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Learning to Teach Prospective Elementary Teachers: A Narrative Inquiry of a Newer Mathematics Content Instructor

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LEARNING TO TEACH PROSPECTIVE ELEMENTARY TEACHERS: A NARRATIVE INQUIRY OF A NEWER MATHEMATICS CONTENT INSTRUCTOR

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Learning to teach mathematics content courses designed for prospective elementary teachers (PTs) is challenging. Mathematics content instructors must be prepared to not only teach mathematics content, but engage PTs in understanding children's mathematical thinking strategies, learning trajectories, and misconceptions (Carpenter et al., 1996; Carpenter & Moser, 1982; I et al., 2020). When considering how instructors might best be prepared and supported, however, one must recognize that there is significant variation in the backgrounds and expertise of mathematics content instructors (Masingila et al., 2012; Yow et al., 2016) and little is known about the preparation, knowledge, and experiences of mathematics content instructors (Even, 2008; Goos, 2009; Masingila et al., 2012; Oesterle, 2011; Zaslavsky & Leikin, 2004). Furthermore, most instructors who are newer to teaching PTs do not feel prepared and additionally report an absence of training, resources, and support at their institutions (Goodwin et al., 2014; Masingila et al., 2012; Yow et al., 2016). Mathematics content instructors, especially those newer to teaching PTs, need preparation and support that account for their background and teaching context.

My dissertation is a narrative inquiry (Clandinin & Connelly, 2000) of one beginning instructor's experiences learning to teach mathematics for elementary PTs. For one semester, I observed all the instructor's classes and meetings with other instructors,

conducted three interviews with the instructor, collected the instructor's teaching and learning autobiographies, and collected the instructor's weekly reflection journals. The findings of my dissertation include narrative themes connecting the instructor's background, preparation, and support to their learning to teach mathematics content for PTs. These findings have numerous implications for the professional development and mentoring of future mathematics content instructors, including how newer instructors might be better prepared or supported to engage in the kinds of newer, more numerous, or more cognitively demanding tasks of teaching associated with mathematics content courses for PTs.

Dedication

To mom, who inspired me to cherish my relationships with people above and beyond everything else.

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CHAPTER 1: INTRODUCTION

Context of the Problem

To improve mathematics education, prospective elementary teachers (elementary PTs) are called to become well-prepared beginning teachers of mathematics (Bezuk et al., 2017; CBMS, 2001, 2012). These calls are grounded in the reality that teaching elementary mathematics is challenging work. First, research repeatedly suggests that teachers are underprepared to teach mathematics effectively in diverse classrooms (Kitchen, 2005; Sleeter, 2001), which some researchers argue is reflective of both a lack of preparation to teach mathematics in ways that build on children's mathematical thinking, as well as a lack of preparation to teach mathematics to an increasingly diverse student population (Howard, 1999; Turner et al., 2012; Wiggins & Follo, 1999). Research suggests that these are not easy skills to learn. They require time and supportive communities (Turner & Drake, 2016), a productive disposition towards mathematics and its teaching and learning (Hand, 2012; Jacobson & Kilpatrick, 2015; Jurow et al., 2012; Turner & Drake, 2016), and a strengths-based approach to elementary students' mathematical work (Kobett & Karp, 2020).

Second, elementary teachers are called to develop students' conceptual understanding of mathematics concepts alongside procedural fluency across mathematics domains (NCTM, 2014; NRC & MLSC, 2001). Providing students with access to opportunities to develop conceptual understanding may be difficult, however, as research suggests that elementary teachers tend to believe students should be taught as they were (Harbin & Newton, 2013; Hiebert, 2003; Maasepp & Bobis, 2014/2015; NCTM, 2014), through memorization of procedures in a drill and practice format. Moreover, research

suggests that elementary teachers' deeply engrained beliefs around mathematics and its teaching and learning do not change easily (Pajares, 1992), especially considering the complex and interrelated nature of beliefs with other affective factors, like mathematics anxiety (Beswick, 2006; Cross Francis, 2015; Philipp, 2007; Polly et al., 2013; Wilkins, 2008).

These challenges are not impossible to manage or overcome, as research shows that teachers' understanding of children's mathematical thinking, as well as their cultural funds of knowledge, supports the learning and identities of diverse groups of students (Carpenter et al., 1996; Turner & Celadon-Pattichis, 2011; Villaseñor & Kepne, 1993). Research also demonstrates that experiencing mathematics as a sense-making activity and engaging in reflection may positively influence elementary teachers' beliefs (CBMS, 2012; Hughes et al., 2019; Liljedahl, 2005; Philipp, 2007; Schoenfeld, 2015; Swars et al., 2009). Mathematics content courses have the potential to develop elementary PTs' understanding of children's mathematical thinking and expose elementary PTs to new ways of teaching and learning mathematics, among other positive support they can provide.

Preparing elementary PTs is made all the more challenging when considering the complexities surrounding the work of teaching mathematics for elementary PTs. First, mathematics content instructors face the unique challenge that elementary PTs are not just learning mathematics content but are developing *mathematics knowledge for teaching* (MKT), which additionally includes pedagogical content knowledge (Ball, Thames, & Phelps, 2008). This means that mathematics content instructors must be prepared to not only teach mathematics content, but also children's mathematical

thinking strategies, learning trajectories, and misconceptions (Carpenter et al., 1996; Carpenter & Moser, 1982; I et al., 2020). Second, mathematics content instructors are teaching a population of students who are re-learning mathematics content that they already have extensive experience with as learners but may struggle to explain (Ball 1988; Ma, 1999; Thanheiser, 2009). Moreover, elementary PTs often believe that they already know elementary mathematics (Thanheiser, 2018), which adds an additional layer of complexity to the work of teaching elementary PTs.

Difficulties in teaching elementary PTs may be exacerbated for newer mathematics content instructors, and in particular, those who have little prior teaching experience. For instance, newer mathematics content instructors may not have a wealth of knowledge of elementary PTs' common conceptions, tendencies, knowledge, and mathematical errors (Greenberg & Walsh, 2008; Masingila et al., 2012; Sztajn et al., 2006). Moreover, newer mathematics content instructors who have little prior teaching experiences may not have a wealth of pedagogical experiences and strategies to draw on to help them elicit and address elementary PTs' mathematical thinking. Having prior *elementary* teaching experiences might be especially useful for teaching mathematics content for elementary PTs. However, according to a national survey of 1,926 higher education institutions in the United States,

> Most instructors [of mathematics content courses for prospective elementary teachers] do not have elementary teaching experience and likely have not had opportunities to think deeply about the important ideas in elementary mathematics, and most institutions do not provide training and/or support for these instructors. (Masingila et al., 2012, p. 357)

My Experiences as a Mathematics Content Instructor

Considering my own experiences as a newer mathematics content instructor, I encountered numerous difficulties that I had not previously encountered in teaching other university mathematics courses. I had bachelor's and master's degrees in mathematics; a few years of experience teaching precalculus at the university level and general education mathematics courses for non-mathematics majors, but I did not have any K-12 teaching experiences to draw on like the mathematics methods instructor who taught the same cohort of elementary PTs did[1]. While I felt that my colleague was able to leverage their extensive experience teaching both school students and elementary PTs to motivate our students, I struggled to find my own way to connect elementary PTs to the mathematics content. Over time, I learned to express myself early on and often as an instructor who understands mathematics deeply and can support elementary PTs in understanding both mathematics content and elementary students' mathematical thinking.

Based on my experiences teaching mathematics content for elementary PTs, thinking deeply about elementary mathematics was important, but thinking deeply about my own teaching practices, how to appropriately elicit and address the expectations and concerns of elementary PTs, and how to assess their mathematical work ended up being far more important for managing the conflicts and tensions that arose in this environment. For instance, assessments in the mathematics content course I taught involved more written explanations and reasoning than in the College Algebra or Calculus I recitation courses I had taught in the past. As such, I needed to learn how to assess my students'

[1] Note that not all mathematics methods instructors have K-12 teaching experiences, and those that *do* have K-12 teaching experiences face their own unique challenges in teaching PTs (Nicol, 1997; Van Zoest et al., 2006).

written explanations and reasoning, and in ways that aligned with both mine and my elementary PTs' expectations for the course. Although I was new to assessing these kinds of questions, I did not think I needed to reach out to my colleagues for support in advance of assessing my students' work. Additionally, none of my colleagues brought any potential challenges surrounding assessments to my attention. It turned out that my expectations for some assessments did not align with some of my students' expectations, and thus it became crucially important for me to figure out how to manage these tensions.

The first time I taught this course, I did not manage the tensions that grew from assessments well. I reached out for support from my colleagues too late and after too much damage had been done, and as a result, the course was a miserable experience for both me and my elementary PTs. The second time I taught this same course, I enacted conflict resolution strategies I learned from members of my supervisory committee, and the conversation I had with my elementary PTs completely changed the dynamic of my classroom in overwhelmingly positive ways. For me, concerns of my own pedagogy, understanding elementary PTs, and my assessment strategies outweighed concerns of the mathematics content I was teaching them. The former concerns made the largest impact on my experiences as an instructor of elementary PTs, and possibly even the experiences of my elementary PTs as well.

Statement of the Problem

I care that mathematics content instructors are able to navigate the numerous challenges that may arise in this teaching environment and engage in positive experiences teaching elementary PTs. When considering how instructors might best be prepared and supported however, one first needs to recognize that there is significant variation in the

backgrounds and expertise of mathematics content instructors (Masingila et al., 2012; Yow et al., 2016) and little is known about the preparation, knowledge, and experiences of mathematics content instructors (Even, 2008; Goos, 2009; Masingila et al., 2012; Oesterle, 2011; Zaslavsky & Leikin, 2004). Furthermore, a majority of instructors who are newer to teaching PTs do not feel prepared and additionally report an absence of training, resources, and support at their institutions (Goodwin et al., 2014; Masingila et al., 2012; Yow et al., 2016). Hence, mathematics content instructors, especially those who have little to no experience teaching PTs, need preparation and support that takes into account their particular background and teaching context. Given how deeply my own concerns for teaching a mathematics content course for elementary PTs were rooted in my background and prior teaching experiences, I believe that attending to instructors' contexts is pivotal in providing appropriate preparation and support.

This conclusion leads me to two main questions: (1) what does the field already know about what it means to be prepared to teach or to learn to teach mathematics content courses for elementary PTs, especially as a newer instructor?; and (2) what does the field already know about supporting newer instructors of mathematics content courses for elementary PTs? I address both questions in the next chapter. Before moving on, I provide an overview of the structure of the dissertation.

Structure of the Dissertation

This dissertation is organized into seven chapters. In Chapter 1, I introduce the educational, research, and personal contexts motivating the study. In Chapter 2, I provide a few definitions and notes on the literature, as well as discuss my positioning. I then provide my review of the relevant literature and introduce the purpose statement and

research questions. In Chapter 3, I describe the research design, theoretical framing, and my reflexivity in the study. I then outline my methods, including the context of the study. In Chapter 4, I present Rowan's background and prior mathematics learning and teaching experiences. In Chapter 5, I present Rowan's preparation and support for teaching their mathematics content course. In Chapter 6, I present Rowan's challenges and successes surrounding teaching and learning to teach their mathematics content course. I then weave the findings from Chapters 4, 5, and 6 altogether, by discussing the ways in which Rowan's background, prior experiences, preparation, and support provide a means of understanding Rowan's challenges and successes. In Chapter 7, I conclude the dissertation with a summary of Rowan's experiences, and discussion of the researcher-participant relationship, implications of the study, and suggestions for future research.

CHAPTER 2: LITERATURE REVIEW AND RESEARCH QUESTIONS

In this chapter, I provide a few definitions and notes on the literature review I conducted, as well as discuss my positioning with respect to the study. I then provide my review of the relevant literature and introduce the purpose statement and research questions.

Definitions & Notes on the Literature

Before reviewing the literature, I define a few terms. First, recall that I earlier defined "PTs" to refer to prospective teachers. When it is necessary to refer to both elementary PTs and secondary mathematics PTs, I may use the term "PTs of mathematics". Second, I use the term "mathematics content instructors" to refer to any professional responsible for teaching mathematics content courses for PTs. I similarly use the term "mathematics methods instructors" to refer to any professional responsible for teaching mathematics methods courses. Such professionals may include faculty members, graduate students, adjunct professors, post-doctoral fellows, or others. For papers that use other terms to refer to mathematics content or mathematics methods instructors, or a subset of them, such as mathematics teacher educators (MTEs), I will use the terms specified in those papers. When it is necessary to refer to both mathematics content and mathematics methods instructors, I may use the term "instructors of PTs" for brevity. Third, I define a "newer mathematics content instructor" to be an individual who self-identifies as a newer or novice instructor of PTs, or who has fewer than 2 years of experience teaching PTs when the former information is unknown or unspecified in the literature.

While the focus of my literature review is on preparation, learning, and support for newer instructors of mathematics content courses for elementary PTs, the literature restricted to this specific population is incredibly scarce. In addition, some papers include both mathematics content and mathematics methods instructors, while other papers include instructors of both elementary PTs and secondary PTs. Nonetheless, studies that diverge slightly from my population of interest (or include instructors from additional populations) still offer insight into the preparation, learning, and support of newer instructors of mathematics content courses for elementary PTs. They may also serve to highlight why attending to the differing backgrounds and contexts of instructors is necessary for future studies in the field. Hence, I did my best to include relevant literature and discuss what might be gained from these papers, all while pointing out their differing contexts.

My Positioning

I am a white, cisgender, female (she/they) graduate research assistant at the University of Nebraska-Lincoln (UNL), a large mid-western university. For the last five years, I have been working towards a Ph.D. in Educational Studies through the Department of Teaching, Learning, and Teacher Education (TLTE). For the first three of those years, I had been working as a graduate research assistant (GRA) in TLTE and a graduate student instructor (GSI) for the Department of Mathematics. This was a unique position to be in, yet one that I wanted to maintain after transitioning from the Mathematics Ph.D. program at UNL following the spring 2018 semester. Doing so allowed me to keep teaching mathematics courses, and in particular, mathematics content courses for elementary PTs.

Before my transition to TLTE, I was a mathematics Ph.D. student for two years. In my first year, I taught two semesters of Calculus I recitations and it was through those teaching experiences--my first teaching experiences--that I started to reflect on issues within mathematics education. In my second year, I taught two semesters of College Algebra and was enrolled in three credits of pedagogy coursework. The pedagogy courses were required for second year Ph.D. students in the mathematics department, spread out over two semesters, and primarily focused on a condensed history of a few learning theories and their applications to teaching undergraduate mathematics. One of my assignments for the course involved observations of other GSIs' teaching, and in doing so I observed a mathematics content course for elementary PTs for the first time. During my second year, I realized that my interests were steadily shifting away from research in mathematics and more heavily towards research in mathematics education. Thus, in spring 2018, I earned my master's degree in mathematics, and began the Educational Studies Ph.D. program in Fall 2018.

Since then, I have been living in two worlds. I have been studying educational and methodological theories and conducting mathematics education research in TLTE, all while continuing to teach courses in the mathematics department, or, as has been the case for the last two years, continuing to discuss mathematics education research and teaching in the mathematics department's seminars. When I interact with colleagues in the mathematics department, I feel seen as that person who cares immensely about teaching. When I interact with colleagues in TLTE, I feel seen as a mathematics person. In spring 2019, I taught my first mathematics content course for elementary PTs. As highlighted in the introduction, this course was one of the most difficult courses I experienced teaching.

Nonetheless, I sought to improve my instruction, and taught this same course again in fall 2019 and in spring 2020. I experienced some important moments of personal growth and reflection in teaching the same course again, as well as in summers 2019 and 2020, and spring 2021, when I assisted teaching courses for practicing teachers of mathematics. To broaden my understanding of the full spectrum of mathematics content courses for elementary PTs offered at UNL, I taught a mathematics modeling course for elementary PTs, in fall 2020 and a geometry course for elementary PTs, in spring 2021.

My own experiences around teaching mathematics content courses for elementary PTs have been pivotal to my identity as an instructor and frame much of my thinking around what it might mean for a newer mathematics content instructor to learn to teach elementary PTs. My views on this are also informed by my prior classroom observations, instructional meeting observations, and interviews with other mathematics content and mathematics methods instructors at UNL, as well as informal conversations with colleagues who have taught mathematics content courses at UNL. I think it is important for me to note that my study is driven and motivated by a personal desire to improve experiences for both mathematics content instructors and the elementary PTs who learn from them. I see room for improvement based on my readings of the literature, but I also see room for improvement based on my own experiences and my understanding of others' experiences through my interactions with them.

Literature Review

Conceptualizing Learning to Teach PTs

One main conceptualization guiding the research on learning to teach PTs of mathematics is the perspective that learning occurs through participation in a community

of practice (CoP; Lave & Wenger, 1991; Wenger, 1998; Wenger, 2008) or community of inquiry (Cochran-Smith, 2003). Many researchers view learning to teach as necessarily occurring through interactions with others, while simultaneously an individual cognitive process occurring through reflection and inquiry into one's own practices and the practices of others (e.g., Konuk, 2018; Masingila et al., 2018; Shaughnessy et al., 2016; Zaslavsky & Leikin, 2004). This emphasis on reflection and inquiry by many researchers likely contributed to the emergence of self-study as more than a means for examining newer instructors' learning to teach PTs (e.g., Nicol, 1997; Chauvot, 2009; Masingila et. al, 2018), but as a theory of instructors' learning to teach PTs in and of itself (Tzur, 2001; Alderton, 2008; Chapman, 2008). The fact that instructors of PTs of mathematics are sometimes far and few between, i.e. not easily accessible to all researchers, has likely also contributed to this developing research practice and theory.

Some researchers use additional theoretical frameworks to conceptualize newer instructors' learning, such as the three-layer model of growth through practice (Zaslavsky & Leikin, 2004) and learning through the tasks of teaching or via problem solving (Masingila et. al, 2018). Nonetheless, there seems to be a consensus that learning to teach PTs is conceptualized as a highly social, collaborative endeavor, involving deep reflection and inquiry, especially into one's own practices. I similarly adopt an integrated perspective on learning to teach PTs, in that I believe that learning is rooted in sociocultural contexts and norms and that it occurs through interactions with others and through reflection and inquiry on the self and others.

The Knowledge (Newer) Instructors (Need to) Learn

What exactly then do newer mathematics content instructors learn or need to learn to teach mathematics courses for elementary PTs? The content of what is to be learned is equally as important as the learning process itself. Some researchers investigate instructors' knowledge, such as mathematical knowledge for teaching teachers (MKTT; Masingila et al., 2018; Zopf, 2010) or broader domains of knowledge that also include knowledge of context (Chauvot, 2009). Primarily, this research has focused on developing theories about the domains of knowledge needed to teach PTs (e.g., Chauvot, 2009), or more specifically the domains of *mathematics* knowledge needed to teach PTs (e.g., Masingila et al., 2018; Zopf, 2010). Alternatively, some studies aim to develop theories regarding *how* instructors develop various knowledge for teaching PTs or current teachers (e.g., Tzur, 2001; Zaslavsky & Leikin, 2004).

For instance, Tzur's (2001) narrative self-study reflected on their growth from a mathematics learner, to a (secondary) mathematics teacher, to a (secondary) mathematics teacher educator, and lastly, to a mentor of (secondary) mathematics teacher educators. For Tzur, becoming a mathematics teacher educator specifically involved an awareness through reflection of: (1) what it means to teach math, (2) how someone comes to know how to teach math, (3) and how someone's activities promote other's learning of mathematics teaching. Zaslavsky & Leikin (2004) took a different approach to developing theory by conducting a grounded theory study. They mostly included experienced mathematics teachers (20 participants total) who had no formal training in teaching teachers. Based on the MTEs' experiences designing and carrying out workshops for in-service secondary mathematics teachers, Zaslavsky & Leikin concluded that: (1) the process of designing tasks promoted the MTEs' growth, (2) the three-layer

model of growth through practice is useful for describing MTEs' growth, (3) and CoPs contribute to MTEs' growth. While I have not found any studies that explicitly adopt and develop either of the specific frameworks posited by Tzur or Zaslavsky & Leikin, both studies have influenced the field's conceptualizations of instructors' knowledge development, as both are cited in subsequent research on instructors' knowledge development (e.g., Chauvot, 2009; Masingila et al., 2018; Zopf, 2010).

Chauvot (2009) examined her own knowledge and the development of her knowledge for teaching both secondary and elementary PTs mathematics methods from her doctoral studies into her third year of a tenure-track faculty position at a large southwestern university. Chauvot acknowledged that the expertise required of teacher educators is complex. Moreover, what this expertise might be and how it might be developed was not well researched or understood, so she conducted a self-study using narrative inquiry to identify the knowledge she used and what she attributed gaining this knowledge to in teaching mathematics methods courses.

Informing her conceptualizations of this knowledge were Shulman's (1986) categories of teacher knowledge. In particular, she drew on Shulman's notions of pedagogical content knowledge, as well as subject matter content knowledge and curricular knowledge to guide her initial framework. Recognizing the added value of a fourth area of teacher knowledge from Grossman's (1990) framework, Chauvot (2009) extended Shulman's categories to include knowledge of context. This domain encompasses such knowledge as the knowledge of the opportunities, expectations, and constraints afforded, set, or imposed by the school districts in which teachers work,

knowledge of school settings and school culture, and knowledge of how higher education operates.

From her analysis of several artifacts and journal entries, Chauvot (2009) constructed a knowledge map for MTEs consisting of the same four categories of teacher knowledge mentioned above. She also extended a number of Ball and colleagues' (2008) domains of mathematical knowledge for teaching (MKT) to make them more particular to MTEs. For example, she specified that the subject matter content knowledge for an MTE is the knowledge of how to develop PTs' specialized content knowledge and pedagogical content knowledge and the knowledge of how to engage PTs with content in ways that connect to teaching students.

One of the main implications Chauvot (2009) discusses in her study is that "different kinds of knowledge is needed to serve different roles such as instructor of university courses and mentor of doctoral students" (Chauvot, p. 369). Among other ideas, Chauvot wonders how instructors of in-service teachers might require different sets of knowledge or skills from instructors of prospective teachers. From my own experience as an instructor of elementary PTs, I wonder how instructors with differing backgrounds and teaching contexts, such as those who teach mathematics content courses and have no K-12 mathematics teaching experience might differ from or relate to Chauvot in the kinds of knowledge they need to develop.

Compared to Chauvot (2009), Zopf's (2010) dissertation more specifically investigated the work of teaching teachers mathematics and the *mathematics* knowledge demands entailed by this work. Two instructors participated in her study. The first is a professor who teaches elementary mathematics for current teachers, while the second is

Professor Deborah Ball, who teaches mathematics methods courses for elementary student teachers. In electing to study instructors with decades of experience teaching teachers, Zopf contributes an understanding of the kinds of mathematics-related knowledge newer instructors might aspire to develop.

Two of Zopf's (2010) primary contributions include (1) a detailed discussion of the ways in which the work of teaching mathematics knowledge for teaching is special, and (2) the distinctive characteristics of MKTT. As examples, MKTT appears to involve knowledge for mathematical knowledge for teaching that is panoramic, connected, and fluent, in addition to knowledge of mathematical structures such as definitions, properties, theorems, and lemmas, and how these are used to do mathematics. Roughly speaking, Zopf describes *panoramic* knowledge for teaching as "a knowledge that captured the broad landscape of mathematical knowledge for teaching and mathematical knowledge foundational to mathematical knowledge for teaching" (p. 195). For instance, Zopf (2010) describes how one instructor's teaching of fractions began with the development of the concept of fraction before diving into operations with fractions. *Connected* knowledge refers to instructors making connections within and across mathematical domains. Lastly, *fluent* knowledge refers to instructors making seemingly effortless transitions between panoramic and detailed mathematical knowledge. One main implication from Zopf's findings is that newer instructors need to develop their own specialized mathematics knowledge for teaching elementary PTs.

Eight years later, Masingila et al. (2018) assert that there remains "a dearth of research literature on preparing MTEs" (Masingila et al., p. 430), as well as on the knowledge needed by MTEs. Thus, their self-study aimed to identify the domains of

MKTT two novice MTEs used and developed while teaching elementary PTs to generate their own MKT in learning mathematics via problem solving. Masingila et al. build on Zopf's (2010) MKTT and findings, asserting that "MTEs require a somewhat different knowledge base than MTs [mathematics teachers], and MTEs need to develop MKTT, in order to support MTs in developing MKT" (Masingila et al., p. 431). The essential idea surrounding teaching and learning via problem solving is that, in the same way that students learn through engaging in tasks (Hiebert & Werne, 1993), MTEs learn through engaging with the tasks of teaching (Zaslavsky, 2005, 2007, 2008). For example, they learn through tasks such as "designing or modifying tasks, supporting learners while they engage in tasks, and reflecting on learners' work" (Masingila et al., 2018, p. 434). In Masingila et al.'s (2018) study, both novice MTEs were prepared as secondary mathematics teachers, but had little or no experience with K-12 teaching. They both had several years experience teaching undergraduate mathematics courses and roughly one year of experience either teaching or observing others' teaching PTs, before teaching the mathematics content course for elementary PTs in the study.

Based on their collective inquiry and reflections into their practice of teaching via problem solving, Masingila et al. (2018) found that both novice MTEs, as well as the more experienced MTE conducting the study, developed MKTT specifically through (a) understanding and deciding on mathematical goals, (b) choosing and facilitating tasks, and (c) using questions to scaffold PTs' learning. Given the significance of their CoP on their learning to teach PTs, they recommend that MTEs form CoPs to support each other in doing the same. Given little experience in schools, but some experience working with PTs, I wonder what additional domains of knowledge the MTEs felt they needed to be

successful teaching PTs more generally and the ways in which those domains may or may not have been a part of their learning in their CoP.

The studies in this section vary with respect to their contexts and participants, and therefore contribute various understandings of knowledge for teaching teachers. For instance, some studies systematically examine the practices, experiences, and knowledge of other instructors (e.g., Masingila, 2018; Zaslavsky & Leikin, 2004; Zopf, 2010), while some studies reflect on the practices, experiences, and knowledge of the authors themselves (e.g., Chauvot, 2009; Tzur, 2001). Hence, the field's current understanding of instructor knowledge is based on a blend of self-studies (Chauvot, 2009; Tzur, 2001), grounded theory research with 20 instructors (Zaslavsky & Leikin, 2004), and close work alongside a few instructors (Masingila, 2018; Zopf, 2010). The field's current understanding of the knowledge newer instructors might need is also based primarily on observations and reflections of more experienced instructors of PTs (e.g., Chauvot, 2009; Tzur, 2001; Zopf, 2010), rather than newer instructors.

Additionally, Masingila et al. (2018) is one of few studies about the knowledge of newer instructors of mathematics *content* courses for *elementary* PTs, while other studies primarily involve *methods* instructors (e.g., Chauvot, 2009; Tzur, 2001) or instructors of *secondary* mathematics teachers (e.g., Zaslavsky & Leikin, 2004). Notably, most of these studies on knowledge also involve instructors with multiple years of experience as high school mathematics instructors (e.g., Chauvot, 2009; Zaslavsky & Leikin, 2004; Zopf, 2010). Few studies involve instructors with elementary teaching experience (e.g., Zopf, 2010) or those without K-12 teaching experience (e.g., Masingila et al., 2018). Overall, while the field has made some progress in conceptualizing instructor learning to develop

knowledge and in understanding the kinds of knowledge that newer instructors of PTs may find useful, it is limited in understanding the kinds of knowledge that newer instructors already possess or need for teaching PTs.

Newer Instructors' Experiences and Challenges

Although researchers consider knowledge, and in particular specialized mathematics knowledge, important for one's preparedness to teach PTs, there are many additional considerations for newer instructors. To uncover these considerations, a number of instructor-researchers have explored newer instructors' experiences learning to teach PTs and the specific challenges these instructors have faced (e.g., Nicol, 1997; Oesterle, 2011; Van Zoest et al., 2006; Yow et al., 2016). Collectively, this literature provides nuanced understandings of the concerns of newer instructors and challenges notions of what may be important for instructor preparation.

An older self-study comes from a book chapter by Nicol (1997), in which she shares detailed reflections on and analysis of her own experiences learning to teach elementary PTs in a mathematics methods course through the lens of tensions or dilemmas. Nicol wrote that while she had seven years of experience teaching mathematics from grades 8 through 12, she felt unprepared for the challenges she met as a beginning teacher educator. The goal of her self-study was to tell her story of experience--the tensions, dilemmas, and challenges surrounding her attempts to teach elementary PTs to adopt a stance of inquiry towards teaching and learning.

Underlying Nicol's (1997) story is her position that elementary PTs ought "to be critical of personal practice, and use [their] deepened insights to move forward" (McNiff, 1993, p. 20, as cited in Nicol). Nicol did not want PTs to merely accept her own advice,

ideas, and techniques as a "bag of pedagogical tricks" (Wineburg, 1991, p.277, as cited in Nicol), but to learn and inquire into their own practice. Being a difficult, but meaningful, philosophy for her to situate her practice in, Nicol discusses (1) the struggles of selecting and using meaningful mathematical and pedagogical activities, (2) the realization of needing to balance both listening for her goals and intentions in PTs' work and listening to the understandings and sense PTs are making of their experiences, and (3) the struggles of creating an authentic and collaborative practice. Nicol's stories resonate with my own experiences as a newer instructor because they suggest that important learning happens when managing the conflicts and tensions that naturally arise in the classroom. Based on my experiences, instructors who have little experience teaching PTs, no experience teaching K-12, and only a few years of experience teaching other undergraduates, but aim to teach PTs in ambitious ways, need to learn how to manage conflicts in the classroom, and in ways that invite PTs and the instructor to investigate together better ways to manage difficult situations.

Similar to Nicol (1997), the second study by Van Zoest et al. (2006) explored the challenges faced by experienced K-12 teachers transitioning into their new role as instructors of middle school PTs. While Van Zoest et al. acknowledge the wealth of knowledge and value that experienced teachers bring, they simultaneously recognize that "experiences alone will not ensure success as a teacher educator" (Van Zoest et. al, p. 134). To assist teachers with this transition, the doctoral program at the participants' university offered a mentored clinical experience, in which the doctoral students work with a faculty mentor in the planning and teaching of a mathematics methods course. Van Zoest et al. share what they learned about the design from their experience.

While the authors do not explicitly frame their study in terms of a community of practice, it is evident from their analysis that the authors view the collaborative components of the experience as essential to the learning process of the doctoral students. In particular, they analyzed interactions among the three novice MTEs and the more experienced MTE during planning and debriefing sessions, as well as their post-experience conversations in order to identify consistent themes. Two main ideas emerged from their conversations: (1) the importance of engaging doctoral students in explicit conversations about what it means to be a teacher educator and (2) the importance of emphasizing experiences that are different from K-12 classroom teaching.

For an example of the second theme, the novice instructors struggled with understanding how to balance sharing their own classroom teaching experience with their goal of encouraging PTs to think critically about teaching and learning. This is extremely similar to the struggle Nicol (1997) described in her attempts to have PTs inquire into teaching dilemmas. In Van Zoest et al.'s (2006) study, some PTs got frustrated when the novice instructors were not sharing their classroom experiences in the ways that they wanted, but the novice instructors only found themselves strengthening their resolve to have the PTs think critically. The authors wrote that "the novice teacher educators did not recognize the conflict between this resolve and their desire to show the preservice teachers examples of effective teaching" (Van Zoest, p. 139). Nonetheless, Van Zoest et al. found these collaborations with experienced teacher educators to be beneficial to novice instructors' learning, and therefore recommend that they be a required component of doctoral programs, so long as these collaborations are a recognized component of the faculty mentor's workload.

While Nicol (1997) and Van Zoest et al. (2006) documented the experiences of newer instructors of mathematics *methods* courses for elementary and middle school PTs, Oesterle (2011) focused her dissertation on the experiences of instructors of mathematics *content* courses for elementary PTs. Motivated by her own experiences and the tensions she felt teaching a mathematics content course for elementary PTs, Oesterle aimed to both understand mathematics instructors' experiences teaching such a course, as well as the major tensions they experienced in doing so. She employed a blend of grounded theory, phenomenological, and case study methods with 10 instructors in mathematics departments in British Columbia.

Considering their backgrounds, all instructors had at least a bachelor's degree in mathematics, and at least a master's degree to teach at the post-secondary level. Only three had formal training in education and only two had K-12 teaching experience--one taught high school mathematics for 1-2 years, the other taught high school mathematics for 6 years. Oesterle (2011) deliberately sought out instructors with a wide range of experience teaching PTs, but not necessarily teaching experience in general. As it turned out, three instructors were teaching PTs for the first time and one instructor was teaching PTs for the second time, while the remaining instructors were teaching PTs for the fourth, sixth, nineth, or twentieth time. However, all instructors had 10-30 years of experience teaching undergraduate mathematics, and so no one instructor was both newer to teaching PTs, as well as teaching in general.

Oesterle (2011) described in nuanced detail the shared experiences and tensions of instructors, as well as ways in which their experiences diverged. Overall, when considering their experiences teaching the course and the students, and how the course is

distinct from other math courses, instructors described concerns that their students' affective and cognitive deficiencies, if allowed to persist, could have repercussions on their future students. These perceptions contributed to instructors' tensions around their priorities for teaching the course, and partially accounted for diversity in instructors' emphases on their affective versus cognitive goals for their students. Instructors also described how their expectations for students to clearly explain their reasoning is higher for these students than might be seen in other math courses. They also experienced tensions around this expectation, in part because of a sociomathematical norm that mathematics does not involve writing, which furthermore pushed some instructors to evaluate students in ways that were unfamiliar to them.

Oesterle (2011) additionally found that many instructors felt unsatisfied with their textbooks, yet primarily relied on them for their learning and instruction. When considering instructors' resources in general, Oesterle noticed that there was little to no mention of consultation with education faculty or academic research in mathematics education. Oesterle points to this issue as one that goes beyond inconvenience, but to a sense of divide between mathematics faculty and education faculty/research. Open lines of communication, however, might have supported those instructors who felt that using manipulatives or facilitating class discussions around pedagogy was beyond their expertise.

The instructors' perceptions, expectations, and beliefs regarding the students, the course, and its resources all factored into the major tensions the instructors experienced, such as instructors balancing their own passions for math with the perceived desires of their students, the level of math proficiency that they should expect students to attain, the

extent to which the mathematics content instructor should address pedagogy and elementary school contexts, and their overall responsibility for ensuring the mathematical preparation of their students. They managed these tensions in one of four ways: (1) sticking with the familiar, (2) deferring to higher authorities or other colleagues, (3) continuing to adjust and experiment, and (4) resigning oneself to lesser goals. Oesterle (2011) points to the significance that norms play in managing these tensions, such as the norm of *covering the content*. Her case study of Simon, an instructor teaching the course for the first time, illustrates how the pressure of needing to cover the content impedes instructors from changing their instruction. In particular, wanting to satisfy affective goals, such as reducing anxiety, influenced Simon to reduce course content and incorporate more of the hands-on activities he thought was helpful for students, while goals for improving arithmetic skills and (more significantly for Simon) covering the syllabus within allotted time influenced him to reduce hands-on activities and lecture more (which Simon believes is efficient and can be effective for students, but is disappointed about doing to the peril of activities).

Oesterle's (2011) dissertation contributes a wealth of knowledge to the field regarding instructors' experiences, perceptions, and tensions around teaching a mathematics content course for elementary PTs. Future studies with similar instructors might additionally examine instructors' preparation, support, and beliefs while teaching the course. Future studies might also involve instructors who are newer to teaching in general, as the experiences and perspectives of these instructors of mathematics content courses for elementary PTs are not well documented, if at all, in the literature. Indeed, Oesterle recommends that future research investigates specifically what mathematicians

need to know in order to be effective instructors of PTs, while taking into account the norms and personal factors of the instructors.

Yow et al. (2016) is one such study that contributed to what is known about instructors of mathematics courses for PTs by investigating perceptions of MTEs' preparation for academic careers (and becoming an MTE) via doctoral programs. Specifically, they examined MTEs' perceptions of the challenges they face in transitioning to a faculty position and MTEs' perceptions of the support, knowledge, and experiences they need to better support preparation and transition of teachers of mathematics. Their participants involved members of a National Science Foundation program that provided mentorship experiences for new PhDs that had recently been hired as MTEs in either Colleges of Education or Departments of Mathematics. 69 instructors completed Yow et al.'s survey, 98% of whom reported being assistant professors. Notably, Yow et al. is one of few studies that discusses the multiple distinct pathways to becoming an MTE. For instance, some MTEs earn a bachelor's degree in elementary education, teach in elementary schools for a number of years, and later on earn a doctorate in mathematics education, which tends to lead towards a career in Colleges of Education teaching mathematics methods courses. On the other hand, some MTEs earn a bachelor's and master's degree in mathematics, then a doctorate in mathematics education, which tends to lead towards a career in Departments of Mathematics teaching mathematics content courses.

Yow et al. (2016) mentions these distinct pathways to highlight the different kinds of borders these instructors cross in their transition to becoming MTEs, e.g., graduate student to MTE versus teacher of mathematics to graduate student to MTE but does not

appear to disambiguate their findings based on these differences. Hence, their findings on MTEs' perceptions of their preparation are an amalgamation of both instructors of elementary and secondary PTs, newer instructors of PTs and those with some experience teaching PTs, instructors with K-12 teaching experience and those without K-12 teaching experience, as well as those with little teaching experience altogether. Indeed, their survey results showcase a diversity in prior teaching experiences with less than a third of participants having previous K-12 teaching experience and over 60% with no teaching experience.

Yow et al.'s (2016) thematic analysis of survey data yielded two main concerns from instructors for transitioning into a faculty position: (1) mentoring and (2) preparation to teach. With respect to support and mentoring, the majority of participants reported having access to seminars and colleagues who openly share assessment materials. 25% of instructors were assigned a mentor that they continue to collaborate with, 20% were assigned a mentor that they stopped working with or found to be ineffective, and another 25% found their own informal mentor. Primarily, instructors described the importance of having a mentor in helping them navigate academic institutions, especially because they felt an expectation to be "on your own" (Yow et al., p. 61), as well as the need for strong research mentoring.

