temperature t on the Celsius scale can be defined in terms of the temperature T on the Kelvin scale as

$$t = T - 273.15. \qquad (14\text{-}11)$$

The relationship between the temperature of a body expressed on the Celsius scale t_c and its temperature expressed on the Fahrenheit scale

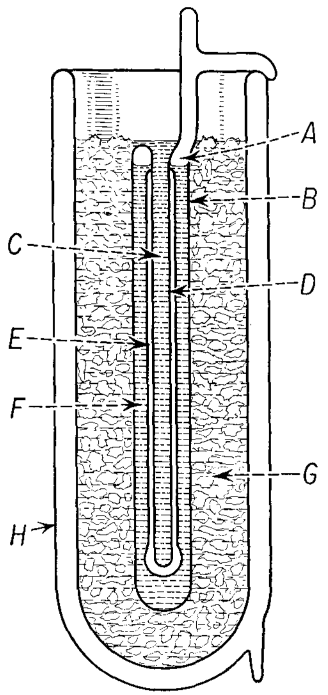

Fig. 14-3 Diagram of the National Bureau of Standards triple-point cell, BD, in use in an ice bath G, within a thermos flask H. A, water vapor; C, thermometer well; E, ice mantle; F, liquid water.

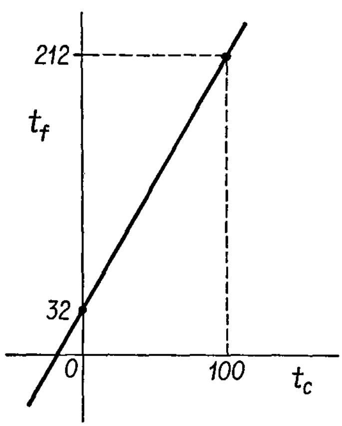

Fig. 14-4 Graph of Fahrenheit temperature versus Celsius temperature.

t_f can be found by plotting the Fahrenheit temperature as ordinate and the Celsius temperature as abscissa on rectangular coordinate paper, as shown in Figure 14-4. The curve relating these two temperatures is a straight line of slope $\frac{180}{100}$, which intercepts the Fahrenheit axis at 32°F. Applying the slope-intercept form of the equation of a straight line

$$y = mx + b,$$

where m is the slope and b is the y intercept, we find

$$t_f = \tfrac{9}{5}t_c + 32. \tag{14-12}$$

This equation may be solved for t_c to yield

$$t_c = \tfrac{5}{9}(t_f - 32). \tag{14-13}$$

Illustrative Example. Find the centigrade temperature at which a centigrade thermometer will read the same number as a Fahrenheit thermometer.

The relationship between the reading of a centigrade thermometer and a Fahrenheit thermometer is always given by Equation (14-12) as

$$t_f = \tfrac{9}{5}t_c + 32.$$

The additional condition imposed by the problem may be expressed analytically as

$$t_f = t_c.$$

Solving these two equations simultaneously, we find

$$t_c = -40.$$

14-4 Thermal Expansion of Solids

A change in the temperature of a substance is nearly always accompanied by a change in its physical dimensions. The expansion which takes place in a perfect crystal when its temperature is increased depends upon the direction in which the expansion is measured relative to the crystal axis. Most crystalline solids are made up of grains within which the crystals are oriented in one direction. The crystal directions are randomly oriented from grain to grain, so that we find that the expansion of most solids is often the same in every direction relative to the crystal axis.

The change in length of a solid rod is a smooth function of temperature and may be represented by an infinite series of the form

$$L_t = L_0[1 + \alpha\,(\Delta t) + \alpha'\,(\Delta t)^2 + \cdots], \tag{14-14}$$

where L_t is the length of the rod at temperature t, L_0 is the length of the rod at some reference temperature t_0, and Δt is the temperature difference $t - t_0$. The coefficients α, α', and so on, are constants which must be evaluated by experiment for each different material. It is common practice to approximate the expansion of a solid by making use of a number of straight lines tangent to the curve which describes the length of a rod as a function of temperature, as shown in Figure 14-5. In this case only the first two terms of the right-hand side of Equation (14-14) are required to represent any one of these tangent lines, and the equation becomes

$$L_t = L_0[1 + \alpha(\Delta t)], \tag{14-15}$$

where α is called the *coefficient of linear expansion* at the temperature t_0. Since the slope of each of the tangent lines in Figure 14-5 depends upon the reference temperature at which it is constructed, the coefficient of linear expansion depends upon the reference temperature at which it is evaluated.

