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A Statistical Approach to IBNR Reserves

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Abstract

This paper develops a three dimensional statistical approach to the estimation of the mean and the standard deviation of pure incurred but not reported (IBNR) reserves. This means that the time of occurrence, the reporting lag, and the claim severity are separately modeled. It is assumed that, beyond any fixed time t, the claim number development process is Poisson and that the severity of loss depends on the length of the reporting lag. Two key assumptions are made to simplify the estimation of model parameters: for a given reporting lag, (i) the conditional mean of the claim size is a linear function of the reporting lag, and (ii) the conditional coefficient of variation of the severity is constant.

Key words and phrases: *stochastic loss development, reporting lag, pure IBNR, conditional distributions, loss reserves*

1 Introduction

The development of losses over time is a key problem for both pricing and loss reserving actuaries. Commensurate with the importance of the problem, there is a large body of actuarial literature (primarily property/casualty, but also health insurance) devoted to loss development.

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In a general textbook on loss reserve estimation, Salzmann (1964) details eight general methods of estimating loss reserves. Each method generally involves relatively simple projections of future loss development from aggregate triangulations of historical data. The methods described in Salzmann are purely deterministic and do not have an explicit mathematical model as an underlying framework. In the 30 years since the Salzmann text was published, these methods have been refined by actuaries and are still widely used today. A basic discussion and documentation of such methods is found in Wiser (1990).

For many years, however, actuaries have recognized two needs of modeling the loss development process: (i) a need to facilitate and improve the estimation process with the application of stochastic models, and (ii) a need to measure probable variations in future loss development. McClenahan (1975), for example, constructs a deterministic model of paid loss development for "analysis of the effects upon reserve adequacy of changes in various exogenous variables and in the testing of the established reserves on a prospective basis." Stanard (1985) simulates loss development triangles under a hypothetical stochastic frequency/severity model to "measure the expected value and variance of prediction errors of four simple methods of estimating loss reserves."

Guiahi (1986) develops a model for IBNR estimation as a stochastic process using the number of claims, severity of claims, and reporting lag to develop the mean and variance of IBNR reserves. In his paper, however, Guiahi assumes that reporting lag and loss severity are independent. This assumption, while convenient, is highly unlikely to be valid for most sets of insurance loss data. It is also likely to produce inadequate IBNR estimates if loss severity increases significantly with reporting lag. Pinto and Gogol (1987) analyze excess loss development by layer using Pareto distributions fitted to casualty loss distributions. Wright (1992) provides an extensive and highly detailed treatment of estimating future paid losses from separate development triangles of loss counts and loss amounts using generalized linear models. Wright deals with many of the issues addressed in this paper in a similar and detailed manner. I recommend Wright's paper to the interested reader.

It should not be inferred from this short list of papers that there is a paucity of literature on the subject. For example, van Eeghen (1981) presents a comprehensive review of the earlier literature on loss reserving. Taylor (1986) provides a detailed description of the component parts of loss development models. More recently, in the *1994 Spring CAS Forum*, for example, there are ten papers devoted to the measurement of variability in loss reserves. There is, however, a need for practical models that readily can be used by practicing actuaries.

For some types of insurance, loss development is sufficiently short-tailed or stable enough to allow relatively simple projections from paid or incurred loss triangles. The development patterns for some coverages, however, are long-tailed and unstable over time. For these coverages, traditional aggregate triangulations or frequency/severity studies may not provide development data of sufficient credibility to reveal the true nature and magnitude of underlying development patterns.

Loss reserves include:

- Case estimates on reported losses;
- Reserves for additional development on reported losses; and
- Amounts carried to reflect liability for losses incurred but not yet reported (IBNR).

This paper addresses the third component, IBNR, of the total loss reserve. The view of IBNR taken here is that of Bornhuetter-Ferguson[1] type methods, which postulate that IBNR is independent of prior loss activity and may be expressed as a function of expected losses and time.