With respect to preparation to teach, instructors described how they felt that they left their doctoral programs without a comprehensive toolkit for teaching, e.g., how to lead a discussion and how to create a syllabus and assignments. They additionally wished that in their doctoral programs they had opportunities for guided reflections of their teaching with a more experienced faculty member. Lastly, they described desires for

more immersion into K-12 mathematics teacher preparation, both in terms of the literature and in teaching methods courses. Given that over 60% of participants had no prior teaching experience, I am not surprised that instructors wanted more for their learning in terms of designing and teaching their own courses. I wonder, however, to what extent desires for immersion into K-12 mathematics teacher preparation were felt by those who teach mathematics content courses versus methods courses. Nonetheless, I find it interesting that these instructors did not voice significant concerns over specialized knowledge for teaching mathematics instruction, yet some researchers prioritize these concerns.

Supporting Newer Instructors

Given what the field knows (and does not know) about newer instructors learning to teach PTs, some researchers have experimented with support systems for newer instructors (e.g., Castro Superfine & Li, 2014; Jackson et al., 2020; Shaughnessy et al., 2016; Suppa et al., 2020). While none of these papers are studies of the experiences of any particular set of novice instructors, all four papers propose their own professional development models or support structures for novice instructors of PTs of mathematics based on their analysis of the model, their work with novice instructors, and the recommendations of the field.

Castro Superfine and Li (2014) proposed a model for the professional development of those who teach elementary PTs mathematics content based on their work with MTEs and the premise that these instructors need to develop their own understanding of mathematical knowledge as it pertains to teaching elementary PTs in mathematics content courses. Castro Superfine and Li describe four key criteria for high-

quality professional development in the literature: (1) the participants (instructors of PTs of mathematics) are currently engaged in the content of teaching and learning, (2) the activities of professional development are chosen to create disequilibrium for the participants, (3) the activities encourage collaboration among participants within a community of practice, and (4) the participants' learning is applicable to the work of teaching teachers. In the remainder of the paper, Castro Superfine and Li describe the details of their model so that others may critique and discuss what structures and knowledge make sense given that instructors of elementary PTs of mathematics come from a diversity of professional backgrounds and often do not have much preparation (Masingila et al., 2012).

In the same way that one might pick and choose what features to adopt (or not) from Castro Superfine and Li's (2014) model into one's own professional development program, Shaughnessy et al. (2016) hope that others select from the variety of support structures they propose for adoption into their communities of practice. In their paper, Shaughnessy et al. discuss what types of knowledge or skills are important to teaching elementary PTs in a mathematics methods course and describe possible structures to support novice instructors' learning of such knowledge and skills. In addition to framing learning as happening through a CoP, Shaughnessy et al. frame their thinking around the work of teacher education through Ball's adapted version of the instructional triangle (Cohen et al., 2003), which emphasizes interactions between and among teacher educators, who are the teachers, preservice teachers, who are the students, and content, which is the knowledge of the instructional triangle for teachers (teachers, children, content).

Of the possible types of knowledge and skill they could have focused on, Shaughnessy et al. (2016) chose to focus on a practice-based approach to teaching called *explaining core content*, which essentially consists of explaining mathematics instruction to elementary PTs and requires one to draw on different types of specialized mathematical knowledge for teaching teachers. The authors provide examples of important skills for novice instructors to develop to support elementary PTs learning to explain core content, as well as structures that have the potential to support novice MTEs development of specialized knowledge and pedagogical skills. The examples of pedagogical skills include MTEs modeling explanations of core content, elementary PTs rehearsing explanations of core content, and MTEs providing feedback on pre-service teachers' enactments. Support structures include participation in a planning group, detailed lesson plans shared and reviewed among the group members, and a planning group structure that involves debriefing, reflecting on how observed pedagogies support elementary PTs learning, and a number of possible activities to prepare for the next class, such as MTEs rehearsing modeling explaining core content or MTEs reflecting on the feedback they would give PTs after watching videos annotated with feedback by experienced MTEs.

Suppa et al. (2020) similarly described a variety of support factors that they believe to be useful to those teaching elementary PTs mathematics content for the first time based on findings in the literature and some of the authors' personal successes with having these support structures in place. Acknowledging the unique challenges associated with teaching PTs, especially for the first time, and viewing collaborative teaching environments as important to the learning of novice instructors, Suppa et al. recommend

three specific supports: (1) consistently observing a more experienced MTE in the classroom and paying attention to the ways in which mathematical content, pedagogy, and elementary PTs' motivations are addressed in the lesson; (2) providing and discussing key features and teaching practices embedded in educative curriculum materials; (3) and regular weekly instructor meetings to discuss and reflect on novice instructors' concerns and anticipate challenges in the near future. These recommendations are similar to Shaughnessy et al. (2016), although Suppa et al. go into far more detail. Nonetheless, both publications offer insight into the kinds of support that educators of novice instructors and the novice instructors themselves are finding useful for learning to teach PTs.

One final paper that addresses professional development for novice instructors of PTs is a bit different from the other three in that it provides a conceptual model for designing your own professional development in the context of novice instructors learning to teach mathematics for elementary PTs. In other words, Jackson et al. (2020) offer a framework that others can use to help them pick and choose the support structures that work for their particular contexts. Two of the key components of their conceptual model is that the designers of professional development for instructors of PTs need to decide on what those instructors need to learn and why, and what constitutes evidence of learning. In this way, designers can be more intentional about what it means for newer instructors to learn to teach PTs of mathematics and how they can facilitate instructors in achieving those goals. Given a seemingly wide array of options, having a process for designing your own professional development program seems helpful.

Conclusion

Based on this literature, not much research has been done with respect to newer mathematics content instructors' preparation and learning to teach PTs. What studies do exist typically focus on a few small slices of the knowledge or skills that may be important for instructors to develop or discuss support structures that may benefit instructors. Moreover, it is not always clear in the literature what preparation and support might be meaningful specifically for mathematics content instructors, as opposed to mathematics methods instructors, for example. In any case, I found Oesterle's (2011) summary of literature on post-secondary mathematics instructors to hold relevance today with respect to *all* instructors of PTs of mathematics:

> although the research is intended *for* instructors (or for course/program designers), there is very little research *about* the instructors… in this context there is a lack of documented research about what happens in ordinary mathematics content courses for preservice teachers, ones that are not undergoing studies for particular interventions. (Oesterle, p. 39)

What literature *about* instructors do exist typically involve one experienced instructor reflecting on their experiences (e.g., Chauvot, 2009; Nicol, 1997; Tzur, 2001), or involve less than a handful of newer instructors (e.g., Oesterle, 2011; Van Zoest, 2006). Additionally, of the studies about newer instructors, most focus on mathematics *methods* instructors with experience teaching high school mathematics (e.g., Chauvot; Nicol, 1997; Tzur, 2001; Van Zoest, 2006). Oesterle (2011) is one of few studies that involves newer instructors of mathematics *content* courses for elementary PTs, as well as instructors with no K-12 teaching experience, albeit with 10-30 years of experience teaching undergraduate mathematics courses. Hence, more perspectives need to be

examined to thoroughly understand the complexities of learning to teach elementary PTs, specifically perspectives from mathematics content instructors who are not only newer to teaching elementary PTs, but newer to teaching more generally.

Studies about newer instructors and their experiences provided me with key insights into instructors' dilemmas, tensions, and challenges, and furthermore, helped me to hypothesize about the connections among an instructor's background, teaching context and experiences. For instance, Nicol (1997) and Van Zoest (2006) both involved mathematics methods instructors with prior experience teaching high school mathematics and both studies revealed common tensions around teaching PTs to think critically about teaching. More studies of instructor experience may reveal further commonalities among instructors with particular backgrounds and in particular teaching contexts. Such studies may also highlight key differences among instructors with similar backgrounds and teaching contexts, as well as previously undocumented challenges for those instructors.

Purpose Statement & Research Questions

Reflecting on this literature and my own experiences as a newer mathematics content instructor, my research puzzle (Clandinin, 2013) is this: how might the backgrounds and teaching contexts of newer mathematics content instructors shape their experiences around learning to teach PTs? Chauvot's (2009) study calls for this kind of exploration: "different kinds of knowledge is needed to serve different roles" (Chauvot, p. 369). Connections between instructors' contexts and their learning to teach PTs do not appear to be well understood in the research literature yet are critical to making progress in understanding newer instructors' learning and growth, the challenges and successes they experience, the expertise they need to develop, and the support structures that might

benefit them. Hence, the purpose of my study was to obtain a more nuanced understanding of a newer mathematics content instructor's experiences around learning to teach elementary PTs in their respective teaching context.

The central question that guided my study is:

How might a newer mathematics content instructor's background and preparation relate to the challenges and successes they experience around teaching and learning to teach mathematics content for elementary PTs?

In particular, I aimed to understand this central question by gaining insight into the following sub-questions:

1. *What challenges and successes does a newer instructor experience around teaching and learning to teach mathematics content for elementary PTs?*
2. *What prior mathematics learning and mathematics teaching experiences provide a continuous and storied understanding of a newer instructor's challenges and successes around teaching and learning to teach mathematics content for elementary PTs?*
3. *How does a newer instructor describe their preparation and support for teaching mathematics content for elementary PTs? Moreover, what experiences surrounding a newer instructor's preparation and support provide a continuous and storied understanding of their challenges and successes around teaching and learning to teach mathematics content for elementary PTs?*

Note that *continuous* refers to an understanding of experience, not as a series of isolated events, but as a thread connected to a larger storyline of who the participant was, is, and

wants to become, while *storied* refers to an understanding of experience as a narratively composed phenomenon--people live by and live in stories (Clandinin, 2013).

In the next chapter, I discuss the meanings of *continuous* and *storied* in more detail, within the description of the research design and theoretical framing. I also describe the ways in which I engaged in reflexivity throughout the study and further outline my methods.

CHAPTER 3: METHODOLOGY

In this chapter, I describe the research design and theoretical framing of the study, including the ways in which I theorize instructor learning, utilized a 3-D inquiry space as a framework for understanding experience and learning, and my rationale for selecting a narrative design. I then describe the ways in which I engaged in reflexivity in the study. Lastly, I outline my methods, including ethical considerations, the context of the study and the participant recruitment, data collection and analysis, and limitations of the study.

Research Design & Theoretical Framing

To address the research questions in the previous section, I chose to engage in narrative inquiry (Clandinin, 2013; Clandinin & Connelly, 2000). I conceive of narrative inquiry as described by Clandinin and Connelly (2000):

> Narrative inquiry is a way of understanding experience. It is a collaboration between researcher and participants, over time, in a place or series of places, and in social interaction with milieus. An inquirer enters this matrix in the midst and progresses in this same spirit, concluding the inquiry still in the midst of living and telling, reliving and retelling, the stories of the experiences that make up people's lives, both individual and social. Simply stated… narrative inquiry is stories lived and told. (p. 20)

Narrative inquiry centers on experience, both as the phenomenon being studied and the method used to understand experience (Creswell & Poth, 2018). Its philosophical roots can be traced back to Dewey's (1938) theory of experience based on notions of continuity of past, present, and future experiences and interactions within situations (Clandinin, 2013; Creswell & Guetterman, 2019). "For Dewey, education, experience, and life are

inextricably intertwined. In its most general sense, when one asks what it means to study education, the answer is to study experience" (Clandinin & Connelly, 1994, p. 415). Embracing Dewey's perspective on education, experience, and life and Clandinin and Connelly's (2000) perspective of narrative inquiry as a way of understanding experience makes this methodology a natural fit for education research in general, and for studying my research problem in particular.

Theorizing Instructor Learning

I draw on social constructivism, situated cognition, and sociocultural learning theories to frame my thinking around a mathematics content instructor's learning to teach PTs. With respect to social constructivism, I view an instructor's learning as necessarily occurring through *interactions* with others, while simultaneously an individual cognitive process occurring through *reflection* and inquiry into one's own practices and the practices of others (Cobb & Bauersfeld, 1995). Integrating this view with situated cognition and sociocultural learning theories, I recognize that an instructor's interactions and reflections are inseparable from *context, time, and place*, and are deeply rooted in sociocultural norms (Gutiérrez, 2008; Lave & Wenger, 1991; Rubel & Nicol, 2020; Vygotsky, 1978). For example, an instructor's classroom at the beginning of the semester may differ significantly from their classroom by the middle of the semester, perhaps in atmosphere or in the norms and mathematical practices that are accepted by the instructor and by their students. Rubel and Nicol (2020) emphasize the significance of place in learning: "People shape place but place shapes us – how we behave, how we interact, who we are, and even who we are becoming" (p. 175). In particular, I view instructor learning as occurring through participation in a community of practice (CoP; Lave &

Wenger, 1991; Wenger, 1998; Wenger, 2008). Investigating an instructor's learning thus requires an understanding of their interactions and reflections on teaching PTs, as situated within their teaching and learning contexts at various points in time.

Focusing on challenges and successes may offer significant insight into instructor learning as well. When an instructor faces a challenge, such as determining how to facilitate discussions and activities around mathematics, this presents an opportunity for disequilibrium in the mind of the instructor (Piaget, 1950). Challenges in the forms of unresolved tensions or dilemmas also offer places for rich cycles of learning (Engeström, 1987 as cited in Gutiérrez, 2008). In overcoming or managing their challenges, an instructor may draw on previously experienced successes to decide what to try, and so might any colleagues that an instructor reaches out to for support. When an instructor experiences success, an instructor may draw conclusions about what might work and what might not for instruction in their teaching context. In this way, an instructor is reinforcing previous learning or updating their previous understandings as a part of the learning process.

Utilizing 3-D Inquiry Space as a Framework for Understanding Experience and Learning

An instructor's challenges and successes around teaching and learning to teach PTs are embedded in their experiences, and are therefore, essential to understanding instructor learning. In this subsection, I describe how I view experience, and how these views are connected to my views on learning. Central to understanding experience is the notion of continuity, that is "the idea that experiences grow out of other experiences, and experiences lead to further experiences" (Clandinin & Connelly, 2000, p. 2). Clandinin

and Connelly's (2000) *three-dimensional* (temporal, contextual, and interactional) *inquiry space* provides a continuous and storied understanding of experience. Hence, I utilized this framework to understand experience, as well as to support me in making sense of my participant's experiences (i.e., the 3-D framework is used both as a theoretical lens and analytical tool).

First, the *interactional* dimension refers to both personal interactions, such as "an individual's feelings, hopes, reactions, and dispositions (Creswell & Guetterman, 2019, p. 522), as well as social interactions, such as "the intentions, purposes, assumptions, and points of view" (Creswell & Guetterman, 2019, p. 522) of other people. Second, the *temporal* dimension refers to time--past, present, and future. Attending to the temporal dimension of experience means that one does not just consider the on-going experiences of the present, but also considers past events, anticipated future events, and how those might be relevant to someone's understanding of current events. Third, the *contextual* dimension refers to place. Attending to the contextual dimension means that one considers "the context, time, and place within a physical setting, with boundaries and characters' intentions, purposes, and different points of view" (Creswell & Guetterman, 2019, p. 522). When considered altogether, these three dimensions allow for a continuous and storied understanding of experience, not as an isolated event, but as a thread connected to the larger storyline of who someone was, is, and wants to become.

The interactional dimension of experience directly relates to an emphasis on learning through interactions and reflections, as discussed in social constructivist learning theories. The temporal and contextual dimensions directly relate to an emphasis on how those interactions and reflections are situated within a specific place at a specific point in

time. Hence, I see Clandinin & Connelly's (2000) 3-D framework as not only key to understanding an instructor's experience, but to understanding their learning to teach PTs.

Rationale for a Narrative Design

Because I view experience as essential to understanding instructor learning and have therefore focused my research on understanding my participant's experiences, their personal backgrounds, and the contexts that surround their experiences, a qualitative approach is appropriate. Moreover, because I aimed to deeply and holistically understand the experiences of one instructor, a narrative inquiry, as opposed to a phenomenology or other qualitative approach, is appropriate. In particular, since narrative inquiry sheds light on the temporal dimension of experience, acknowledging that participants' understanding of people and events change over time (Bell, 2002), then narrative inquiry is an appropriate methodological choice. Other research methodologies might only capture participants' experiences at certain points in time, whereas narrative inquiry captures the fluidity of participants' changing lives. In this way, narrative inquiry is more apt to depict the transitional nature of learning to teach PTs.

Another significant reason narrative inquiry is a good fit for this study is that I additionally view experience as stories we live by and live in (Clandinin, 2013). Connelly and Clandinin (2006) describe their Dewey-inspired view of human experience in terms of living storied lives: "People shape their daily lives by stories of who they and others are and as they interpret their past in terms of these stories" (Connelly & Clandinin, 2006, p. 375). In this way, I understand experience to be a narratively composed phenomenon, and therefore did not merely collect stories of experience from my participant and retell

these stories but entered and lived alongside my participant. Being in and around my participant's spaces was important to gain insight into my research problem because it offered me opportunities to see in fuller context the myriad, connected, and nuanced impacts of my participant's prior experiences, background, and preparation on their current experiences, as well as the experiences they hoped to have.

My Reflexivity

Because narrative inquiry, and qualitative research more generally, relies heavily on the relationship between the participant and the researcher, as well as the interpretations of the researcher (Creswell & Poth, 2018), the role of the researcher is necessarily important to consider for my study. Crabtree and Miller (1999) describe reflexivity as follows:

> Reflexivity refers to self-reflection, self-criticism, and is based on the premise that the engaged field researcher is an active part of the setting, relationships, and interpretations. Knowing yourself and how you affect and are changed by the research enterprise are central to field research and, ideally, occur throughout the research process. (p.14)

Part of knowing yourself involves understanding that the qualitative researcher brings their own biases, values, and assumptions into the research (Creswell & Guetterman, 2019). These biases, values, and assumptions are ever-present in the mind of the researcher and are positioned in the researcher's many writings, fieldnotes, memos, or jottings. These biases, values, and assumptions can also change as the researcher learns and grows throughout the research process. Thus, throughout my study, I reflected on and documented how I might be affecting the research process and how the research process

might be affecting me (Creswell & Poth, 2018). These writings took the form of reflexive memos (Watt, 2007). In this way, I had opportunities to grow in my awareness of my thought processes, as well as my experiences with the phenomenon being studied, and better understand how they shaped the research and were shaped by the research.

For a study involving instructors of PTs of mathematics, it mattered that I am a graduate student instructor with a mathematics background studying in a department of teaching, learning, and teacher education and that I have taught mathematics content courses for elementary PTs for the past 3 years. Because I have taught mathematics content courses for elementary PTs before, I have my own experiences and stories that have shaped my ideas about what it means to learn to teach PTs of mathematics. Hence, I documented my biases, values, and assumptions around teaching elementary PTs to grow in my awareness of them and how these might be changing throughout the qualitative research process.

As part of being reflexive, I also took into consideration various aspects of the research context. For example, Weis and Fine (2000) suggest that all qualitative researchers consider the question "Has my writing connected the voices and stories of individuals back to the set of historic, structural, and economic relations in which they are situated?" (Creswell & Poth, 2018, p. 228). This encouraged me to attend to the political contexts of the study and how I may have positioned my participant within these contexts in my analyses and writing.

An additional consideration is the extent to which I was an insider or outsider in my research context and how my engagement with my participant influenced the research. This is important because a researcher who shares membership with their

participants along some dimension of their identity may interpret events very differently from a researcher who does not share those same memberships. For example, when I shared that I had prior experiences in a mathematics Ph.D. program with my participant, and because my participant was enrolled in a mathematics Ph.D. program at the time of the study, I anticipated that they might perceive me (and I them) as one who is familiar with and accustomed to similar norms and lived realities of a mathematics graduate student. My participant and I may have also been quicker to establish trust, as we were both graduate students who could commiserate with each other with respect to this aspect of our identities. Thus, in my journaling and in my writing, I reflected on the ways in which I might have assumed or taken for granted the meanings of my participant's words and actions, as well as my own. I also occasionally reminded my participant during interviews and other informal conversations to do their best not to assume familiarity with similar experiences when telling stories, but to assume instead that I might have had very different experiences, and therefore include more details.

Along similar lines of thought, a researcher who has power or authority (or perceived power or authority) with respect to those involved in the study may influence the trust between the researcher and participants (Creswell & Poth, 2018). This is essential to contemplate in mathematics education research because power and status often come along with identifying with mathematics (Gutiérrez, 2013). Being the primary researcher of a study may come along with its own perceptions of power and authority as well. Hence, I often reflected on various dimensions of my identity with respect to power and how I might have influenced my relationship with my participant. Specifically, I anticipated that my position in a department of teaching, learning, and teacher education

would result in my participant perceiving me as an authority on teaching and learning, or at the very least, as someone who is especially critical or judgmental about teaching. While I intended to observe my participant's teaching for the sake of understanding their experiences, I did not intend to determine the extent to which I think they were a good instructor. Therefore, to mitigate this power dynamic, I explicitly outlined my intentions (and anti-intentions) to the instructor at the beginning of the study and reminded them of these intentions on occasion.

Methods

Ethical Considerations

Before conducting this study, I obtained approval from the Institutional Review Board at the University of Nebraska-Lincoln (UNL), the university with which I am affiliated. I also obtained approval from the institution in which my participant was affiliated. Before recruiting my participant, I informed them of my research plan and my intentions to share the content of this study publicly. I received their written consent to participate in the study. One anticipated benefit to my participant was that reflection on one's own teaching through our conversations and interviews might support them in their future teaching endeavors. In addition, by conveying to the participant my interest in their experiences, another anticipated benefit for my participant was feelings of validation and being heard. To thank my participant for participating in my study, I offered my participant a choice between one of two gifts, which they could redeem as soon as they provided their consent: (1) a National Council of Teachers of Mathematics (NCTM) membership ($59 for first-time members or $94 for returning members) along with a book of their choice from nctm.org (up to $41); or (2) a $100 visa gift card.

While there were little to no known risks associated with the study, maintaining the privacy and confidentiality of the data were integral. I discussed with my participant whether they wanted to be identifiable or remain anonymous for the study. Given that they chose to remain anonymous, I consulted with them regarding a choice of pseudonym for me to refer to them throughout the study, including in my fieldnotes of classroom observations and observations of instructor meetings, the notes I took during the interviews, and the interview transcripts. All data were (and are) stored on my personal OneDrive storage account, which was (and is) password protected (and only I know the password).

The Context of the Study & Participant Recruitment

The setting for my inquiry was Cardinal University (a pseudonym), a large mid-western university. Elementary PTs are required to enroll in three mathematics content courses at Cardinal University once admitted into the elementary education program. Operations & Number Systems (O&NS; a pseudonym), the mathematics content course of interest for this study, is taken during the STEM semester, an integrated effort to connect the contents and pedagogies of mathematics, science, and technology. O&NS aims to develop elementary PTs' understanding of some of the earliest grades' mathematics content, such as the base-10 number system, the operations of addition, subtraction, multiplication, and division, fractions, and the properties of arithmetic. Because O&NS is taken alongside Elementary Mathematics Methods (EMM), as well as a practicum that meets at elementary schools for a full day twice a week, the hope is that elementary PTs will be able to see the value in what they are learning and apply it to

teaching elementary mathematics. During the semester of my study, there were three sections of O&NS, and my participant's section enrolled 18 students.

O&NS is offered through the mathematics department at Cardinal University, while the elementary practicum, EMM, and other elementary methods courses in technology and science are offered through the teaching and learning department at Cardinal University. As such, the primary candidates for recruitment from those teaching O&NS consisted of mathematics faculty, mathematics post-doctoral fellows, mathematics graduate students, or adjunct instructors. For the past eight semesters leading up to and including the semester of the study, 16 out of 24 sections of O&NS were taught by graduate student instructors (GSIs) enrolled in the Mathematics Ph.D. program in the mathematics department or the Educational Studies Ph.D. program in the teaching and learning department[2]. Furthermore, every semester for the past eight semesters leading up to and including the semester of the study, at least one GSI taught a section of O&NS, though typically 2 out of 3 sections were taught by GSIs. A typical structure that has presided over O&NS for many years at Cardinal University is that one faculty member teaches one section of the course and acts as convenor, while the remaining two sections are taught by two graduate student instructors.

The mathematics department at Cardinal University is home to between 50 and 100 graduate students in both master's and Ph.D. programs. In their first year in the Ph.D. program, most GSIs are assigned as recitation instructors for Calculus I or Calculus II. In year two, GSIs are given a choice of pre-calculus courses to teach, which includes Intermediate Algebra, College Algebra, Trigonometry, or a liberal arts mathematics

[2] Note that GSIs enrolled in the Educational Studies program do not typically teach O&NS.

course. After that, GSIs may request teaching assignments from a larger variety of mathematics courses, including all the courses previously mentioned, the mathematics content courses for elementary PTs, and other mathematics courses, such as linear algebra. Few mathematics GSIs enter the Ph.D. program with prior teaching experience. Therefore, most GSIs who teach mathematics content courses for elementary PTs at Cardinal University might reasonably be expected to have around 2-6 years of prior teaching experience and may likely view themselves as a newer instructor to teaching elementary PTs, if not teaching in general.

I recruited one instructor, Rowan (she/they), at Cardinal University who was teaching O&NS and who identified as both a newer instructor of PTs and a newer instructor more generally. I recruited an O&NS instructor, rather than an instructor for the other two mathematics content courses for elementary PTs, because most elementary PTs' first mathematics content course is O&NS. Hence, an instructor teaching O&NS may have been more likely to experience additional challenges related to elementary PTs being new to mathematics content courses compared to an instructor teaching the other two mathematics content courses. Given multiple potential participants, I asked the instructor teaching O&NS with the least experience teaching mathematics content courses to participate in the study. Rowan had not taught O&NS prior to the semester of the study and had never taught any mathematics content course for PTs of mathematics. I describe Rowan's background and prior mathematics learning and teaching experiences in detail in Chapter 4.

Other participants in my study included Rowan's students (18 total), as well as the other O&NS instructors (2 total) and EMM instructors (3 total). Although these students

and instructors were not the main focus of the study, they played an integral role in my Rowan's experiences. For example, knowing the content of what students shared with the entire classroom, along with the conditions under which they were sharing this content, was important for understanding the interactional dimension of Rowan's teaching experiences. As such, students were included in my observations of Rowan's classroom. Other instructors were included in my observations of Rowan's instruction-related meetings, as well as Rowan's classroom, as applicable.

Data Collection

In my narrative study, I closely observed and inquired into the lived experiences of one newer instructor, Rowan (a pseudonym) (she/they), of a mathematics content course for elementary PTs, Operations & Number Systems (O&NS). In order to deeply understand my participant's experiences and learning, as well as the roles that their background, preparation, and support played within their experiences and learning, I spent considerable amounts of time with Rowan and collected several types of data (Creswell & Poth, 2018). In this section, I describe each type of data I collected, how and when it was collected, and the purposes of its collection. A summary overview of each type of data I collected is contained at the beginning of the section, along with a summary overview of how each type of data I collected connects to Clandinin & Connelly's (2000) 3-D inquiry space. A timeline for data collection is contained at the end of the section.

Table 1

Summary of Data Collection

Type of data (research question (RQ) it relates to)	Participant(s) involved	Frequency	Purpose

Demographic survey (central RQ)	Rowan	1	To gather data on Rowan's background and prior teaching experience.
Mathematics learner autobiography (RQ2)	Rowan	1	To gather data on Rowan's prior experiences and beliefs surrounding mathematics learning. This informed my understanding of the impact that Rowan's background had on their experiences teaching O&NS.
Mathematics teaching autobiography (RQ2)	Rowan	1	To gather data on Rowan's prior mathematics teaching experiences. This informed my understanding of the impact that Rowan's background had on their experiences teaching O&NS.
Teaching and Diversity, Equity, and Inclusion (DEI) Statement (RQ2)	Rowan	1	To gather data on Rowan's background and prior mathematics teaching experiences. This informed my understanding of the impact that Rowan's background had on their experiences teaching O&NS.
Classroom observations (central RQ; RQs 1-3)	Rowan; All students in the classroom	30 classes	To gather data on Rowan's experiences teaching O&NS.
Observations of O&NS instructor meetings (central RQ; RQs 1-3, primary focus on RQ3)	All O&NS Instructors (3 total)	14 meetings	To gather data on Rowan's current and anticipated challenges and successes, and the nature of Rowan's support.
Observations of Joint O&NS-EMM instructor meetings	All O&NS Instructors (3 total);	3 meetings	To gather data on Rowan's current and anticipated challenges and successes, and the nature of Rowan's support.

(central RQ; RQs 1-3, primary focus on RQ3)	All EMM Instructors (3 total)		
Observations of instructor meetings between Rowan and corresponding EMM instructor (central RQ; RQs 1-3, primary focus on RQ3)	Rowan; Corresponding EMM Instructor	1 meeting	To gather data on Rowan's current and anticipated challenges and successes, and the nature of Rowan's support.
Early semester interview - preparation (RQ3)	Rowan	1	To gather data on Rowan's preparation for teaching O&NS.
Mid-semester interview - support (RQ3)	Rowan	1	To gather data on Rowan's support for teaching O&NS. To triangulate data gathered from observations of instructor meetings.
End of semester interview - reflections on learning (central RQ; RQ1-3, primary focus on central RQ)	Rowan	1	To gather data on Rowan's cumulative learning to teach O&NS; overall impacts of Rowan's background, preparation, and support on their experiences and learning; perceptions of idealized preparation and support for them and future instructors.
Instructor reflection journal entries (central RQ; RQ1-3)	Rowan	13 weekly journal entries (no entries for weeks 11 and 12)	To gather data on Rowan's challenges and successes; Rowan's learning about mathematics, students, and teaching; impacts of background, preparation, and support on their experiences and learning. To support me in generating questions for interviews.

			To triangulate data gathered from observations and interviews.
Other artifacts (central RQ; RQ1-3)	Rowan	1 syllabus; 1 textbook; Variety of assignment descriptions	To gather data on Rowan's experiences teaching O&NS.
Descriptive memos (central RQ; RQ1-3)	Me	47 memos (30 classroom, 14 O&NS meetings, 3 Joint meetings)	To gather contextual data surrounding observed interactions throughout the week.
Reflexive memos (central RQ; RQ1-3)	Me	47 memos (30 classroom, 14 O&NS meetings, 3 Joint meetings)	To gather data on my thoughts and reflections and how they may be influencing my interpretations of prior collected data.

Table 2

Connecting Data Collection to 3-D Inquiry Space

Type of data (research question (RQ) it relates to)	Interactional	Contextual	Temporal	Other
Demographic survey (central RQ)		x	x	x
Mathematics learner autobiography (RQ2)	x	x	x	
Mathematics teaching autobiography (RQ2)	x	x	x	
Teaching and Diversity, Equity, and Inclusion (DEI) Statement (RQ2)	x	x	x	x
Classroom observations (central RQ; RQs 1-3)	x	x		
Observations of O&NS instructor meetings	x	x	x	

(central RQ; RQs 1-3, primary focus on RQ3)				
Observations of Joint O&NS-EMM instructor meetings (central RQ; RQs 1-3, primary focus on RQ3)	x	x	x	
Observations of instructor meetings between Rowan and corresponding EMM instructor (central RQ; RQs 1-3, primary focus on RQ3)	x	x	x	
Early semester interview - Preparation (RQ3)	x	x	x	
Mid-semester interview - Support (RQ3)	x	x	x	
End of semester interview - reflections on learning (central RQ; RQ1-3, primary focus on central RQ)	x	x	x	
Instructor reflection journal entries (central RQ; RQ1-3)	x	x	x	
Other artifacts - syllabus (central RQ; RQ1-3)		x		x
Other artifacts - textbook (central RQ; RQ1-3)	x			
Other artifacts – variety of assignment descriptions (if applicable) (central RQ; RQ1-3)		x		x
Descriptive memos (central RQ; RQ1-3)	x	x	x	
Reflexive memos (central RQ; RQ1-3)	x		x	

Demographic Survey

Towards the beginning of the semester, Rowan completed a brief demographic survey to gather initial data on their background and prior teaching experience.

Specifically, Rowan's responses to the survey questions provided a timeline of their teaching experiences, which supported me in understanding their autobiographies and their responses to later interview questions (see sections below). Rowan's demographic survey responses primarily provide contextual (the contexts of previous teaching and learning environments) and temporal (past) data. See Appendix A for the demographic survey questions.

Autobiographies & Teaching and DEI Statement

Towards the beginning of the semester, I asked Rowan to write two autobiographies: (1) a mathematics learner autobiography, and (2) a mathematics teaching autobiography. Rowan completed both autobiographies around week nine of the semester. The mathematics learner autobiography contained Rowan's most salient experiences learning mathematics and provided me with insight into the ways in which their prior experiences learning mathematics may have influenced the ways in which they taught O&NS, including their beliefs about mathematics and what it means to learn mathematics. The mathematics teaching autobiography outlined the breadth and trajectory of Rowan's experiences teaching mathematics content courses, as well as highlighted pivotal events in their teaching experiences. The mathematics teaching autobiography similarly provided me with insight into the ways in which Rowan's prior experiences teaching mathematics may have influenced the ways in which they taught O&NS, including their beliefs about what it means to teach mathematics. Both autobiographies primarily provided interactional (personal), contextual (the contexts of previous teaching and learning environments), and temporal (past and current) data. The prompts for both autobiographies were adapted from the mathematics autobiography

prompt created by McCulloch et al., (2010) for elementary teachers. See Appendix B for each autobiography's prompts.

At the same time Rowan sent me their autobiographies, they additionally sent me a copy of the teaching and diversity, equity, and inclusion (DEI) statement they had been using for academic job interviews. Their teaching and DEI statement provided another piece of data with which to better understand Rowan's most salient experiences and beliefs surrounding teaching and learning mathematics. Their teaching and DEI statement provided interactional (personal), contextual (the contexts of previous teaching and learning environments), and temporal (past and current) data.

Observations & Additional Artifacts

Throughout the semester, I observed all of Rowan's O&NS classes (30 observations), all of their meetings with the other two instructors of O&NS (14 observations), all of their joint meetings with the O&NS and EMM instructors (3 observations), and all of their meetings with the corresponding EMM instructor teaching the same section of O&NS students (1 observation). The purpose of observing Rowan's classes was to regularly gather data on their experiences teaching O&NS. When possible, I additionally checked in with Rowan after class to ask them how they thought the class went. These informal conversations (or debriefs) held in passing occurred following almost every class. These debriefs provided additional insight into the personal thoughts and feelings of Rowan during specific interactions with others in the classroom. These debriefs also provided opportunities for me to gather additional information on other collected data, such as observations of their instructor meetings or weekly reflection journals, as well as Rowan's general state of being: how were they doing and how was

that changing over time? Observations of Rowan's O&NS classes primarily provided interactional (social) and contextual (the context of their classroom) data.

The purpose of observing any and all instructor meetings that Rowan engaged in was to regularly gather data on their current and anticipated challenges and successes surrounding teaching O&NS, along with the nature of Rowan's support for teaching O&NS. These instructor meetings primarily provided interactional (personal and social), contextual (the context of their meetings and classrooms), and temporal (past, current, and future) data.

I did not audio- or video-record any of the observations of the classroom, planning, or instructor meetings. Instead, I wrote detailed descriptive fieldnotes during each observation. These fieldnotes included descriptive notes about what I saw and heard. I then wrote reflexive notes regarding what I thought or felt about what I saw and heard in a separate document, alongside descriptive and reflexive memos (I describe these memos below). I additionally gathered artifacts related to their teaching of O&NS, including their syllabus, textbook, and a variety of assignment descriptions throughout the semester. These artifacts provided additional data on Rowan's experiences teaching O&NS and occasionally supported me in triangulating the data from classroom observations. The textbook provided interactional (social) data, the syllabus provided contextual (the context of the classroom) data, and the variety of assignment descriptions provided contextual (the context of the classroom) data.

Interviews

To gather data on Rowan's preparation and support for teaching O&NS, I interviewed Rowan on their experiences around being prepared to teach O&NS, as well

as undergraduate mathematics courses more generally, once towards the beginning of the semester (week two). I then interviewed Rowan on their experiences around being supported for teaching O&NS once near the middle of the semester (week eight). Observations from Rowan's meetings with various instructors were helpful in triangulating Rowan's responses from both interviews on preparation and support. I also interviewed Rowan once at the end of the semester (week seventeen, the week after finals week) to gather data on Rowan's cumulative learning to teach O&NS and the overall impacts of Rowan's background, preparation, and support on their experiences and learning to teach O&NS. In this final interview, I additionally asked about Rowan's idealized preparation and support for themselves and future instructors with similar backgrounds and in similar teaching contexts. For all three interviews, I added or modified questions to my original interview protocols to gather additional information on Rowan's prior experiences, preparation, support, challenges, or successes based on-going data collection (time-permitting). These additional questions supported me in making sense of and triangulating data across sources. All three of these interviews primarily provided interactional (personal), contextual (the context of their classroom and meetings), and temporal (past, current, and future) data. All three of these interviews were audio-recorded and transcribed by me. See Appendix C for the early semester interview questions focused on preparation, Appendix D for the mid-semester interview focused on support, and Appendix E for the end of semester interview questions focused on the overall impacts of their background and preparation on their challenges, successes, and learning.