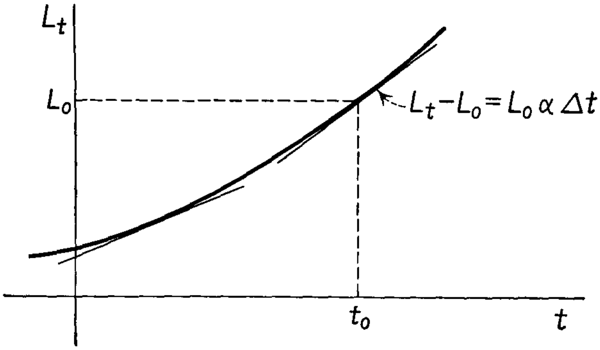

Fig. 14-5

The slope of the tangent line at temperature t_0, at which the length of the rod is L_0, is given by $L_0\alpha$. If we write the change in length of the rod as

$$\Delta L = L_t - L_0,$$

we may express the coefficient of linear expansion as

$$\alpha = \frac{\Delta L/L_0}{\Delta t}. \tag{14-16}$$

In other words, the coefficient of linear expansion is the fractional change in length of the rod per degree of temperature change. The units of α are therefore $(\text{degrees})^{-1}$. Since a Fahrenheit degree represents a smaller temperature interval than a centigrade degree, the coefficient of linear expansion per degree Fahrenheit is five ninths of the coefficient of linear expansion per degree centigrade. The linear-expansion coefficients of several solids are given in Table 14-1.

If two metals, say brass and steel, are welded or riveted together so that they form a single straight piece at room temperature, then, when the temperature is raised, the strip will bend in the form of an arc with the brass on the outside, as shown in Figure 14-6. This is due to the fact that brass has a greater coefficient of expansion than steel. A bimetallic strip of this kind is frequently used in thermostats. The strip is fixed at one end, and the bending of the free end may be used to actuate a switch at some predetermined temperature.

A solid in which the expansion is the same in every direction is said to be *isotropic* with regard to this property. When the temperature of an

TABLE 14-1 COEFFICIENTS OF LINEAR EXPANSION AND VOLUME EXPANSION

Substance		α
Aluminum	(20–300°C)	25.7×10^{-6} per °C
Brass	(0–100°C)	19.3
Copper	(25–300°C)	17.8
Pyrex glass	(21–470°C)	3.6
	(550–570°C)	15.1
Invar	(20°C)	0.9
Lead	(18–100°C)	29.40
Platinum	(40°C)	8.99
Steel	(40°C)	13.2
Tungsten	(0–100°C)	4.3
	(1000–2000°C)	6.1
Substance		β
Mercury	(0–100°C)	181.8 per °C

isotropic solid is changed, the length of each linear dimension is changed. The area of each element of area is changed, and the volume of each volume element is changed. We may represent the expansion of area and the volume expansion by mathematical series analogous to Equation (14-14) and may approximate the expansion by use of the first two terms of the series, as before. To represent the area expansion, we write

$$A_t = A_0[1 + \sigma(\Delta t)], \tag{14-17}$$

where σ (sigma) is the coefficient of area expansion at the temperature t_0. Similarly, to represent the volume expansion we write

$$V_t = V_0[1 + \beta(\Delta t)], \tag{14-18}$$

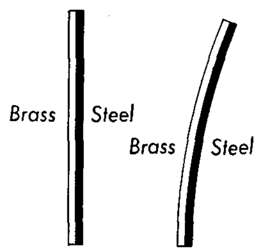

Fig. 14-6 Bending of a bimetallic strip when heated.