The expected losses are a function of expected loss counts and expected severity of loss. Thus, it makes sense to look at the development of both frequency and severity over time. The key variables in the emergence of reported loss counts are the occurrence date and the report date, each represented on a time line with the beginning of the accident year set equal to time zero and all subsequent dates represented as the time elapsed from the beginning of the accident year to the respective date. The time, Z, elapsed between the occurrence date and report date will be referred to throughout this paper as the continuous reporting lag.

The first task is to identify a reasonable representation of the underlying severity of loss distribution for the losses in general. [See Hogg and Klugman (1984) for a thorough discussion of choosing loss distributions.] The size of loss, however, may be a function of the reporting lag Z defined above. For each value of Z there may be separate loss distributions referred to as *conditional distributions* to distinguish them from the marginal loss distribution.

The final component of IBNR development is the manner in which claim occurrences arise over an exposure period. The most common assumption is that occurrence dates are uniformly distributed throughout the accident year; this assumption is adopted in this paper. It should

[1]Bornhuetter-Ferguson type methods are loss reserving methods that are based on the work of Bornhuetter and Ferguson (1972).

be noted, however, that the results of this paper easily can be extended to include a more general claim occurrence distribution.

This three dimensional approach to modeling the loss development process (occurrence of loss, loss count development, and loss severity) is the framework for the model developed in this paper. Note that the primary data to be used in the model development need not be limited to loss triangulations. When such triangulations are highly volatile and involve a limited number of claims, a compilation of each individual claim by accident date and reporting date may facilitate the actual modeling process.

2 The Mathematical Model

Following Guiahi (1986), IBNR will be modeled as a three dimensional stochastic process based on the number of claims, severity of claims, and reporting lag. The assumption of independence between reporting lag and loss severity, however, will be discarded and replaced with a model describing the dependence structure between these two variables using conditional mean severities.

The time interval $(0,1)$ is assigned to the accident year. There are three basic random variables: occurrence date X, reporting date R, and size of loss S. A fourth random variable, denoted by Z, is the continuous reporting lag and is defined as $Z = R - X$. Table 1 summarizes the notation that will be used in connection with the variables X, Z, and S.

Table 1
Definition of Random Variables

Variable	Space	PDF	Mean	Variance
X = Time of Occurrence	$(0,1)$	$u(\cdot)$	$1/2$	$1/12$
Z = Continuous Lag	$(0,\infty)$	$g(\cdot)$	m	ζ^2
S = Loss Size	$(0,\infty)$	$f(\cdot)$	μ	σ^2

Note: Space = Sample Space; and PDF = Probability Density Function.

In the most general model, the three random variables (S,Z,X) would be interdependent. It does seem likely, however, that the length of the reporting lag should not depend on the occurrence date. That is, Z and X should be independent in the statistical sense. It is not clear, however, that S and Z or S and X need be independent pairs. If there is a significant underlying loss trend by accident date, then S

and X should be dependent. It is well-known that loss size tends to increase with settlement lag, and it may be true that loss size also tends to increase with reporting lag.[2] If so, then S and Z also will be dependent. Put another way, trend may have a two dimensional effect on loss size. The first dimension is the effect of trend on loss size by accident date, and the second is trend over time elapsed since accident date. In some cases, however, underlying trend in either dimension may be either nonexistent or extremely difficult to establish or quantify. This may be true, for example, in some liability coverages where large losses play a significant role.

In this paper, the assumption is made that there is no trend on loss size by accident date within an accident year. For any given accident year, loss size and accident date are assumed to be independent. Moreover, as has been previously stated, the uniform distribution of occurrences assumption often will be appropriate. It follows that S is dependent only upon the continuous lag Z.