Instructor Reflection Journal Entries

Corresponding with regular observations of Rowan's teaching and meetings with instructors were weekly (weeks one through fifteen) reflection journal entries from Rowan. Once per week during the semester and for the first eight weeks of the semester, I sent Rowan an open-ended survey link to reflection journal prompts. For the last seven weeks of the semester, I sent Rowan a spreadsheet containing the reflection journal prompts for Rowan to return to me with their responses. The reason for the mid-semester shift was to flexibly support Rowan. Rowan was jotting weekly notes on what they wanted to write for their responses (knowing that the prompts would be the same) but did not have time towards the end of the semester to turn in responses to me on a weekly basis. Thus, Rowan typically submitted each of their responses for weeks 1-8 the weekend following the week of interest, or about one week after the week of interest, whereas they sent me the remainder of their responses for weeks 9-15 in bulk at the end of the semester. Rowan also did not submit responses for weeks 11-12 because they were preparing to defend and subsequently defending their dissertation those weeks and did not have time to take notes on their responses.

The reflection journal prompts provided data on Rowan's internal thoughts and perspectives on how their teaching of O&NS was going, what they were learning about teaching O&NS, and the impacts that their background, preparation, and support were having on their learning. These prompts were the same each week starting in week two (I added one new prompt in week two), so that I could see how the instructor's responses changed over time. Reflection journal entries provided interactional (personal), contextual (the context of their classroom and meetings), and temporal (past and current) data. See Appendix F for the weekly reflection journal prompts.

Descriptive and Reflexive Memos

Following nearly every observation of a class or instructor meeting, I wrote one descriptive and one reflexive memo. Hence, I wrote both types of memos approximately 3-4 times per week throughout the semester, for a rough total of 47 descriptive memos and 47 reflexive memos. Along with fieldnotes taken during observations, descriptive and reflexive memos assisted me in interpreting situations and writing rich descriptions of themes (Emerson et al., 2011; Spradley, 2016), as well as restorying events with rich contextual detail. More specifically, descriptive memos assisted me in gathering contextual data surrounding observed interactions throughout the week, whereas reflexive memos assisted me in gathering data on my own thoughts and reflections surrounding observed interactions throughout the week and how they may be influencing my interpretations of prior collected data. When writing my reflexive memos, I additionally considered the ideas described earlier in the section on my reflexivity. The descriptive memos primarily provided interactional (social), contextual (the context of the classroom and meetings), and temporal (past and current) data, whereas the reflexive memos primarily provided interactional (personal) and temporal (past, current, and future) data.

Data Analysis & Trustworthiness

Ultimately, my goal for data analysis was to utilize the various data sources I collected to support me in restorying Rowan's experiences around their teaching and learning to teach PTs. Restorying is a process of analyzing and reorganizing participant's stories according to some chosen framework. Because I was primarily interested in understanding the content and underlying meaning of Rowan's experiences, I utilized a thematic framework (Creswell & Poth, 2018) to assist me in restorying. In this case, I

used Clandinin & Connelly's three-dimensional inquiry space, as it is not only a framework for conceptualizing participant's experiences, but for analyzing and reorganizing them.

During each of my observations of Rowan's O&NS classes and instructor meetings, I composed detailed fieldnotes on what I saw and heard, as well as my interpretations of what I saw and heard. I analyzed these fieldnotes, along with the early, mid-semester, and end of semester interview transcripts, Rowan's reflection journal entries, Rowan's mathematics learning and mathematics teaching autobiographies, and Rowan's teaching and DEI statement using cycles of open-coding (Saldaña, 2016) based on broad categories related to my research questions: (1) challenges, (2) successes, and (3) prior experiences, preparation, and support. As I re-read each and every piece of data and coded ideas, excerpts, or interactions within the above categories, I organized ideas/interactions from (a) classroom fieldnotes and memos, (b) instructor meeting fieldnotes and memos, and (c) reflection journal responses into tables based on these categories chronologically and contextually. For example, an idea/interaction about a success provided in Rowan's week one reflection journal entry was organized into the table containing ideas/interactions for successes—in the row for week one and under the column for reflection journal entries. See Table 3 for an example of Rowan's (anticipated) successes for week one. Organizing these ideas/interactions into tables based on research questions supported me in narratively understanding and triangulating emergent themes. In particular, organizing these ideas/interactions chronologically and contextually supported me in situating interactions according to the contextual and temporal dimensions of the 3-D framework, which further supported me in looking

across time and/or context for themes related to Rowan's challenges, successes, prior experiences, preparation, and support.

Table 3

Rowan's (Anticipated) Successes for Week 1

Classroom Fieldnote/Memo	Instructor Meeting Fieldnote/Memo	Reflection Journal Entry
Rowan said to students that they don't know yet if the final is cumulative: I'm wondering how much the ways in which they communicate transparently with students may be challenging for them or successful for them in communicating expectations to students. (Researcher Memo—Class 1)	Anticipating: Robin asked if there were any burning issues first. I think it's good to take care of pressing matters first and let other instructors voice concerns. (Researcher Memo—Meeting 1) …they spend time talking about the important concepts, activities to do, and misconceptions students might have. I feel that these things were rarely present in my meetings from previous semesters... It helps to know what to emphasize, and which activities do a good job emphasizing those points, and especially knowing how students might deal with those concepts or activities… (Researcher Memo—Meeting 1)	Successfully prepped their first class. (Question 1) Textbook was a major resource (definitions, topics, suggested exercises), but so were the other instructors (class activities, physical manipulative recommendations) (Question 1) Anticipating: "I think the students will be fun and easy to work with. Since they want to be teachers, they'll have an appreciation and understanding for both the teaching methods I use and the teaching methods for mathematics they're learning." (Question 4)

As part of these cycles of coding, but also as I collected data and wrote my reflexive and descriptive memos, I additionally wrote analytic memos. These analytic memos supported me in hypothesizing about themes or narrative threads that highlighted important aspects of Rowan's experiences (Saldaña, 2016). Taken altogether, my collective readings and re-readings of the data, cycles of coding, analyses of the tables based on my research questions, and analytic memos assisted me in identifying rich themes and narrative threads with which to write about Rowan's experiences.

To enhance the restorying process and the trustworthiness of my narrative retelling of Rowan's stories, I invited Rowan to provide feedback on my ideas for themes and my writing. One of the main reasons to involve Rowan in the restorying process is that the participant's perspective on the chronology and writing of their stories offers invaluable insight towards conveying the meaning of their experiences in ways that feel authentic to the participant. Hence, I invited Rowan to provide feedback on any identified themes or narrative threads, including whether they felt resonant of their experiences or whether they might be reframed to better resonate with their experiences. Currently, however, Rowan is a first-time faculty member at a new institution. Therefore, out of respect for their time, I invited Rowan to provide as much or as little feedback as they felt they had the time and energy for providing, with an understanding that providing no further feedback or suggestions to me is also a way to signal their general acceptance of the version written. As of the time of this dissertation, Rowan provided no further feedback or suggestions.

As I analyzed the data and wrote about Rowan's stories, I looked for patterns, but also kept in mind that lives are messy. Clandinin (2013) suggests that authors be flexible in their writing structures, but also emphasize a few core components:

> We must in the composing, co-composing, and negotiation of interim and final research texts, make visible the multiplicity, as well as the narrative coherence and lack of coherence, of our lives, the lives of participants, and the lives we co-compose in the midst of our narrative inquiries. (p. 49)

Thus, the narrative researcher takes care to make visible the ways in which lives are complex and multifaceted (multiplicity), as well as the extent to which the written stories make sense of lives lived (narrative coherence). Using multiplicity and narrative coherence to guide me, I did my best to avoid crafting stories that were too neatly packaged and hid the uncertainty and fluidity that existed in Rowan's or others' lives.

Creswell and Poth (2018) discuss the reliability or trustworthiness of a study in terms of three different lenses: the participant's, the researcher's, and the reader or reviewer's. Although there are many ways in which one might evaluate the trustworthiness of a study, I view a high-quality study as one that leverages a multiplicity of validation strategies that span across these three lenses. Looking through the participant's lens, my study involved prolonged engagement and persistent observation in the field, as well as member-checks with the participant. Observing each and every class and meeting that Rowan taught over the course of a semester, as well as regularly asking Rowan for their reflections and interpretations of various interactions, increased the reliability of my study from the participant's lens. Looking through the researcher's lens, I collected and analyzed multiple kinds of data, including observation and interview data,

and triangulated these sources. For instance, the reflection journals were triangulated with classroom observation data, and the interview surrounding support was triangulated with observations of instructor meetings. I additionally wrote reflexive memos to clarify researcher bias. Looking through the reader or reviewer's lens, I utilized the full breadth of my data to assist me in generating rich, thick descriptions of the participant's experiences. I also engaged in peer reviews of my writing with members of my committee to support me in writing reliable, thick descriptions.

Limitations of the Study

First, it is both a strength and a limitation of my study that only one participant was the focus of my study. The strength of limiting my focus to one participant was that I was able to dedicate incredible amounts of time and energy to getting to know them exceptionally well. The limitation of only focusing on one participant, however, was that others who might have also contributed to my understanding of the issue received far less attention. Another dual strength and limitation of my study, and narrative studies more generally, comes from the fact that, ultimately, the researcher imposes meaning on participants' lived experience. Although I utilized a plethora of validation strategies, like external audits and member checks (Creswell & Poth, 2018), the re-storying process was filtered through my interpretations. This is a limitation of the study, in that the question of what stories were told and what conversation these stories contributed to, was largely up to me. I am necessarily limited to my own perspective—another researcher might piece together a different narrative, for the purposes of contributing to a different conversation. Yet, the re-storying process I engaged in is a strength of the study because an outside researcher searching for meaning in someone else's life can result in powerful

illustrations and have powerful effects (Josselson, 1996). Lastly, it is important to note that although these stories are not generalizable (nor are they intended to be), much can be gleaned from an in-depth examination of one participants' contexts and experiences. The themes surrounding Rowan's experiences allow for rich inferences into the social processes of individuals within complex social structures and for one to imagine how these experiences might exist or manifest within similar contexts (Riessman, 2008).

CHAPTER 4: ROWAN'S BACKGROUND AND PRIOR EXPERIENCES

In this chapter, I describe Rowan's prior experiences surrounding learning and teaching mathematics. First, I provide a brief overview of Rowan's background as a graduate student instructor. I then elaborate on Rowan's prior experiences learning mathematics and what motivated them to attend graduate school. In addition, I mention a special person in Rowan's background—Rowan's mom—due to her influence on Rowan's beliefs, trajectory into teaching, and (as is more apparent in Chapter 5) ongoing development as a mathematics instructor. Next, I focus on Rowan's experiences teaching mathematics prior to Operations & Number Systems (O&NS), including how they came to teach O&NS, as well as the story of their preparation for learning to teach in general. To craft this piece of Rowan's narrative, I mainly draw on experiences highlighted in Rowan's autobiographies, demographic survey, teaching and DEI statement, and preparation interview.

Instructor Background Overview

At the time of this study, Rowan (she/her and they/theirs) was a mathematics graduate student instructor in their fifth and final year in a mathematics Ph.D. program at Cardinal University. Throughout those five years, Rowan taught calculus recitations and lecture and Intermediate Algebra, as well as assisted teaching a mathematics content course for practicing teachers during a two-week summer session. In some semesters, Rowan also taught these courses from the position of associate course convenor, a role which I describe in more detail in the sections that follow. In the semester in which this study occurred, Rowan was teaching Operations & Number Systems (O&NS) for the first

time. They were also in the process of defending their dissertation and interviewing for academic jobs at several institutions.

Prior Mathematics Learning Experiences: "Social Outcast to Struggling Student"

Rowan described their experiences learning mathematics throughout most of their K-12 experience as something that usually came easily to them, but in which they often felt like a social outcast. In elementary school, they felt like an outcast because they learned mathematics procedures at a slower pace than their peers, who, as Rowan remembered, asserted that anyone still stuck on multiplication during the division unit, like Rowan, "must be so bad at math." However, Rowan's elementary teachers thought Rowan was good at math, and in middle school Rowan enrolled in advanced mathematics classes with students in higher grades. Unfortunately, this only exacerbated Rowan's feelings of being a social outcast. This time it was not because Rowan worked on mathematics at a slower pace, but because they were placed in mathematics classes without their grade-level peers; Rowan did not know anyone and often worked alone. In high school, Rowan returned to being in classes with their grade-level peers again, but for each success Rowan experienced, they additionally experienced discomfort from being teased about their success.

Feelings of being a social outcast finally started to dissipate for Rowan in their mathematics courses in college, especially in their junior and senior years. For instance, Rowan worked with a small group of peers on a research team and collaborated with a small group of peers to do homework outside of their topology class, and these were the first times in which Rowan experienced mathematics as a social activity, or at least an enjoyable one in which they felt seen and valued as a productive member of the group.

Rowan was proud of their mathematics accomplishments in college, and they did not follow up this memory with associated feelings of discomfort or embarrassment. Just pride.

In graduate school, Rowan felt the tables turn. For a long time, mathematics used to be relatively easy for them to do and socially challenging. By the time they got to graduate school, however, mathematics had fully switched to being easy to socialize about but very difficult to do. Rowan developed close bonds with the peers in their cohort, but also felt that graduate school was "a perpetual state of running to stay in place" and had since learned to accept that not every concept will make sense all the time. Rowan stated, "Every day I learn more things, but I also learn that there are so many things that I do not know."

"Getting my Ph.D. to teach"

Teaching mathematics was Rowan's main motivation for applying to mathematics Ph.D. programs, even though they were somewhat unsure about the level of mathematics—secondary or tertiary—they wanted to teach at the time. Hence, when Rowan was accepted into the mathematics Ph.D. program at Cardinal University, they were excited to learn that the university had programs for post-baccalaureate students to pursue mathematics teacher certification at the K-12 level. They reasoned that they could always apply for a master's program in education if the mathematics Ph.D. program did not make them happy. Nonetheless, Rowan was certain that they wanted to teach mathematics and knew that getting a Ph.D. was one way to do it. Rowan explained, "I'm getting my Ph.D. to teach. People think that's ridiculous because you don't need a Ph.D. to teach. But, I'm getting my Ph.D. to teach."

Rowan's Mom

Teaching had been on Rowan's mind long before applying to graduate school however, in large part, because their mom was an elementary teacher and made a significant impact on Rowan's career aspirations and their perceptions about teaching.

> My mom impacts just my teaching generally, not even in relation to this course, because my mom was a grade-school teacher and then was an administrator of preschool. And she, she loved it. My mother loved teaching. And I think that that really has impacted the entire trajectory of my life... I do know that when I was thinking about what I want to do with my life, I was like, Oh, maybe I will go into chemistry because I like chemistry, and then I'll come back to academia and I'll teach chemistry... or, oh, you know, math is really neat and with math it's like a little bit, even expected that you have to teach the whole time. So that's pretty cool... So, definitely my mom has had a big influence on like wanting to teach, and I think a lot of that comes from her passion for it, and also all the discussions we had as I was growing up about like what teaching is and this, that, and the other. So, it definitely has pushed me into, into that a lot. (Final Interview)

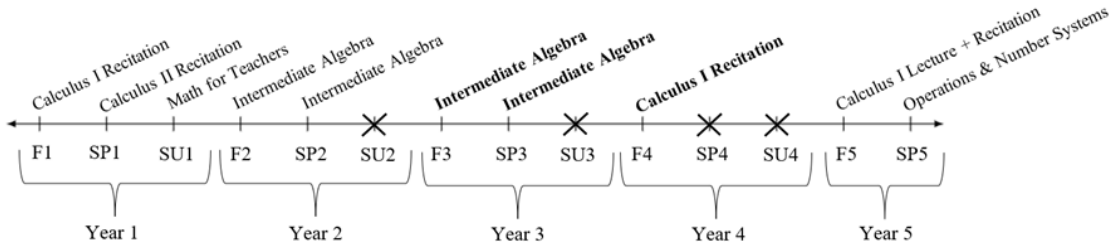
Hence, Rowan's positive relationship with their mom supported Rowan as they developed similarly positive feelings about teaching and aspirations for careers involving teaching. Rowan's mom has also had a long-term impact on Rowan's beliefs about mathematics learning, specifically the idea that, as long as the person wants to, "Everyone can learn math. Everyone can do this."

Prior Mathematics Teaching Experiences

See Figure 1 for a timeline of Rowan's prior teaching experiences—all of which were *mathematics* teaching experiences, and all of which occurred as a graduate student instructor at Cardinal University. In Figure 1, note that F, SP, and SU refer to the fall, spring, and summer terms, respectively. A term that is marked with an "X" (crossed out) indicates a term in which Rowan did not teach. Bolded course titles indicate a semester in which Rowan additionally served as an associate convenor for the course.

Figure 1

Overview of Rowan's Teaching Experiences



First Experiences—Treating Calculus Recitations Like Tutoring

Rowan described tutoring as their earliest experiences influencing their teaching. When they were in high school, they tutored students in mathematics at a low-income middle school and when they were in college, they tutored other college students in mathematics courses. From these early experiences, Rowan learned that getting students to feel comfortable enough to show their work can be "half the battle", as well as how difficult it was for Rowan to tutor people who were essentially their peers.

Rowan's first teaching experience was in their first year of graduate school at Cardinal University. In their first semester, they were assigned to instruct Calculus I

recitation[3], a course in which students are required to enroll alongside Calculus I lecture. Whereas Calculus I lecture has a large class size (around 100 students) and is where students are introduced to new material for most of the class, Calculus I recitation has a smaller class size (around 25 students) and is where students work on calculus tasks for most of the class. At first, Rowan believed they could treat Calculus I recitation similarly to how they tutored students in college. "Let the students work and just field questions." But when Rowan received a harsh comment from a student on their teaching evaluation at the end of the semester, Rowan started to feel like they should have exhibited more authority and worried that they ultimately lacked the authority needed to lead a classroom effectively.

Mathematics Content Courses for Teachers—They "remind me of my mom"

In the summer following their first year teaching Calculus I and Calculus II recitations, Rowan was an assistant instructor for a mathematics content course for practicing teachers. Rowan encountered some stressful and challenging situations in this two-week course because they needed to make a lot of decisions in a short amount of time. (See later section on Rowan's preparation for teaching in general for a detailed example.) Amid these challenges however, Rowan enjoyed working with practicing teachers.

> The students were a treat though. Since they teach, they have an empathetic understanding of the complexities of teaching. Several of the students also reminded me of my mom—my mother was a grade-school teacher, and they had similar spirits and mentalities around teaching and

[3] First-year graduate student instructors at Cardinal University are typically assigned to teach either Calculus I or Calculus II recitations.

> learning… we ended up having a lot of really productive conversations where I would give them advice on mathematics, and they would give me advice on teaching. (Rowan, Autobiography)

Through this course, and perhaps because of their relationship with their mom, Rowan developed an asset-oriented mindset towards practicing teachers—practicing teachers not only know how to teach their students but have valuable knowledge relevant to an instructor of undergraduate mathematics courses—and used this opportunity to develop their teaching practices.

Intermediate Algebra—Falling in Love with Teaching

Rowan taught Intermediate Algebra as the instructor of record for the next four semesters[4]. At Cardinal University, Intermediate Algebra does not count towards core requirements for students to graduate but is required to enroll in College Algebra in subsequent semesters (and College Algebra does satisfy core requirements for most non-mathematics majors to graduate). Rowan stated that the students in these courses, for the most part, did not have a good experience learning mathematics in high school. Rowan elaborated in their teaching autobiography that they felt that Intermediate Algebra was challenging to teach because they had to learn what their students knew so far and figure out how to get them from there to a place where "they not only understood the material but also had the confidence to realize they could do mathematics." At the same time,

[4] Second-year graduate student instructors are typically assigned to teach either Intermediate Algebra or College Algebra (with their preferences between the two considered), or on occasion a liberal arts mathematics course for non-mathematics majors. In their third year and beyond, graduate student instructors may provide preferences for a wider variety of mathematics courses; third-year instructors are typically assigned to teach precalculus courses, while fourth-year instructors and beyond run the gamut from teaching more advanced mathematics courses like Linear Algebra to returning to teaching Calculus recitations.

teaching Intermediate Algebra fostered Rowan' passion for teaching: "I always knew I liked helping students with math, but this is where I fell in love with teaching."

Convening Courses—Sometimes Invaluable, Sometimes Leads to Burn Out

For the last two semesters in which they taught Intermediate Algebra (their third year of graduate school), Rowan was additionally an associate convenor for the course. In their role as associate convenor, Rowan was responsible for coordinating all sections of Intermediate Algebra, which included duties such as setting the course schedule and syllabus, setting up the online homework system, holding weekly meetings with instructors, observing instructors' classes, and writing assessments[5]. Rowan was also mentored by a faculty course convenor and would meet with them regularly for advice on various issues related to convening and teaching. Rowan viewed the relationship with their mentor as invaluable and overall expressed a positive experience with associate convening for Intermediate Algebra. When Rowan was associate convenor for Calculus I in the following semester (the beginning of their fourth year of graduate school) however, their negative experiences engaging with a different faculty course convenor, along with dealing with the COVID-19 pandemic, led them to feel burned out. These negative experiences centered around the faculty convenor's perceptions that the Calculus I students were not doing enough and their related decision to add a large group project on to the syllabus in the middle of the semester. As a result of burn out from teaching, Rowan did not teach the following semester (though they remained a graduate student).

Calculus I Lecture—Focusing on Relationships with Students

[5] All precalculus and calculus courses, as well as other mathematics courses, such as the liberal arts mathematics course for non-mathematics majors, typically have one or two convenors assigned to coordinate all sections of the course with similar duties.

Having taken one semester off, Rowan returned to teaching in the subsequent semester (the beginning of their fifth year of graduate school). That semester, Rowan taught a special section of Calculus I lecture and recitation, in which all their students were enrolled in a scholarship program designed to increase their success in college. The scholarship program was created for students who identify with or come from groups that have not experienced as much success as other historically privileged groups (e.g., students from low-income families, first-generation college students, students who are not white, etc.). Knowing this information about their students, Rowan went into teaching this course mentally prepared to "be more of a support" for their students than they might have if Rowan was teaching a typical Calculus I lecture and recitation.

> You have to be more of a support than you would be for a classical class in terms of all sorts of things, like, students may not know what the word office hours mean, or students are coming in with, with more varied mathematical backgrounds, even if on their transcripts it says they've passed up through whatever class. (Rowan, Preparation Interview)

Overall, Rowan enjoyed their experiences teaching Calculus I lecture and recitation because of how the students in the scholarship program built their own community and knew each other really well. Rowan felt that they got to know their students really well as a result and appreciated "getting to focus on what is causing my students to maybe not be successful in the first few weeks of class or up until the first exam" and fostering relationships with students. Although Rowan yet again encountered issues surrounding how Calculus I was being convened, teaching the students in their course reinvigorated Rowan's love for teaching.

Why Teach Operations & Number Systems—Enjoying Teaching Teachers and Fixing the Pipeline

For the following semester, Rowan knew that they would be busy interviewing and finishing up their dissertation, so their first choices for courses to teach were ones that they had taught before, specifically Calculus I or II lecture or recitations. However, they also included mathematics content courses for prospective elementary teachers, because they enjoyed teaching teachers and felt that teaching future teachers is one way to contribute towards fixing issues in the STEM pipeline.

> What made me want to teach [O&NS]? Yeah, so I. So, I do really care about like teaching and teacher education...these are the people who are going to be teaching the grade-schoolers who then go on to be middle schoolers, then high schoolers, and then my college students. And if I can help them to support the students to be, like, solid in their mathematical background from the get-go. Like I'm sort of creating this machine, where instead of waiting until students get to college... students tend to come in with either a bad feeling about math or a lot of misconceptions about math even though they've probably seen calculus before. Um, so like sort of helping that pipeline so that students aren't coming to college and paying for courses that they're failing and in particular coming to college, paying to take [Intermediate Algebra], which does not meet the requirement of math classes they have to take and then having to pay to take another math class, all to get like an English degree. So that's sort of one side, another side of it is teaching... I enjoy teaching teachers and also, I feel like we can

> do better by teaching teachers mathematics, so that students like mathematics. (Rowan, Preparation Interview)

Hence, Rowan wanted to prioritize their efforts to graduate, but, at the same time, also felt multiple internal motivations to teach mathematics content courses for prospective teachers based on their prior experiences, including enjoying teaching teachers and thinking they could support systemic change in how K-12 students feel about mathematics. In any case, Rowan was assigned to teach O&NS in their final semester of graduate school.

Distinguishing Preparation from Prior Experiences and Support

In the sections and chapters that follow, I describe various aspects of Rowan's preparation and support, along with their experiences teaching O&NS. Hence, I think it is important to provide some working definitions. Essentially, the distinguishing feature between preparation and support is temporal in nature. I use the term *preparation* to refer to those areas and activities specifically intended to prepare the instructor for getting ready to teach a course *before* they start teaching it, whereas I define *support* to refer to those areas and activities intended to support the instructor *while* they are teaching the course. The distinction is important for this dissertation, in so far as it helps to differentiate what Rowan was looking back on and drawing from while teaching O&NS, as opposed to what they were currently experiencing as a part of teaching O&NS.

To understand how preparation is related to prior experiences, I use the term *prior experiences* to broadly encompass not only the preparation an instructor engaged in more directly related to learning to teach O&NS, but any of the many possible experiences that the instructor or I felt were relevant to their learning to teach O&NS, including their

preparation for teaching more generally. Hence, preparation is a part of prior experiences, but it more specifically deals with those experiences in which Rowan was intentionally being prepared to grapple with and accomplish the tasks of teaching. For instance, I categorize Rowan's mathematics learning experiences and their relationship with their mom as prior experiences, but not preparation for teaching. This distinction can be somewhat nebulous however, because opportunities to teach courses in the past are not necessarily preparing Rowan for teaching O&NS per se. However, those experiences were preparing Rowan to learn to teach, more generally—which they needed to be able to teach O&NS—and, moreover, were part of a systemic plan for graduate student instructors to be prepared to teach any of the mathematics courses that are available for them to teach at Cardinal University. What is meaningful about this distinction is that it separates those prior experiences that were specifically intended to prepare Rowan for teaching mathematics courses at Cardinal University from those experiences that occurred outside of programmatic expectations. In this way, preparation can be used to describe that which might be within the realm of possibility for professional development, as opposed to the experiences that an instructor already brings with them to their academic institution.

In the next section, I describe Rowan's preparation for teaching in general, and in the following chapter, I weave Rowan's preparation and support for teaching O&NS throughout the narrative of their experiences teaching the course.

Preparation for Teaching in General—The Backburner

According to Rowan, most of their preparation for teaching in general developed from their prior teaching experiences, particularly from teaching Intermediate Algebra

and Calculus I recitations, but also from being an associate convenor for both Intermediate Algebra and Calculus I. Additionally, their preparation to teach was a gradual process that Rowan compared to cooking, in particular shifting pots to the backburner. In their first year of teaching, they participated in a five-day orientation at the outset of the semester and taught Calculus I recitations but did not feel like they learned a lot about teaching yet. “We would have a lot of discussions about group work and how should you lead recitations? But since we were just leading recitations, I really put it on the backburner.” End of semester evaluations from students put into perspective the idea that they needed to take more responsibility for teaching and to be more prepared than they were for tutoring. Nonetheless, there were aspects of teaching and being prepared for recitation that did not come up for them in their first year. “Mostly it was, you know, which problems tripped you up, which ones were difficult to show your students and not, you know, how are you making groups? How are you starting class? What activities are you doing?”

Their preparation for teaching may have been light in their first year, but, according to Rowan, it was barely there when they were getting ready to assist teaching a mathematics content course for practicing teachers in the summer. Rowan did not receive the instructional materials for the course until the first day of class and did not have a lot of time to prepare lectures on the occasions when the lead instructor would ask Rowan to lecture:

> So... dropped into that class with no idea what’s going on, no preparation. I was a TA so it mattered less, but sincerely, like the first day of classes, [lead instructor] and I met before class and was like, Here’s this binder.

> Here's what they're doing. I will be asking you to teach a couple of the lectures. I'll let you know which ones. He would tell me, like the morning of. He'd be like, this is the one. Sometimes he'd tell me the night before, but a lot of times it was just like, alright, [Rowan] you've got this, you're going to teach this class. And I was like, amazing. *laughs* So after having only taught *laughs* recitations, I was essentially given a sheet that said, this is what we're learning today. And I had, like I had to teach the students how to do proof by induction with pretty much no preparation. A whole bunch of people that teach elementary school, not math majors, not people with master's in mathematics, just elementary school teachers. No preparation to teach them how to do proof by induction. (Rowan, Preparation Interview)

From this experience, Rowan had to prepare lectures with short notice. Because Rowan felt that the lectures went okay in this course, Rowan learned that lecturing on the fly was a skill that they could rely on in their future teaching if they needed to. The absence of preparation and support for preparing lectures in this course subtly shifted their mindset towards believing that it might be acceptable to deprioritize or even forgo preparing lectures in advance, or at least that it might be acceptable on occasion and with some notes already available.

In their second year of teaching, Rowan was enrolled in the pedagogy course required for second year graduate student instructors. The idea of the course is to provide instructors with an introduction to theories around teaching and learning mathematics, as well as to be a practical support for issues that arise as first-time instructors of record.

Note that the instructor of Rowan's course was a mathematician and may not have had K-12 teaching experience or an extensive background in mathematics education research. Rowan felt that the pedagogy course was "a big eye opener in terms of really preparing for classes." Specifically, Rowan began to think through aspects of preparing to teach a lesson, such as how to form student groups during group tasks. However, Rowan had access to many resources that made it possible for them to not necessarily need to think through developing a lesson plan. For instance, in precalculus courses and calculus recitations students purchase a workbook full of problems that are designed to line up with the course material and are expected to be completed in class as a part of lesson activities. These workbooks are created well in advance of the course by other graduate students and faculty members who are not necessarily planning to teach the course in subsequent semesters. Moreover, Rowan had access to the department's online suggested lesson plans (via a wiki) for precalculus courses and calculus recitations.

> ...the course wiki for the page, which is sort of a place where it says: Today you teach this material, here is a proposed way to set it up. Make sure you present these items. Make sure you do these problems in the workbook. So not only structure in terms of like I have all the materials I need, the students have the workbook, I have the textbook, but someone sincerely sitting down and just saying, Do. This. In. Class. Did I always follow that? No, I didn't always agree with them. But on days when I didn't necessarily have a ton of time or a lot of thought to prep class, I could just print off the wikipage. I was ready, I was done, someone set it for me, so I didn't really have to prepare for class realistically. So,

> preparation for teaching was, was minimal. I mean, I read through the notes and when I had the time I would read for math class, but I didn't always have the time, so I just didn't bother. (Rowan, Preparation Interview)

Hence, there was no pressure for Rowan or other instructors of precalculus courses and calculus recitations to learn to select and sequence tasks or to even write their own lesson plans. In this way, learning to plan lessons was largely left up to instructors who felt a need to deviate from or supplement what was already available to them. It is a task of teaching that, especially for someone newer to instruction and simultaneously trying to pass graduate-level mathematics courses, qualifying exams, and comprehensive exams, is easily shifted on to the backburner.

Other teaching responsibilities were not even cooking on the stove yet, as they were taken on by the course convenors for precalculus and calculus courses, such as selecting homework problems and setting up the online homework system for students (which would also grade students' solutions automatically), creating a shared syllabus and schedule of assignments, and writing exams. Thus, Rowan's responsibility and engagement in the tasks of teaching significantly shifted when they became associate convenor for Intermediate Algebra in their third year of teaching.

> So in that role, I'm setting the schedule, I'm writing the exams, I'm leading other people through how to teach this class and write the exams. So definitely like this big jump up in responsibility, but still a lot of structure. I mean, I wasn't starting from scratch. I was taking the last

> semester schedule and revamping it. I am putting it into the new classes. I was writing, like fresh exams. (Rowan, Preparation Interview)

Convening courses pushed them to engage in activities and grow in areas that they might not otherwise have done. In addition to writing exams and modifying a shared syllabus and schedule of assignments for the course, Rowan would talk with the faculty convenor for Intermediate Algebra almost daily, which supported them in reflecting on their teaching practices. Because Rowan's colleagues were also graduate student instructors with few years of teaching experience (or at least few years of experience teaching Intermediate Algebra), Rowan additionally observed their colleagues' teaching and provided them with feedback. Although any graduate student instructor could engage in these activities, regularly interacting with the course convenor and observing colleagues' classes are only expected of course convenors. Hence, Rowan's role as associate convenor pushed them to reflect on teaching practices and to learn how to accomplish more of the tasks of teaching than they would have if they were just teaching a few sections of a convened course like they did the previous year.

Rowan believed that the backburner mentality they experienced in their second year of teaching, especially around learning to plan lessons, hindered their growth as an instructor. Specifically, they felt that the suggested online lesson plans did them a disservice in terms of learning to teach, and as a result, emphasized providing guidance and support for the other instructors as associate convenor, as opposed to uncritically relying on available resources.

> I also did away with the concept of... Relying on the wiki for the people teaching my class because I thought it did me a disservice, like it was nice

> for a few class periods. To see that, to see how classes should be prepped, but I think it does a disservice in terms of making a teacher to say, here's exactly what you do. You are just there to enact these practices for like… Just follow, follow the list. So I did away with that and sort of I was always willing to help instructors if they needed more guidance, more support in terms of how to lead their class, but definitely sort of pushing them into thinking about it themselves. Because I was doing that for the first time as the third year I was teaching a class and I felt like that is a long time to wait before having to think about, like the timing of your class. What are you going to present? That kind of thing. (Rowan, Preparation Interview)

Hence, Rowan began taking more responsibility for planning their own lessons—shifting it off the backburner—in their third year of teaching and encouraged other instructors to do the same.

When Rowan returned to teaching a Calculus I recitation in their fourth year, however, preparing for class and planning lessons shifted to the backburner again. "I'm not teaching a lecture. I'm just doing a recitation." The decrease in responsibility coupled with Rowan's familiarity with teaching recitations meant that Rowan was not presented with much in terms of new and major challenges in learning to teach. In this way, Rowan's learning to teach more generally had also shifted on to the backburner. What had been cooking more in their third year of teaching was only kept warm in their fourth year.

Likewise, in their fifth year as a Calculus I lecturer, Rowan did not experience much in terms of their preparation to teach this course, nor much in terms of their expectations around learning to teach more generally while in this role. The structure of a convened course was still there, and as a result, Rowan did not have as much responsibility. Moreover, Rowan similarly had access to an entire semester's worth of lesson slides, provided by the course convenor. Nonetheless, continuing to grow as a reflective instructor, Rowan did not "blindly follow" them as they felt they did in their first two semesters of teaching Intermediate Algebra. Instead, Rowan used them "as a basis" for setting up their lesson and would "pretty aggressively change them." But, as a Calculus I lecturer for a section of student scholars, Rowan also noticed a silver lining to the decrease in responsibility around the more structural components of teaching.

> I didn't write the schedule. I didn't figure out the homework assignments. I didn't have to do hardly anything. All of that structure was there. I just had to think through, how do I want to teach this material to my students the best that I can? I thought that was actually really productive in terms of I could focus on what I thought mattered, which was that, and in particular, I was teaching the [anonymized] scholars. In particular, it mattered to me that I made connections with my students and that I taught them the material the best I thought I could. And so not having to think about the homework sets and the scheduling and all of that was nice. (Rowan, Preparation Interview)

Thus, because Rowan did not have to worry about setting up schedules and assignments, they felt they could use their time to focus on fostering relationships with their students and enacting lessons to the best of their abilities.

Summary of Rowan's Preparation for Teaching in General

In summary, Rowan felt that their preparation for teaching in general prepared them well for thinking through aspects of planning instruction, especially for lecturing and structuring group work. This area of preparation was primarily developed through their prior teaching experiences and the pedagogy course in their second year of teaching. Rowan additionally felt that they were well prepared for reflecting on their teaching through observing colleagues' teaching and being observed as a result of their experiences as an associate convenor. Overall, convening courses supported them in learning to teach in general because of their engagement in these activities along with regularly interacting with a supportive faculty convenor and the increase in their responsibilities, such as setting schedules and writing assessments.

Yet Rowan often experienced little preparation for teaching a course in advance of her teaching it. At the same time, they often had access to many resources during teaching, such as colleagues' lesson plans and slides. Out of necessity, Rowan learned that they could plan instruction on the fly, but they simultaneously recognized the importance of preparing instruction in advance; when Rowan stopped convening courses, they came to appreciate the additional time to focus on developing positive relationships with students and enacting instruction. For Rowan, learning to plan instruction appeared to be one of many areas related to their preparation for teaching in general that shifted on and off the backburner the most.

Although convening courses cultivated Rowan's growth and expanded their skillset, there remain tasks of teaching that they never experienced in their years leading up to teaching O&NS (i.e., some pots never made it to the stove). For example, Rowan never wrote homework problems or graded them, because for all precalculus, Calculus I, and Calculus II courses, faculty course convenors managed an online system that stores a repository of homework problems and automatically grades students' solutions. Therefore, Rowan's experiences of creating assignments were restricted to writing exams (only as associate convenor) in which they had previous semesters' exams to support them in thinking through appropriate test questions. And Rowan's experiences of grading assignments were restricted to grading quizzes or exams in which they would assign or be assigned a few questions from the exam to grade across all sections. Although Rowan gained some additional experience with evaluating and assessing students' work through experimenting with mastery-based grading in their fourth year of teaching Intermediate Algebra, the extent to which Rowan was prepared for this area of teaching appeared to be minimal.

An important consideration in this backburner metaphor is the fact that stoves, like instructors, have limited space for cooking and keeping pots warm. Moreover, at any given moment, one pot might need more attention than another, perhaps because the food inside is raw or too cold to serve. In the following chapter, I shift from considering Rowan's preparation for teaching in general to their preparation and support for teaching O&NS, as these are the main foci of research question three.