where β (beta) is the coefficient of volume expansion at the temperature t_0. The coefficient of area expansion and the coefficient of volume expansion may be expressed by equations analogous to Equation (14-16) as

$$\sigma = \frac{\Delta A/A_0}{\Delta t}, \tag{14-19}$$

and

$$\beta = \frac{\Delta V/V_0}{\Delta t}. \tag{14-20}$$

We may relate the coefficient of linear expansion to the coefficient of area expansion by considering the expansion of a square plate of dimensions

$L_0 \times L_0$ at temperature t_0. The area of this plate at temperature t_0 is L_0^2. When the temperature of the plate is t, the length of a side is L_t, given by Equation 14-15, and the new area of the plate is

$$A_t = L_t^2 = \{L_0[1 + \alpha(\Delta t)]\}^2$$
$$= L_0^2[1 + 2\alpha\,(\Delta t) + \alpha^2\,(\Delta t)^2],$$
$$A_t = A_0[1 + 2\alpha\,(\Delta t) + \alpha^2\,(\Delta t)^2].$$

In general, the coefficient of linear expansion is a small number. Over the limited temperature interval for which the coefficient of linear expansion represents the true expansion of the body, we may neglect the term in α^2 as being small in comparison with the other terms on the right-hand side of the equation. Thus we have

$$A_t = A_0[1 + 2\alpha\,(\Delta t)].$$

Comparing the above result with Equation (14-17), we see that the coefficient of area expansion σ is numerically equal to twice the coefficient of linear expansion α. In the form of an equation we have

$$\sigma = 2\alpha. \tag{14-21}$$

Similarly, we may find that the coefficient of volume expansion β is numerically equal to three times the coefficient of linear expansion, or

$$\beta = 3\alpha. \tag{14-22}$$

Coefficients of areal expansion and of volume expansion of solids are not usually tabulated in compilations of technical data, for the reason that they may be readily obtained from the tabulated coefficients of linear expansion.

Illustrative Example. A hollow copper sphere has an inner radius of 4 cm and an outer radius of 5 cm at a temperature of 20°C. Find the change in volume of the spherical cavity when the temperature is raised to 420°C.

The change in volume of a copper sphere is the same whether it is solid or hollow. We may think of a solid sphere as consisting of a solid central core and a hollow spherical shell whose inside diameter is the same as the diameter of the core. The two parts fit perfectly at all temperatures. Thus the volume of a spherical cavity at any temperature is the same as the volume of a solid sphere of copper at that temperature. To find the change in volume of a spherical cavity in a hollow copper sphere, we find the change in volume of a solid sphere of copper subjected to the same temperature change.

The coefficient of linear expansion of copper valid in the range 0–625°C has been measured as 16.07×10^{-6}/°C. The change in volume may be obtained from Equation (14-20). The numerical values are

$$\beta = 3\alpha = 48.21 \times 10^{-6}(°\text{C})^{-1},$$
$$V_0 = \tfrac{4}{3}\pi r^3 = 268\ \text{cm}^3,$$
$$\Delta t = 400°\text{C}.$$

Thus
$$\Delta V = \beta V_0 \Delta t,$$
$$\Delta V = 5.17 \text{ cm}^3.$$

Thus the spherical cavity expands from a volume of 268 cm³ to a volume of 273.2 cm³.

14-5 Thermal Expansion of Liquids

As a general rule, a liquid expands when its temperature is raised. The notable exception to this is water, which contracts when its temperature