Let $\mu_k(z)$ be the kth conditional mean of $[S \mid Z = z]$, i.e.,

$$\mu_k(z) = E[S^k \mid Z = z], \quad \text{for } k = 1, 2, \ldots, \tag{1}$$

and let the coefficient of variation of $[S \mid Z = z]$ be denoted by $\varphi(z)$. Assuming that $g(z) \geq 0$, it follows that

$$\mu(z) = \mu_1(z) \tag{2}$$

$$\sigma^2(z) = \mu_2(z) - (\mu(z))^2 \tag{3}$$

$$\varphi(z) = \frac{\sigma(z)}{\mu(z)}. \tag{4}$$

The density $g(z)$ describes how loss counts will develop over time. Although other forms are possible, it is common to assume that ultimate claim frequency is either Poisson, negative binomial, or binomial; see Panjer and Willmot (1992, Chapter 6). In this case, however, I will assume that the ultimate number of claims is a Poisson random variable. In addition, given any fixed point in time $t \geq 0$, the number of counts reported after time t is assumed to be Poisson.

At this point, the basic assumptions used to develop the model are listed:

Assumption 1: The conditional mean $\mu(z)$ is a linear function of z. In particular,

$$\mu(z) = \kappa(z - m) + \mu. \tag{5}$$

[2] This relationship between loss size and settlement lag dates back at least as far as Salzmann (1964, pages 5–6).

Note that a linear function is used for convenience, but it is used only after it became apparent that more complex models would not provide a superior fit.

Assumption 2: The coefficient of variation of $[S \mid Z = z]$ is independent of z, and hence is a constant c, i.e.,

$$\varphi(z) = c. \tag{6}$$

Assumption 3: The continuous reporting lag Z has known probability density function $g(z)$ and has mean m and variance ζ.

Assumption 4: The number of claims and time of reporting are independent.

Assumption 5: Claim occurrences within the accident year are uniformly distributed on $(0, 1)$.

Assumption 6: Loss size S and lag Z are independent of the time of occurrence X.

Assumption 7: Given a fixed time t, the number of counts reported after time t, $N(t)$, has a Poisson distribution with mean and variance denoted by $\lambda(t)$. Clearly, the aggregate ultimate loss count is Poisson with mean and variance given by $\lambda(0)$.

One consequence of Assumptions 1 and 2 is the following:

$$E[S^2] = \mu^2 + \sigma^2 = (1 + c^2)(\kappa^2\zeta^2 + m^2). \tag{7}$$

Equation (7) can be established easily as follows:

$$\begin{aligned}
E[S^2] &= E[E[S^2 \mid Z]] \quad \text{from Bowers et al. (1986, eqn. (2.2.10))} \\
&= E[\mu_2(Z)] \\
&= E[(\mu(Z))^2 + \sigma^2(Z)] \\
&= (1 + c^2)E[(\mu(Z))^2] \quad \text{from Assumption 2} \\
&= (1 + c^2)E[(\kappa(z - m) + m)^2] \quad \text{from Assumption 1} \\
&= (1 + c^2)(\kappa^2\zeta^2 + m^2).
\end{aligned}$$

3 The Main Results

Now fix a time $t > 0$, and let $S_i(t)$ denote the size of the ith claim reported after time t. If IBNR(t) denotes total loss dollars reported after time t, then

$$\text{IBNR}(t) = \sum_{i=1}^{N(t)} S_i(t). \tag{8}$$

From Bowers et al. (1986, Chapter 11) or Panjer and Willmot (1992, Chapter 6), IBNR(t) is a compound Poisson random variable under the assumption that the number of claims and the loss sizes are mutually independent random variables. The mean and variance of IBNR(t) are thus given by

$$E[\text{IBNR}(t)] = \lambda(t)E[S_i(t)] \tag{9}$$
$$\text{Var}[\text{IBNR}(t)] = \lambda(t)E[S_i^2(t)]. \tag{10}$$

The problem of estimating the mean and variance of the IBNR reserve at any given time t thus is reduced to finding the first two moments of $S_i(t)$. There are two cases to consider: $t < 1$ and $t \geq 1$. The first case is needed for incomplete accident years and seems rarely to be addressed. Unfortunately, it is this case that usually will generate the largest expected values of IBNR and, therefore, cannot be ignored.