CHAPTER 5: ROWAN'S PREPARATION AND SUPPORT FOR TEACHING OPERATIONS & NUMBER SYSTEMS

In this chapter, I describe Rowan's preparation and support for teaching Operations & Number Systems (O&NS). Although some of their experiences surrounding preparation and support emerged as challenges or successes in and of themselves, these descriptions further contribute towards a continuous and storied understanding of other challenges and successes Rowan experienced. Moreover, I describe the evolving nature of Rowan's support, which provides context to understanding the challenges and successes I describe in the following chapter. I portray Rowan's experiences with preparation and support in both chronological and thematic ways and depict illustrative moments from their meetings with instructors. I describe the nature of the preparation and support that were most salient to Rowan or identified by me, as predominately expressed in Rowan's preparation interview, support interview, and reflection journals, as well as my fieldnotes and the analytic and reflexive memos I wrote following observations of Rowan's meetings with instructors.

Preparation for Teaching Operations & Number Systems

"Pretty much nothing"

When I interviewed Rowan about their preparation for teaching O&NS two weeks into the semester, they stated that it was "pretty much nothing" because they did not obtain access to key instructional resources, such as a working draft of the syllabus, in advance of their first joint meeting with all the instructors teaching O&NS, including graduate student instructors Rose and Robin, and all the instructors teaching Elementary

Mathematics Methods (EMM), including graduate student instructors Willow and Cypress and faculty instructor Juniper[6].

> Okay. The preparation. Okay then. Pretty much nothing, we had a meeting with everybody, which was more of like a, get to know everybody, do some, some basic like, make sure we're on the same page about different activities. But because I didn't get a copy of the syllabus until that meeting, I had no idea what activities they were talking about. So, I really had to lean on, on [Robin] and [Rose], who have taught the course before and are teaching it with me again. They are teaching it again. I'm teaching it for the first time this semester, so at least like, they knew what those things were, but I just sort of blindly was like, hey, whatever feels good. And in an email chain, when we were setting up that primary meeting, [Robin] even said, they were like, "I taught this course probably the most out of everybody. And it's like, if you need me to, I will sort of take the lead" and I'm going… "[Robin] you do, you do you. You do it. And I, I will take that support. You figure out the scheduling syllabus. I will let you." We based the schedule off of. [Rose] took the fall schedule and she started to like, reorient it for our course and then she sent it out so that we could talk about it in the big group. And essentially, [Robin] and I just said, "yeah, this looks good. Let's do it." I have no idea if I should be agreeing with the schedule, realistically. But I have a schedule. I suppose technically, I have a schedule that was given to me, which is similar to

[6] All names are pseudonyms.

> previous preparations, but I didn't get them until the very start of this semester. (Rowan, Preparation Interview)

The timing of Rowan's (in-)access to instructional resources was significant in part because it impacted their ability to contribute to curricular decisions during the pre-semester meeting among the O&NS and EMM instructors. Without access to materials like a draft of the syllabus and assignment descriptions, Rowan was not well-positioned to discuss (de-)constructing the schedule of assignments with colleagues. Yet the structure of this first meeting was to (1) take turns introducing yourself, including naming your favorite shape, (2) choose which assignments to add to or remove from the joint syllabus, and (3) decide on the frequency of future meetings involving all O&NS and EMM instructors. The sudden and immediate need to create a schedule without resources and with less than a week before classes started made Rowan feel unprepared.

Obtaining Access to Resources—"I just feel like more time would have been useful"

In addition to the syllabus and schedule of assignments, Rowan did not have access to a repository of instructional resources, such as various instructors' lesson plans or assignments from prior semesters, nor did they obtain access to a large repository later. Towards the end of the first instructor meeting, Rose asked Rowan if there was anything they needed to get started. Rowan responded that they thought they had enough to get started, especially since they had managed to obtain an online version of the textbook, but Rowan seemed to not want to bother the other instructors. As the meeting continued, it occurred to Rose to share information about online manipulatives and physical manipulatives they personally purchased, and Rowan could use, if desired. But access to

resources beyond those that came up naturally through conversation were not offered during the meeting.

Typically, when Rowan received access to instructional resources, they were shared in small batches for a specific, timely purpose. For instance, Rowan asked Rose to share a couple of lesson slides with them as a support to plan their own first lesson. And at various points later in the semester, usually a few weeks in advance of the assignment due date, Robin and Rose shared with Rowan a few assignments from previous semesters, so that the instructors could collectively decide on which assignment all O&NS students would complete. These instances occurred somewhat sporadically however, as Rowan did not usually ask for supplementary instructional materials, and Robin and Rose did not usually share resources unless a specific necessity for sharing occurred. Hence, Rowan's access to resources often relied on either Rowan knowing what to ask for and feeling comfortable asking for it, or the other instructors having the foresight to offer Rowan various resources unprompted.

Rowan recognized their own agency in obtaining access to resources, as well as some of the ways in which their choices hindered potential for communication. Specifically, prepping lessons the night before class made communicating with other instructors difficult. At the same time, Rowan felt that the night before was when it worked in their schedule to plan lessons, and also that there was not enough time for them to reach out to instructors given the time they needed to spend unpacking unfamiliar mathematics content.

> I just feel like more time would have been useful. It's not that I, it's not that I don't or couldn't get the resources that I need. A lot of times it's

> more just. In terms of my schedule, the way it works out, I didn't get the materials until right before the semester started, I couldn't backlog prepping a few courses. I am prepping them the night before. I'm not going to email the other [O&NS] instructors after 10 p.m., expecting them to respond, before my class at 9am, so I'm just sort of doing it. A lot on my own right, and I talk… it's so important to have a conversation about teaching, so important to be familiar with the research, to communicate, do all this communication. And I'm not doing that. And I feel like a lot of it just has to do with that time crunch. It's not that I don't want to, or that I don't think those activities would be useful. It's that because of having to teach, you know, material that I'm not super familiar, I mean, I'm familiar with the counting numbers and the construction of the real line, but I'm not super familiar with like. How do you want to construct this for the students? (Rowan, Preparation Interview)

Rowan went on to explain how, recently, they had been reconciling their understanding of the construction of the number line with the presentation of material in the textbook.

> In fact, I was actually taken aback by the construction of the real line because I am so used to. Counting, okay, you count things whole with the zero and then you do it all based on operations, so, you know, constructing the number line is this idea of you have the number one you add to get to, you know, infinity positive. You subtract to go to infinity negative, you use multiplication in sort of a weird way and then you use division to construct the fractional or rational numbers. And that that is an extension.

> And then like the activity we did today, is there a number between each decimal? Yes, that is the extension to irrational numbers as well. And that is. We say that we want students to think like mathematicians, but saying, hey, we're just gonna kind of exclusively focus on base-ten is not thinking like a mathematician, so it's like a weird, I'm following the textbook because I trust the textbook discourse. I trust that it's all very important stuff. But I find this like weird disconnect between how I learned material and how it's like widely accepted that that material was constructed…
>
> And like, it's hard… hard to prep for that. (Rowan, Preparation Interview)

Overall, Rowan's point was that, even though they had the ability to reach out to other instructors, they did not necessarily feel like they had the time to do so, given that they prepped for classes the night before and given that they needed to spend their time reconciling their perspective on mathematics with the perspective of mathematics presented in their textbook. As a result, Rowan wished that they had time before the semester started to engage in this work of unpacking the curriculum of the textbook.

Relatedly, obtaining access to the textbook was a struggle for Rowan, which additionally prevented them from examining the curriculum in advance of the semester's start.

> I had to actively reach out to figure out what is the textbook for this course... I need to bug [staff] about this because I honestly haven't even asked for a copy of the, they realistically don't even have a physical copy of the textbooks, which the department is supposed to provide... I haven't actually gone and bugged [staff] about it. I don't. I don't go to the office.

> This is kind of the thing. This is kind of the issue with ’his seme’ter is, because of Covid, I don’t go to my own building. I don’t go to my office. I don’t have any reason to, so I don’t do it. And so not only did I not come to campus the week before classes, I even couldn’t because I didn’t get my re-entry test. Right, so I couldn’t have physically gotten a copy of the textbook until, Thursday before classes started at the earliest. And then they didn’t have one. So, *laughs* I’m sitting here in this class where I have no materials, I have minimal support. I don’t even have the textbook, which it turns out once I got a copy of the textbook, it turns out the textbook is very good at sort of telling you how to run the course, what material to cover, but I’m sitting here a few days before classes start and I have no idea what’s going on. I ended up getting the textbook in a very roundabout way... I have a friend whose mom teaches grade-school, and her school provides her with the subscription-based education textbook service. So I am logged in to my friend’s mom’s account to read this book online. (Rowan, Preparation Interview)

Although Rowan felt a responsibility to “bug” someone to acquire the textbook, some of the challenges they faced were beyond their control. Restrictions on entering buildings due to Covid-19 obstructed Rowan from checking in with staff in-person, but even then, no textbook was available. Rowan likely anticipated that they would be contacted about the availability of their teaching materials but did not expect that they could not obtain them and with only a few days before the start of the semester.

Similar to Rowan's experiences in Intermediate Algebra and calculus courses, one instructor, Robin, acted as an instructional leader even though there was no formal convenor for O&NS. What is particularly unique about this circumstance is that Robin was not a faculty member (neither was Rose), and therefore broke precedence with a structure that typically presided over the course for many years at Cardinal University. Recall from Chapter 3 that, usually, one faculty member taught one section of the course and acted as convenor, while the remaining two sections were typically taught by two graduate student instructors. Hence, Rowan's preparation for teaching O&NS was likely somewhat different from a typical graduate student instructor teaching this course, though in some ways similar to what might be expected. Specifically, Rowan's prior teaching experiences indicate that, even if a faculty member lead this course with Rowan, they would likely still have only met with fellow instructors one week or less in advance of teaching the course. However, it is also likely that Rowan would have obtained access to a large repository of instructional materials (though not necessarily a full semester's worth of lesson plans or slides) directly from a faculty convenor at some point during the semester, if not right from the beginning.

"I'm piecing together course content as I go"

In addition to the inadequate timing and absence of resources, Rowan felt that their preparation was "pretty much nothing" two weeks into the semester because, according to Rowan, they were not provided with a structured introduction to the purposes and goals of O&NS. Indeed, in the pre-semester meeting with all the O&NS and EMM instructors, no one provided such an introduction, either to the shared purposes and goals between the two courses or specifically for O&NS, and no such introduction came

in any of Rowan's subsequent instructor meetings. The day after the pre-semester meeting, I had independently wondered if Rowan had discussed overarching goals of O&NS with anyone in an analytic/reflexive memo.

> Have they met with anyone outside of this meeting to discuss things like, the purpose of the course they are teaching, the most important things to emphasize to help prospective teachers understand, or how one might go about structuring a lesson? Does [Rowan] understand that they are not just teaching math content, but student strategies and misconceptions about content, as well as the tensions between conceptual and procedural understanding? Has [Rowan] chatted with other GSIs about this course yet or are they coming in dark? (Kelsey, Researcher Memo)

As a result of the absence of an introduction to the course, Rowan developed an unsteady and uncertain mentality about teaching the course. In their week one reflection journal, Rowan emphasized their growing uncertainty, "I feel like I'm piecing together course content as I go instead of having a clear view of the course and the assignments." In week two, Rowan's growing uncertainty seemed to fuel Rowan's displeasure towards the ways in which mathematics GSIs are generally assigned to teach courses:

> More of an understanding for me about what I'm teaching and why I'm teaching it would be nice because I was just sort of tossed into the course with everyone expecting that I kind of know what I'm doing, and I *don't*. Not everyone who knows math is a good math teacher. Now, I don't think that's true for me. I like to believe that I'm a fairly good math teacher, but it's sort of, it's. It's really an unfair assumption to make that you can have a

> graduate student randomly picked out of the pool assigned by [faculty member] to [O&NS] and that you can drop them in this environment and that they are going to create a class that is *good* for the students. The way that you can kind of expect, like, a calculus course—it's really hard to, like, mess up a calculus recitation, right? But it would be so easy to mess up this class. I feel like. Even though it's so prescribed in the textbook, I feel like it could be really easy to mess up a lot. I worry about that. (Rowan, Preparation Interview)

Rowan continued to stress this uncertainty as the semester unfolded. For instance, as Rowan was considering the various resources they had access to during week eight of the semester, Rowan noted again that they did not fully understand the course objectives.

> There are course objectives and learning goals. There's a syllabus, I would say that I have no idea what the objectives of this class are. No one has really made that clear to me, but not like… There's a vibe in the book. Learn the four basic functions, but that's not really a course objective or a learning goal. (Rowan, Support Interview)

Less Prepared for Providing Feedback on Student Work

Along with identifying lesson or course objectives/goals, Rowan identified creating assignments, adapting and modifying the course, and providing feedback on student work as areas that they felt less prepared for with respect to teaching O&NS during the preparation interview. Rowan felt only slightly more prepared to evaluate and assess student work than the aforementioned areas, though overall still wary about it. Two weeks into the semester, part of the wariness manifested because Robin and Rose

were assigning homework differently from each other (e.g., Rose opted for homework to be completed in groups, while Robin opted for homework to be completed individually). More so though, Rowan had never graded homework. The combination of no prior homework grading experience and low perceptions of unity between the other two instructors regarding homework decisions, made Rowan feel undirected and unsettled about how they wanted to create and grade their own section's homework assignments. However, Rowan explained that they were in the middle of testing out multiple rubrics based on grading systems they had tried as an instructor for Intermediate Algebra, and because of their prior experiences with trial and error, Rowan seemed slightly more optimistic about evaluating and assessing students' work. At this point however, Rowan felt less prepared for writing feedback on their students' work, and overall unsure whether Rowan could successfully adapt or modify the course during the semester, if they needed to.

The primary reason Rowan identified providing feedback on students' work as an area they felt less prepared for with respect to teaching O&NS is that, even without a structured introduction to the course, Rowan perceived this course as new and different. Two weeks into the semester, Rowan was struggling to figure out how to grade and write feedback on student work because it involved more variety than the assignments they were used to in the past. Specifically, Rowan was just starting to realize that many of the solutions to the exercises in students' O&NS assignments involved explanations with math drawings, whereas in the past, students' assignments typically involved calculations leading to a single answer. For instance, at Cardinal University, a standard question from an Intermediate Algebra exam might involve finding all possible solutions to an equation,

whereas a standard question from an O&NS exam might involve considering possible reasons a K-6 student might identify an incorrect mathematical relationship, along with possible ways to illustrate to a K-6 student the correct relationship (see Table 4).

Table 4

Sample Exam Questions for O&NS and Intermediate Algebra

O&NS	Intermediate Algebra
Consider the following problem: Kelsey wrote on her paper that 1.24 > 1.3. What is a possible misconception that Kelsey may be reflecting in her work? Make an accurate base-ten drawing which you would use to help explain to Kelsey the correct relationship between the two numbers. How would you use the drawings to help Kelsey understand the correct relationship between the two numbers?	Solve the following equation and state the number of solutions. $\frac{x+3}{3}+\frac{3x}{2}=\frac{5x+7}{6}$

Moreover, perceived differences in their own content expertise further contributed to their sense of uncertainty in providing feedback on student work.

> I have not graded the Homework 1; I am not ready to provide feedback on their work. I, I can tell them if it's right or wrong, but like a lot of them are like think-through questions and there's a good chance that they're going to think it through in a way that's totally valid, but I just am not going to know how to provide feedback…I'm pretty good at it for, like say, a calculus course, something where I feel like I'm an expert in the material. I'm a little nervous about providing feedback on student work where I am not necessarily an expert on the material… I don't have expertise on, how are students going to think incorrectly about comparing decimal values? I

> can tell you which one's bigger and I can tell you why it's bigger and I can correct my students' understanding of why it's bigger. But it's hard to correct my students saying, "this is how I think someone would think about it incorrect(ly)." Like that is, at so many levels, like it's. Oof. Um. I will get more experience with this, but evaluating, providing feedback on student work, I will get more experience. Hopefully that goes well. (Rowan, Preparation Interview)

Rowan seemed to accept the dual problem of grading and writing feedback as a challenge, but a productive one. As the semester continued, however, Rowan came to recognize the differences in their experiences more fully and felt more and more overwhelmed by their situation, as I describe in a number of sections in Chapter 6. In the next two sections, I describe a few areas and activities that Rowan identified as beneficial and supportive with respect to their preparation for teaching O&NS.

Listening to and Learning from K-12 Teachers

Although Rowan emphasized more strongly the areas and activities surrounding their preparation for teaching O&NS that felt insufficient, Rowan also identified areas and activities that felt beneficial and supportive. Specifically, enrolling in graduate-level education courses through Cardinal University's teaching and learning department offered Rowan opportunities for structured conversations around teaching from the perspectives of K-12 teachers. Rowan expressed a strong desire to listen and learn from K-12 teachers' experiences and viewed these opportunities as potentially supportive of their growth as an instructor of prospective teachers.

> It's a little interesting taking the [education] courses because it is like grade school, I'm working with grade-school teachers for the most part… I think it's important and I always wish that I knew more about how my students were learning math up until I see them. And it's hard to get like a good view on that without just like finding grade-school teachers to ask. (Rowan, Preparation Interview)

Although hearing about their peers' lived experiences was not always a part of their coursework, Rowan felt that their additional coursework in education prepared them to engage in reflecting on their teaching practices in O&NS.

Optimistically Prepared for Planning Instruction

Rowan lastly highlighted planning instruction as an area they thought they might be well prepared for with respect to teaching O&NS, albeit with some hesitancy and self-doubt, but also optimism.

> So, in terms of knowing how to plan instruction for previous courses and then having this textbook that does sort of outline the expected way to teach it, I feel prepared for it. I wish that I had more time to think critically about it, but it's not that I can't do it… I haven't had, not that I'm not prepared to do this, I haven't put into my planning of instruction a lot of chances to communicate meaningfully with students yet. I haven't hit very many good activities to insert myself in their groups… I'm good at a lot of things, but I feel like I'm not. I don't know. Yeah, this feels different, and I'm not great at it. I would say I'm pretty good at selecting the tasks and participation structures with the material that I have been given. I have

> done a pretty good job, I think, at picking the right things to do and how to time them. I've been getting a little better. (Rowan, Preparation Interview)

When Rowan reflected on their preparation for teaching O&NS at the end of the semester, they felt that they were really "grasping at straws" in terms of what they could actually do. By the end of the semester, Rowan instead emphasized that they had more to learn and could be much better at planning instruction.

Support for Teaching Operations & Number Systems

Eight weeks into the semester, I interviewed Rowan about their support for teaching O&NS. They described all the ways in which they were supported to teach O&NS, including the support they found most and least useful, as well as the support they wish they had. In this section, I describe Rowan's support, including the nature of their support, the benefits and limitations of their support, and their ideas for additional support.

Meetings with Operations & Number Systems Instructors

Rowan first and foremost identified the weekly meetings with O&NS instructors as their "biggest" and most useful support. Before specifically describing what Rowan found useful about these meetings, I describe the meetings themselves.

Robin always led the meetings and started each one by asking Rowan and Rose how things were going or whether they had "anything super bad happening", "crises", or "concerns" that they needed to "vent" or "chat" about. Sometimes, Rowan and Rose gave short answers, for instance, that things were going well, that they were a bit behind schedule, that they had not graded exams yet, that some students were out with COVID, or, later in the semester, that students were getting a bit burned out. Otherwise, Rowan

and Rose shook their heads with nothing to say, especially if Robin opted for a question like, “any crises?”

Immediately following their brief check-in with Rowan and Rose, Robin stated their collective plans for the next week, including which sections of the textbook they all aimed to cover or whether they had an exam review or exam planned for the next week. Robin then proceeded to describe critical mathematics concepts or connections among concepts, as well as in-class activities (usually from the textbook) that they thought were important for each of that week’s sections. Meetings were typically 35 minutes long, and a little less than half of that time was spent with Robin discussing concepts and activities.

When discussing concepts and activities, Robin often commented on what they thought the textbook did or did not do well, and which activities they thought were most beneficial or could be skipped and why. Robin additionally articulated their memories of how their previous O&NS students interpreted certain activities or mathematical ideas. In the first O&NS instructor meeting, Robin invited Rose to “pitch in” on these discussions whenever they had anything to add, and usually, Rose chimed in with their own perspective on what they thought was important or how their prior O&NS students interpreted mathematics concepts. For instance, Robin’s discussion around base-10 activities characterized many of their discussions, both in terms of who spoke, and in that the content of the discussion emphasized identifying certain concepts or activities as important or challenging for students.

> Robin says the number line stuff is the most important and that decimals are the natural extension of base-10. Robin says the number line stuff always comes up on exams, so make it clear that it’s important. Robin

> suggests that it might be worth covering the 1.1 and 1.2 number line content and activities at the same time. Rose adds that students really have a hard time with the number line question from prior exams and on the final. Robin says Activity 1D is fairly important because it connects the idea that objects are bundled in groups of 10 in a really natural way. Robin says let me warn you about Activity 1D #3 and about the fact that students forget to write down their key (e.g., 1 dot = one; 1 stick = 10; 1 square = 100). Robin shares their screen and says that some students go backwards, as in they might say that 7.31 is represented by a group of a hundred sticks, 30 sticks, and 7 sticks. (Researcher Fieldnotes, Week 1 Meeting)

Most of the time, Robin and Rose agreed with each other about which concepts and activities were important and why, and built off each other's ideas, though sometimes Rose would disagree with Robin and provide an alternative perspective. For example, consider their discussion around teaching equivalent fractions, in which Rose convinced Robin that repeating vertical divisions of equal sized parts on a fraction strip is useful for understanding magnitude and unit fractions:

> Robin talks about how they "don't 100% like [textbook author]'s treatment of equivalent fractions." Robin says let me show a different drawing to use instead. Robin opens up a Zoom whiteboard and draws a rectangle vertically split in 3 equal parts. Robin says this is [textbook author]'s treatment: they shade 2 out of 3 parts, and then 2/3 = 4/6 by vertically subdividing each of the 3 parts into halves. Robin says that is a reasonable thing to do, but Robin personally prefers to do something

> where instead of doing vertical subdivisions, they do horizontal subdivisions. Robin says this is because this diagram helps with doing multiplication of fractions, so getting students used to this kind of drawing is useful. Rose says "I'm going to go into the reason that [textbook author] does this. It relates to Common Core." Robin says "Really?" Rose says yes, the unit fraction emphasis is the reason that [textbook author] does this. Rose says it's a magnitude concept. Rose says they are measuring the same strip with 1/6—you can see the doubling of both the number of total pieces and the number of pieces that you have out of the total pieces. Rose says it helps students to see the idea of multiplying by equivalent units. Robin says they will probably need to do that then. Robin says, for equivalent fractions, sticking with strip diagrams apparently has a good pedagogical reason. (Researcher Fieldnotes, Week 3 Meeting)

Note that Rose was a mathematics graduate student instructor with 11 years of mathematics teaching experience in grades 7-12 and was specializing in mathematics education in their Ph.D. program. Specifically, Rose had spent time reviewing research literature related to students' learning of fractions, among other K-12 mathematics concepts. As a result, Rose occasionally discussed connections between research literature and mathematics concepts in meetings. For instance, Rose shared a method for multi-digit multiplication they found in a research journal:

> Rose says that they think the partial products method is actually trying to get the structure into students' heads. Rose says then students don't have to think, "oh I have to put a zero here." Rose says they think that's part of

> what we're seeing, but it's also international… Rose says here, this is a journal, and they can send Robin and Rowan a copy of it. Rose says they're showing the method this way. Rose shares their screen. Rose says this is out of a journal. Rose says they're saying this is the thinking—they're saying you can do it from left to right, but you can go from right to left. (Researcher Fieldnotes, Week 9 Meeting)

As previously illustrated in Robin's discussion around base-10 activities, both Rose and Robin often drew on their prior experiences working with students, usually prospective elementary teachers, as they discussed teaching mathematics concepts. Because Rose had experience teaching 7-12 mathematics, as well as homeschooling their own kids and tutoring other families in K-6 mathematics, Rose sometimes drew on their experiences teaching K-12 mathematics concepts to K-12 students. For instance, Rose demonstrated a way to think about dividing a fraction by another fraction that came "from years and years of tutoring with students" (Rose, Week 13 Meeting). Their method involved thinking about ½ divided by 2/3 as equivalent to multiplying both the numerator and denominator of ½ by 3/2 (i.e., a clever way to multiply by 1).

When the O&NS instructors were not discussing mathematics concepts or the selection of in-class activities, they were, for the most part, either selecting and editing the roughly bi-weekly assignments for students to complete (from an electronic folder containing assignments from previous semesters), or they were collectively designing exams from scratch and editing them for clearer wording, student-friendly formatting, consistency in points/values assigned across problems and across prior exams, and length

considerations, such as adding or removing problems based on desired content coverage versus time constraints.

Because Robin led these meetings, their structure and content largely depended on Robin's agenda. Furthermore, Robin intentionally designed these meetings to provide support they thought would be useful for instructors based on their own prior experiences in O&NS instructor meetings. On one occasion when Robin and I were waiting for Rowan and Rose to join the Zoom call, I asked Robin what they remembered about their previous meetings with O&NS instructors and Robin responded with how they were doing things differently:

> I remember [a previous O&NS course convenor] rushing through things and not really talking about content and (not) talking about here's the sections we should be covering, here's what we talk about for Habits of Mind (assignments). I have tried when I'm convening things to be a little bit better than that. (Robin, Week 7 Meeting)

Moreover, Robin intended to support Rowan as a newer instructor. For example, when Rowan was out of town interviewing for jobs, and, as a result, missed the week two meeting with O&NS instructors, Robin decided to send Rowan a curated list of recommended in-class activities. Part of Robin's inspiration for sending Rowan this list, as well as more generally discussing content and activities in the meetings, was to ensure that Rowan did not miss out on covering certain content or activities that might show up on an exam. Robin experienced this in a prior semester with another course convenor and did not want Rowan to have a similarly frustrating experience.

While Robin specifically named incorporating conversations about content and Habits of Mind assignments into the meetings, it also seemed that asking about instructor concerns was on Robin's mind from the start of the semester as well. At the beginning of the week one meeting, Robin stated that "the most important thing right now is crisis management if anyone is having a crisis" (Robin, Week 1 Meeting), and, as previously mentioned, Robin led every meeting thereafter with a question intended to briefly check-in on the other instructors.

Another key observation from these meetings was that, from the first meeting until the fourteenth and final meeting, Robin and Rose were the primary contributors to every discussion. Rowan was often quiet during these meetings, but especially during discussions of mathematics content and activities. At some point, Robin likely only expected Rose to contribute, as this interaction indicates:

> Robin says, as far as activities go, they think all three activities are worthwhile. They also think it's worth having them write their own word problems, which they don't think any of the activities do. Robin asks if Rose has anything to add on. Rose shakes their head. Robin says, no. Okay. So, rounding up or down. It is worth having students write word problems… (Researcher Fieldnotes, Week 11 Meeting)

During the support interview, Rowan described their own participation during instructor meetings as them "observing a discussion." According to Rowan, the main reason they did not contribute to discussions was because they had not previewed the next week's content or activities in advance of the weekly meeting. Rowan explained this as they elaborated on the variety of support they had access to:

> Colleagues I go to, to discuss the textbook. Well, it's technically what we do in the meetings. I wouldn't call it a discussion, but I am observing a discussion during that. But I could probably contribute if I read those sections before the meetings. (Rowan, Support Interview)

Nonetheless, Rowan found Robin and Rose's discussions around content and activities useful, especially when they involved students' actions and interpretations of the material.

> So, in terms of like them telling me sort of things to expect or things that is going on in their class, I think is a helpful support. So, right, they tell me whether or not they're behind in class, how their students, sort of an idea of how students are doing on things. And then also. Sort of like tips and tricks for you know, this material is going to go maybe this way or I've had students misinterpret this this way, that kind of stuff, that's been really useful. (Rowan, Support Interview)

In alignment with Robin's intentions, another reason Rowan found these discussions useful was because they often provided insight about which activities Robin and Rose were going to emphasize on the exam, and Rowan wanted to make sure that their students were well prepared.

> They suggest which activities to do and not do. I've been keeping an eye on those, mostly because they then pick those activities to make exam questions off of, and I want to make sure that my students aren't disadvantaged. But I don't necessarily use it as a way to like structure my class. I definitely use it as like a, "Oh I need to make sure we cover this,"

> because I'm pretty sure we, when we sit down to talk about Exam 2, someone's going to say, "we should do a question like this". And I want to make sure my students are ready. (Rowan, Support Interview)

Indeed, the connections among content, activities, and exam questions was often explicit. Recall that during the week one meeting Robin stated, "the number line stuff always comes up on exams, so make it clear that it's important" (Researcher Fieldnotes, Week 1 Meeting). Similarly, during the week eight meeting, Robin concluded a discussion saying, "I think we've covered what gets talked about in section 4.5. Activities 4K, 4L, 4M—that's basically practice in doing math with mental things. That is a common exam question in my experience" (Researcher Fieldnotes, Week 8 Meeting).

During the support interview, Rowan described having colleagues with whom and communities in which to talk about teaching as some of her most useful support for teaching O&NS. Rowan explained that "teaching is just such a social thing, and sometimes you need different perspectives on how that's going" (Rowan, Support Interview). Indeed, Rowan valued these meetings so highly because they offered opportunities for Rowan to listen to others' experiences and obtain a variety of perspectives. Rowan further elaborated:

> the access to those communities and colleagues has been very important. Like a lot of times, it's even just a, "this is what happened in class today". Oh, my goodness, and then them being like, "yeah [Rowan] that's fine, that happens in class." *laughs* I feel this is really useful, actually for this class (Rowan, Support Interview)

The Textbook

Rowan secondly pointed to the textbook as their other main support for teaching O&NS during the support interview.

> Besides that, my biggest resource, I would say, is the textbook, which I truly structure my entire class off of—what is in the textbook. I sit down and read the chapter, I read the exercises. I put the slides together with a lot of texts adapted from the textbook. The exercises are just screenshots of the exercises put on the slides for us to do in the class together. So that truly structures like my day to day like what? What material am I gonna cover? That kind of stuff. (Rowan, Support Interview)

Rowan further emphasized why they found the textbook to be such a significant support—it was an essential resource, without which Rowan "could not teach."

> The textbook, I mean, I followed, my lesson planning has come straight from this textbook. I could not teach it. I cannot conceptualize of what I would do to put this class together, if not for the textbook. Except for something in place of the textbook that is essentially the textbook. Right, activities and material. I need something to go off of. There is not a comprehensive file or resource sharing to provide those materials to me in any other way. (Rowan, Support Interview)

Ultimately, Rowan recognized the importance of a "comprehensive" set of curriculum materials in their support for teaching the course. Hence, they noted that a digital repository, much like ones they used in the past, could function in place of the textbook. However, it may be because Rowan did not have access to a comprehensive digital repository that the textbook stood out as so starkly important to them.

> This textbook is really important just because it is the only consistent material that I have when it comes to this class. I know that the entire textbook is there. I know that it has everything right there. I can access it without going through any social interaction. It's there. It's static. I have exercises. I don't nece(ssarily) like, this is the kind of class where, maybe you don't need this textbook or maybe you don't need *a* textbook, but you need some comprehensive set of materials, and the textbook provides that. (Rowan, Support Interview, emphasis original)

In addition to the textbook's comprehensiveness and consistency, Rowan likely also developed a sense of reliance on the textbook due to their inclination to not want to bother the other instructors. Rowan appreciated that they could access the textbook "without going through any social interaction." Yet, Rowan also wished to reach out to the O&NS instructors for additional planning support at the beginning of the semester.

> I wish I had someone to go over my notes and lesson plan before I teach a section since I don't have any experience with this particular material. I think the other [O&NS] instructors would if I asked but it's a lot to ask for. I'm trying to trust my gut and the textbook. (Rowan, Week 1 Reflection Journal)

Desired Support

A "Static" Resource-sharing Repository

Although Rowan found inherit value in both the O&NS instructor meetings and the textbook, they also valued them out of necessity. One of the main reasons they were both so valuable was because Rowan relied on them for creating assignments and lesson

plans. Eight weeks into the semester though, Rowan lamented not having access to a "static" repository of materials that could support them in doing these activities.

> We write our own exams, and I don't have access to the old ones. And databases, I don't know, does this course have a wiki page? I don't—if it does, I don't have access to it… We have some resource sharing. We share some files. We share some stuff. I wish that there was something more like, more like how there, for a lot of courses you do like a full Dropbox with all the old exams, all of the old Habits of Mind… all of the old syllabi. I don't know why this class doesn't have that. I mean, clearly someone has to motivate that and do that. But like, something static, that's not necessarily just teacher-to-teacher… There should be like an instructor-devoid sense of file management that does not rely on, for instance, what would have happened this semester if there were three "me"s teaching this class? Something that has crossed my mind. What if there are three people who have never taught [O&NS] teaching the class? We would have no idea what we're doing. Literally zero clue. And no. Who? Who do you go to for the materials? Who do you ask for this and that? What if? What if all the files were only accessible by [faculty member] and then [they] left? Where does the class go from there? You know, like, so something a little more comprehensive and a little less socially gathered in terms of file resource sharing. (Rowan, Support Interview)

Note that Rowan's desire for a resource-sharing repository was explicitly connected to their prior experiences teaching other courses. In the past, Rowan had full access to wiki pages containing a semester's worth of lessons and file sharing systems with old assignments and syllabi from multiple semesters. But in this semester, Rowan needed to ask Robin and Rose for materials as the occasions arose. As a result, Rowan wanted access to a bit more resources than they had.

Grading Discussions—"A structure that just should be there."

Again, Rowan generally found meetings with O&NS instructors useful, but there was one primary support that they wished the meetings could have provided. Specifically, Rowan wished that the O&NS instructors discussed their grading choices during the meetings.

> Yeah, I want to put like opportunity to discuss assessment, to discuss assessment. I know that this feels like. It's sort of here, like colleagues I go to for assessment materials. I have them, but I feel like we run out of opportunities to discuss assessment, because none of us have graded our exams by one of the Thursday meetings and then we all pass exams by the next (meeting). And so, I didn't have like opportunity to discuss, like, how I graded my exams, how it is going. Again, that's one of those things where it's like—could I motivate myself to ask or to do that? Probably, but I feel like that's a structure that just should be there, discussing grading, like in particular, grading. I know that this aspect can mean like writing exams and that kind of stuff. I don't know if I graded as harshly or less harshly than everyone else or if I missed an important part of the

> question. Very *inaudible* I just wish that I had more time. (Rowan, Support Interview)

In the same way that time emerged as a limiting factor for Rowan in obtaining access to resources and communicating with instructors towards the beginning of the semester, timing and scheduling once again limited Rowan's abilities to discuss grading. And, like how Rowan did not want to "bug" staff about obtaining access to the textbook or bother Robin and Rose about gaining access to other resources, Rowan pondered the extent to which they could or should be responsible for ensuring that grading was a topic of discussion. Just as strongly as they did towards the beginning of the semester, Rowan continued to navigate grading challenges. But because Rowan was used to conversations about grading in their prior teaching experiences as a "structure that just should be there," their descriptions of support revealed these underlying tensions.

Teaching Observations

A support Rowan did not have access to but stressed multiple times as an important issue for teaching O&NS was opportunities to observe and be observed.

> This is a big one that bugs me so much. Opportunities to observe and be observed. We all teach at the same time. I suppose I can be observed by someone else, but it's not the same... I would love to see what Rose's class looks like; I would love to see what Robin's class is like. But it is like pulling teeth, finding people to substitute for [O&NS]… I don't have an opportunity to observe or someone who I can actually truly observe right, because we all teach at the same time. So, I don't have that, I wish I had that. I don't have time; I wish I had time. (Rowan, Support Interview)

Although Rowan might have been able to record their class or watch recordings of others' classes, Rowan did not feel like they had the time to set this up nor did they believe it would offer a "truly" authentic experience like it did when they observed courses as an associate convenor. By the end of the semester, Rowan imagined alternative ways in which future instructors could observe or be observed teaching O&NS.

> This, this combination of, opportunities, opportunities to observe and to be observed… it's just a mess of the way that, like I get why the department schedules the STEM block the way that they do. But it is. If this is a course you're going to make graduate students teach—and so they don't have, like several semester's worth of opportunity to like, figure that out and plan for it—having to be at the same time can be really difficult. An idea that I just thought of, it would actually be super useful, would be if they just got a faculty member who has taught the course before… [they could] pick like one topic that that faculty member's going to teach, or like a week's worth of topics that that faculty member feels comfortable subbing in for and just like set up a structure, whereby it's like, on this day the faculty member is going to sub for this person and they are going to go observe… (Rowan, Final Interview)

Rowan pointed out issues with their idea, however. They realized that their own faculty mentors have not taught O&NS before, and therefore would be limited in their usefulness. Indeed, most faculty members at Cardinal University have not taught O&NS or other mathematics content courses for prospective elementary teachers. Hence, finding

someone who was not already teaching O&NS but had expertise in teaching the course would be difficult, if not occasionally impossible.

A key reason Rowan especially emphasized classroom observations as a support for teaching O&NS was because they felt that O&NS was "run very differently" than other classes they taught previously. Furthermore, Rowan valued teaching observations because of their prior experiences as an associate convenor. They explained this connection at the end of the semester,

> And I think we mentioned this in one of the previous interviews, being able to observe someone teach [O&NS] would be great. That is actually, like, based on my prior experiences. Observing someone teaching the thing that I want to teach, or I am teaching is super useful. But they all run at the same exact time. So, I have no opportunities unless I am finding a sub, which we've already just discussed, the pitfall of trying to find a sub. I have no opportunities to observe their classes or see what, what does classroom management look like to them? What does that look like? Because they can talk about, you know, you can talk about, and be like "I lecture for so long", "this is material I use", "this is how I have my kids do group work". But it's so different to actually see that, right? Like, every [Intermediate Algebra] class technically has the same set-up, but every [Intermediate Algebra] class is run so differently in a way that's like. Not totally tangible until you see it. (Rowan, Final Interview)

Although Rowan felt they had "no opportunities" to observe someone else teach O&NS this semester, they nonetheless believed that, with sufficient logistical planning, a

solution could (and should) exist to the problem of learning to teach O&NS from a more experienced individual. They benefited from observing others' teaching in the past and believed that it would benefit future instructors of O&NS to have that same support.