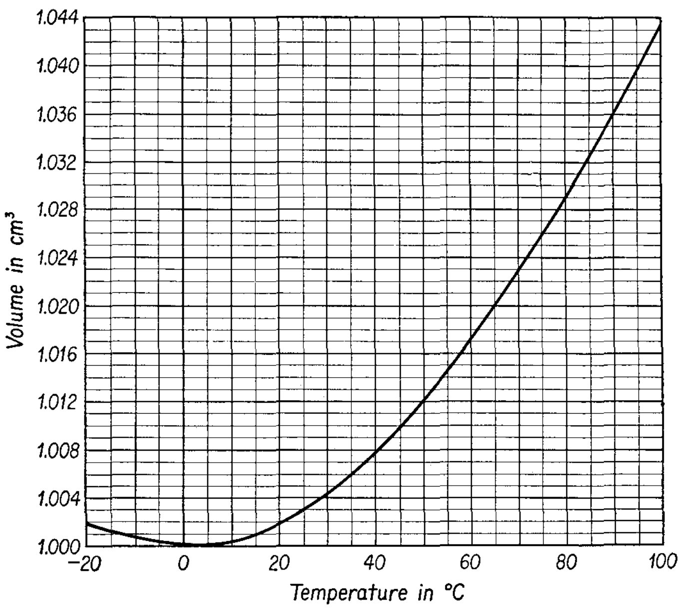

Fig. 14-7 Curve showing the volume of water as its temperature is raised from −20°C to 100°C with its minimum volume or maximum density at 4°C.

is raised in the limited region from 0°C to 4°C. Above 4°C water expands with an increase in temperature. The behavior of water at atmospheric pressure in the range from −20°C to 100°C is shown in Figure 14-7. (NOTE: Water is a supercooled liquid from −20°C to 0°C.)

One method for determining the coefficient of volume expansion of a liquid is to put the liquid into a container of known volume provided with a narrow tube at one end, as shown in Figure 14-8. A glass container is usually employed. The level of the liquid in the tube is observed at the initial temperature t_0 and at the final temperature t. The surface of the liquid is exposed to the atmosphere so that the pressure of the liquid remains constant. Since liquids generally have greater coefficients of expansion than glass, the level of the liquid will rise as the temperature is raised. Only the relative expansion of the liquid with respect to the container can be directly determined by this method. If the coefficient of volume expansion of the container is known, the coefficient of volume expansion of the liquid

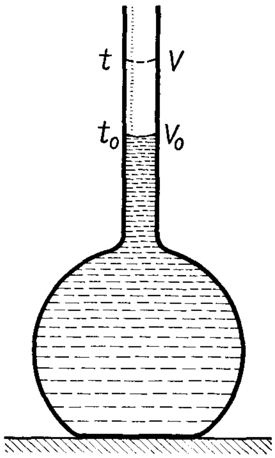

Fig. 14-8 Expansion of a liquid in a container when heated.

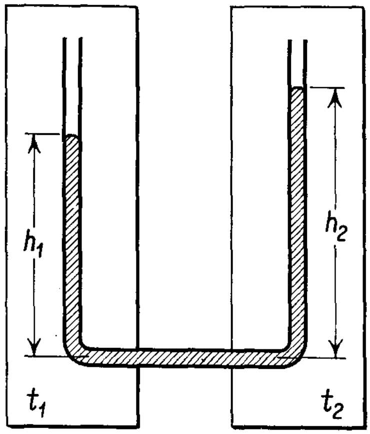

Fig. 14-9 Dulong and Petit apparatus.

can be determined. The apparent change in volume of the liquid is the difference between the change in volume of the liquid and the change in volume of the container.

The expansion of a liquid may be determined by a method, introduced by Dulong and Petit in 1817, in such a way that the measurement does not depend upon the expansion of the container. In its most elementary form the apparatus consists of two vertical tubes maintained at different temperatures. The two vertical tubes are connected by a horizontal tube, as shown in Figure 14-9. At the level of the horizontal tube, the pressure must be the same in both columns. The observed difference in the height of the liquid in the two columns is due to the difference in density of the liquid at the two temperatures.

14-6 Expansion of Gases

In studies of the expansion of solids or liquids, the substance studied is generally maintained in an open system, exposed to the atmosphere, so that the expansion coefficient is determined at constant pressure—the pressure of the atmosphere. Since a gas must be studied in a closed container, a change in temperature may involve a change in both the volume and the pressure of the gas. It is customary to study the behavior of the gas at constant pressure, where the volume changes with temperature, or at constant volume, where the pressure changes with tempeature.