Case I ($t < 1$): If a loss that occurs at time X is reported after time t, it follows that $X + Z > t$. Thus $S_i(t)$ and $[S \mid X + Z > t]$ are equivalent random variables, i.e., they have the same probability distribution. In other words,

$$\begin{aligned} a_1(t) &= E[S_i(t)] = E[S \mid X + Z > t] \\ a_2(t) &= E[(S_i(t))^2] = E[S^2 \mid X + Z > t]. \end{aligned}$$

Now as X and S are independent, then

$$a_1(t) = \frac{\int_{x=0}^{t} \int_{z=t-x}^{\infty} \mu(z)g(z)dz\, u(x)dx}{\int_{x=0}^{t} \int_{z=t-x}^{\infty} g(z)dz\, u(x)dx} \tag{11}$$

$$a_2(t) = (1 + c^2)\frac{\int_{x=0}^{t} \int_{z=t-x}^{\infty} \mu(z)g(z)dz\, u(x)dx}{\int_{x=0}^{t} \int_{z=t-x}^{\infty} g(z)dz\, u(x)dx}. \tag{12}$$

Next, let $I_j(t)$ denote the indicator random variable for the reporting of the jth claim after time t, i.e.,

$$I_j(t) = \begin{cases} 1 & \text{if the } j\text{th claim is reported after time } t; \\ 0 & \text{otherwise.} \end{cases}$$

Clearly, $N(t) = \sum_{j=1}^{N(0)} I_j(t)$, and

$$\Pr[I_j(t) = 1] = \int_{x=0}^{t} \int_{z=t-x}^{\infty} g(z)dz\, u(x)dx.$$

Hence, from Assumption 4, the $I_j(t)$s are independent, so

$$\begin{aligned}\lambda(t) &= E[N(0)]E[I_j(t)] \\ &= \lambda(0)\int_{x=0}^{t}\int_{z=t-x}^{\infty} g(z)dz\,u(x)dx. \qquad (13)\end{aligned}$$

Case II ($t \geq 1$): Here the maximum value that x can take is one so,

$$a_1(t) = \frac{\int_{x=0}^{1}\int_{z=t-x}^{\infty}\mu(z)g(z)dz\,u(x)dx}{\int_{x=0}^{1}\int_{z=t-x}^{\infty}g(z)dz\,u(x)dx} \qquad (14)$$

$$a_2(t) = (1+c^2)\frac{\int_{x=0}^{1}\int_{z=t-x}^{\infty}(\mu(z))^2 g(z)dz\,u(x)dx}{\int_{x=0}^{1}\int_{z=t-x}^{\infty}g(z)dz\,u(x)dx}. \qquad (15)$$

Again,

$$\lambda(t) = \lambda(0)\int_{x=0}^{1}\int_{z=t-x}^{\infty} g(z)dz\,u(x)dx. \qquad (16)$$

Note that in these equations, $\mu(z)$ is given by equation (5) and $u(x) = 1$ for $0 \leq x \leq 1$. Thus in order to estimate the mean and variance of IBNR at any time t for a given accident year, one needs to know the density $g(z)$, overall mean severity μ, conditional mean severities $\mu(z)$, and conditional coefficient of variation c.

If there is no trend across accident years and the conditional means and severities apply to all accident years, then the aggregate expected value of IBNR simply will be the sum of the IBNR, as calculated in this paper, for each of the accident years. If there is trend across accident years, but all other aspects of the model (e.g., parameters for $g(z)$ and c) are assumed to hold across accident years, one only need adjust the value of μ for each accident year, calculate the model's expected IBNR, and sum the results over the accident years. Although the model assumes no trend by accident date within the accident year, the effect of any trend on the calculated IBNR results is probably minimal and safely can be ignored.