Physical Manipulatives

Alongside meetings with O&NS instructors and the textbook, Rowan, to a lesser extent, identified physical manipulatives as useful for them in teaching O&NS.

> These concepts are difficult to conceptualize, and they so naturally lend themselves to physical manipulatives. And I think that it is a good practice to provide multiple ways of learning for different students. Right. Some students learn from lectures, some students learn from reading, some students learn from physically manipulating their environment. It lends itself so nicely to that. I don't do that as often as I (want to) do. (Rowan, Support Interview)

However, Rowan additionally viewed physical manipulatives as a support they wished they had more of, even if they were not sure which manipulatives (such as 1-unit-by-1-unit-by-1-unit cubes) might best support their endeavors.

> More manipulatives would be great here – being able to make arrays and rotate them and things like that would be neat here. Also, some kind of manipulative for the area and volume definitions I think would be good for clarification, but I don't know what that would look like. (Rowan, Week 8 Reflection Journal)

Note that while Rowan often spoke of wanting physical manipulatives more generally, they specifically wanted pattern tiles.

> …hexagons and equilateral triangles and parallelograms. And I think like one other shape or something, and you can use those to create like tessellations and do shapes and stuff like that. And you can talk about ratios—like, three of the parallelograms make a hexagon, and six of the triangles make the hexagon as well, and stuff like that. Some of the class activities I've had to skip needed those as well, which they were like, you can technically make these out of paper, but could you imagine cutting out hundreds of these shapes? And also, it would be useful if they were in color; the blocks are in color. And then do, there's a lot of like colored strips that they want you to print out and that kind of stuff that I just. I both don't have access to color printing and don't have the time to *pause* print and cut out all of those shapes, and I'm not going to waste my students time making them take 10 minutes of class to cut out their own shapes. (Rowan, Support Interview)

Part of the reason Rowan lacked access to physical manipulatives, like pattern tiles, was because they were unsure who to ask for manipulatives, but also because Rowan, once again, did not want to bother others.

> Physical manipulatives are a huge part of this class. Like, like, do I have access to a printer? Yes. Do I have access to the beans? Sure. But they don't have all the things that I want, like color printing is actually a big thing that comes up in the textbook and I'm. And then even if I print them myself, I have to cut them out, and that's like a. At some point you have to decide that you're done paying out of pocket for a class that you're

> teaching since you're not supposed to be doing that anyways. So physical manipulatives, I have access to some, but definitely not the ones that I want all the time. And I don't know if there's even anyone I could ask, so I could probably email [staff], but I feel like it's not worth the hassle for everyone involved. (Rowan, Support Interview)

Ironically, this interview occurred one door down from a room containing multiple buckets of pattern tiles among dozens of other physical manipulatives for teaching elementary mathematics courses. Rowan was not aware that such a room existed. Personally, this is one of few moments in which I grappled with a bit more tension than usual during the study, as I weighed my desire to support Rowan against my intention to understand Rowan's challenges and successes in learning to teach O&NS as naturally as possible. Ultimately, I decided not to intervene, but to wait and see whether Rowan would reach out to other instructors or obtain additional physical manipulatives in some other way. I was especially curious as to whether Rowan's future interactions with their corresponding EMM instructor might lead to greater access to physical manipulatives. Unfortunately, as I discuss in the sections that follow, by this point in the semester, Rowan was fairly disconnected from the other EMM instructors and disenchanted about establishing productive relationships with them.

Access to Elementary Teachers

Hardly any time had passed before Rowan started developing their desire for access to elementary teachers as a support. By the second week of the semester, at least one of Rowan's students (PTs) was asking them specific questions about elementary students' mathematics learning trajectories that Rowan did not know the answers to. For

instance, Rowan recalled one PT asking them "when a student would learn about or be able to conceptualize the idea of decimals in base-ten notation" but Rowan said they were not "able to provide anything useful" (Rowan, Week 2 Reflection Journal). Because their PT asked this question, Rowan realized that they did not have a good sense of the timeline in which elementary students learn mathematics content and thought elementary teachers might be an appropriate community from which to seek support. Eight weeks into the semester, Rowan continued to imagine that elementary teachers might offer support, and in ways similar to how EMM instructors could have offered support during joint O&NS and EMM instructor meetings.

> I would imagine it being like what we want the meetings with the [EMM] instructors looking like. We meet with them before the semester so they can give us any thoughts that they have in terms of: how do they think teachers need to be prepared; like, a quick overview of what their classes look like, that kind of thing; like, maybe, like a couple of sample grades, you know, like maybe a couple of different ones—just some vibes in terms of resources their students have, issues they've seen come up with their students; like, just some overarching general stuff at the beginning of the semester. And then, being able to meet every few weeks just to be like, "Hey, this came up in my class," or "Hey, this is an example that I think that I want to make about student work. Would you think that this is reasonable for a student to have done?" or like, "Hey, this came up. Would you ever consider, like my textbook says, to teach this this way? Is that something that you would consider doing? Or is my textbook a little bit

> crazy?" So, stuff like that. Just sort of, like, a reality check for the class. Right? My students sometimes give me little mini reality checks because they're doing their practicum. Yeah, but it's definitely different from the perspective of, like, a full teacher who has to prep their own classes… I don't have access to teachers, but I wish that I did *inaudible*. Wish that was different. Even just someone that they're like, here is a list of, like, a teacher from each grade that you can reach out to if you have a question. And send an email. (Rowan, Support Interview)

Rowan likely thought that elementary teachers would be an ideal support early on because of their positive experiences learning from and with elementary teachers both in the summer mathematics content course they taught as a teaching assistant and in the graduate-level methods courses in which they enrolled at Cardinal University's department of teaching and learning. (I explain this connection in detail in later sections in Chapter 6.) However, Rowan's desire for access to elementary teachers was likely also bolstered and sustained by unproductive joint meetings with O&NS and EMM instructors (as I explain in the next section). In the second week of the semester, Rowan considered asking EMM instructors for resources related to elementary students' mathematics learning, but by the fifth week of the semester, Rowan seemed to no longer believe they could be relied on for support.

Meetings with Elementary Mathematics Methods Instructors

After the third and final joint meeting with EMM and O&NS instructors, Rowan lost faith in EMM instructors as a reliable source of support. Rowan recounted their experience at the last meeting:

> And then we pretty much never meet everybody all together. I actually was quite irate at the time. We said at the last meeting that was supposed to be [O&NS] and [EMM] (instructors), everyone together, which we scheduled for roughly once a month. One [EMM] instructor showed up and it was my teaching partner who had no idea what they were doing. So, we weren't able. To my knowledge, unless someone gave it out and no one told me, to my knowledge, we have still not given the students the assignment that we were supposed to figure out in that meeting. That was supposed to go out, like, the day after the exam… Like, I don't mind if the students don't have the assignment yet' That doesn't bother me. What bothers me is that we scheduled a meeting all together. Pretty much no one showed up. (Rowan, Support Interview)

This meeting stood out in Rowan's memory as one of the most upsetting experiences of the semester. At the start of the meeting, Juniper, the EMM instructor who led the meetings, joined the Zoom call with their camera off (normally it was on), promptly said they needed to leave to go to another meeting, made Robin host of the meeting, and left the call. After Juniper left, Robin said they would wait for the other EMM instructors because they thought they were going to talk about the rough draft assignments. Robin asserted that they were not sure what they could do at these meetings without EMM instructors.

In the remaining 27 minutes of the meeting, Robin and Rose discussed specifications such as instructions and point values for upcoming joint assignments, while Willow and Rowan listened (Cypress did not attend). Halfway through the meeting,

Robin invited Willow to share their opinions, asking the EMM representative of the meeting, “What are your thoughts?... Do you want that to be a hybrid-grading sort of thing?... I don’t think it makes sense to have [O&NS] grade that thing.” However, Willow seemed uncomfortable making decisions without Juniper present, as did the other instructors. A few minutes later, Rose noted, “I meet with Juniper tomorrow and can make sure that it has their stamp (of approval) on it.” The meeting concluded after the instructors exhausted their ideas for discussing joint assignments, and, although a recurring time for future joint meetings had already been set at the first meeting, no further meetings occurred.

What was especially upsetting about this event for Rowan was the precedent Juniper had established at previous meetings. The other instructors clearly looked to Juniper for leadership and support in making decisions. Hence, Juniper’s abrupt exit shattered these expectations. But it also shattered Rowan’s developing notions that teaching O&NS would be a collaborative experience with EMM instructors.

> Sometimes I feel like as [O&NS] instructors, we are not worth their time, which feels weird because we’re all supposed to be co-teaching this together. At least that’s how it was presented to me was we are all working on this together. And then that’s not happening, which again is fine, but it was the way it was presented. And I keep feeling like I’m supposed to have this resource that I don’t have. (Rowan, Support Interview)

As a result of the joint instructor meetings, Rowan concluded that they lacked a resource that they were supposed to have. But what about meetings with Rowan’s

corresponding EMM instructor Willow? Perhaps Rowan might not have come to this conclusion if they worked with Willow to not only navigate their own challenges as a newer instructor, but support Willow in navigating their challenges as a newer instructor. However, Rowan did not think being paired up with a new instructor was beneficial.

> I suppose technically there is support from the [EMM] instructors. I don't really meet with my [EMM] instructor, they are also teaching their class for the first time. So, we're sort of at this place where neither of us have any expertise on what's going on, which I'm going to be honest, I think is, I think someone made a mistake in scheduling that. For instance, [Rose] has taught this class several times and so has [Juniper], and they have really productive meetings every week. And I think that it would have been more useful if the support were split. That the [EMM] instructor that's never taught before has a [O&NS] instructor who's taught before and vice versa, as opposed to putting us both together to kind of flounder. (Rowan, Support Interview)

Indeed, Rowan only ever met with Willow once during the second week of the semester. Their 10-minute Zoom meeting consisted of Rowan and Willow each sharing the state of their classes, how teaching via Zoom compared to in-person, what students were doing that seemed successful or challenging, which assignments their students would be completing next, and how light they anticipated their students' current assignments being. After this meeting, each instructor only occasionally communicated with the other instructor, usually about joint assignments or student attendance and participation. Hence, a combination of broken expectations and a desire to be apprenticed by (a) more

experienced instructor(s) led Rowan to conclude that seeking support from EMM instructors was not a viable option.

Emergent Support: Debriefs, Reflection Journals, and Conversations with Mom

"It is not a failure on my part to need to ask for more resources." (Rowan, Final Interview)

It was not until my final interview with Rowan that I learned of a few activities and interactions that emerged as supportive (some of which were significant) for Rowan. First, Rowan found that their informal conversations (or debriefs) with me following my observations of their classes, as well as their weekly reflection journals, were supportive. Both the debriefs and the reflection journals were designed to offer opportunities for me to learn about Rowan's experiences, but they further supported Rowan in reflecting on their teaching. Rowan described how my asking them to explain their actions in class during debriefs raised their awareness around their instructional patterns and decisions. They additionally described how regularly responding to questions about support in their reflection journals increased their own sense of agency around seeking support.

> I think that if I hadn't taken the time to reflect with you after class every day, I wouldn't have thought as much about, "Oh, like this is a thing you're doing. Do you know why you're doing that?" Like, I didn't even notice at first that I really wasn't circulating my tables until you had mentioned it. And I was like, "Oh my gosh, why am I doing that?" Or, you know, when the reflection journals are asking, like, "What? What has supported you? What supports would you maybe need?" It's like, "Oh, well, what supports would I need?" I st… that didn't go super well

> because I still don't know, like, what supports I needed. But it did highlight to me the. Something that I had not thought about a ton before. Is that it is not, it is not a failure on my part to need to ask for more resources. Throughout this whole experience here at [Cardinal University] I have been given everything I could possibly ask for before I could ask for it until this semester. And so, I was always like, well, if this is what everyone else has been given and they seem to do just fine, then it's a failure on my part that I need something more or something different. So, it definitely helped reframe that mindset. Now, a lot of reflection and then some things that you brought up about observations were all pretty helpful. (Rowan, Final Interview)

Second, when I asked Rowan what prior experiences they thought impacted their learning to teach O&NS the most, Rowan revealed that they had been reflecting on their teaching of O&NS and learning of elementary mathematics with their mom throughout the semester:

> I feel like there are, like, two big things that I would bring up in my journals a lot… all the reflecting I had done with my mom about my experience in grade-school, which is something we've done on and off throughout the years and that I've done actually quite a bit during this time because I. So I, I come to campus, I teach my class, you and I debrief for a moment, and then I drive home, and on the way home I call my mom. And as part of that, we don't always talk about what I just taught, but sometimes we do, and we talk about sort of, how I did things and stuff like

> that. So that has been, like, a key thing, especially because I would, like, talk about different methods and things we were learning in this class. You know, I'll be like that, that is not. We were never told to teach like that at all and like, these were the things that we did instead. And these are the things you learned in your class. And don't you remember how you could never pass the multiplication tables timed tests, [Rowan]? (Rowan, Final Interview)

Indeed, Rowan's reflections with their mom and the questions in their reflection journals that asked them about their support were identified among a few factors that Rowan attributed to their most significant successes:

> Oooo, it's on every, it's everybody. Right? It took everybody. I mean, it took me as the instructor, right? I clearly put some effort into this class, but it took, you know [Robin] sort of being more organized and being able to keep things together, and it took [Rose] always interjecting with, you know, how things have gone previous semesters and tips that she has for stuff like that. And it took being able to reflect with my mom about how my experience went and how I'm connecting that to the experience of having a classroom. It took you and I reflecting on whether or not I had supports, which then would prompt me to ask for more support sometimes. And. No, it's. You take out any one of those pieces and who knows what would have happened. (Rowan, Final Interview)

Although these three supports: debriefs, reflection journals, and conversations with mom were not provided by the mathematics department or Cardinal University at-large, they

stood out to Rowan as significant to their successes in teaching O&NS. In the chapter and sections that follow, I specifically discuss the progression that led to Rowan seeking out support from their mom, and how conversations with their mom empowered them to navigate their teaching experiences, and specifically the challenge of addressing student questions in class.

CHAPTER 6: CHALLENGES AND SUCCESSES

In this chapter, I describe the most salient challenges and successes that Rowan experienced around teaching Operations & Number Systems (O&NS). These challenges and successes are portrayed through themes, and each theme is organized to capture the nature and progression of these challenges and successes chronologically. Note that Rowan's challenges and successes were often directly identified as such by Rowan, and predominately expressed in Rowan's reflection journals and interviews, as well as my fieldnotes and the analytic and reflexive memos I wrote following observations of Rowan's classes and meetings with instructors. In the sections following these themes, I describe how Rowan's background, prior experiences, preparation, and support provide a continuous and storied understanding of their challenges and successes around teaching and learning to teach O&NS.

What Counts as a Challenge or Success?

Successes and challenges are sometimes viewed in tandem with each other. Often, we experience success after navigating a challenge, whereas other times, we experience success after accomplishing our goals without running into major obstacles. Occasionally, navigating a challenge proves so difficult, that even if we overcome it, we continue to perceive it as more of a challenge than a success because it seemed unnecessarily difficult. As I present and at times conveniently label Rowan's challenges and successes as such, I keep this duality in mind.

I write Rowan's challenges to include experiences that Rowan explicitly identified as challenges, as well as experiences that Rowan or I viewed holistically as recurring struggles of significant difficulty. These include experiences or activities that

Rowan overall felt they had not yet learned how to manage or overcome by the end of the semester or were more cumbersome to navigate than they imagined it would have been with sufficient preparation or support. I write Rowan's successes to include experiences that Rowan explicitly identified as successes, as well as experiences that Rowan or I viewed holistically as an accomplishment of significance. These include instances in which Rowan accomplished their goals or successfully managed teaching tasks with few issues. These also include experiences or activities that Rowan or I thought easily could have resulted in negative consequences but did not or resulted in more overall benefits than drawbacks for Rowan. Because of the interconnected nature of challenges and successes, I nonetheless provide alternative perspectives or limitations to viewing these challenges and successes as such, especially when these views seem meaningful for unpacking Rowan's experiences and their implications.

The Semester-long Progression of Rowan's Challenges and Successes

Rowan started the semester excited but also a little "nervous" about teaching O&NS. Specifically, Rowan told me after their first class that older students might expect their instructors to be more experienced and more confident. Despite Rowan's concerns, a few students seemed excited for O&NS; at the end of the second class one of the students at the later self-proclaimed "boy's table" said in a low and relaxed voice to their tablemates that this class actually seemed really cool and that they were excited for this class.

Developing Beliefs about Prospective Teachers and Teaching O&NS

Much of Rowan's excitement towards the beginning of the semester seemed to stem from Rowan's beliefs about their O&NS students as prospective teachers. Rowan anticipated the following success in their reflection journal:

> I think the students will be fun and easy to work with. Since they want to be teachers, they'll have an appreciation and understanding for both the teaching methods I use and the teaching methods for mathematics they're learning. (Rowan, Week 1 Reflection Journal)

Although Rowan thought that their O&NS students would appreciate their teaching methods, they were simultaneously a bit worried that prospective teachers would be "critical" of their teaching methods because "they are studying teaching and teaching methodologies and could be critical of the ways I lead class" (Week 1 Reflection Journal). In a debrief following the first class, Rowan further described that teaching older students, as opposed to freshman, made them a bit nervous because they believed that older students are more likely to expect their instructors to have more experience and be more confident. Hence, Rowan started off the semester excited but also just a bit nervous.

Rowan also reflected on their O&NS students' assets towards the beginning of the semester, though sometimes in light of the deficits of other populations of students they taught in the past.

> The [O&NS] students are very forgiving when life gets in the way of class. I had to tell a few students that I would have to get back to them the next day or the day after, but that I'd extend the homework deadline to compensate for my shortfall. I also had to cancel office hours the day

> before the first homework assignment being due. They were all very considerate of the situation. I've had students in first-year math courses be much less considerate in similar situations. (Rowan, Week 2 Reflection Journal)

In addition to "forgiving" and "considerate," Rowan described their O&NS students as "committed" based on their attendance and participation in class:

> I'm hopeful that my students will still do well on their first homework assignment! They have been really committed to attending and participating in class, and I hope that will be able to come through in their work despite my inability this week to provide proper support. (Rowan, Week 2 Reflection Journal)

In week three, Rowan similarly described their O&NS students as "always very kind and understanding" in situations when, for example, Rowan forgot to add a due date to a quiz or an upload file button for their assignments, as compared to students in the past who became "frustrated" or lost "faith" in Rowan due to missing adding these necessary features to assignment submission portals. Rowan furthermore felt that their O&NS students were more "committed to the coursework" than other students they taught in the past:

> I'm hopeful that I'll get to experience my students doing well on their exam soon! I know the first one will probably be a little rough, but I have a lot of faith in these students. They're clearly committed to the coursework and getting things done. I've never had every student turn in the first homework or have every student find a group to work in without

> issue before – I think these are good omens for the rest of the class. (Rowan, Week 3 Reflection Journal)

Rowan continued to develop beliefs about O&NS students' commitments to learning in week four, elaborating on O&NS students' "eagerness to learn" as compared to Intermediate Algebra students' "timidness and hesitation":

> I've been thinking a lot about my [Intermediate Algebra] students this week. It's interesting comparing how the groups in the two classes behave – for the most part both sets of students struggle a bit with some of the basic math concepts, but the eagerness to learn of the [O&NS] students makes facilitating group work so much smoother compared to the timidness and hesitation of the [Intermediate Algebra] students. (Rowan, Week 4 Reflection Journal)

By the end of the semester, some of these early semester beliefs about prospective teachers still resonated with Rowan. When I asked Rowan about their thinking around teaching prospective teachers in a classroom debrief during the fifteenth and final week of classes, Rowan responded that "it's different" both in terms of the students and the content of the course and proceeded to comment on prospective teachers' "appreciation for school" and active learning teaching styles:

> [Rowan] says that it's really nice that when you are teaching PTs, you are teaching other students that have an appreciation for school. They are academics in the K-12 world, in the same way that we are academics. [Rowan] says that teaching PTs is also different from teaching calculus students or other students because PTs have an appreciation for doing

> group work and student presentations, and an active learning style. [Rowan] says that other students will push back on active learning and just want to do things on their own. [Rowan] says that even if PTs are not always jumping in to participate in group work or student presentations, they still have an appreciation of it and understand its benefit. [Rowan] says that this is really nice. [Rowan] says it's also true that PTs have the ability to critique my teaching, however. And the content is quite difficult. But [Rowan] says that it's really nice. (Researcher Fieldnotes, Week 15 Classroom Debrief)

Similar to earlier in the semester, Rowan still believed that prospective teachers had a greater appreciation for the benefits of active learning styles in comparison to other populations of students they taught in the past. They also still believed that prospective teachers had the ability to critique their teaching and overall, felt that teaching prospective teachers was pleasant. Rowan went on to explain how their thinking was influenced by their mom and their previous experiences teaching a summer math content course for teachers:

> [Rowan] says that it's really easy for them to think about what it's like to be a PT just because their mom was a grade-school teacher, and she shared with [Rowan] what it was like learning to teach. [Rowan] says, although that was 35 years ago and things are different now. [Rowan] says that they also taught teachers when they were assisting with [summer math content course for teachers] and that was the greatest experience they ever had.

> [Rowan] says that the teachers were really great. (Researcher Fieldnotes, Week 15 Classroom Debrief)

Although Rowan did not explicitly name their developing beliefs around teaching prospective teachers or O&NS as a challenge or success, I include the progression of Rowan's developing beliefs early on because reflecting on the nature of their students was so salient for Rowan towards the beginning of the semester, and because Rowan's developing beliefs about their O&NS students contributes towards understanding the following section on addressing student questions (which I further explain in the following chapter).

Addressing Student Questions

By the middle of the semester, the freshness of Rowan's bubbling excitement for teaching O&NS started to simmer and Rowan fell into more routine patterns that they described as "business-as-usual". From the beginning of the semester, one of the classroom events that Rowan came to rely on as the usual "business" was their students (or rather one of Rowan's students) regularly asking them questions not just about mathematics content, but questions at the intersection of mathematics content and knowledge of K-6 students' mathematical thinking and/or K-6 schools. Early on, Rowan found these intersectional questions about mathematics and K-6 students/schools especially challenging.

Consider the question that one of Rowan's students, Bruce, asked during the second week of classes and Rowan's response, in which they said that they were not sure of the answer to their question and not "the best person to ask":

> [Rowan] clicks to show next slide that says, "Decimals on a Number Line." [Rowan] talks about zooming in on a number line. [Rowan] talks about how it would be hard to suss out where your number is if you were trying to put hundredths on a number line labeled with whole numbers without a lot of space, so, if we are just looking between 2 and 3, we can zoom in on the number line. [Rowan] clicks to show the next slide and asks if there are questions. [Bruce] asks in what grade number lines would be attainable. [Rowan] says that they aren't sure. They think counting numbers are Kindergarten and first grade. And they think it would be attainable for 2nd or 3rd graders, but it would depend on the curriculum in the district. [Rowan] says that they aren't really the best person to ask because they are more about the math from the math department and if they were from the teaching department they might know better. (Researcher Fieldnotes, Week 2 Classroom Observation)

In Rowan's journal response to whether there was anything they wished they were more prepared for related to teaching O&NS, Rowan reflected on Bruce's question and their ability to "provide anything useful," saying that they wished they had a better sense of the timeline in which elementary students learn mathematics content:

> I wish I had a better sense of the timeline on which elementary students would learn this material. A [O&NS] student asked when a [K-6] student would learn about or be able to conceptualize the idea of decimals in base-ten notation and I wasn't able to provide anything useful, but I think it would help the [O&NS] students to be able to consider the age of their [K-

> 6] students while they learn the material. (Rowan, Week 2 Reflection Journal)

During the preparation interview (also in the second week), Rowan additionally reflected on the extent to which they "have the authority" to anticipate K-6 student thinking and worried about missing important ideas behind student thinking:

> Anticipating student thinking questions and actions. Not necessarily my [O&NS] students in class, although I am doing that, but the [K-6] students that they would teach to. I am anticipating that thinking in a way that maybe I don't have the authority to do, and possibly I could miss a lot of important. Important ideas behind [K-6] student thinking. (Rowan, Preparation Interview)

Thus, Bruce's question challenged Rowan to reflect on their ability to support their O&NS students' inquiries about K-6 students' mathematical thinking early on.

Perhaps addressing questions involving knowledge of or experience with K-6 students' mathematical thinking might not have presented itself as a challenge for Rowan if they were asked these questions infrequently. However, Rowan was asked such questions during lecture or whole group discussions almost every week starting from the second week of the semester through at least week ten of the semester (see Table 5 for sample questions from each week in which a question about mathematics content with respect to K-6 students and/or schools was asked during lecture). And as the semester continued, Rowan continued to reflect on their ability to discuss K-6 students' mathematical thinking:

I wish I was more prepared to speak about proposed student work. The textbook has a couple examples, but I can't come up with additional ones on the fly and feel confident it is something a student might do. (Rowan, Week 7 Reflection Journal)

Table 5

Questions Asked to Rowan about Mathematics Content with Respect to K-6 Students or Schools during Lecture or Whole Group Discussion

Week	Questions Asked to Rowan	Rowan's Response (and Subsequent Conversation)
2	In what grade would number lines be attainable? (Bruce)	I'm not sure. I think counting numbers are Kindergarten and 1st grade. I think it would be attainable for 2nd or 3rd graders, but it would depend on the curriculum in the district. I'm not really the best person to ask because I know more about math being from the math department, but if I was from the teaching department I might know more. (Rowan)
3	I did 5th grade practicum and the cooperating teacher said not to say improper fraction, but to say fraction greater than 1. I don't understand. Why did the teacher say that is how [district] does things? (Sarah)	I've only briefly talked to an education person about this, but I think improper sounds like "incorrect", like "wrong fraction", but there's nothing wrong with 6/5. Fraction greater than 1 emphasizes more how we've been defining fraction. Someone sometime decided that it was better to do mixed fractions. The improper fraction sounds like it's wrong even though it's not. (Rowan)
4	Problem: Anna thinks 2/3 = 6/7 because $\frac{2+4}{3+4} = \frac{6}{7}$. "I'm trying to put myself into the headspace of a child... The fact that you can't add four to the bottom. How can you convey that to a 9-year-old?" (Bruce)	Physical manipulatives are better than drawings, but you could take 1/3 and 1/7 and show that you can't cover the 2/3 with the 1/7 pieces properly. You've fundamentally changed the fraction. (Rowan) You can take four copies of the fraction to show this as well. (Bruce) Yes. (Rowan)

	How do you help a student lost in the process of doing long division? (Bruce)	You can make lists by 3s, then lists by 5s and find the first number that matches. That is a process that I can see how you get lost. What they think about is really just that the 3 needs a 5, so I do 2/3 x 5/5 and this 5 needs a 3, so 3/5 x 3/3. (Rowan) Oh, I see. So you can see why it works from the procedure. (Bruce) Yes. (Rowan)
5	What is the purpose of knowing this (types of addition problems) for us as teachers? Do we say to our kids, hey, we're going to do change unknown problems today? (Bruce)	I'll bet you don't say that. But actually, I don't know. Your math methods instructor would know that better. What I think is that you want to be able to ask your students every different kind of subtraction question you can. (Rowan)
7	(With respect to learning addition strategies) "Do you know if we're going more over the natural way that children deal with numbers?" (Bruce)	I'm going to say a little bit of yes to both… (Rowan) I think I understand what you're saying... Maybe you see what the student is doing and meet them where they are at by knowing these methods. (Bruce) This is a skill I am teaching to the class, rather than this is something you should be necessarily teaching. (Rowan)
	(With respect to writing decimals as sums of fractions) "This 1/3 does not play nice at all. What happens when a kid asked me that?" (Bruce)	"Yes, we have this notion of finite versus infinite fractions..." We have 1/3 where the 3 repeats infinitely many times. This process only works for finite fractions and not infinite fractions. You'd have to introduce this process with fractions as compared to decimals. (Rowan) "I see, you'd have to incorporate the looming question into the base of it in the first place." (Bruce)
8	"Any questions about multiplying by 10 in base-ten in this class?" (Rowan) "I consider this a shortcut (moving the decimal point when	I wouldn't start with the shortcut, but maybe recommend starting with toothpicks and then doing problems with multiplying by 10 so that students can see that multiplying by 10 is special. (Rowan)

	multiplying by a power of 10)... I can't remember how I learned it when I was 5." (Bruce)	
9	Any questions or confusions about order of operations? (Rowan) I know how to do this. But instead imagine me as a child. Why can't I add, then divide? (Bruce)	Here's an example: $f(x) = \frac{2x+2}{3x+7}$. You can't really simplify this equation anymore, so you don't really need to consider order of operations here. Something incorrect that I imagine happening is crossing off both twos in the equation $f(x) = \frac{2x+2}{3x+2}$. That is a misunderstanding of division. Oh, well actually there are hidden parentheses here. *writes f(x) = (2x+2) ÷ (3x+7) on the board* We would need to write these. We couldn't simplify this equation anymore. (Rowan) Simplifying and solving are two different things and I need to understand that. (Bruce) *nods head* Yes, you need to understand that those are two different things, which is challenging for calculus students. (Rowan)
10	What is the most common way we do this (2-digit by 2-digit multiplication) today? My understanding is that [City] public school is moving towards partial products method. I've noticed that some students skip carrying a step, and that you say thirteen tens rather than a hundred and thirty. Does that make sense? (Kenny)	That makes sense. "I know from the outset that the 2 is in the tens place and allowing that mindfulness of place from the outset..." I want to understand what you mean about carrying the one. (Rowan) Can you do an example, maybe with 98 + 98? (Kenny) Let's look at 98 + 98. You have 18 but then you write 19. It's like you're skipping a step. (Rowan) I understand it but want to know what's right for the process for kids to understand. (Kenny) Rowan does another example with 38 x 6 and writes 228.

> I think I understand why my cooperating teacher emphasizes it, but I also don't. (Kenny)
>
> Yeah, I think the partial products just emphasizes more of the steps. Have you ever seen kids who want to add up all the numbers from left to right and end up with too many digits? It's trying to prevent that. (Rowan)
>
> Yeah, I think that makes sense. Sorry for taking up the time. (Kenny)
>
> Oh, no, its fine. Are there any more questions? (Rowan)

From Table 5, there are a few situations in which Rowan emphasized asking these questions to their corresponding EMM instructor. In many situations however, Rowan drew on their developing proficiencies with physical manipulatives that K-6 students use (e.g., toothpicks for base-ten numbers being special and fraction strips for fraction equivalence), their prior and developing mathematics knowledge (e.g., "finite versus infinite fractions"), or their prior teaching experiences (e.g., mistakes calculus students make with rational functions). When having these "on the fly" conversations, Rowan additionally demonstrated empathy for multiple perspectives when making decisions as a teacher, rather than asserting a superior perspective. For instance, consider the following conversation between Rowan and one of their students, Kenny, in which Rowan offered a possible rationale for the actions of Kenny's cooperating teacher and pivoted the conversation away from judging those actions as right or wrong. Instead, Rowan discussed the cooperating teacher's actions as a possible approach to supporting students learning:

Rowan: [Rowan] clicks to next slide and says let's do 45 times 23. [Rowan] writes $45 \cdot 23 = (40+5)(20+3) = 40 \cdot 20 + 5 \cdot 20 + 40 \cdot 3 + 5 \cdot 3$ (FOIL). [Rowan] says, "Partial products is essentially a shorthand of foiling... the common method ends up being a short-hand of the partial-products method... I just have to make sure I do every step I would have done in the common method... In the partial products method, we have to be very, very cognizant of place value. 2 is really 20, so it's 20. Any questions about how to do the common method or partial products method for 2 digits?"

Kenny: [Kenny] asks: What is the most common way we do this today; my understanding is that [City] public school is moving towards partial products method. [Kenny] says they've noticed that some students skip carrying a step, and that you say thirteen tens rather than a hundred and thirty. [Kenny] asks if that makes sense.

Rowan: [Rowan] says that makes sense. [Rowan] says, "I know from the outset that the 2 is in the tens place and allowing that mindfulness of place from the outset..." [Rowan] says they want to understand what [Kenny] means about carrying the one.

Kenny: [Kenny] gives an example.

Rowan: [Rowan] does an example given by [Kenny]. [Rowan] says let's look at 98 + 98. [Rowan] says you have 18 but then you write 19; It's like you're skipping a step.

> **Kenny:** [Kenny] says they understand it but want to know what's right for the process for kids to understand.
>
> **Rowan:** [Rowan] does another example with 38 x 6 and writes 228.
>
> **Kenny:** [Kenny] says they think they understand why their cooperating teacher emphasizes it but also doesn't. (I'm thinking [Kenny] feels a tension between their cooperating teacher wanting kids to write out steps with place value and kids wanting to jump to the next step).
>
> **Rowan:** [Rowan] says, yeah, I think the partial products just emphasizes more of the steps; Have you ever seen kids who want to add up all the numbers from left to right and end up with too many digits? The partial products method is trying to prevent that.
>
> **Kenny:** [Kenny] has been nodding their head up and down and says something like yeah, I think that makes sense. [Kenny] says sorry for taking up the time.
>
> **Rowan:** [Rowan] reassures [Kenny] that its fine and then asks if there are more questions. (Researcher Fieldnotes, Week 10 Classroom Observation)

Rowan navigated the question of "what's right for the process for kids to understand" by suggesting that the partial products method "just" emphasizes more of the steps and that the method is one way to prevent kids from losing track in the addition process.

Even though Rowan did not feel they initially had the preparation or the authority to address questions around K-6 students' experiences and mathematical thinking, their offering of mathematical perspectives through an empathetic lens provides one reason to view Rowan's navigation of this challenge as a success, in addition to their drawing on

their developing proficiencies with K-6 mathematics content, physical manipulatives, and anticipated K-6 student thinking. Rowan's navigation of their O&NS students' questions around K-6 students and schools seemed even more successful to me in light of Robin's experiences with their O&NS students. Towards the end of the semester, Robin began lamenting during instructor meetings about "getting a lot of pushback from a few of my students" and students "complaining" about misalignment between Robin's teaching and what O&NS students are seeing in [City] public schools for their practicum. Thus, navigating students' questions and maintaining a classroom environment of openness, curiosity, and mutual respect for mathematical perspectives, but especially teaching and teachers, is not a given. Rather, this may be a skill Rowan developed through reflective practice and a productive disposition towards teachers and teaching, as I discuss in later sections.

Learning "Different" and "Hard" Content as an Instructor

Recall that by the fifteenth and final week of the semester, Rowan emphasized O&NS as a course that was "different" and with content that was difficult. I described the following classroom debrief earlier in the section on Rowan's developing beliefs about prospective teachers and teaching O&NS:

> I ask them what they think about teaching PTs. [Rowan] says pre-service teachers? I say, yes, prospective teachers. [Rowan] says "it's different." [Rowan] says that it's different both in terms of the students and the material… And the content is quite difficult. (Researcher Fieldnotes, Week 15 Classroom Debrief)

That O&NS was different and its content difficult emerged as a salient challenge for Rowan around early to mid-semester. Some of these challenges were related to reconciling Rowan's mathematics knowledge with the perspectives offered in their textbook. For example, Rowan did not anticipate that "in this class we really only consider the number line as a visual representation of base-ten" (Rowan, Week 2 Reflection Journal), as opposed to thinking "like a mathematician" (Rowan, Preparation Interview) about how the number line is constructed. Most of the time however, Rowan found themselves challenged to develop their own conceptual understanding of elementary mathematics. In week four, Rowan started to recognize the difficulty in learning O&NS course content for themselves:

> Basic math is hard! I'm learning better what material the students are going to struggle with (for instance a fraction of a fraction was a big struggle this week, I had to come up with several different ways to describe it which was a learning experience for me). (Rowan, Week 4 Reflection Journal)

Moreover, Rowan felt that their lack of familiarity with course material in O&NS hindered them from teaching more flexibly, especially in comparison to their prior teaching experiences:

> [Rowan] feels that in other classes, like Intermediate Algebra, but especially Calculus, they had more flexibility to be able to do what they wanted, because they feel that they can come up with problems themselves. They said that in Calculus, they are given that freedom to teach the course however they want, knowing what they know, whereas in

> this course [O&NS], they feel that they do not have that same flexibility because they don't know the material as well. They aren't as familiar with different ways of doing a fraction problem, for example. Hence, [Rowan] explains that they feel more "locked-in" to the material and the exercises in [O&NS]… [Rowan] mentions that with Calculus they have a textbook, a workbook, and they feel like they know the material. For example, they said they could write their own problems about limits. For them, that would be more intuitive than writing their own problems about fractions. [Rowan] mentions that with [O&NS] they just have the textbook to rely on, they don't have other resources. And if they were to look stuff up online, they wouldn't really know if it was well-vetted or reliable. (Researcher Fieldnotes, Week 6 Clarification on Week 4 Reflection Journal)

Rowan continued to emphasize learning new material, as well as the challenges in utilizing new O&NS content. They additionally compared their re-learning of elementary mathematics content to their own memories of learning mathematics and emphasized changes in their thinking. For instance, Rowan described their learning around comparing fractions through nonstandard methods as "hard to write a good problem" for and "hard to put that process into words", as well as "something I don't remember ever being taught":

> I've learned a lot about comparing fractions through nonstandard methods (e.g. not common denominator, common numerator, etc.). It's hard to write a good problem where you have to use nonstandard reasoning –

> where the relationship is not overly obvious, but you can still reason through it. This is something I don't remember ever being taught, but it's definitely something I do all the time mentally without thinking about it. It was hard to put that process into words. (Rowan, Week 5 Reflection Journal)

In week seven, Rowan similarly described changes in their thinking about elementary mathematics. For instance, they "hadn't thought about how many steps are really involved in a traditional 2-digit numbers addition problem" (Rowan, Week 7 Reflection Journal). In week eight, Rowan described challenges surrounding their teaching of multiplication, as they had not realized the significance of the multiplier and multiplicand until after their teaching of the lesson:

> I think I brushed off the idea that we don't care very much about the order of multiplication since multiplication is commutative. I should've thought more about how much the book stresses the difference between the multiplier and multiplicand… I hadn't thought much about the fact that the order of multiplication is something that students may need as a way to conceptualize multiplication before knowing that multiplication commutes. (Rowan, Week 8 Reflection Journal).