An apparatus suitable for studying the change in pressure of a gas at constant volume is shown in Figure 14-10. The gas under investigation is kept in the bulb A and in the narrow tube leading to the manometer. The mercury in tube C of the manometer is always kept at the same level by being raised to the index point I by raising or lowering the tube B. The U-shaped section of the tube R is frequently a flexible rubber hose.

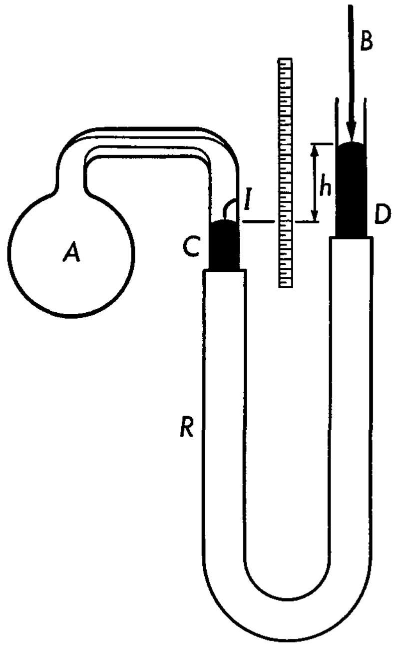

Fig. 14-10

Let us define the *coefficient of pressure change at constant volume* β' as

$$\beta' = \frac{\Delta P/P}{\Delta t} \cdot \qquad (14\text{-}23)$$

When the coefficient of pressure change at constant volume of hydrogen is measured at 0°C, this coefficient has the value of approximately 0.00366 per degree centigrade. A value very close to this is obtained with other gases, provided that the pressure is not too high and the temperature is significantly above the temperature at which the gas liquefies. The interpretation of this numerical value is that the pressure of hydrogen at constant volume will change by 0.00366 or 1/273.2 of its pressure at 0°C for each centigrade degree change of temperature. The use of a constant-volume gas thermometer has already been discussed in Sections 14-2 and 14-3.

By a slight modification of the apparatus pictured in Figure 14-10, the pressure of the gas may be kept constant, and its volume may be measured as a function of the temperature. The *coefficient of volume change at constant pressure* β may be defined as

$$\beta = \frac{\Delta V/V}{\Delta t} \cdot \qquad (14\text{-}24)$$

When the coefficient β is measured for hydrogen at 0°C, its value is again found to be 0.00366 per degree centigrade. The coefficient of volume change at constant pressure of other gases is close to this value, provided that the pressure is not too high and the gas is far from the temperature at which it becomes liquid.

14-7 Absolute Scale of Temperature

The fact that the constants β' and β have the same numerical value for hydrogen, and that these constants are approximately the same for other gases, has led to the introduction of a scale of temperature known as the *absolute gas scale of temperature.* From Equations (14-23) and (14-24) we see that, if hydrogen is maintained at constant volume, its pressure should be equal to zero at a temperature of $-273.2°C$. Similarly, if hydrogen is kept at constant pressure, its volume should be zero at a temperature of $-273.2°C$, provided that it remains a gas.

Let us rewrite Equation (14-23) in terms of the initial pressure P_i, the final pressure P_f, the initial temperature t_i, and the final temperature t_f. We find

$$\beta' = \frac{(P_f - P_i)}{P_i(t_f - t_i)}\,.$$

If we take the initial temperature as

$$t_i = 0°C,$$

and set

$$\beta' = 1/273.2 \text{ per degree centigrade,}$$

we find, after some algebraic manipulation,

$$\frac{P_f}{P_i} = \frac{t_f + 273.2}{273.2}\,.$$

Let us define the absolute centigrade gas temperature T as the centigrade temperature plus 273.2, or

$$T = t + 273.2.$$

For the case chosen the initial absolute temperature is $T_i = 273.2$. Thus we have

$$\frac{P_f}{P_i} = \frac{T_f}{T_i},$$

or

$$P = KT, \tag{14-25}$$

where K is some constant for a particular quantity of gas maintained at a constant volume. From Equation (14-24) we may find, by a similar derivation, that