Finally, it may be necessary to have parameters that vary by accident year to reflect changes in the reporting lag distribution and/or the conditional coefficient of variation to calculate expected IBNR in each accident year and sum the results.

In all cases, the expected value of total IBNR is the sum of the expected values of the individual accident year expected values. Moreover, if one can assume independence of losses by accident year, the variance of the aggregate IBNR will be the sum of the individual accident year IBNR variances.

4 An Application of the Model

An example using an actual data set will illustrate practical use of this model. The data are loss counts and incurred losses by accident year and report year for a group errors and omissions (E&O) program. This program is selected because it is long-tailed and extremely volatile. The basic data in Table 2 and Table 3 show the exposures, reported loss counts, and reported losses for report years 1990-1993 on accident years 1980-1993.

Table 2
Loss Counts by Report Year

		Loss Counts				
AY	Exposed	1990	1991	1992	1993	1990-93
1980	2,599.9	0	0	0	0	0
1981	2,473.3	0	0	0	0	0
1982	2,597.6	1	0	0	0	1
1983	2,646.7	0	0	0	0	0
1984	2,537.0	0	0	0	0	0
1985	2,673.4	0	0	0	0	0
1986	2,911.6	0	0	1	0	1
1987	3,055.2	1	0	1	0	2
1988	2,810.8	2	4	2	0	8
1989	2,887.2	25	6	3	1	35
1990	2,907.6	47	46	9	2	104
1991	2,922.6		64	40	3	107
1992	3,018.1			50	26	76
1993	3,034.2				41	41
Totals	39,075.2	76	120	106	73	375

Note: AY = Accident Year.

The first step is to determine the ultimate frequency and the density function $g(z)$ from the loss counts and exposures. There are many possible choices of the form of the density, and several are tried. Because there are some claims with extremely long reporting lags, the two parameter Pareto is selected, truncated somewhat arbitrarily at 15 years. Thus for $0 \leq z \leq 15$,

$$g(z) = \frac{\alpha\beta^{\alpha}(\beta + z)^{\alpha-1}}{1 - (\frac{\beta}{\beta+15})^{\alpha}} \quad \text{for} \quad \alpha, \beta > 0. \tag{17}$$

Table 3

Loss Amounts (in $s) by Report Year

AY	1990	1991	1992	1993	1989-93
1980	0	0	0	0	0
1981	0	0	0	0	0
1982	5,000	0	0	0	5,000
1983	0	0	0	0	0
1984	0	0	0	0	0
1985	0	0	0	0	0
1986	0	0	500	0	500
1987	26,207	0	599	0	26,806
1988	8,453	72,329	905	0	81,687
1989	81,196	169,133	9,935	16,136	276,400
1990	282,473	151,367	98,812	20,271	552,923
1991		748,559	319,906	78,260	1,146,725
1992			245,260	122,702	367,962
1993				318,316	318,316
Totals	$405,319	$1,143,379	$677,909	$557,678	$2,776,319

Note: AY = Accident Year.

Occurrences are assumed to be uniformly distributed, with no trend in loss size. The parameters (α and β) and frequency λ are determined using discrete unweighted least squares with loss counts tabulated by discrete lag n. If E_z is the exposure that underlies the counts observed for lag n, the problem is to find the parameters λ, α, and β that minimize the sum of squares $\mathcal{L}$:

$$\min \mathcal{L}(\lambda, \alpha, \beta) = \sum_{n} (\lambda E_z p(n) - N_n)^2 \tag{18}$$

where

$$\begin{aligned} \lambda &= \lambda(0) \text{ is the ultimate annual claim frequency;} \\ n &= \text{Report year} - \text{Accident year} = \text{ the discrete lag;} \\ N_n &= \text{Number of reported loss counts for lag } n\text{;} \\ E_n &= \text{Associated exposure; and} \\ p(n) &= \text{Portion of ultimate loss counts reported for lag } n. \end{aligned}$$