Towards the end of the semester, Rowan emphasized themselves as a learner of this challenging content alongside their students.

> I'm afraid the students will ask a lot of questions during the review that I am unprepared to answer. It's been a while since we learned the material from the beginning of the semester (and I was learning it right alongside

> them) so I'm afraid I'll have forgotten something. (Rowan, Week 14 Reflection Journal)

Rowan's challenges in learning course material resulted in Rowan wishing they had learned the material before the semester began:

> I wish I had learned this material before to give it a chance to set in. Maybe a crash course before the semester begins? [Robin] gives us a weekly overview of the material but I think a time gap could be useful… Wish there was a tl;dr for the course. Could have used the chapter recaps to get a preview of the entire course, but they tend to gloss over some of the important nuances that are stressed in the class even if not in the text. (Rowan, Week 14 Reflection Journal)

Although learning new and unfamiliar course content can be viewed as a success, their experience surrounding their learning was often perceived as a challenge that contributed to feelings of being uncertain and unsteady, as I describe in the section that follows. Moreover, Rowan's need to learn new course content was connected to their experiences with their preparation, with which they felt discontent and tension. However, Rowan's learning of new content was also connected to their later learning regarding communicating expectations to students, as I describe in a subsequent section.

"Uneasy" and "Unsteady"

By the end of the semester, Rowan described their experiences overall as a "whirlwind" and had developed an "uneasy" and "unsteady" orientation towards the O&NS course material and their teaching of it.

> Whirlwind… I felt like I was flying by the seat of my pants this whole time… like uneasy or unsteady. I'm uneasy with the course material, I'm unsteady in my ability to set aside time to plan instruction. I'm uneasy about being able to answer my students' questions, there was just like this sense of unease and there was no one thing going on in class that I felt stable in. (Rowan, Final Interview)

Rowan's uneasy and unsteady orientation became more apparent around week nine. In week nine, Rowan expressed some difficulty with making sense of the course material, as well as hesitancy to ask students to present on nuanced material in class. Consider Rowan's week nine reflection journal entry in response to anticipated challenges followed by my fieldnotes following the sixteenth class in week nine:

> It's hard enough for me to make sense of the material from the textbook so that I can teach my students – I have minimal hope they will fully understand the material from the textbook when they miss class. (Rowan, Week 9 Reflection Journal)
>
> [Rowan] says they wish they had gotten to do student presentations today but that they weren't sure about doing it for this topic because it is so up to interpretation and nuanced. Like, one student could do one thing wrong, and it could be really confusing for everyone. (Researcher Fieldnotes, Week 9 Classroom Debrief)

Hence, some of Rowan's challenges in learning course material may have further unsettled them into feeling less certain about providing O&NS students' opportunities to learn the O&NS content. Rowan similarly discussed feeling less confident in their

students' abilities to understand "weird, nuanced interpretations" of the course content in week thirteen:

> [Rowan] thinks about earlier in the semester compared to now and how they're asking for less student input than they used to. [Rowan] talks about how they could have elicited more student input today but that they didn't because they were kind of afraid of a student having an incorrect answer/interpretation that other students aren't able to understand why it's incorrect or what a correct answer/interpretation might be. [Rowan] talked about how there are these "weird, nuanced interpretations" in the book, and they didn't want to spend time on any activities that might seem like a waste of time to students getting at these nuanced interpretations between the definition of fraction and interpretations of division. [Rowan] doesn't feel confident that the students would have understood the nuance or that it would have been viewed by students (and perhaps [Rowan] themselves) as a good use of time… (Researcher Fieldnotes, Week 13 Classroom Debrief)

In week ten, Rowan reflected on their challenges, writing that "I have been dealing with a lot of confidence issues this week. I feel unprepared and unqualified for this course, and I worry that that attitude is becoming apparent to my students" (Rowan, Week 10 Reflection Journal). Rowan additionally conveyed uncertainty regarding their enactment and how students would react to activities:

> I ask [Rowan] how this class is going for them compared to previous courses they've taught at this point in the semester. [Rowan] says that

> usually they are confident about how their students will react to certain kinds of activities, but for this class [Rowan] still feels uncertain about how students will react to activities. But also, and maybe more primarily, [Rowan] still feels uncertain about what they are doing as an instructor. [Rowan] voiced a few times how they feel both uncomfortable with what they are doing, but also uncomfortable with how they are going to run the class, especially due to the lifted mask mandate… (Researcher Fieldnotes, Week 10 Classroom Debrief)

Rowan similarly worried about projecting their own discomfort with the material onto their students in week fourteen. This time, Rowan emphasized not learning the content themselves in elementary school as the main reason for feeling unsteady:

> I ask [Rowan] how they think it went. [Rowan] said that it went fine but that they were uncomfortable with the material and worried that their discomfort showed or was projected onto their students. Division with fractions is not material they are super comfortable with teaching… [Rowan] says that they didn't remember learning about division with fractions when they were in school. They said that they remembered learning about the division algorithm and doing division, but there really wasn't a focus on dividing with fractions. [Rowan] says they learned to multiply by the reciprocal, but the reasoning behind dividing fractions is a foreign concept… [Rowan] mentions that they didn't feel comfortable with dividing fractions as equivalent to dividing numerators and dividing denominators. [Rowan] says that they don't remember learning this and

> feels like the most they could really do with this in class today was to show PTs that you CAN divide the numerators and divide the denominators, but not really get them to any level of comfort in understanding that. [Rowan] says that it doesn't sit well with even themselves yet—it's not something they fully believe works and so this makes them feel a bit uncomfortable and are a bit worried that their students might feel similarly uncomfortable because of their discomfort. (Researcher Fieldnotes, Week 14 Classroom Debrief)

Other times, Rowan's uncertainty centered around figuring out what their main learning goals for their students should be. Consider my fieldnotes following the seventeenth class in week nine:

> They think that the material feels a bit strange to present because it's not really new material but just applying the things they already know to multiplication facts and percents, so they aren't really sure what their main goal for their students is. They think it would be easier to determine the main goal if they were presenting a new concept, but with this it just feels like a collection of stuff. (Researcher Fieldnotes, Week 9 Classroom Debrief)

Furthermore, just as Rowan felt "locked-in" to the textbook due to their unfamiliarity with O&NS course material (see previous section on Rowan learning different and hard content as an instructor), Rowan felt restricted in what they could do in the classroom because they felt uncertain about what they were teaching:

> We also, Oooo, we also talked about. This is something that made me feel like a bad teacher while I was teaching, even though I know that it's like a totally fine thing to do while you teach because people write textbooks. And if they write a good textbook, people keep using it. But my mom would talk about the fact that they could never get a new curriculum instated for any of the classes at her school because the teachers just wanted to continue teaching out of the same textbooks that they have already prepped. And "we're already ready", and the textbooks they have, have the solutions and they just, like, wouldn't deviate at all during class from what was in the textbook and the solutions in the back of the textbook. And I feel like that's how I, like, had to teach this semester since I didn't know what I was teaching. I feel like I would need to teach this class one or two more times to really feel like. That's all I could do. I think the first time out, that's what I needed to do. (Rowan, Final Interview)

One final consequence of Rowan's unease was something Rowan considered their "biggest failure for the semester" during the final interview. Rowan prided themselves on learning everyone's name, but this semester, Rowan approximated only learning twelve out of eighteen students' names, and felt "really torn up about it." This was something Rowan worried about in their week two reflection journal, and was made additionally difficult throughout the semester because of COVID and mask-wearing. When I asked Rowan if there was anything they would do differently to support them in learning their students' names, Rowan explained how their unease with the material made it difficult to learn names:

> But there is something about the stress of this class and my unease with the material that made it so that I was so focused on that. That like I couldn't pick up my students' names, like, as I was learning them, while I was there, and stuff like that. *inaudible* So, yeah, something I would change is to be. More mindful of the way that the stress this course causes affects other aspects, other things that I think are a given. Won't necessarily be able to do in a stressful situation. (Rowan, Final Interview)

Note that, occasions existed in which Rowan expressed feeling more confident. For instance, in week thirteen, I asked Rowan after class if any successes had come up for them lately, to which they responded that they were "feeling more confident in presenting material," though still unsure about what they were doing for class. Overall, however, Rowan felt less confident and less certain about O&NS mathematics content and their teaching of it towards the end of the semester.

Guilty Grading Baggage

One of the most salient challenges for Rowan revolved around grading. Rowan started the semester grading every homework problem assigned to students on every homework set. Note that each of Rowan's 18 students completed homework sets individually on an almost weekly basis (there were 12 homework sets total for the semester), and each homework set contained an average of 10 problems. Thus, Rowan planned to grade 180 homework problems for each of 12 weeks over the course of the semester, in addition to exam problems and other assignments. At first, Rowan thought their plan for grading homework was reasonable, but quickly realized it was too

ambitious. In week three, Rowan described feeling successful about having graded their students' first homework set, but accomplishing this was a challenge:

> Even though the grading schema was a success, it was definitely a challenge to grade this first assignment. It's rare for me to have to grade written out thought processes so it was a bit rough grading the first few assignments. I actually graded the first couple a few different ways until I settled into a grading schema that felt right. (Rowan, Week 3 Reflection Journal)

During the final interview, Rowan stated that they only ended up grading "every single problem on every single homework set" for two homework sets before they "got super far behind grading homework". After the second homework set, Rowan only graded what they considered their "five most important questions—the ones that I thought might be on exams."

It was around week five when Rowan started falling behind on their expectations for grading homework. Because their students' first exam was the following week, Rowan started worrying about students going "into the exam without feedback on some of the material" (Rowan, Week 5 Reflection Journal). Specifically, Rowan was concerned that students might repeat similar mistakes on exams:

> I anticipate running into an issue where a student gets something wrong on the exam that they got wrong in the same way on a homework assignment I didn't finish grading before the exam. Then it will be my fault for not getting them the feedback they needed. (Rowan, Week 5 Reflection Journal)

This concern weighed on Rowan; at the beginning of class during the first exam, I greeted Rowan and asked them how it was going, to which they responded that they were good, but disappointed that they didn't get all of their homework graded before the exam. And later in their week six journal, Rowan reflected on their concern with students making similar mistakes on the exam in light of students' exam performance:

> I didn't finish grading all of the students' homeworks leading into the Exam, and it caused students to mess up on the same material they missed in homework (which they should've gotten feedback on). I wish I had more experience with or tools for grading this type of assignment so they could get quicker feedback. If I could do it over, I wouldn't have any Canvas file uploads, maybe Gradescope (a digital grading system) or paper homework. (Rowan, Week 6 Reflection Journal)

Another consequence of being behind in grading was a struggle to keep up to date with their students' understanding of the material: "Since it's taking so long to grade assignments, I'm not particularly well in tune with their progress yet." (Rowan, Week 5 Reflection Journal) Rowan additionally viewed the technology they were using as a barrier to efficient grading in their week five and week six reflection journal, and wished they had more experience with or current access to tools that could support them in grading. Furthermore, Rowan started to wish they had more experience with grading homework in general, as well as more experience grading homework involving drawings and explanations.

> More experience grading homeworks would have been useful. Not only were homework assignments all online and auto graded for all of my

> previous courses so I don't have much experience, but also the types of homework they do in this class (a lot of drawings and explanations) are less familiar to me and take a while to grade. (Rowan, Week 5 Reflection Journal)

In week six, Rowan further elaborated on why they felt less prepared to grade work in O&NS in comparison to other courses they taught in the past:

> I wish I was more prepared to assign point value to incorrect reasoning. It's not like computational math (e.g. a lot of the problems you would see on a calculus exam) where there are a limited number of wrong things students tend to do. Here, I have to weight different incorrect reasonings against one another to determine which ones are more or less correct than others. It feels quite uncomfortable. (Rowan, Week 6 Reflection Journal)

Hence, assigning point values to what Rowan viewed as more diverse reasoning in O&NS students' solutions was one reason why grading was more time-consuming and challenging for Rowan. They similarly emphasized this aspect of the challenge of grading again during a classroom debrief in week fourteen:

> I ask [Rowan] what one of the most challenging assignments or specific problems they've graded for this class has been. [Rowan] says that grading questions in which there is nuance in the question and trying to decide between how much that nuance matters and whether it should have value assigned to it versus just writing feedback and leaving it is one of the biggest challenges they run into when grading. So, there are problems in which there is nuance in understanding the material, and the nuance being

> (or not being) conveyed in the answer, and [Rowan] has to start "splitting hairs" over what's worth it to take points off for or not. (Researcher Fieldnotes, Week 14 Classroom Debrief)

As the semester progressed, the weight of having unfinished grading grew heavier and heavier for Rowan. With their students in class, in our debriefs after class, and in their reflection journals, Rowan seemed to feel a need to admit to having large quantities of grading to do. And as more and more ungraded assignments piled on, Rowan seemed to express more and more guilt around their grading baggage. In week seven, Rowan immediately began one of their classes by getting how far behind they were in grading off their chest:

> [Rowan] says, alright, good morning, everybody. I wanted to have your exams graded today but I am only part way done and didn't want to hand out only some of the exams. Based on how far I am, I should have the exams back on Wednesday. Some context, "searching for jobs in academia is 10 times harder". [Rowan] explains that students can always email and ask for them to look at something if they really want feedback on something right away. [Rowan] says, not the way I want to be doing feedback (doing it late), but I will get caught up definitely by Exam 2, during spring break. (Researcher Fieldnotes, Week 7 Classroom)

Note that Rowan identified another reason for being behind in grading here; they were interviewing for academic jobs at the same time they were teaching O&NS. In their week seven reflection journal, Rowan also anticipated continuing to fall behind on grading: "I'm worried about continuing to fall behind on grading. I'm already a couple behind.

I'm hoping to catch up over spring break but I'm so busy." Note that in week eight—the week before spring break—Rowan expressed some optimism about completing grading; In a debrief following class, I asked Rowan if they were ready for break, to which Rowan responded that they were ready to get stuff done, like the grading and their dissertation. However, in week nine—the week after spring break—Rowan returned to feeling dissatisfied with their progress:

> [Rowan] waits for me by the door while I grab my stuff. I ask them how they are doing. They say they're doing okay. They aren't as caught up as they want to be, but they are doing alright. [Rowan] says that they've graded 4 homework sets and that they only have 3 homework sets left to grade. (Researcher Fieldnotes, Week 9 Classroom Debrief)

At this point, Rowan was primarily concerned with staying on top of grading their O&NS students' homework assignments. However, homework was not the only assignment that eventually contributed to Rowan's grading baggage. Following the sixteenth class in week nine, I asked Rowan about their grading for Habits of Mind assignments. Rowan was not yet worried about grading their students' Habits of Mind assignments, because they felt those had a lower priority than getting the homework graded. The reason Rowan felt Habits of Mind assignments were a lower priority was because they thought those assignments were "tangential," "extra," or outside of the core O&NS course material. Moreover, Rowan mentioned peeking at their students' Habits of Mind assignments, and felt unconcerned with their students' performance on them, and additionally unconcerned that they might have to assign low grades. Rowan stated that they expected to give everyone a 90% on each of their assignments using a "bullseye"

grading system: students could earn a 100, 95, 90, 50, or 0% on the assignment with scores of less than 90% reserved only for students who really missed the mark. Although Rowan had at least 4 out of the 8 total Habits of Mind assignments for the course ready to grade, Rowan wanted to prioritize grading homework sets, and therefore, anticipated waiting until near the end of the semester to provide their students with feedback.

When I asked Rowan about grading Habits of Mind assignments again in week twelve however, they shifted from not being too worried to feeling bad about their progress:

> I ask [Rowan] if they've gotten to grading Habit of Mind assignments yet. [Rowan] says no and they feel bad about it. [Rowan] says that they intend to grade the soccer task first so they can get feedback on that and then do the rest. They are hopeful about being able to get caught up on that grading soon (perhaps after they finish their defense). [Rowan] says that it isn't so much about having the time to grade them as it is the mental energy to grade them. [Rowan] says they are just so outside of the material students are learning. [Rowan] says maybe they could have incorporated them into class more, but they didn't. (Researcher Fieldnotes, Week 12 Classroom Debrief)

Here, Rowan started to express some of the same guilt over grading Habits of Mind assignments as their students' homework sets. Rowan seemed remorseful about their progress, as well as not incorporating Habits of Mind assignments in class. At the same time though, Rowan understood these assignments as so "removed from the rest of the course" (Rowan, Week 15 Reflection Journal) that they could not prioritize them more.

In the remaining weeks of the semester, the stress of grading both homework and Habits of Mind assignments weighed heavily on Rowan's mind. They reemphasized some of the same connections they made earlier as to why grading was such a struggle for them—they were not used to grading their own homework—and additionally connected the idea that O&NS course content is challenging to their struggles with grading:

> [Rowan] says that the grading for this course is a "double whammy" because not only is the content more challenging, but you do have to grade it yourself. There's no auto-grading in this course like there is in other courses. (Researcher Fieldnotes, Week 12 Classroom Debrief)

Rowan moreover described their grading progress in terms of an unachieved ideal—what they should have done. In week thirteen, Rowan told their students in class that they should have been giving them feedback on Habits of Mind assignments sooner:

> [Kenny] asks, for HoMs, are you going to do participation, or are you going to do something else? [Rowan] says that it is really going to be graded on effort. [Rowan] says I really should have been giving you feedback on those sooner. They will be graded based on: Did you sincerely attempt to answer all of the questions? [Rowan] says I think that is fair given how hard some of the problems are and that you aren't allowed to use outside resources. (Researcher Fieldnotes, Week 13 Classroom)

An additional dimension of Rowan's guilt manifested around conversations about breaks. After the following class in week thirteen, I asked Rowan if they were looking forward to

anything over the weekend, to which Rowan responded that they looked forward to getting the second exam graded, similar to how Rowan looked forward to getting grading and their dissertation (or thesis) finished over spring break—albeit with less optimism. Hence, Rowan conveyed a seemingly constant awareness and pressure (and possibly even shame) to complete grading responsibilities they perceived as pressing or overdue.

In week fourteen, I asked Rowan about their current successes or challenges, to which Rowan deferred to speaking about their challenges and again named getting grading done as a challenge:

> I ask [Rowan] if any successes or challenges have come up for them lately. [Rowan] says successes, *pause* well, I have challenges. [Rowan] says that getting grading done is a challenge. Because they had to turn in their thesis revisions over the weekend, that meant that exam grading didn't get done. (Researcher Fieldnotes, Week 14 Classroom Debrief)

Rowan additionally identified Habits of Mind assignments as potentially the most challenging assignments in the course to grade, but because they had not yet graded any of these assignments for their students, they planned on grading them with an easier-to-use system: how much effort did they think the students put into the problem?; did they try to do it or not try to do it? Rowan further explained why they thought Habits of Mind assignments might have been more challenging for them to grade, had they managed to start grading them sooner:

> [Rowan] says if they had graded them sooner, then they might have done what [Robin] and [Rose] did; they saw that students hadn't been turning in solutions that dealt with all of these things they wanted, like understanding

> the nuance of an N+1 proof, and so that would have involved work in helping students get that nuance of solving challenging problems. Because these are challenging problems. (Researcher Fieldnotes, Week 14 Classroom Debrief)

Rowan further noted the struggle of grading baggage. Having so many assignments of one type to grade made the task too daunting:

> [Rowan] says that it's also been a challenge to get these Habits of Mind assignments graded and part of the challenge is that they want to just get all of a certain kind of grading done in one sitting and they rarely have time to get something done in one sitting, so maybe they just kind of have to get over that. (Researcher Fieldnotes, Week 14 Classroom Debrief)

Because grading weighed so heavily on Rowan's mind, they occasionally described their successes in terms of grading too. After the following class in week fourteen, I asked Rowan how it was going, and Rowan said that "they stayed up really late getting exams graded, but now they are done and so that counts as a success for this week" (Researcher Fieldnotes, Week 14 Classroom Debrief). Nonetheless, the burden of grading stood out solidly for them as the primary challenge in weeks fifteen and sixteen. In our debrief after class, I asked Rowan if any other successes or challenges have come up for them lately, and Rowan again responded in terms of their grading progress:

> I ask [Rowan] if any other successes or challenges have come up for them lately. [Rowan] says that they still have to catch up on their homework grading. They hope to get caught up with that by this weekend. [Rowan] says that if they grade one homework a day, they can get there. It will be a

> lot, but they feel a need to do it, especially since the exam got moved up to Monday. [Rowan] says that a lot of their projects still haven't been graded either. I ask [Rowan] how many homeworks they have left to grade. [Rowan] says that there are 6 homeworks left. [Rowan] says that the last homework hasn't been turned in yet and that material isn't on the exam, so they won't have that graded yet. So [Rowan] says they still have about 5 homeworks left to grade. (Researcher Fieldnotes, Week 15 Classroom Debrief)

In their final journal in week fifteen, Rowan reflected on the "mountain of grading" as a challenge and named time management as a skill they wished they had learned in their preparation for teaching O&NS:

> Still struggling a little with getting grading finished. I want the students to not only have feedback on their homework but also a clearer idea of their grades and I'm pretty sure I won't be able to provide that. Having more experience grading (or less of the project grading dumped on me) could have been helpful… I wish I was more prepared to allocate my time appropriately for grading – now I'm left with a mountain of grading to complete in a few days. (Rowan, Week 15 Reflection Journal)

And in week sixteen, the guilt over grading that had come to be characteristic for Rowan still shone through after their students' completion of the final exam. Consider our debrief following the final exam, in which Rowan derailed my congratulatory message on finishing and graduating to focus on the grading they had left to accomplish:

> I say to [Rowan] congratulations! How does it feel to be finished and graduating soon? [Rowan] looks at me with a bit of apprehension and says well, I don't quite feel finished just yet. [Rowan] says that they still have a lot of grading to get done. I ask them how the grading is going. They accomplished their goal last week with grading the homework and got all of the grading for homeworks that had stuff that was going to be on the exam done by Saturday. They say that was their goal last week and they accomplished it. So [Rowan] still has 1 or 2 homework sets left to grade, along with all of the Habit of Mind assignments (8 total), and along with all of their portions to grade from the projects shared across the EMM course (and the final exam students just finished). [Rowan] says that they want to get all of the grading done by Thursday this week because that is when their parents are coming for graduation. [Rowan] says that is ambitious and probably won't happen but that is what they are aiming for. (Researcher Fieldnotes, Week 16 Final Exam Debrief)

In Rowan's mind, they were far from "finished" and in a place where they felt like celebrating, despite their accomplishments in meeting their most recent goals for grading. At the end of the semester, Rowan had to deal with the grading baggage that had been accumulating throughout the semester. It weighed heavy.

"TIME" and Being a Graduating Ph.D. Candidate

One of the biggest challenges for Rowan throughout the semester surrounded time. Specifically, Rowan struggled to manage their time attending to the numerous demands of teaching O&NS: grading and providing feedback on numerous multiple

assignments, which typically involved drawings and explanations, learning different and difficult course content, and addressing student questions around K-12 student thinking. While doing or preparing for these tasks are challenging in and of themselves, Rowan found that being a Ph.D. candidate—defending their dissertation, preparing for graduation, and interviewing for academic jobs—often took away time they felt was necessary for them to feel more comfortable (and less uneasy and unsteady) with teaching new material and finishing their grading assignments in a timely manner.

Note that some of Rowan's unease and uncertainty was related to a general sense of a lack of time in their preparation for teaching O&NS. In week one, Rowan wrote: "I wish I had more time to start prepping the course before it began. I feel like I'm piecing together course content as I go instead of having a clear view of the course and the assignments" (Rowan, Week 1 Reflection Journal). For the majority of the beginning of the semester though, interviewing for academic jobs was especially time-consuming and taxing for Rowan. In total, they interviewed at five academic institutions, three of which were virtual and two of which were in-person, and all of which involved multiple interviews spread across several days. In week two, Rowan described how interviewing for jobs "really got in the way of [O&NS]":

> This week life really got in the way of [O&NS]. I'm currently applying to jobs and had an in-person interview across the country. Not only did I completely miss the [O&NS] prep meeting (and forgot to tell anyone I wouldn't be able to make it! I knew I was missing something) but I also wasn't able to support my students as well as I had wanted to for their very first homework assignment. (Rowan, Week 2 Reflection Journal)

Interviewing for jobs not only disrupted Rowan's office hours and ability to attend instructional meetings, but they disrupted Rowan's progress towards finishing grading. Recall from the previous section that in week seven, Rowan stressed that searching for academic jobs was part of the reason they were behind on grading:

> [Rowan] says, alright, good morning, everybody. I wanted to have your exams graded today but I am only part way done and didn't want to hand out only some of the exams. Based on how far I am, I should have the exams back on Wednesday. Some context, "searching for jobs in academia is 10 times harder". (Rowan, Week 7 Classroom)

Rowan had hoped, however, that accepting a job offer would allow them to finally catch up with their grading, as they shared with me in a debrief following that same class:

> [Rowan] says they are excited to make a decision (about which job to take) and finally get some grading done and get things back on track. I say and you (Rowan) have a spring break too. [Rowan] says yes, they will get grading done, they want to be grading all break, but if they have to use all of their spring break to get it done, they will. (Researcher Fieldnotes, Week 7 Classroom)

In week eight, Rowan additionally hoped that during their spring break, they might "get stuff done, like the grading and their thesis" (Researcher Fieldnotes, Week 8 Classroom Debrief). Thus, with the prospects of a break in the near future, Rowan started to postpone their grading in hopes of a more ideal time—when they did not have other pressing obligations like their thesis to complete.

The stress of not having enough time was strong for Rowan in the middle of the semester. As highlighted above, they often felt the pressure of time against grading, but they also felt the pressure against their other teaching responsibilities. In week six, Rowan anticipated the challenge of not having enough time to both plan their lesson and finish grading: "I'm racing against the clock in terms of getting class material prepped and homeworks graded, I'm afraid one of these days I'll show up without 75 minutes of material prepared" (Rowan, Week 6 Reflection Journal).

Rowan especially emphasized the significance of time during the support interview in week eight. At various points throughout the interview, Rowan interjected to express the overall sentiment of a lack of time or desire for more time, saying, "I just wish that I had more time" or "I just don't have enough time." In addition, Rowan added time as a desired support to the table of possible support, writing "TIME" in all-caps, saying "it is quite important, this class takes a lot of time, this class takes a lot of time. Rowan felt that they were so busy that when I asked Rowan if there was any support for teaching O&NS they might change, Rowan described choosing the "wrong semester" to teach O&NS and elaborated on how the collective commitments of the class led them to "an insurmountable amount to catch up on," especially with respect to grading:

> I picked the wrong semester to teach this class. I don't have the time to do it. I think that I could have really enjoyed and had a better experience in a semester where I wasn't so busy. Whatever the, it's like so many hours per class teaching. I surpass that every single week and I'm behind in everything. I have always consistently spent quite a bit more time teaching than what I am contracted for. This semester I don't have the time to

> overwork, to the extent that I need to to make this class run smoothly, I'm hitting usually just above the allotted time every single week, and that's all that I am capable of committing to this class and I'm behind. I've graded two of their homework assignments, none of their Habits of Mind. I have no idea how my students are doing in this class. Struggling to just like get prepped for class every day, so. So, between office hours and prepping for class and answering student questions. I hit time on the class, on that alone, and then I have to introduce grading, and writing quizzes, and grading quizzes and exams. Putting, how much that is, *inaudible* hours, and just all sorts of things that now that I've gotten behind on them, it's like an insurmountable amount to catch up on, since I already don't have time to do like the normal amount of grading. It's a lot of grading. I definitely was not prepared to grade this class. (Rowan, Support Interview)

Unfortunately, spring break offered little reprieve for Rowan from the stress of time and the pressing need to complete their grading and their dissertation. Instead, some of Rowan's initial fears around running out of time to prepare for class started becoming prevalent. In week eleven, Rowan had not examined their students' exam review guide in advance of the class in which they asked their students to complete it, and therefore had to figure out how to explain problems involving hypothetical K-12 student work on the fly. Consider Rowan's whole class discussion, in which a student asks about hypothetical student work and Rowan apologizes for not having figured out the solution in advance:

> (O&NS) Student says I have no idea where she's (hypothetical student) getting that. [Rowan] says I had to think about it for a while as well. [Rowan] writes 2336 – 1779 = ?. [Rowan] writes 1779 + ? = 2336. [Rowan] writes a number line and explains: "Essentially, we're trying to group up two values that are nicer... so I'll go up by 21 to get to 1800. 1800 to 1836, I go up to 36. But I don't want 1836... I disagree with where they put the 36 in the problem, I think they should have saved it for last, but following the student reasoning... which is all of these numbers added together... I apologize to groups who I explained this problem to..." [Rowan] explains that they were figuring out the solution in class today. [Rowan] says they didn't write the problem, but they had figured it out in the moment. (Researcher Fieldnotes, Week 11 Classroom)

In week twelve, Rowan similarly struggled to plan for their class ahead of time. On the few occasions in which Rowan felt less sure about their solution, they apologized profusely, as illustrated in Rowan's whole class discussion involving identifying types of division problems and solving them:

> [Rowan] explains the problem, saying, "This is a how many in a group problem... So, this last problem... how many can you buy for $1... *mumbles very lightly* should have thought this one through..." [Rowan] pauses before the whiteboard. [Kenny] says, "Do you want me to tell you my thinking?" [Rowan] says, "Yeah, absolutely, I should have done this one before class." [Kenny] provides an idea. [Rowan] thinks about what [Kenny] says. [Rowan] says sorry I should have solved this one before

> class. Another student at the back left says I think it should be the other way, because 3 is the product. [Rowan] says I really want to think on this one before I sign on to this. A student at middle left says I just solved it like the last problem. [Rowan] says that makes sense. [Rowan] says I think that makes a lot more sense. [Rowan] says, apologies for not being prepared for this one before class. (Researcher Fieldnotes, Week 12 Classroom)

Note that Rowan's in-class struggle to explain solutions to their students in weeks eleven and twelve was uncharacteristic of their explanations in previous weeks. When I debriefed with Rowan after class, Rowan explained that they had just defended their dissertation the previous day, and as a result, was not prepared for teaching their lesson.

> I look to [Rowan] and ask them how's it going? [Rowan] says something along the lines of: well, I was not prepared for class today; I just threw the slides together and didn't really look over those problems; This is not really a surprise after the day I had yesterday defending my dissertation. I ask [Rowan] how the defense was. [Rowan] says I basically crashed after my defense and woke up at midnight and then planned my lesson for today. I laugh and say I don't blame them at all and ask, how was the defense? [Rowan] says that their advisor answered half the questions that the committee posed. [Rowan] also said that they just got comments on their defense yesterday, so they have little time to make those changes and submit them, as well as getting caught up with their other coursework. I ask [Rowan] how they think the class went. [Rowan] talks about how they

> threw the slides together and didn't really feel like they understood the material, but just enough to sort of surface-level understand the material. They say that they didn't really prepare for those problems like they usually do. They usually know the solutions to the problems ahead of time but didn't here. (Researcher Fieldnotes, Week 12 Classroom Debrief)

Notice that even though Rowan successfully defended their dissertation, they were already concerned about having little time to submit corrections to their dissertation, as well as catch up with the coursework they were enrolled in as a graduate student. Moreover, Rowan was so busy with defending their dissertation that they completed no reflection journal entries for weeks eleven and twelve. Hence, Rowan's dissertation defense thoroughly exhausted them and impacted their planning, among other O&NS teaching responsibilities.

As briefly mentioned in the previous section, Rowan finally experienced some optimism and relief because of their perceived progress on their dissertation (thesis): "Now that I'm done with my thesis defense, I can finally take a deep breath when I have a spare moment instead of having to stress about my defense when I have a spare moment" (Rowan, Week 13 Reflection Journal). Rowan further emphasized the impact of rest on their teaching:

> Teaching this course after a full night's rest is so much better. I definitely have had classes where I was tired and misspoke a lot (like calling the commutative property the distributive property several times over by accident) and I know that those slip ups on my part make it that much more difficult for my students. (Rowan, Week 13 Reflection Journal)

However, Rowan anticipated that their relief would be temporary: “I still haven’t completed the final document for my dissertation so it’s possible that this well rested optimistic version of me will be short lived... my thesis document isn’t done so I may get too busy for class again.” (Rowan, Week 13 Reflection Journal)

In week fourteen, Rowan was indeed “burnt out” (Rowan, Week 14 Reflection Journal) and challenged to plan their lessons, get comfortable with learning the O&NS material, and finish grading, as well as revise and submit their dissertation. Consider our debrief after class in week fourteen, in which Rowan explains that they prioritized finishing their revisions over lesson planning and grading, and felt that they lacked the time to get comfortable with the O&NS material given their other priorities:

> I ask [Rowan] how they think it went. [Rowan] said that it went fine but that they were uncomfortable with the material and worried that their discomfort showed or was projected onto their students. Division with fractions is not material they are super comfortable with teaching. I ask [Rowan] how planning for today went. They say that they didn’t spend a lot of time on it. Part of the reason for that was that their advisor wanted corrections on their thesis today, so they had to get that done. [Rowan] re-iterated that they don’t feel comfortable with the division sections and might have wanted more time to get comfortable with the material… I ask [Rowan] if any successes or challenges have come up for them lately? [Rowan] says successes *pause* well, I have challenges. [Rowan] says that getting grading done is a challenge. Because they had to turn in their

> thesis revisions over the weekend, that meant that exam grading didn't get done. (Researcher Fieldnotes, Week 14 Classroom Debrief)

At the end of the semester, Rowan reflected on the importance of time. In their week fifteen reflection journal, Rowan thought, "I'm not as good at organization and time management as I thought I was" (Rowan, Week 15 Reflection Journal). During the final interview (the week after the final exam), I gave Rowan the table they edited and used to discuss their support for teaching O&NS and asked them if there was anything they said back then that especially resonated with them now; Rowan responded in laughter with the "TIME" they wrote in all-caps: "This all-caps 'TIME' that I put and highlighted and put red and blue next to. *laughs* Definitely. Definitely resonates with me" (Rowan, Final Interview).

Rowan especially reflected on the impact of being a graduating student in teaching O&NS. In response to my question about the most significant challenges they experienced as an instructor, Rowan first and foremost identified time and elaborated on how their teaching responsibilities "fell by the wayside because of the lack of time." They, moreover, emphasized how there was not enough time to prep lessons after taking care of other teaching responsibilities and that prepping lessons also took time because they needed to learn the O&NS material:

> Yeah. It was really time, and then just sort of everything that fell by the wayside because of the lack of time. Most pressingly, grading fell by the wayside. Quite a bit. Sometimes I wouldn't get back to my students as quickly as I wanted to. Was also sort of a point of contention for me. I would have to miss office hours in order to fit other meetings into my

> schedule, which I would always feel really bad about. I, you know, just like this, that, and the other. There was not enough time to do everything. And then it also left, like, minimal time to prep the class, right, like, even just, like, because I had to learn the material myself. I had to read through the textbook, learn the material myself, go through the exercises, make sure I knew how to do the exercises, and then I needed to put them together to teach them. And by the time I had done that, we're already looking at an hour, hour and a half, two hours of, of prep time, and I don't have time to then think about, well, what if I want to do something different than how the textbook says? What if? What if I need physical manipulatives? Well, I was usually prepping the night before, so if I didn't have it on hand myself, we weren't doing it. Um, you know, that kind of stuff. Yeah. Just time. Could have gone better with just a little more time. (Rowan, Final Interview)

When I asked Rowan about the extent to which they had been able to manage or overcome these challenges, Rowan described how they were always burdened by another priority for graduating: first, interviewing for jobs, next defending their dissertation, and last revising and submitting their dissertation. They learned that "there always is a next thing, and to deal with the situation that I'm in instead of like hoping for a better situation in the future":

> But it was always. There was always a next thing. I think I had kept convincing myself, like when I'm done with the job search, then everything will be better. When I turn in my dissertation, then everything

> will be better. When I defend it, then everything will be better. But there just always is a next thing. And I think that I could have done better to, to realize that there always is a next thing, and to deal with the situation that I'm in, instead of, like, hoping for a better situation in the future, where suddenly I'll be able to catch up on all my grading, and suddenly I'll be able to prep classes for longer, and stuff like that. That came and went here and there, but really didn't stick. (Rowan, Final Interview)

Time and being a graduating Ph.D. student provided constant challenges for Rowan. They always presented themselves as barriers to being a better O&NS instructor in terms of supporting students, learning the O&NS content, prepping for class, and providing timely feedback and grades for students. Although Rowan learned that their old mindset—just get x done, "then everything will be better"—was unproductive and planned to deal with situations as they arise in the future, they concluded that graduating students are too busy to be teaching O&NS. When I asked Rowan about the kinds of prior experiences they imagine would be most helpful for managing the challenges they experienced or anticipate that future instructors might experience, Rowan responded, "Don't ever let a graduating student teach this class. Never again. Never again. You're too busy, you're out of town all the time." (Rowan, Final Interview)

Passing and Surviving

Even though Rowan faced seemingly "insurmountable" challenges and ultimately concluded that graduating Ph.D. students should not teach O&NS in the future, Rowan managed to identify a few successes during the final interview. They described their

successes in terms of students passing the final exam, as well as expressed the overall sentiment that they "didn't mess anything up too bad":

> Significant successes. All of my students that were attending class passed the final exam. *inaudible* a success. I wouldn't say that I think that it's like an important measure necessarily, I think you can be successful in a course and still fail the final exam. I've been successful in courses and still failed the final exam, but that through all of this, even though I didn't always give their homeworks back on time. Even though the Habits of Mind were in flux, even though this, even though that. We still got to a place where the students understood the material well enough to pass that final exam. *inaudible* measure of understanding. Like, I didn't mess anything up too bad. (Rowan, Final Interview)

Hence, by the end of the semester, Rowan defined success in terms of their O&NS students' achievement (passing), as well as their assessment of the extent to which they might have messed up a class (surviving). Note that students' passing exams was one measure of success Rowan identified throughout the semester, whereas Rowan surviving teaching seemed to emerge more as a success towards the end of the semester. Table 6 shows reflection journal entries in which Rowan identified students' performance as a success or anticipated success.