$$V = K'T. \tag{14-26}$$

Equation (14-26) is called *Gay-Lussac's law.*

From the basic conception of the absolute gas scale of temperature, we see that negative numbers for the absolute temperature would lead to such absurd results as the existence of negative pressures and negative volumes for a gas. Hence the temperature of a body expressed on the absolute scale must always be a positive number. It is not possible at this point to say whether there is a physically achievable absolute zero of temperature. All substances become liquids at temperatures above the absolute zero. The lowest temperature measured with a gas thermometer is about −272°C, or 1° abs, using helium at low pressure.

It is apparent from the preceding discussion that what is needed is a temperature scale which is independent of the properties of a particular substance. There is such a scale, known as the *absolute thermodynamic scale,* or *Kelvin scale of temperature,* which will be discussed in Chapter 19. There we shall see that temperatures on the thermodynamic scale are in agreement with temperatures on the absolute gas scale for a perfect gas, and are very close to temperatures on the absolute gas scale achieved with a constant-volume hydrogen thermometer.

Although the temperature 273.2° abs is sufficiently accurate for our purposes, it should be noted that the average value of the best experimental determinations of the ice point is

$$T = 273.165^\circ \text{ abs}.$$

The lowest temperature which has been obtained experimentally is 18×10^{-6} ° abs. This has been done by a process involving the magnetization and demagnetization of certain magnetic materials which were previously cooled to a temperature of about 1° abs.

The absolute temperature can also be expressed in terms of the Fahrenheit degree by means of the equation

$$T_F = \tfrac{9}{5}T, \tag{14-27}$$

where T_F is the absolute temperature in Fahrenheit degrees, and T is the absolute temperature in centigrade degrees. When absolute temperature is measured in Fahrenheit degrees, the temperature scale is called the *Rankine* scale of temperature. This scale is used in engineering in the United States and in Great Britain.

14-8 Thermal Stresses

If a rod is heated while its ends are confined, the rod is thereby put into a condition of internal compressive stress. If the rod is cooled when its ends are confined, it assumes a condition of internal tensile stress. To calculate the stress within the rod, we assume that the rod has acquired its final configuration in a virtual two-step process, in which the first step is the change in temperature of the free rod, while the second step is the application of forces to the ends of the rod to return it to its initial length.

When the temperature of a rod of length L, made of a material whose coefficient of linear expansion is α, is altered by an amount Δt, the change in the length of the rod is given by Equation (14-16) as

$$\Delta L = \alpha L \, \Delta t.$$

The fractional change in length of the rod is given by

$$\frac{\Delta L}{L} = \alpha \, \Delta t.$$

Let us suppose, for definiteness, that the temperature of the rod has increased, and that the rod has increased in length. We may return the rod to its original dimensions by applying a compressive stress sufficient to produce a decrease in the length of the rod equal to ΔL, or to produce a strain in the rod equal to $\Delta L/L$. From the definition of Young's modulus of elasticity, we have

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{\text{stress}}{\alpha \, \Delta t} \cdot$$

Thus the stress produced in a rod whose modulus of elasticity is Y will be given by

$$\text{Stress} = \alpha Y \, \Delta t, \tag{14-28}$$

when the rod is confined so that it cannot expand or contract when it is subjected to the temperature change Δt. The state of thermal stress in a rod is determined by its coefficient of linear expansion, Young's modulus, and the temperature change to which the rod has been subjected. The force which must be applied to the ends of the rod to produce this stress is determined from the definition of stress as the force per unit area of the rod, as given in Equation (13-1).