Note that $p(n)$ is given by the equation:

$$p(n) = \begin{cases} \int_{x=0}^{1} \int_{z=n-x}^{n+1-x} g(z)dz\, u(x)dx & \text{if } n \neq 0; \\ \int_{x=0}^{1} \int_{z=0}^{1-x} g(z)dz\, u(x)dx & \text{if } n = 0; \end{cases}$$

The least squares estimated values are

$$\hat{\lambda} = 0.0315, \quad \hat{\alpha} = 9.4274, \quad \text{and} \quad \hat{\beta} = 4.8475. \tag{19}$$

From equation (17), the estimated density $\hat{g}$ gives mean and variance of Z as

$$\hat{m} = 0.5752 \text{ and } \hat{\zeta} = 0.4195. \tag{20}$$

The next step is to model severity of loss S, both globally and conditionally. A study of all reported mature losses results in the following selected global mean and standard deviation:

$$\hat{\mu} = 8,807 \text{ and } \hat{\sigma} = 28,637. \tag{21}$$

The severity by discrete lag is volatile due to a paucity of data. Most counts are at discrete lag zero, however, so that value of κ is selected for which the observed discrete lag zero severity would be reproduced, i.e., so that

$$7,894 = \frac{\int_{x=0}^{1} \int_{z=0}^{1-x} (\hat{\kappa}(z - \hat{m}) + \hat{\mu}) g(z)dz\, u(x)dx}{\int_{x=0}^{1} \int_{z=0}^{1-x} g(z)dz\, u(x)dx}.$$

This gives

$$\hat{\kappa} = 2,707. \tag{22}$$

Once $\hat{\kappa}$ is known, the conditional coefficient of variation, c, is calculated using equation (7). The model is now complete, and all of the quantities of interest mentioned in this paper can be determined. For the sake of brevity, however, only the development of the year end 1993 IBNR expected value and standard deviation is shown as Table 4.

Table 4
Year End 1993 IBNR Reserve Estimates

AY	Mean	Std. Dev.
1993	\$429,898	\$222,206
1992	107,018	126,463
1991	31,453	76,055
1990	10,997	48,984
1989	4,303	32,941
1988	1,808	22,730
1987	912	17,054
1986	428	12,255
1985	202	8,791
1984	102	6,481
1983	58	5,027
1982	30	3,755
1981	15	2,691
1980	7	1,887
1980-1993	\$587,231	\$275,253

Note: AY = Accident Year, and Std. Dev = Standard Deviation.

The importance of the value of κ, which determines the slope of loss sizes by lag, in the reserve estimates is illustrated by two alternative calculations. If $\kappa = 0$, the total IBNR reserve decreases from \$587,231 to \$480,489. At the other extreme, $\mu(0) = 0$, and $\kappa = 15313.59$ increases the reserve to \$1,084,335.

The actual fit of the model to the observed data, especially in the case of loss size, is not of great importance, given the strong variation in the observations. Moreover, the value of this example lies not in the discovery of the true underlying forces operating on the development of losses but in the illustration of the model concepts when applied to the data. Table 5 does, however, give an indication of goodness of fit.

5 Some Closing Comments

The central loss development model described in this paper is designed to provide a logically consistent statistical approach to pure

Table 5
Goodness of Fit

Discrete Lag	Total Exposed	Actual County	Model Counts	Actual Dollars	Model Dollars
0	11,882.5	202	202.36	$1,594,608	$1,597,632
1	11,735.5	137	136.01	675,171	1,246,532
2	11,528.2	20	23.75	354,658	284,644
3	11,660.8	10	5.51	128,742	81,421
4	11,664.8	3	1.52	17,041	26,678
5	11,451.0	1	0.48	599	9,749
6	11,177.2	1	0.17	500	3,920
7	10,768.7	0	0.06	0	1,548
8	10,454.7	1	0.03	5,000	856
9	10,254.6	0	0.01	0	313
10	10,317.5	0	0.01	0	340
11	7,670.8	0	0	0	0
12	5,073.2	0	0	0	0
13	2,599.9	0	0	0	0
ALL		375	369.91	$2,776,319	$3,253,633