Table 6

Reflection Journal Responses Related to Students' Performance as a Success or Anticipated Success

Week	Reflection Journal Response

6	"I'm hoping my students all pass the exam, which would be a big win. I especially hope they perform comparably to [Robin] and [Rose]'s students. I possibly am less anticipating this and moreso am fearful that their grades will be significantly lower."
7	"I passed back exams and they had gone fairly well and the students didn't have any questions/concerns about them (at least none they told me)."
10	"I think my students will do okay on the partial products part of the exam! [Rose] warned that students would be very much against having to learn and use the partial products method of multiplication but it seemed like my students were very open to it and several had even seen this method or something similar before."
15	Based on the grades I have so far and my students' performance during review, I am anticipating all of my students that have engaged with all of the course material to pass the class.

Now, what might it mean to mess up a class? Rowan elaborated on what it might mean to survive teaching "without any catastrophic failures" in their week fifteen reflection journal:

> I managed to finish the classes without any catastrophic failures! I was definitely nervous throughout the semester that I wouldn't get slides prepped in time or would sleep through my alarm or would not be able to answer a students' question but now that we've made it through all of the class periods there's no chance of that happening anymore. (Rowan, Week 15 Reflection Journal)

Teaching O&NS without "catastrophic failures" was not a trivial success for Rowan. Recall from previous sections that Rowan expressed a great deal of concern and unsteadiness over prepping for their class and addressing students' questions as a busy, graduating student. Moreover, a lack of "catastrophic failures" was not necessarily a given for teaching O&NS, especially when considering Rowan's concerns in light of the issues that Robin experienced, such as "pushback" and "complaining" from students—tensions that Cypress negotiated with their shared students' in their EMM course (see end

of addressing student questions section to further recall these issues). Because of these issues, Robin finished the semester with an overall negative experience teaching O&NS, whereas Rowan, despite their challenges, finished the semester with an overall positive experience teaching O&NS. Thus, surviving teaching O&NS with overall positive views about their students' experiences and their own experiences was a critical way in which Rowan experienced success.

Learning to Seek External Support and Ask for More Resources in the Future

Despite having little preparation for learning to teach O&NS, as well as limited support through their institution, Rowan sought out external forms of support and persevered. And although Rowan did not identify their learning to seek external support or ask for more resources in the future as successes, I identify them as ones in addition to those explicitly named by Rowan. First, Rowan's unique sources of support, at least in part, explained some of the successes Rowan experienced throughout the semester. As I discuss in later sections, Rowan's decision to seek support from their mom impacted their successes in navigating O&NS students' questions and conversations surrounding K-6 students and schools.

Second, Rowan's eventual learning to ask for more resources was a form of growth that Rowan experienced through teaching O&NS. Recall that weekly reflection journals increased Rowan's sense of agency around seeking support and led Rowan to realize that "it is not a failure on my part to need to ask for more resources." Regularly responding to questions about the support they had and wished they had made Rowan realize that they *could* ask for resources whenever they needed them. Moreover, Rowan

planned to incorporate asking for additional resources into their practices as a faculty member in the future, even though they were uncertain about what they might be:

> This is something I've actually been talking through with the person who's going to be my new boss in the fall because he asked me all the time at the end of our meetings, "What do you need? What do you need? What do you need?" And we finally have to have the discussion that I, I need to be told what I need in this junction. Like, I, I know that I need something. I don't know what it is, and I sort of need to be told. I need to be told what I need to say that I need because I'm honestly not sure. (Rowan, Final Interview)

Hence, the culmination of Rowan's experiences teaching O&NS influenced them to become a more agentive advocate for themselves, even in uncertain situations.

Not Necessary to Evolve—A Metaphor for Rowan's Learning to Teach O&NS

While not explicitly connected to any particular challenges or successes, Rowan related their experiences learning to teach O&NS to their identity as a mathematician. When I first asked them to provide a metaphor describing their experiences learning to teach O&NS, Rowan thought of the metamorphosis of the caterpillar, because they needed to grow or transform for this course. They stated that "the caterpillar is a mathematician who knew all this math and was fine, but then they encountered this course and needed to cocoon-up for a little while, and when they came back out, they had some new features" (Researcher Fieldnotes, Week 16 Final Exam Debrief). However, Rowan also immediately emphasized that their metaphor was not a perfect analogy, as they felt unsatisfied with the fact that caterpillars are expected to transform into

butterflies, whereas mathematicians do not have to transform into teachers of mathematics content courses for PTs.

During the final interview, Rowan revised their metaphor to capture their identity more closely. They chose Eevee's evolution process in Pokémon (a Japanese media franchise involving fantastical creatures who can grow stronger through the use of evolution stones) as their metaphor instead. What made Eevee a better metaphor was that Eevee has "several different options of ways to go," as opposed to a caterpillar, who has one way to go. Moreover, Eevee does not have to evolve at all—they can remain Eevee—whereas a caterpillar necessarily undergoes transformation into a butterfly as a part of their life cycle. Rowan described how these ideas were connected to their learning:

> I know, like, everyone is growing in mathematics and learning different things in mathematics, and that's just sort of, like, one way you can flourish. And there are other options that are equally valid that you can flourish in… I think I like that better than the caterpillar, because the caterpillar kind of implies that you will only hit your final form if you do this specific thing. You can do different things, and if you do a different thing, you're going to look a little different. (Rowan, Final Interview)

Notice that earlier Rowan described the caterpillar as a mathematician, and during the final interview, they highlighted learning and growing in mathematics, as opposed to being a teacher, educator, or growing in instruction. The transformation of Rowan's *metaphor* is important because it signaled to me that Rowan's identity as a mathematician is a fundamental basis for who they are. They were unsatisfied with the caterpillar

metaphor because they did not want to portray their identity as a mathematician as somehow lost or shed through the learning process. Rather, Rowan wanted a metaphor that would align with their view of themselves as a mathematician. Rowan saw themselves squarely as a mathematician, and teaching mathematics content courses for PTs (which they saw as more specialized) as an additional skill that can (but does not have to) add on to being a mathematician.

I incorporate Rowan's metaphor for their learning to teach O&NS because it showcases Rowan's identity (or background) as an important aspect of understanding their experiences. That Rowan sees themselves primarily as a mathematician coincides with their background and is important to keep in mind when reading the following sections aimed towards understanding the connections among Rowan's background, prior experiences, preparation, support, challenges, and successes. As I describe in later sections, Rowan's identity as a mathematician provides insight into the ways in which Rowan engaged in lesson planning and support, and why preparation for lesson planning, in particular, matters for mathematicians like Rowan.

Understanding Rowan's Challenges and Successes

"This math class is different"—Moving Beyond Learning How O&NS is Different as an Instructor to Communicating Expectations for Students

As discussed in the previous sections, much of the work surrounding teaching O&NS was new, less familiar, and at times "uncomfortable" for Rowan. Towards the end of the semester, Rowan realized that the O&NS course, including its content, purposes, and learning goals, was not just new and different for Rowan as an instructor, but new and different for their students as well. Specifically, Rowan learned that the O&NS

course is not just about focusing on “how to do a skill”, but “how to navigate all its intricacies”:

> I’m not sure that I made the distinction between the different methods clear enough or the fact that everyone needs to know all the different ways to solve a division problem. In this course, sometimes the students get too caught up in thinking they just have to know how to do a skill, not how to navigate all its intricacies. (Rowan, Week 13 Reflection Journal)

What did Rowan mean by “intricacies” and how did Rowan come to be concerned about them? Rowan wanted O&NS students to navigate these “intricacies” and “nuances” in their homework solutions, especially around problems involving interpreting hypothetical K-6 students’ mathematical reasoning. Rowan noticed that their O&NS students would only solve problems a correct way, rather than addressing the K-6 students’ reasoning (e.g., the mathematical ideas the K-6 students likely already know and/or how they might adjust their solution(s) to be correct). Rowan’s reflection on this issue made them realize the importance of communicating their expectations for interpreting K-6 students’ solutions early in the semester. Specifically, they wished they had emphasized O&NS as a course in which O&NS students are not just re-learning mathematics content but learning as much of the nuances surrounding a particular mathematical concept as possible, such that the O&NS students could examine any K-6 student’s work and understand what they might have been thinking. Rowan described these ideas to me in a debrief after class:

> I ask [Rowan] to give an example of a problem in which they have been “splitting hairs” over the nuance of understanding course material.

[Rowan] says that it's usually problems in which PTs are being asked to correct (K-6) student reasoning. So, there are a few things they see (O&NS students doing on assignments). One is that some PTs will look at the student work, and then just solve it a correct way without addressing what the student knows and did correctly (i.e., meeting the student where they are at). Another is that (O&NS) students might be trying to interpret (K-6) student work, and then, in the process, they trivialize the problem and write a solution to that problem. [Rowan] says, but I'm not really qualified to assess that work; How do I know what the student was thinking? [Rowan] says, it's not really the PTs' fault for interpreting the problem that way, in fact, the (K-6) student could interpret it that way. But that's sort of a nuanced thing that we need to teach PTs. So, like, in the book, there are these problems that usually provide student work with their thinking behind it, and then you have to decide if the (K-6) students' reasoning is correct or not. But there's not really problems that are devoid of the context and you are trying to figure out what's going on. And those are two separate skills really. [Rowan] says, we want PTs to be able to determine whether or not a (K-6) students' reasoning is correct, to meet them where they are at, and help them get on the right track. But we also want PTs to be able to think through the possibilities for what (K-6) students might be doing. [Rowan] says, and that's not something that I really think I feel like I can help them with outside of the classroom; I don't feel qualified to assess that and assign value to the nuance of

> interpreting student work. [Rowan] says that they feel like this is something that they would want to spend more time on at the beginning of the semester if they could go back through and do it over again. They want to teach PTs how to be a student in this class. They want to talk about how they need to learn how to understand and interpret student work and respond appropriately to it. [Rowan] says that they think (O&NS) students feel like they are just learning about what multiplication is—as evidenced by what (O&NS) students do in their homework sets—which, for some of them, that's fine if they are re-learning what multiplication is, but really, they are learning how to teach multiplication to (K-6) students, which is completely different. [Rowan] says, you are learning about the nuances of multiplication so that you can convey those nuances to (K-6) students and you can look at any (K-6) student work and know what's going on. [Rowan] says that there isn't enough time for this in the class to talk about this though. (Researcher Fieldnotes, Week 14 Classroom Debrief)

By the final interview, Rowan had firmly cemented the idea that "this math class is different" from "the way that it has been in all of their (O&NS students) other math classes," and as a result, learned that setting up classroom expectations so that O&NS students can not only do K-6 mathematics but "teach it to someone else with all of its proper nuance" is important:

> Something I learned is more important than I thought it was, is setting up classroom expectations. Not necessarily just, like, what you do in the class—I feel like we did pretty well learning that together—but in terms

of, like, what does a good homework look like? What does it look like if you've done well? What does it look like to engage with this kind of math?... Just giving my students feedback on their assignments is not enough to correct their behavior for a lot of students. To get to a place where we're turning in good assignments, but, like, being a little more up-front about, like, this is exactly the type of thing I expect. I think that I did not have to approach it so aggressively in my other courses because in all of my other courses, students have been answering questions to answer the question, right? Factor this thing. "Well, I'm going to have finished if I can factor it myself", whereas. I think a lot of my students treated this class that way: Do this multiplication problem and draw it. And they feel like if they can do the multiplication problem and draw the array, that that should be it. Right? Them being able to demonstrate that skill is the end of the line, the way that it has been in all of their other math classes. At least that's how I've been running a lot of my math classes. "I can demonstrate this skill, I'm done". I'm sort of ignoring the fact that in this course, it's not just can you do this skill, it's do you understand the skill, and could you teach it to someone else with all of its proper nuance? And even beyond that, when your student does it wrong, can you correct it properly?... I definitely had several homework sets where there would be an explanation, like a request for an explanation at the end of the question, and my students would do everything up until that point and just stop the problem. Sort of a misunderstanding of how this math class is different.

> I'm not teaching them how to multiply numbers. Hypothetically, they should know how to multiply numbers because a grade-school teacher taught them. I'm teaching them how to teach how to multiply numbers. (Rowan, Final Interview)

"Uneasy" and "Unsteady" Disposition Developed through Preparation, Support, & Prior Experiences

Recall that Rowan developed an "uneasy" and "unsteady" orientation towards the O&NS course material and their teaching of it. Difficulties in making sense of less familiar course material contributed towards their developing sense of uncertainty, but this sense originally manifested because of the sudden and immediate need to start teaching O&NS without an introduction to the purposes and objectives of the course or the kinds of resources they were accustomed to in their prior teaching experiences, such as a repository of instructional materials from previous semesters. As a result, Rowan wished for more time at the beginning of the semester to prepare for and be oriented to teaching O&NS.

Rowan's uncertainty also developed through their support and prior teaching experiences. Rowan primarily relied on the O&NS instructors and the textbook, feeling "locked-in" or restricted in what they could do because of their unfamiliarity with new and different course material and limited resources in comparison to teaching Calculus and Intermediate Algebra. Moreover, Rowan described feeling more comfortable with teaching about mathematics concepts in Intermediate Algebra or Calculus than in O&NS. For example, recall that writing problems about limits was more intuitive to Rowan than writing problems about fractions. Rowan's prior mathematics learning experiences

influenced their uncertain disposition as well, especially on occasions when they did not remember learning the elementary mathematics they were teaching. For example, recall that Rowan did not feel comfortable teaching ideas around dividing fractions with fractions. As a result, Rowan focused more on the procedure of dividing numerators and dividing denominators with their students than understanding why it works.

One reason Rowan's discomfort is important to understand is because they have a strong identity as a mathematician, *and* they struggled to make sense of elementary mathematics content. Their experiences illustrate how challenging a content course focused on understanding elementary mathematics can be for mathematicians to teach, and therefore important not to underestimate. About halfway through the semester, Rowan described how their perspective on the O&NS curriculum shifted; at first, they were surprised to learn they would spend an entire semester on four operations, but later felt they barely had enough time to cover the core content. Hence, additional support in learning and growing comfortable with O&NS course material might have benefited Rowan, especially in moments when they encountered understanding elementary mathematics at a deeper level (and subsequently engaging PTs in deepening their understanding) as challenging or different from what they learned or how they learned it in elementary school.

"we didn't learn how to make sure that we had *time*"

For Rowan, having enough "TIME" was a problem of multiple, overflowing responsibilities. As mentioned previously, around mid-semester, Rowan adopted the mindset that if they finished each of their tasks within a certain set of responsibilities, "then everything will be better" enough to work on the next set of responsibilities.

Although Rowan later found this mindset unproductive, they might have felt like they had enough time if, at the same time they were engaging in teaching and grading new material, they were not also trying to graduate. However, related to the newness of the course and their preparation for teaching O&NS, Rowan also felt uncertain and unsteady, which contributed to moments of indecision (e.g., deciding on a grading scheme for homework assignments), thereby making each responsibility consume more time than it might have otherwise. Moreover, Rowan struggled to manage their time all semester long, not solely because they were graduating or because their preparation did not necessarily support them in learning certain skills (e.g., how to select homework grading schemes in alignment with your values), but because they were not necessarily taught how to manage multiple responsibilities at the same time. Consider Rowan's reflections on their prior experiences and preparation for teaching in general in light of their challenges to prioritize responsibilities:

> there's a lot of good time put into teaching kids how to lecture and how to get student feedback… there's a lot of these things and activities that we are taught how to do and to do well. But what we are not necessarily taught is that in order to do all of these things at the same time, you have to be quite a bit better at time management and deciding what things you're willing to give up… When I was taking pedagogy, I could sit around and chat with my colleagues for hours about precisely how we want to do groups. That is not a luxury I have this semester… I had all of this *time* to reflect on and change and implement… part of it is because a convenor or associate convenor was taking over a lot of the brunt of

> things, like setting up schedules, setting up the homework assignment… That kind of thing, where we learned how to do a lot of these great activities, but we didn't learn how to make sure that we had *time* to do these great activities… That probably also feeds into why I'm feeling more like a failure in this class than in previous classes is because I don't. I suddenly. I've learned all these things and I've been adding all these things to my teaching practice. And now, suddenly, I'm incapable of doing them, not because I've lost the skill, but because I just don't have the time. (Rowan, Final Interview, emphasis original)

Unlike in their pedagogy course or in their prior teaching experiences, Rowan was challenged to prioritize their responsibilities, as opposed to taking their time reflecting on each one. Specifically, teaching courses with a convenor or associate convenor afforded Rowan the "luxury" of reflection time. Recall that this was the silver lining from Rowan's fifth year of teaching: having fewer responsibilities meant more time to reflect on fostering relationships with students and enacting lessons to the best of their abilities. However, even though Rowan had experience as an associate convenor, having fewer responsibilities throughout their years of teaching overall often meant pushing off learning to accomplish certain tasks or hone certain skills to the backburner. Hence, Rowan's experiences highlight a significant conundrum in their preparation for teaching: when and to what extent does decreasing instructors' responsibilities support (or hinder) their learning to teach? I consider this question in Chapter 7. For now, I provide further explanations as to why teaching O&NS was a challenge for Rowan considering their background, prior experiences, preparation, and support for teaching O&NS.

Less Prepared and Supported for Grading

One of the main challenges Rowan experienced throughout the semester involved grading. First, recall that Rowan had little experience with grading homework in general because all the courses they taught in the past utilized auto-grading systems for students' homework assignments. Second, recall that Rowan had less familiarity with grading and providing feedback on solutions involving explanations. As a result, Rowan felt less prepared for grading homework and wished they had more practice grading homework in their prior experiences and preparation for teaching in general. Recall that Rowan described these connections as early as week five of the semester.

> More experience grading homeworks would have been useful. Not only were homework assignments all online and auto graded for all of my previous courses so I don't have much experience, but also the types of homework they do in this class (a lot of drawings and explanations) are less familiar to me and take a while to grade. (Rowan, Week 5 Reflection Journal)

By the end of the semester, Rowan continued to emphasize these same connections between their prior experiences (little experience grading homework) and challenges grading (difficult to grade explanations), as well as the same concluding sentiment towards their preparation for teaching (they wished they were more prepared to grade homework). Consider Rowan's response to my question: "What kinds of prior experiences, do you imagine, would be most helpful for managing the challenges that you experienced or anticipate that future instructors might experience?":

Definitely more practice hand-grading. Right? I, in all my previous classes my students did [auto-grading system] homework, so I actually didn't have to grade them, right. I had to supplement before the semester started. I didn't have to grade their homework sets, and a lot of exam grading has gone completely digital as well. Whereas the (this semester), I mean, even though technically what I did was I pulled up their assignment as I was starting it on Canvas, like a picture of their assignment on Canvas, and I wrote my notes on an iPad. Really, it's more of like a hand-grade. Like, I'm reading every single question because that's what every single question. I'm doing it in a manually handwritten way. That kind of stuff, that is a skill that I was not provided with through my four years of experience here. Either that or change the way [O&NS] things are graded, but I think that that would be difficult as well. I mean, honestly even just putting them into Gradescope (a digital grading system) would have been, well it at least would have been faster, but more experience hand-grading or grading that type of word explanation type work. Would be good. I was really left floundering trying to decide what, what are different explanations worth, right? If I'm given five technically incorrect explanations, which one of them are worth most of the credit, which one's worth none of the credit? And nothing's ever worth none of the credit in this class, at least in my opinion. So, like what's worth only half, what's worth, you know, only a couple of points because they tried? You know and having to parse that all out is. Hard to do when you've never had to do

> it before. It's hard enough to do when you've never had to do it before, it would be hard enough to do if someone were writing an explanation in a calculus class. But on top of that I've never learned this material before. (Rowan, Final Interview)

Notice that Rowan emphasized "hand-grading" as a skill they were "not provided with through my four years of experience" and that they were "left floundering" when trying to assign point values to students' explanations. Deciding the value of explanations was "hard enough to do when you've never had to do it before," and thus, Rowan imagined that more practice might have supported them in managing this challenge.

Additionally, when I asked Rowan what they thought were the most important things they learned as a newer instructor of O&NS, Rowan first and foremost centered the challenges of grading, stating that: (1) O&NS problems were more difficult to grade than calculus problems because O&NS problems were less "straightforward" and because "it's not just important for them (O&NS students) to know that they've done the problem incorrectly, but it's very important for them to understand exactly why"; (2) and that "grading takes forever" but could have been less "bad" with more preparation:

> Some important things. I definitely learned that you have to be cognizant of sort of what you're going to choose to do in terms of grading. Like calculus problems, you can grade pretty quickly because it's kind of like, right or wrong. And if it's right, you can quickly scan to see that they did the right process and you're done. And if it's wrong, you can probably pretty easily see like a common mistake people make or sort of where that's gone wrong. And I don't know if it's because I'm more in tune with

> calculus than with the [O&NS] type materials. But, like, it is, it is sort of quick and straightforward to grade, for instance, something from calculus, whereas something from this [O&NS] course. It's not just important for them to know that they've done the problem incorrectly, but it's very important for them to understand exactly why. And you can sort of lose all time that you have during the semester just by trying to write like comments about all of the different things that are slightly wrong in the work that they've done…So, big things I've learned: Grading takes forever, but there are ways that you can make it take less time while still giving them the things that they need and I wish I had known on the front end how bad of a time grading was going to be so I could have properly prepared at the beginning of the semester instead of having to switch gears mid-semester. (Rowan, Final Interview)

Hence, Rowan wished both for more experience with grading homework, especially students' explanations of their solutions, as well as more preparation at the beginning of the semester for ways to "make it (grading) take less times while still giving (O&NS students) the things (feedback) they need." Notice from the quote above and recall from prior sections that time management issues exacerbated Rowan's grading challenges. In both the final interview (see previous quote above) and their reflection journal, Rowan felt that having more time to prepare at the beginning of the semester may have supported them in having a "smoother semester overall":

> I'm being reminded of my experience as an associate convener – in those semesters, I was able to spend a chunk of time before the semester

> preparing schedules and auto-graded assignments and it let me have a much smoother semester overall (especially with respect to grading). (Rowan, Week 15 Reflection Journal)

Although Rowan predominately connected their grading challenges to their prior experiences and preparation for teaching, their support for teaching O&NS was also intertwined. First, note that Rowan was accustomed to sharing grading assignments and making collective grading decisions with other instructors when they were teaching Intermediate Algebra and Calculus. They strongly believed that justifying teaching decisions to others was integral to their instructional practice, which they learned from being an instructor in coordinated/convened courses:

> I think it (coordinated courses) serves so many good purposes in terms of supporting the graduate students and also making sure that students get an equitable shot at passing a class. It's definitely a unique thing to have expertise in. It's different to have to justify all of your teaching decisions to another person. Justify teaching. This is not like a needed activity, because plenty of. Plenty of instructors don't pass any of their teaching decisions by anyone for better or worse. But for instance, like this idea of, so, adapting, modifying the course. I brought up a new assessment practice for [Intermediate Algebra], right? Because we do coordinated courses, I can't do that in a bubble. I have to say, you know, what does the research say? Does the research say it's good? What are people doing in practice? Has it been working out? And then I have to go to my convenor and the other instructors and say, I want to do this thing, but I need you on board

> with that to make sure it's going to work out. So, sort of a lot of cool, native efforts. So, different than like reflecting and discussing, but like sincerely working together with others about teaching is like a very specific. And different from these things. (Rowan, Preparation Interview)

Moreover, recall from the previous chapter on support that Rowan viewed grading conversations as "a structure that just should be there." However, additionally recall that Rowan did not discuss grading deliberations with Robin and Rose. For instance, Rowan grappled with dissonance between their expectations for equitable exam grading and the nature of their O&NS course, but did not raise these concerns at instructor meetings:

> [Rowan] mentions that they may not go into a lot of detail about how to grade every problem at this [O&NS] instructor meeting like they have done for other courses they have taught. [Rowan] says that things are different now because you have your own section where you do homework your own way, whereas before it was like these are the homework problems. And the grading was done together, but now the grading is not. [Rowan] says, and this is equitable? Uh... (Researcher Fieldnotes, Week 6 Classroom Debrief)

Thus, grading assignments largely independently also contributed to Rowan's struggle, because it conflicted with how they learned to grade in the past—collectively. The nature of Rowan's support hindered them from collaboratively learning to assign point values to students' explanations or write feedback more efficiently.

O&NS is Less "aggressively organized and aggressively supported"

"But just this vibe of like, 'you don't want to do that'. And then I feel like, though almost no one—the people who say not to teach [O&NS] have never actually taught [O&NS]. The people who have taught [O&NS] never said not *to teach [O&NS]. *laughs* So that's like, what? What do people think is bad about? Like, I don't know what seems to be bad about it. I'm having a fun time." (Rowan, Preparation Interview)*

"I think I get now why people say that they don't like this class—teaching this class. Because it's not the teaching part. Teaching it's fine. I enjoy my students. I enjoy the experience. It's everything else that's difficult. Lack of access to materials. Lack of access to resources. The huge time commitment. Everything else is the problem, the class itself is, is fun." (Rowan, Support Interview)

In a general sense, Rowan attributed many of their challenges in teaching and learning to teach O&NS to an absence of support. At the end of the support interview, Rowan experienced a profound moment of clarity. After reflecting on the areas and activities related to their support for teaching O&NS, they gazed upon the documents they had just marked-up in silence. Only moments ago, they were bursting forth with stories and descriptions; their emotions varied, but they maintained an excited energy. Once they had finished however, they finally allowed themselves to pause and release. After all, Rowan had also finished teaching just minutes before the interview. In the brief quiet, they let the weight of their tiredness slump their shoulders forward, and Rowan slowly asserted what they realized made teaching O&NS difficult—the lack of support.

Overall, Rowan felt that they encountered more challenges or that their challenges were exacerbated because they lacked some of the same support they were accustomed to in their prior teaching experiences, such as a "static" repository of resources. In addition

to resources, the nature and structure of their support for teaching O&NS was different enough from their experiences in convened courses to create tensions or struggles for Rowan. For instance, recall that Rowan strongly believed in the learning potential of opportunities to observe and be observed because of their prior experiences as an associate convenor. Rowan wished on multiple occasions that this same opportunity was available for O&NS, especially because "these classes are run very differently I feel like than other classes I've taught before" (Rowan, Support Interview).

During the final interview, Rowan cemented their desire for support consistent with the "aggressively organized and aggressively supported" convened courses they were accustomed to in all their previous teaching experiences. Specifically, Rowan described their experiences as an associate convenor with a supportive faculty member lead convening as among their most impactful prior experiences in learning to teach O&NS:

> any experience I had where [faculty member] was convening, which includes when I was associate convening [Calculus] and when I was lecturing [Calculus] for the first time. Just this sense of. Like aggressively organized and aggressively supported. Right? It's, I, I know that I could have used [Robin] and [Rose] more. I know that they've taught it before. I know that they have more resources. I know that they have more expertise. But there's always this barrier of, like, "Well, but I don't have to bother them, so I don't want to bother them cause I know they're busy". Whereas on the flip side, the way [faculty member] convenes is to just shotgun things at you. Essentially, like, to give you all the sides, to give you all this

> material, to make sure they're setting things up, to set up the meetings, to do all those kinds of things. So, where, even though I probably could have gotten similar levels of support from both of them, it being offered instead of me needing to request it created this barrier that sort of created a very different experience. (Rowan, Final Interview)

Notice that Rowan emphasized support "being offered instead of me needing to request it" as creating a "barrier" and "very different experience" for Rowan. They wondered about the extent to which their challenges were due to this "loss of structure."

> (O&NS is) probably one of the first courses that you teach outside of the very structured convened courses… I wonder how many of these issues would just arise naturally from that loss of structure. There are definitely certain things that are different as well, like, the having to grade the explanations thing is like a very [ON&S] thing… but some of it does just come from the nature of this being like a class where you, you know, do calculus recitations and then you teach as instructor of record while taking pedagogy, and while being convened, and then you probably move into one more convened course, and then you're just sort of let loose. (Rowan, Final Interview)

Furthermore, for future instructors newer to teaching O&NS, Rowan believed that it might be helpful to encourage the mathematics department's administration to even out the amount of support that occurs or better prepare instructors for the transition to teaching courses without convenors:

> maybe the administration should be evening out the amount of support that occurs for courses, that or better prepare them. Right. We take a whole pedagogy course when we're being the instructor of record for the first time. And that is, like, a big and scary change, but someone who is going from a convened course to a course where I'm told all the time to just do, what, what I want to. Here's what I'm doing, here's 'hat they're doing, and you do whatever you want to. (Rowan, Final Interview)

Hence, Rowan viewed an "aggressively organized and aggressively supported" course as one in which an instructional leader is ready "to just shotgun things (resources) at you" and provide "everything I could possibly ask for before I could ask for it" (Rowan, Support Interview). From these experiences, Rowan developed an expectation of not needing to ask for resources and was not as accustomed to making decisions independently and/or diverging from other O&NS instructors' choices (i.e., being "let loose").

Why Not Ask for More Support from O&NS Instructors?

Given how strongly Rowan emphasized a lack of preparation and a desire for more support, one might wonder: why not ask for more support from O&NS instructors, especially during instructor meetings? After all, recall that Rowan identified O&NS instructor meetings as one of their most useful supports because of the opportunities they provided to obtain a variety of perspectives on course activities, content, and a general sense of "what happened in class today" from other instructors. Furthermore, Rowan identified instructor meetings as a means to "support anyone who is looking for additional support or help":

> I'm used to the course convenor structure. I'm used to there being a weekly meeting with all of the instructors, that it's led by sort of someone in charge and that we discuss class and that that is just its purpose. Right, I've led these meetings. And so, in my mind, the purpose of those meetings is to make sure everyone's on the same page and to support anyone who is looking for additional support or help. I don't always take advantage of it the way that I probably should, but I think that having access to that and being able to do that thing is quite important. Right? Just straight up every week, having someone say, "How did it go?" It's just a good leeway. It's, it's definitely easier than having to like actively reach out to someone to be like, 'something went bad', cause then you have to decide: How bad is bad enough to set up a meeting? You know? (Rowan, Support Interview)

"How bad is bad enough to set up a meeting?" offers some insight as to why Rowan did not raise many of the concerns they shared with me all semester long, beyond the few reasons I provided or alluded to in previous sections. For instance, Rowan's belief that grading discussions were "a structure that just should be there" hindered Rowan from asking for this support. Recall that Rowan also might not have had enough time to read the next textbook section(s)'s material in advance of their meeting with O&NS instructors, as they described during the support interview:

> I feel like it would go totally fine if I came in and I was like, "here is my concern relating to teaching." Part of the issue with meeting at three on Thursdays is that I have not looked at what we're teaching the next week.

> I have not started prepping it or anything, so I do have it totally open to discuss my concerns. But I don't know that it would be a productive discussion beyond me just being like, "I'm flying by the seat of my pants, and I don't know what I'm doing". So, the opportunity is definitely there. But it is. It is there on technicality. (Rowan, Support Interview)

Both these perspectives on structure and scheduling suggest that Rowan felt constrained by the structures and routines established that semester. But even though Rowan may not have had enough time to prepare discussing the following week's content, perhaps Rowan could have voiced their various concerns about grading, access to resources, or the timing of instructor meetings, etc. Hence, I further elaborate on why Rowan might not have asked for various support, especially during instructor meetings. Notably, Rowan's background and prior experiences continued to offer a means of understanding why they did not usually ask for support.

Not Worth Bothering Others Beyond Assumed Structures

One of my main concerns for Rowan throughout the semester was that their inclination to not want to bother others might hinder them from reaching out for support when they needed it. Indeed, recall that Rowan did not want to "bug" staff to obtain access to the textbook at the beginning of the semester, nor did they seem to want to bother the remaining O&NS instructors or staff to obtain access to resources throughout the semester. For instance, recall that towards the beginning of the semester, Rowan thought that O&NS instructors would provide support for lesson planning if they asked, "but it's a lot to ask for." Also recall that during the support interview, Rowan thought it was "not worth the hassle for everyone involved" to inquire about obtaining physical

manipulatives. Rowan additionally extended this sense of not having concerns worth others' time beyond the O&NS instructors. Recall that when Juniper abruptly left the final joint EMM-O&NS instructor meeting, Rowan felt that "as [O&NS] instructors, we are not worth their time."

Rowan explained during the final interview that, in part, they "didn't have the confidence to ask for them (resources)," but Rowan also expected that any resources they needed would already have been given: "they were not provided to me. And so, from my perspective the resources don't exist, but it's quite possible that they do." Moreover, Rowan wanted to respect others' time, believing that other instructors were likely to be similarly busy throughout the semester. Recall from the previous section that Rowan intertwined notions of being offered resources versus asking others for resources with the idea that "I don't want to bother them (O&NS instructors) cause I know they're busy." Because Rowan believed that support structures should already be in place, then asking for support or resources beyond what was already provided was equivalent to asking others to go above and beyond their time allotted for responsibilities as an O&NS instructor. Hence, Rowan's belief that their concerns were not worth others' time coincided with their beliefs about support structures, and ultimately hindered Rowan from reaching out for support.

Pride as a Good Mathematics Instructor

Another reason Rowan often did not ask for support similarly involved Rowan's dispositions, but also connected to their identity as a mathematics instructor. During the final interview, Rowan identified "personal pride" and their "inability to let the other

[O&NS] instructors know that I was struggling" as a possible contributing factor towards their overall challenges in teaching O&NS:

> On reflection, possibly a lot of my issue with this course was my own personal pride and my inability to let the other [O&NS] instructors know that I was struggling. I think that if I was able to put my pride aside, a lot of these things could have gone better. (Rowan, Final Interview)

At another point during the final interview, Rowan described themselves as "a good teacher" and stated that their faculty mentors "know that I'm good at teaching." Hence, Rowan thought of themselves as a "good" mathematics instructor and wondered if their beliefs ("pride") surrounding this identity might have influenced them to stay silent about their struggles. For example, Rowan underestimated the challenge of grading because they positioned themselves as an overall better instructor than the average mathematics graduate student instructor:

> I also just like to think… maybe the people who say it takes forever to grade, just aren't good graders, which is not a fair assessment… when I heard that from a whole bunch of other graduate students who I know don't like to teach… I started to get the mindset that it wouldn't actually take that long. I should have trusted it more when I heard it from Robin and Rose. (Rowan, Final Interview)

Thus, Rowan's notions of what it meant to be a "good" mathematics instructor also hindered Rowan from reaching out for support.

Good Mathematicians Do Not Ask for Mathematics Solutions

Remember that Rowan first and foremost viewed themselves as a mathematician, whereas their learning to teach O&NS was a secondary skill, or add-on, to their profession. More generally however, Rowan viewed themselves as a "good" mathematics instructor and, at times, this identity intersected their identity as a mathematician in ways that hindered them from reaching out to others for support.

During the final interview, additional reasons Rowan gave for feeling unable to ask for support was that they were uncertain about what exactly they needed to be more fully supported and that they had "no concept of what is appropriate and inappropriate to ask for." For instance, Rowan situated this tension around mathematics solutions:

> I'm not sure what I need. And I also have no concept of what is appropriate and inappropriate to ask for, like. Do they have solutions to the Habits of Mind? I don't want to ask for them and then have everyone be like, "[Rowan] these are easy problems you should be able to do them". You know, and it's like, are there solutions to the Habits of Mind? I don't know, they weren't provided. I don't really want to ask for them though, because, like, I can solve them. But do I have to solve them? And all that kind of stuff. (Rowan, Final Interview)

Hence, Rowan, at least in part, felt uncomfortable asking for mathematics solutions as a support for grading because they believed that doing so would conflict with their identity as a mathematician. In Rowan's mind, mathematicians do not ask for solutions to "easy problems," yet Rowan wondered if having access to solutions would have better supported them as a mathematics instructor.

"How bad is bad enough"?

One final consideration is that perhaps nothing Rowan viewed as "bad enough" had occurred throughout the semester. What might constitute "bad enough" can be understood through both Rowan's prior teaching experiences and their experiences teaching O&NS. Recall that Rowan's experiences as associate convenor for Calculus I in the beginning of their fourth year were so negative that they decided not to teach the following semester. These negative experiences centered around the faculty convenor's negative perceptions of calculus students as well as their related decision to add a large group project to the syllabus in the middle of the semester. Furthermore, Rowan described "a very hostile environment that was created in those (instructor) meetings," which "culminated in having to have a structured discussion" to resolve conflicts surrounding graduate student instructors "feeling bullied."

Rowan did not describe hostility in their experiences teaching O&NS, however. Thus, in comparison to their prior teaching experiences, Rowan might not have thought that any of their concerns merited discussion. Rowan might not have wanted to voice their concerns in light of Robin's experiences either. Recall that Rowan survived the semester without any "catastrophic failures," such as not finishing preparing their lessons or being able to address students' questions, whereas Robin lamented the "pushback" and "complaining" from a few of their students, and occasionally voiced their laments during instructor meetings. Indeed, Robin's experiences provide insight into why Robin typically framed their questions about how things were going as whether anyone had "anything super bad happening" or "crises" that anyone needed to "vent" about. Hence, Rowan likely also did not view their concerns on the same level of "crises" that Robin was experiencing with their students.