Thermal stresses are widely encountered in practical engineering problems. Thus the steel rails of a railroad are commonly provided with expansion joints, but it is possible to lay continuous steel rail to any convenient length, provided that the rails are securely anchored to a roadbed with spikes and crossties capable of exerting a force sufficient to produce the stress given by Equation (14-28). It is not adequate to exert the force at the ends of the rails, for the rails would fail by buckling. The continuous rail must be spiked to the roadbed at suitable intervals. In laying concrete road or curbing, it is common practice to provide expansion joints because of the greater expense and difficulty of securing the road or curb to the ground. Similarly, it is difficult to provide large, long buildings with sufficient restraint, and expansion joints must be provided to keep cracks from forming in brick walls. Steel tires are fastened to the cast-iron wheels of a railroad car by a process of heating the steel rim and allowing it to cool and contract onto the wheel. The stress in the tire exerts forces on a short element of length of the tire which have a normal component, holding the tire to the wheel by frictional forces between the tire and the wheel, as shown in Figure 14-11. Such a tight fit between two members is called a *shrink fit.*

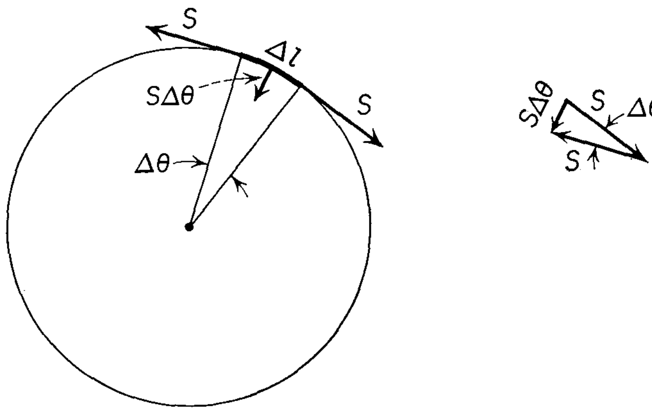

Fig. 14-11 The radial force exerted by a circular hoop under tension S upon a section of length Δl is given by $S\Delta\theta$, where $\Delta\theta$ is the angle subtended by the section Δl at the center of the circle.

When a solid is heated and subsequently cooled, as in heat treatment and quenching, or in a welded member, internal thermal stresses are often generated through the irregular cooling of the member. When hot, a solid body may easily be deformed without development of internal strains, but when the solid has cooled sufficiently, any further deformation is accompanied by the generation of internal stresses. Thus an irregularly shaped forging being quenched after heat treatment is cooled first where it is thinnest. As the thicker sections subsequently cool and contract, they are restrained by the cooler sections of the forging, and a state of internal stress is produced which may be greater than the rupture strength of the material of the forging, so that cracks develop. A drinking glass into which boiling water is poured will often crack from the thermal stresses developed at the boundary between the hot and cold portions of the glass.

14-9 The Meaning of Temperature

In the discussion of the concept of temperature in Sections 14-1 to 14-3, we have defined the meaning of temperature through the operation of measuring temperature with the aid of thermometric properties of macroscopic systems. Thus the measurement of temperature requires that matter be present in bulk. The concept of temperature is a macroscopic one. It is meaningless to ask the question, "What is the temperature of a proton moving at a speed of 100 mi/hr?"

The definition of temperature was also based upon the concept of equilibrium; that is, it is assumed that any changes in the temperature of the system are taking place sufficiently slowly so that the state of the system could be considered constant during the time of measurement. If the state of the system changes rapidly, so that it does not have sufficient time to come to equilibrium, the meaning of temperature is somewhat ambiguous. This is the case in a flame, or in the exhaust gases of an engine, where the processes of combustion take place with great rapidity, and the gases do not have time to come to equilibrium. In such cases it is often found that different methods of measuring temperature yield different results, for the measured temperature is a function of the method of measurement as well as the condition of the object whose temperature we wish to determine, and the number assigned as the temperature has meaning only in comparative terms.

Problems

14-1. At what centigrade temperature will the reading of a Fahrenheit thermometer be numerically equal to twice the reading of a centigrade thermometer?

14-2. How long must a steel rod be in order that its length will increase by 0.02 in. as a result of a temperature change of 10°C?

14-3. The distance between two markers is measured with a steel tape at 25°C. The reading of the tape is 80 ft. If the calibration of the tape is correct at 0°C, determine the distance between the markers.