IBNR estimation. There are strong advantages to this approach, however, beyond logical consistency:

- The model allows for reserve valuation at any time $t > 0$.
- The model can be used to estimate unreported future losses that can be checked against actual future emergence of such losses within a statistical framework. This is because the model not only forecasts expected values but also the expected variation in such losses.
- The model allows for the valuation of incomplete accident years. Incomplete accident years pose a serious problem for traditional claim run-off triangle methods.

There are still many basic unanswered questions about this model, some of which lie primarily in the apparent arbitrariness of the assumptions that have been made. For example,

- Do loss sizes increase by reporting lag? If so, is a linear model appropriate? With modern hardware and software, the simplification of a linear model may not be necessary.

- Is it realistic to assume that the developments of loss counts and loss sizes over time arising from a fixed accident period are mutually independent?

- How robust is the model? The whole question of model parameter errors, which are critical pricing and reserving considerations for estimating needed security margins, is ignored. I believe that the model is sufficiently robust to be used when loss development is highly volatile and the process variance[3] is expected to be large enough to play a significant part in estimation of the reliability of the expected value estimates of IBNR. The example given in Section 4 illustrates this point.

It may be tempting to try to apply the concepts in this paper to the claims-made environment, which is especially suited to the development of reported losses, for the estimation of claims-made pure premium components. This temptation may be particularly strong because Marker and Mohl (1980) show that an occurrence basis pure premium can be decomposed into a sum of claims-made components, with adjustment for differences in reporting patterns that arise from the two coverage types. Moreover, McClenahan (1988) includes the cost of extended reporting tails as a component of the occurrence basis pure premium. This temptation, however, must be dampened severely by two major considerations. First, claims-made coverages have arisen largely out of concern for strong and unpredictable loss trends. Second, the model is based on accident years, while the claims-made environment is defined in terms of policy years. For these reasons, I somewhat reluctantly have overcome this temptation to produce nice formulations

[3]As used by Herzog (1985), the term *process variance* refers to "the variance of the frequency, severity, or aggregate claim amount of an individual combination of risk characteristics," and is, therefore, a conditional variance. The context in Herzog is one of a population consisting of a collection of different individual combinations of risk characteristics, so that the total variance is the sum of (i) the expected value of process variance, and (ii) the variance of the hypothetical means. Here, the context used for the term *process variance* is somewhat different in that it refers to the total variance arising from the model, but is also conditional upon the parameters employed in the model. In this sense, the term *process variance* is employed analogously to the term *process risk*. Variance arising from error in the selection of parameters (which is not estimated in this paper) is analogous to parameter risk. (See McClenahan (1990, p. 61) for definitions of process risk and parameter risk.)

from the model using assumptions that will be unrealistic in most real world situations.

I do suggest, however, that if one can devise a more general model that incorporates trend, shifts in reporting patterns, and distributions of policy inception dates, then Monte Carlo simulations may be used to estimate both the expected values and the process variances needed to determine IBNR reserves for occurrence basis coverages and claims-made pure premiums without having to deal with extremely complex mathematical formulae. With the powerful desktop computers and commercial software readily available today, I believe firmly that practical results could be obtained at minimal cost.

Specific technical questions as to forms of distributions or functional ways in which loss sizes vary are wide open. This paper makes no attempt to answer such questions. Rather, this paper is designed to build a practical framework or approach for the practicing actuary to develop his or her own model to produce IBNR estimates that can be tested scientifically from emerging experience.

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