14-4. A copper ring has an inside diameter of 4.98 cm at 20°C. To what temperature must it be heated so that it will just fit on a shaft 5.00 cm in diameter?

14-5. What will be the stress in the ring of Problem 14-4 after it has shrunk in place onto the shaft? Assume that the dimensions of the shaft remain constant.

14-6. A glass flask having a volume of 1 liter at 20°C is filled with mercury at that temperature. What volume of mercury will run over the lip of the flask when the temperature is raised to 100°C?

14-7. The pressure of a volume of gas at 27°C is 546 mm of mercury. What will be its pressure, expressed in millimeters of mercury, if the temperature is increased to 28°C, the volume being kept constant?

14-8. A constant-pressure air thermometer contains a mass of air whose volume is 300 cm^3 at 0°C. What will be its volume at 50°C?

14-9. The following data were taken in an experiment with a constant-volume air thermometer, such as that shown in Figure 14-10. Barometric pressure 754.0 mm; height of column C 48.4 cm and height of column D 44.7 cm at the ice point; height of column C 48.4 cm and height of column D 71.0 cm at the steam point; height of column C 48.4 cm and height of column D 62.3 cm when the bulb is surrounded with warm water. Determine (a) the coefficient of pressure change of the air and (b) the temperature of the warm water.

14-10. A clock regulated by a seconds pendulum made of brass has a correct period of 2 sec when the temperature is 70°F. Determine the gain or loss, in seconds per day, when the temperature rises to 97°F. Assume it to be a simple pendulum.

14-11. Derive the equation

$$\beta = 3\alpha$$

relating the coefficient of volume expansion β of an isotropic solid to its coefficient of linear expansion α.

14-12. A steel rod 1 m long and 0.5 cm in diameter is clamped between two fixed supports at its ends. The temperature of the rod is raised 30°C. Determine (a) the stress in the rod and (b) the force exerted by each support. Young's modulus = 20×10^{11} dynes/cm^2.

14-13. A steel bomb is filled with water at 10°C. If the system is heated to 100°C, determine the increase in pressure of the water (a) neglecting the thermal expansion of the steel and (b) taking into account the thermal expansion of the steel. Neglect the change in dimensions of the steel bomb due to the tension it experiences. Take the bulk modulus of water as 2×10^4 atm^{-1}.

14-14. Derive a formula for correcting the reading of a mercury barometer calibrated at 20°C when the barometer is read at a temperature of t°C. Neglect the expansion of the scale attached to the barometer. Express your result in terms of the correction Δh, the reading of the barometer h, the coefficient of volume expansion β of mercury, and the temperature. Does the expansion of the glass container affect your result?

14-15. A block of aluminum is immersed in water at a temperature of 20°C, and the buoyant force on the aluminum is observed to be 10 lb. What will be the buoyant force on the block of aluminum at a temperature of 4°C?

14-16. A steel tire 2 in. in width whose inner diameter is 0.999 ft and whose outer diameter is 1.05 ft is to be heated so that it may be placed on a cast-iron wheel 1.000 ft in diameter. (a) Assuming that the cast-iron wheel is perfectly rigid, find the tensile stress in the tire when it has cooled. (b) If the coefficient of friction between the tire and the wheel is 0.2, find the force which must be applied to the tire to pry it from the wheel. Take Young's modulus for steel as 29×10^6 lb/in.^{2}.

14-17. A steel rod 1 ft long is welded at one end to a copper rod of the same diameter and length, and the two bars are mounted between rigid supports. Find the stress in each bar when their temperature is increased by 50°C. Young's modulus for steel is 30×10^6 lb/in.^{2}; for copper: 18×10^6 lb/in.^{2}.

14-18. A steel cable 0.5 in.^{2} in cross-sectional area is tightened to a tension of 20,000 lb/in.^{2} when the cable is at 0°C. What is the tension in the cable when the temperature is increased to 20°C? Young's modulus is 30×10^6 lb/in.^{2}.

14-19. Find the coefficient of volume expansion of water at (a) 40°C and (b) 70°C.