In summary, Rowan expressed directly or alluded to multiple reasons for remaining silent about their various concerns and desires for more support. First, Rowan often experienced a lack of time surrounding course structures. For instance, they did not have enough time to read through course material in advance of instructor meetings to know what support they might have needed regarding the next week's lesson plans. Second, Rowan's belief that grading discussions were "a structure that just should be there" hindered Rowan from asking for this kind of support, but also coincided with Rowan's belief that their concerns were not worth others' time. Moreover, Rowan likely did not believe that any of their experiences were "bad enough" compared to their past experiences or Robin's experiences to merit taking away from others' time. Third, Rowan believed and later grappled with notions that "good" mathematics instructors and mathematicians do not reach out for certain support, particularly mathematics solutions. In the next chapter, I discuss the significance that these reasons hold for their implications on support for newer instructors.

Learning to Adapt and be Flexible from Prior Experiences

Although grading was a major challenge for Rowan, they experienced some success along the way. Towards the beginning of the semester, Rowan realized that they should not continue to grade their students' homework assignments the same way the entire semester. Because of their prior experiences experimenting with grading systems in Intermediate Algebra and because of the ways they needed to flexibly respond to teaching challenges during the beginning of the COVID-19 pandemic, Rowan learned to adopt a more flexible mindset towards making mid-semester changes. Rowan explained

these connections during the final interview, in response to my question asking what preparation or support impacted their learning to teach O&NS the most:

> I think teaching [Intermediate Algebra] during the pandemic is what taught me the most. That one semester when everything shut down over spring break. Because it put me in a position where I had to be able to think on my feet and adapt to the situation. And I was also not only teaching that semester, but I was the associate convenor that semester, and the convenor was a little busy with meetings in several other classes. So, really me and the other instructor were left to our own devices to pull the whole thing together from scratch, to have to work closely together to just get things figured out as quickly as possible. And I think that that's. That ability to be adaptable and also the confidence that I could do that. The confidence that it is okay to switch things up in the middle of the semester, the confidence that it's okay to say, "Hey, this is how I graded the first two homework sets, it wasn't working. We're doing something else". And sort of all that kind of stuff. I think that prepared me the most because if I had just stuck with the plan I had for this semester, the one that I made before the semester started, I probably would still be grading right now. So, sort of that, that ability to be adaptable, right? During the pandemic I was forced to be adaptable, but it gave me sort of that skill set and that confidence that adaptability is a good thing. (Rowan, Final Interview)

Had Rowan not developed the confidence to "switch things up in the middle of the semester" prior to teaching O&NS, perhaps their grading challenges might have been

exacerbated. For instance, Rowan may have been even further behind in grading homework assignments, in addition to Habits of Mind assignments. Hence, being "put in a position where I had to be able to think on my feet and adapt to the situation" developed Rowan's "confidence that adaptability is a good thing," which they in turn applied to challenging situations in O&NS. Even though Rowan's grading challenges were at times "insurmountable," Rowan's learned adaptability made them more manageable than they might have been otherwise.

The Progression of Support and Beliefs Leading to Rowan's Empathetic Navigation of Student Questions

Recall that addressing O&NS students' questions about mathematics content with respect to K-6 students' mathematical thinking or K-6 schools was a regularly occurring event for Rowan. One student in particular, Bruce, asked questions that challenged Rowan to reflect on their ability to support O&NS students' inquiries about K-6 students' mathematical thinking right from the beginning of the semester. Although Rowan did not typically identify addressing O&NS students' questions as either a challenge or a success, navigating these conversations in class proved to be a regular concern for Rowan and had a significant impact on their teaching practices. Previously, I offered a few perspectives through which Rowan's navigation of their students' questions could be understood as a success. Specifically, I argued that, while navigating students' questions, Rowan maintained a classroom environment of openness, curiosity, and mutual respect for mathematical perspectives, but especially for teaching and teachers. I now argue that Rowan's background, prior experiences, and conversations with their mom were critical to Rowan's success in developing this skill and mindset about teaching and teachers.

Furthermore, I discuss the progression that led to Rowan seeking out support from their mom, and how conversations with their mom built their confidence to navigate the challenge of addressing students' questions in class.

The challenging questions asked by Rowan's O&NS students, especially Bruce, influenced Rowan to reflect on their ability to support O&NS students' inquiries. Remember that in week two, after feeling unable to "provide anything useful" to address Bruce's question, as well as concerned that they lacked the "authority" to anticipate K-6 student thinking and worried about missing "important ideas behind [K-6] student thinking", Rowan felt a need to develop "a better sense of the timeline on which elementary students would learn this material." Hence, Rowan decided to seek out an "authority" on K-6 student thinking.

Further recall that Rowan thought that elementary teachers would be an ideal support as early as week two of the semester. Growing up with their mom, who had K-6 teaching experience, may have influenced Rowan to consider elementary teachers as a support, as well as their positive experiences learning from and with elementary teachers both in the two-week mathematics content course they taught as a teaching assistant and in the graduate-level methods courses in which they enrolled at Cardinal University's department of teaching and learning. Remember from their prior experiences that Rowan expressed a strong desire to listen and learn from K-12 teachers' experiences and viewed these opportunities as potentially supportive of their growth as an instructor of prospective teachers.

> It's a little interesting taking the [education] courses because it is like grade school, I'm working with grade-school teachers for the most part…

> I think it's important and I always wish that I knew more about how my students were learning math up until I see them. And it's hard to get like a good view on that without just like finding grade-school teachers to ask. (Rowan, Preparation Interview)

Thus, Rowan was driven to learn "more about how my students were learning math" and considered those with K-6 teaching experiences as potentially helpful resources.

Additionally recall that Rowan's desire for access to elementary teachers was likely also bolstered and sustained by unproductive joint meetings with O&NS and EMM instructors. Remember that in the second week of the semester, Rowan considered asking EMM instructors for resources related to elementary students' mathematics learning, but by the fifth week of the semester, Rowan seemed to no longer believe they could be relied on for support. As a result, Rowan felt a need to look beyond the support accessible through their institution. Given that Rowan did not have access to practicing K-6 teachers, Rowan reached out to their mom to regularly debrief their classes and reflect on their mathematics learning experiences.

Note that Rowan likely emphasized having access to practicing elementary teachers as a desired support, both because not all instructors have access (personal or otherwise) to people with K-6 teaching experience, but also because Rowan viewed their mom's perspective as limited.

> I make a lot of assumptions about what goes on in grade-school classrooms. Honestly, most of what I talk about comes from my personal experience as a child, which is a fuzzy thing. I mean, I've also discussed a lot of my, like, childhood schooling with my mom as an adult. Like, as an

> adult, I've discussed it with my mom… which helps a lot. But I definitely don't have access to like a modern perspective on grade-school teaching. I have access to my mother's perspective, and she taught '90, she taught '89 to '92. That was a while ago. (Rowan, Support Interview)

Nonetheless, Rowan's regular conversations with their mom stood out to them as an activity significant to their successes in teaching O&NS. One of the main ways in which discussing their prior experiences as a mathematics learner supported Rowan was that it built their confidence to discuss K-6 students' mathematical thinking with prospective teachers:

> But definitely was impactful was thinking about, thinking about and being able to talk through what I had done (as a K-6 student) in comparison to what I am teaching. And it also helped because then I could more confidently tell my (O&NS) students, "Oh, like, when I learned it, and possibly when you learned it, because you're only, like, a little bit younger than I am. Like, this is. I remember this being a thing". My mom said that this was a thing. And I think that connection is useful to me. I wish I had more of it, but it would be so hard to track down because, like, I don't fully remember my experience, right? I did have to talk it through with my mom, and my mom was very involved in my schooling, which is why she remembers. (Rowan, Final Interview)

Relatedly, Rowan might have also found success in navigating these unpredictable classroom discussions, because they had been developing positive orientations towards teaching and teachers prior to teaching O&NS and seemed to want

to develop positive beliefs about their students when they started teaching O&NS. For instance, recall that Rowan started the semester believing that "the students will be fun and easy to work with" because "they want to be teachers":

> I think the students will be fun and easy to work with. Since they want to be teachers, they'll have an appreciation and understanding for both the teaching methods I use and the teaching methods for mathematics they're learning. (Rowan, Week 1 Reflection Journal)

Moreover, during the support interview Rowan described the students as the enjoyable part of their experiences teaching O&NS, rather than their challenges with limited support:

> I think I get now why people say that they don't like this class—teaching this class. Because it's not the teaching part. Teaching it's fine. I enjoy my students. I enjoy the experience. It's everything else that's difficult. Lack of access to materials. Lack of access to resources. The huge time commitment. Everything else is the problem, the class itself is, is fun. (Rowan, Support Interview)

In summary, when Rowan was challenged by questions from their O&NS students about K-6 schools or K-6 students' mathematical thinking, Rowan actively sought support from their mom, as they did not have access to practicing K-6 teachers or believe that EMM instructors could be relied upon for support. Conversations with their mom supported them in reflecting on their own K-6 mathematics learning experiences, as well as some of their mom's experiences teaching K-6 mathematics. In addition to building their confidence, Rowan continued to develop positive orientations towards

teaching and teachers. Previously, Rowan developed these orientations both from living with their mom and from assisting a two-week mathematics content course for practicing teachers. A resulting influence of their developing beliefs and their reflective work was that, in some instances, as with the prior illustration involving their O&NS student, Kenny, Rowan managed to pivot prospective teachers' questions into thoughtful conversations centered around empathizing with the perspectives of multiple educational figures, e.g., cooperating teachers, K-6 students, and mathematicians.

Understanding Passing and Surviving

Recall that, by the end of the semester, Rowan defined success in terms of O&NS students' achievement (passing), as well as their assessment of the extent to which they might have messed up a class (surviving). Previously, I outlined a few of the ways in which passing and surviving without "catastrophic failures" could be understood as successes. Primarily, Rowan persevered despite their unsteadiness over preparing lessons and addressing students' questions as a busy, graduating student. But also, the absence of "catastrophic failures" was worth celebrating in light of both Robin's challenges in teaching O&NS, as well as Rowan's negative experiences teaching Calculus I with an unsupportive course convenor who created a "hostile environment" for graduate students. In this section, I illuminate Rowan's perspective on what made their successes surrounding passing and surviving possible.

During the final interview, I asked Rowan to what or whom they attributed their successes around passing and surviving and why. Rowan responded with a holistic answer:

> Ooo, it's on every, it's everybody. Right? It took everybody. I mean, it took me as the instructor. Right? I clearly put some effort into this class. But it took, you know, [Robin] sort of being more organized and being able to keep things together, and it took [Rose] always interjecting with, you know, how things have gone previous semesters and tips that she has for stuff like that. And it took being able to reflect with my mom about how my experience went and how I'm connecting that to the experience of having a classroom. It took you and I reflecting on whether or not I had supports, which then would prompt me to ask for more support sometimes. And. No it's. You take out any one of those pieces and who knows what would have happened. (Rowan, Final Interview)

Rowan's response is a reminder that the connections among and between their prior experiences, preparation, support, challenges, and successes are deeply intertwined. Rowan's response also pointed out to me how invisible successes can often be. Notably, Rowan had much to say about their challenges and the myriad ways that their preparation or support might have been improved, but at times seemed to struggle to identify successes, and not usually with the same assuredness or emphasis on singular events as their challenges. Perhaps passing and surviving is a more long-term success, and therefore more holistic in nature. But perhaps successes are often contingent on multiple interconnected pieces, such that "you take out any one of those pieces and who knows what would have happened."

CHAPTER 7: CONCLUSION AND DISCUSSION

Overview

Mathematics content instructors, especially those who are newer to teaching prospective elementary teachers (elementary PTs) need to navigate numerous challenges and engage in positive experiences teaching elementary PTs. The ways in which instructors learn to navigate challenges and create success depends on their contexts, the specific challenges and successes they experience, the expertise they have or need to develop, and the internal and external support available to them. The purpose of my study was to obtain a more nuanced understanding of a newer mathematics content instructor's experiences around learning to teach elementary PTs in their respective teaching context. The central question that guided my study was:

How might a newer mathematics content instructor's background and preparation relate to the challenges and successes they experience around teaching and learning to teach mathematics content for elementary PTs?

Specifically, I aimed to understand this central question by gaining insight into the following sub-questions:

1. *What challenges and successes does a newer instructor experience around teaching and learning to teach mathematics content for elementary PTs?*
2. *What prior mathematics learning and mathematics teaching experiences provide a continuous and storied understanding of a newer instructor's challenges and successes around teaching and learning to teach mathematics content for elementary PTs?*

3. *How does a newer instructor describe their preparation and support for teaching mathematics content for elementary PTs? Moreover, what experiences surrounding a newer instructor's preparation and support provide a continuous and storied understanding of their challenges and successes around teaching and learning to teach mathematics content for elementary PTs?*

To address the central and sub-research questions, I engaged in narrative inquiry (Clandinin, 2013; Clandinin & Connelly, 2000), utilizing Clandinin and Connelly's (2000) three-dimensional (temporal, contextual, and interactional) inquiry space to provide a continuous and storied understanding of my participant's experience. In the previous chapters, I provided a continuous and storied understanding of the numerous connections among this instructor's background, their prior mathematics learning and teaching experiences, their preparation and support for teaching Operations & Number Systems (O&NS), and the challenges and successes they experienced teaching O&NS.

In this chapter and in accordance with my research questions, I first provide a summarized narrative of Rowan's semester-long challenges and successes, highlighting key aspects of their background, preparation, and support along the way. Second, I discuss my relationship with my participant and how it interacted with the study. Third, I discuss several implications, related research puzzles, and suggestions for future research based on these findings. Note that although these findings and implications are grounded in my participant's experiences, the themes surrounding their experiences allow for rich inferences into the social processes of individuals within complex social structures and for one to imagine how these experiences might exist or manifest within similar contexts (Riessman, 2008).

A Summary of Rowan's Experiences

Rowan started the semester excited but nervous. They felt ready to build on the positive orientations about teaching and teachers as an assistant instructor for a two-week mathematics content course for teachers but felt rushed into teaching a course with "pretty much nothing" in terms of their specific preparation for it and with access to fewer resources than they were accustomed to in their prior teaching experiences. From the onset of the semester, Rowan encountered challenging questions about K-6 students' mathematical thinking and K-6 schools. Although Rowan described having helpful support with respect to anticipating O&NS students' thinking from their meetings with O&NS instructors, they did not believe the elementary mathematics methods (EMM) instructors thought the O&NS instructors were "worth their time" nor did they have access to "a modern perspective" from elementary teachers on K-6 students' mathematical thinking. Hence, in response to this recurring challenge, Rowan sought their mom, a former elementary teacher, to regularly unpack their own mathematics learning experiences as well as their mom's mathematics teaching experiences. These conversations, in addition to Rowan's disposition to listen and learn from K-12 teachers, may have contributed to Rowan's learning to navigate O&NS students' questions with empathy and foster a classroom environment of respect for a variety of mathematical perspectives from multiple educational figures, such as cooperating teachers, K-6 students, and mathematicians.

Towards the middle and end of the semester, Rowan struggled to learn "different" and "hard" content as an instructor, as well as struggled to stay on top of their grading responsibilities. Both challenges impacted the development of Rowan's overall "uneasy"

and “unsteady” disposition, but especially the absence of an orientation to O&NS’s course goals at the beginning of the semester. As the semester progressed, the weight of having unfinished grading grew heavier and heavier for Rowan. There were two main reasons grading proved to be an “insurmountable” task for Rowan. (1) Rowan lacked the “TIME” as a graduating Ph.D. student interviewing for academic jobs, as well as the time management skills that they could have developed in their prior teaching experiences and (2) Rowan had little prior experience with grading homework or with assessing and providing feedback on mathematics assignments in which students explain their mathematical reasoning and/or incorporate visual representations in their solutions. Although Rowan could have reached out to O&NS instructors for more support, multiple concurrent factors conflicted with their ability to do so: (1) their time to examine lessons in advance of instructor meetings, (2) their belief (developed through being an associate convenor in “aggressively organized and aggressively supported” convened courses) that discussing grading is “a structure that just should be there”, (3) their belief (developed through both prior and current teaching experiences) that their concerns were not “bad enough” or worth others’ time, and (4) their belief that “good” mathematics instructors (who are also mathematicians) do not ask for “easy” mathematics solutions as a support.

Nonetheless, Rowan found success based on their students’ achievement on exams and their having survived the semester without the kinds of “catastrophic failures” they experienced with an unsupportive course convenor in the past or that an instructional leader for O&NS, Robin, experienced with their students during the semester. Rowan additionally applied their learning to adapt and be flexible with respect to making mid-semester changes to their grading due to their prior experiences teaching during a

pandemic. In the end, Rowan realized that “this math class is different” not only for them as an instructor, but for their students. Hence, they learned the importance of communicating expectations to students regarding their mathematics solutions early on and often.

The Researcher-Participant Relationship

Narrative inquiry is a relational methodology, in that researchers investigate their relationship with their participant alongside the research questions centered on their participant’s experiences (Clandinin & Connelly, 2000). In this section, I elaborate on the relationship I established with my participant and how it shaped the ways in which I engaged in this inquiry and came to understand my participant’s experiences. In other words, who was I in Rowan’s story and who were they in mine?

Shared Identities and Experiences Work Towards Establishing Trust

Throughout the study, Rowan and I engaged in a collegial and friendly relationship. Rowan often felt comfortable sharing a range of affective experiences with me, including joys and frustrations. During our interviews, we would frequently laugh together to commiserate over experiences with which I could identify or easily imagine, as both of us experienced at least 2 years in a mathematics Ph.D. program, both of us were graduate students near completion of our degrees, and both of us had experience teaching similar undergraduate mathematics courses under similar convened structures. Rowan and I had also bonded through a close professional relationship as camp counselors a few years ago during a week-long summer mathematics program for high school girls. Overall, I believe that our many shared or similar experiences and identities supported me in establishing their trust. I sincerely cared about Rowan and their

experiences and believe that Rowan knowing about my own background and desire to improve preparation and support for newer instructors made establishing this trust easier.

Reflection Journals and Debriefs Shift Rowan's Mindset about Support

Over time, Rowan and I's relationship grew to be mutually beneficial. Before the start of the study, I anticipated benefiting from our relationship in many ways and wondered to what extent the activities I asked Rowan to engage in might benefit them. Although I wanted to engage in as naturalistic an inquiry as possible, I simultaneously hoped that regularly collected data for the sole purposes of my study might offer some benefit to Rowan as well. As it turned out, the debriefs between Rowan and I supported Rowan, as well as the weekly reflection journals. Recall from the chapter on Rowan's support that, during the final interview, Rowan reflected on the extent to which our debriefs after class and the weekly reflection journals impacted their experiences:

> I think that if I hadn't taken the time to reflect with you after class every day, I wouldn't have thought as much about, "Oh, like this is a thing you're doing. Do you know why you're doing that?" Like, I didn't even notice at first that I really wasn't circulating my tables until you had mentioned it. And I was like, "Oh my gosh, why am I doing that?" Or, you know, when the reflection journals are asking, like, "What? What has supported you? What supports would you maybe need?" It's like, "Oh, well, what supports would I need?" I st… that didn't go super well because I still don't know, like, what supports I needed. But it did highlight to me the. Something that I had not thought about a ton before. Is that it is not, it is not a failure on my part to need to ask for more

> resources. Throughout this whole experience here at [Cardinal University] I have been given everything I could possibly ask for before I could ask for it until this semester. And so, I was always like, well, if this is what everyone else has been given and they seem to do just fine, then it's a failure on my part that I need something more or something different. So, it definitely helped reframe that mindset. Now, a lot of reflection and then some things that you brought up about observations were all pretty helpful. (Rowan, Final Interview)

Primarily, the debriefs and reflection journals raised Rowan's awareness surrounding their own support and may have contributed to Rowan's decision to seek external support during the semester. Ultimately, both activities contributed to Rowan's overall shift in mindset about support, particularly their learning that "it is not a failure on my part to need to ask for more resources."

Negotiating Our Relationship & Tensions in Maintaining My Distance as a Naturalistic Researcher

For the most part, my role in this study was that of a naturalistic observer. As such, I did my best to avoid interacting in ways that might have influenced Rowan to act and think differently than they might otherwise. I truly wanted to capture Rowan's experiences as they were, for the sake of my own learning and the field's learning about what *really* happens as a newer instructor of a mathematics content course for elementary PTs. During the final interview, Rowan essentially summarized the nature of our "one-sided" relationship:

> I mean, you are observing me teaching and that has been useful, but I feel like it's not quite the same (as being observed by a peer instructor) because I feel like we are in a sort of one-sided situation where I'm supposed to tell you everything as, and you're supposed to like, as unbiased as possible, take in that information, is like the vibe that I get so that your study can go well. (Rowan, Final Interview)

Indeed, most of our interactions could be described as Rowan telling me everything and me taking it all in. For most of our time together, I listened to Rowan and asked Rowan questions to elicit their thinking and ideas, rather than interject with my own stories. Nonetheless, maintaining my distance and prioritizing the one-sidedness of our relationship was somewhat difficult at times. To further illuminate the nature of our relationship and some of the ways in which Rowan and I negotiated it, I highlight a few key moments in which tensions surfaced or I felt challenged to maintain distance.

One of the earliest examples of these tensions occurred two weeks into the semester, during the preparation interview. I knew that Rowan had obtained access to a digital version of the textbook for O&NS, but I did not know for certain whether they were using a student or teacher version of the textbook. After Rowan finished regaling me with the details surrounding the roundabout way in which they obtained access to a digital copy, I decided to ask them if they were using a student version. Rowan then asked me point-blank: "Is there a teacher's version?" My brain short-circuited for a moment, but then I replied, "yes." Rowan then asked me, "Is that the one that [staff member] is going to give me if they can ever find it," to which I also replied, "yes." Lastly, Rowan asked me whether I thought the textbook contained anything more helpful

than answers to exercises, to which I responded sincerely from my perspective as an instructor: “the primary thing that I remember is that there are sort of, like, very small, terse answers to solutions…”

This sequence of interactions opened a well of emotions for me. I was shocked and conflicted by the problematic nature of Rowan’s situation. I wanted Rowan to have reliable access to the same textbook any other GSI would have access to under normal circumstances, but I did not want to influence Rowan to use different resources than they were already planning on using. I also recognized that *I* had been the one to inform Rowan about the existence of a resource by the nature of my question, and given the directness of Rowan’s questions, I felt compelled to answer their questions sincerely. Throughout the study, I was not typically in a situation where I needed to decide how much information to reveal (when I had more information than they did), but on such occasions, I usually gave my succinct, honest answer. At the same time that I was a naturalistic observer, I was also a fellow GSI willing to answer some of the same questions Rowan could have asked to any other GSI with experience in teaching a similar course.

Rowan did not usually test the boundaries of our relationship beyond its “one-sided” nature. This may have been because I expressed to Rowan at the beginning of the study and on various occasions that my intention was to capture the reality of their experiences as much as possible. As previously mentioned, the idea that I was really just there to “take in information” seemed to characterize how Rowan viewed my role throughout the study. There were many, many instances when Rowan could have asked for my opinions or ideas surrounding teaching O&NS but did not. There were times when

Rowan needed a substitute for their classes, but they never once mentioned me as a possible option. And they almost never asked me for support during their classes. The one rare moment in which they did was to ask for more information regarding an assignment on the O&NS syllabus. This was a memorably awkward moment for me because Rowan had never cast a spotlight on me during any of their classes, but in this one instant, Rowan chose to re-direct their student's question to me along with the attention of their entire class:

> [Bruce] says they have one more question. What's the Map MP assignment that's due April 3rd? [Rowan] looks at the ground, says umm..., then looks up at me, and asks me what that stands for. I say that it might stand for modeling project. [Rowan] says they will look for it. They scroll through their Canvas (learning management system) page, then pull up the syllabus. [Rowan] says, sorry they are behind on keeping up with projects for this class. I say that it could also stand for mathematical practices. [Rowan] looks it up and says that if this is due Monday, then they will make sure that it's actually due at a better time and will look into it. (Researcher Fieldnotes, Week 10)

Beyond this question, Rowan did not usually ask much of me. I, however, asked for Rowan's engagement in activities beyond what they would normally do, and as a result needed to negotiate those activities. The main negotiations surrounded the collection of data, primarily the weekly reflection journals. As I discussed in a previous chapter, towards the end of the semester, Rowan fell behind in submitting their reflection journal responses. I knew that Rowan was exceptionally busy, and that managing their

time was challenging. At the same time that I wanted data, I did not want to exacerbate Rowan's challenges and stretch them too thin. Thus, I kept my expectations surrounding the completion of the journals flexible, as Rowan and I discussed following a class in week eleven:

> I ask [Rowan] if there is anything I can do to make the journals easier. [Rowan] says yes, they are behind on those. They say that they have just been taking notes and putting them off because it is not a high priority, but if it is making my job difficult, then they can make it a higher priority. I say that I feel a difficulty here, because I don't want this to be a higher priority than other things they are doing, but at the same time it is hard for me to get immediate feedback on how they are experiencing things compared to what I'm observing when the reflections are many weeks overdue. I say that a week isn't so bad, but 3 weeks is hard for both of us to remember. [Rowan] says that they will defend their thesis in about a week and a day from now, and that's after the exam too, and that they intend to turn them in every week from that point on. I ask [Rowan] if it would be helpful to turn in reflection journals in another format. Like, would it be helpful if it wasn't via Qualtrics (data management service), but they just sent them to me some other way? [Rowan] says they take notes somewhere, but really, it's just that it's lower on the to-do list. [Rowan] says that maybe I could ask some of the journal questions after class when we meet up in the hallway so that you can get a sense of the

> things that are most important. I say that could work. (Researcher Fieldnotes, Week 11)

As a result of this conversation, I asked Rowan more questions during debriefs to elicit responses related to reflection journal prompts. This meant that our debriefs usually lasted longer. However, not having to type up responses likely saved Rowan's overall time during the semester. I believe this was a reasonable compromise.

Implications & Suggestions for Future Research

Instructor Backgrounds and Mindsets Matter

Instructor backgrounds and mindsets matter. An implicit assumption of my study was that I *could* find connections between an instructor's background and their challenges or successes teaching and learning to teach O&NS. Indeed, both Rowan and I identified several connections throughout the semester. Sometimes these connections were rather explicit. For instance, Rowan's struggle to manage their time and complete grading responsibilities was often directly connected to being a graduating Ph.D. candidate. Frequently going out of town for job interviews or defending their dissertation took away a noticeable amount of time from Rowan's schedule.

Other times these connections were more implicit, but the search for them revealed key insight into some of the mindsets that contributed to Rowan's decisions. For instance, Rowan's choice *not* to seek support from O&NS instructors with respect to grading illuminated a plethora of insights into the ways in which Rowan's prior teaching experiences shaped their beliefs about support structures within their department. Considered alongside their access to various resources, they provided a deeper

understanding of why Rowan was overall less agentive about reaching out for support within their institution.

In considering which of Rowan's experiences other mathematics graduate student instructors (GSIs) might find relevant, it may help to consider the ways in which Rowan might be similar or different from the larger mathematics GSI population. One main way Rowan was similar to other mathematics GSIs was that they viewed themselves squarely as a mathematician, as opposed to, say, a mathematics teacher educator. And some of the ways in which Rowan approached mathematics instruction was through their identity as a mathematician. For example, one of Rowan's first grapples with teaching O&NS involved reconciling how they learned mathematics as a mathematician with their textbook's sequencing of course content. Another example came from the end of the semester, when Rowan expressed concern that asking for "easy" mathematics solutions to support instruction might be viewed as inappropriate. Hence, there may be space for mathematicians to see themselves in Rowan's experiences and consider the ways in which a similar background may lend themselves to more or less similar experiences in one's own context.

One finding connected to Rowan's background that I especially urge mathematicians to take seriously was that a mathematics GSI with a strong identity as a mathematician found learning O&NS course content difficult. This is important because one take-away from this study is that mathematicians do not necessarily have a strong mathematics background for teaching mathematics content courses for elementary PTs. Mathematicians need preparation and support focusing on developing mathematics content knowledge. Rowan found understanding division with fractions challenging, and

because of their discomfort with their own understanding of the content (but also because of the pressure they felt to finish teaching new material at the end of the semester), decided to focus their lesson on procedural rather than conceptual understanding of the content. Hence, it may be important for mathematics content instructors, including mathematicians, to develop their knowledge around mathematics content they might not have learned in elementary school. I see this as an important means of ceasing the seemingly endless cycle of individuals never developing conceptual understanding of certain mathematics content like division with fractions.

One main way in which Rowan may differ from a typical mathematics GSI was that they came into teaching O&NS with a strong desire to build on previously developed positive notions of teaching and teachers. For instance, Rowan started the semester believing O&NS students would be "fun and easy to work with" and quickly viewed them as empathetic, much in the same way they had viewed practicing teachers in a summer mathematics content course as having an "empathetic understanding of the complexities of teaching." Mathematics GSIs who teach mathematics content courses for elementary PTs for the first time may not have prior experiences developing positive orientations from teaching teachers in low-stakes settings like Rowan (a two-week course as a teaching assistant). Furthermore, Rowan may be set apart from typical mathematics GSIs in that Rowan's relationship with their mom played a significant role in their learning to address O&NS students' questions and maintain a classroom environment empathetic to mathematical perspectives from multiple educational figures (e.g., cooperating teachers, K-6 students, mathematicians). Not all mathematics GSIs may have

close, personal or professional relationships with current or former teachers whom they can turn to for regular support.

Rowan may also differ from other mathematics GSIs in that they did not find "the teaching part" or the students to be "the problem" with teaching O&NS. Recall the following pair of quotes from the preparation and support interview, respectively:

> But just this vibe of like, 'you don't want to do that'. And then I feel like, though almost no one—the people who say not to teach [O&NS] have never actually taught [O&NS]. The people who have taught [O&NS] never said *not* to teach [O&NS]. *laughs* So that's like, what? What do people think is bad about? Like, I don't know what seems to be bad about it. I'm having a fun time. (Rowan, Preparation Interview)
>
> I think I get now why people say that they don't like this class, teaching this class. Because it's not the teaching part. Teaching it's fine. I enjoy my students. I enjoy the experience. It's everything else that's difficult. Lack of access to materials. Lack of access to resources. The huge time commitment. Everything else is the problem, the class itself is, is fun. (Rowan, Support Interview)

Rowan described other mathematics GSIs as normally expressing dissatisfaction (sometimes even "hate") with teaching O&NS. Robin, the lead instructor, also experienced problems with teaching O&NS that centered around their students. Anecdotally speaking, although I have spoken to a handful of mathematics GSIs who taught a mathematics content course like O&NS and had satisfying experiences with their students, I have also spoken to just as many mathematics GSIs who taught a mathematics

content course like O&NS and had overall negative experiences centered around their students. Although I cannot speak to the population of mathematics GSIs at large, the probability of finding mathematics GSIs with overall positive experiences teaching mathematics content courses like O&NS at any given institution should be better than 50%. Developing positive and productive orientations to teaching, teachers, and students does not happen overnight. Rowan's experiences suggest that mathematics GSIs may benefit from early experiences developing positive mindsets towards teachers and teaching. Specifically, an explicit introduction to and discussion of asset-based frameworks early in mathematics GSIs' preparation for teaching may support instructors in fostering more productive beliefs in the long-run.

Making Time and an Inviting Space for Instructor Concerns Matters

At various points in time, Rowan expressed directly or alluded to multiple reasons for not sharing their various concerns and desires for more support with O&NS colleagues or other faculty and staff at their institution. The idea that their concerns were not worth others' time or "bad enough" for voicing beyond pre-existing structures, along with the idea that "good" mathematics instructors and mathematicians do not reach out for certain support, particularly mathematics solutions, were especially insightful because they highlighted some of the less visible cultural and sociomathematical norms that hindered Rowan from engaging in or advocating for various support. Considered altogether, these ideas suggest that instructional leaders need to make genuine time and inviting spaces for instructor concerns and minimize stigma around asking for support.

Exactly *how* instructional leaders make time and space is an important nuance to discuss. Some researchers have already mentioned the importance of instructors having

opportunities to raise their own concerns (e.g., Suppa et al., 2020). However, Rowan's experiences suggest that the field needs to investigate these opportunities more deeply. Recall that the main O&NS instructional leader, Robin, asked Rowan and Rose how things were going or whether they had "anything super bad happening", "crises", or "concerns" that they needed to "vent" or "chat" about at the beginning of every weekly O&NS instructor meeting. Given that Rowan did not likely believe they had "anything super bad happening" that would be worth others' time (and their own) to discuss, Rowan often remained silent. Thus, the ways in which instructional leaders frame their questions matters. Perhaps instructors may be more likely to share their thoughts and concerns if norms have been constructed that encourage instructors to do so. Perhaps time might also be intentionally set aside so that instructors' thoughts and concerns are inherently worth others' time to listen to, even if they have not reached the level of "crises" or "catastrophic failures."

Rowan's experiences further demonstrate the complexities of power dynamics in instructors' opportunities to voice their thoughts and concerns. Before the semester began, I wondered whether Rowan might feel more inclined to share their thoughts during O&NS instructor meetings because the other two instructors were also mathematics GSIs, as opposed to faculty members who might make them feel more judged (especially over challenges in understanding mathematics content) or more pressured not to waste the time of those in higher positions of power. But I also anticipated Rowan might be more likely to share their thoughts because Robin seemed to care about instructor experiences and was personally driven to enhance O&NS instructor

meetings for newer instructors. Recall that Robin intentionally planned meetings to be more supportive in terms of discussing mathematics content:

> I remember [a previous O&NS course convenor] rushing through things and not really talking about content and (not) talking about here's the sections we should be covering, here's what we talk about for Habits of Mind (assignments). I have tried when I'm convening things to be a little bit better than that. (Robin, Week 7 Meeting)

As Robin explained that they wanted to "be a little bit better than that," I felt a warmth and personal connection to the changes they had made to make teaching this course easier to navigate for newer instructors. Indeed, Rowan found discussions about the course material supportive, especially when those discussions intersected with O&NS students' experiences. Yet having peers for colleagues nonetheless pressured Rowan to not take up the time of instructors who they perceived to be just as busy as they were.

As I continue to reflect on support for newer instructors of courses like O&NS, a research puzzle I extend to mathematics education researchers, as well as instructional and department leaders is this:

How might instructional leaders (with their own diverse backgrounds, experiences, and positions of power with respect to their colleagues) foster spaces in which mathematics content instructors, especially those who are newer, feel genuinely empowered to discuss their challenges, successes, hopes, and needs?

Perhaps one of the first norms to establish within collaborative teaching communities is what Rowan learned by the end of the semester—that "it is not a failure on my part to need to ask for more resources." But more research is needed. Specifically, more research

that carefully attends to instructors' backgrounds and teaching contexts within collaborative teaching communities for mathematics content courses is needed to understand what works and what does not to support newer instructors.

Little Preparation and Support for Lesson Planning Establishes "Wing-it" Culture

Earlier, I argued that Rowan's preparation for learning to teach more generally was akin to a slow-cooking process, in which various aspects of teaching, such as learning to plan lessons, were shifted on and off the backburner. Recall one of the reasons learning to plan lessons more generally was on the backburner for most of Rowan's preparation: Rowan had access to many resources that made it possible for them to not necessarily need to think through developing a lesson plan. After teaching O&NS without some of those same resources, Rowan emphasized having more to learn and that they could be much better at planning instruction. Thus, Rowan had little preparation for learning to lesson plan. Instead, Rowan came to rely on the skills and mindsets they developed around lesson planning based on their prior experiences.

One of those mindsets included lecturing on the fly. Previously, I argued that the absence of preparation and support for preparing lectures for their summer mathematics content course subtly shifted their mindset towards believing that it might be acceptable to deprioritize or even forgo preparing lectures in advance, or at least that it might be acceptable on occasion and with some notes already available. Although Rowan doubted the extent to which learning to plan lessons "on the spot" was "good" for them, they nonetheless learned that "I could pull together a full 50-minute lecture on the spot if I needed to." Rowan expressed this in response to my question about which prior experiences impacted their management of their challenges the most:

> But [faculty member] decided that what would be good for me—sometimes I doubt how much it was good for me, and how much it was that then [same faculty member] didn't have as much to do—um, was that I taught a few of the lectures. But the thing was that I would maybe get told after evening office hours the night before. One time I was actually told the morning of class that I was going to be teaching it. And like, do I have all the materials in front of me? Yes. Have I had a second to think about how I'm going to teach it, what I'm going to do, is it going to go well, what examples do I want? No, none of that. And so, I think. Having to do that process taught me that. Think. I can, I can do that. It wasn't unsuccessful to do that, right? I could pull together a full 50-minute lecture on the spot if I needed to. But then it also reminded me of how much better things go, if I don't do that. And so, I think. I think part of that is what pressed me so much to always time, prioritize prepping class. I didn't always prep class to the extent that I wanted to. But I had some nights where I was working real hard on other stuff, and I was not in the mood to get class prepped and I was reminded those times that I was thrown in the front of the class with no preparation and a couple of notes. And it went *fine*. It did not go *well*. (Rowan, Final Interview)

In Rowan's telling of this story, I felt a strong sense of absurdity, alongside the very real stress and tension surrounding producing lectures with so little time to do so. I also felt Rowan's relief and guilt surrounding their reminder of "how much better things go, if I don't do that (lecture on the spot)." Rowan reluctantly wielded a double-edged sword. On

the one side, lessons might go "much better" if they spent more time planning, but then they might not accomplish as many tasks, like preparing for interviews, preparing for their dissertation defense, grading homework, doing homework for their own classes, etc. On the other side, knowing that things might turn out "*fine*" but not "*well,"* spending less time planning might mean accomplishing their responsibilities both as an instructor and as a graduating Ph.D. student. Based on their prior experiences, Rowan seemed to find the latter side of the sword to be a more realistic option to survive teaching O&NS.

One reason that lecturing on the spot may be related to math culture is because Rowan's notion of acceptability around winging-it also seemed to be connected to Rowan's identity as a mathematician. In week twelve, Rowan described how "math-I-know-how-to-do" problems were not problems they would spend very much time thinking through as part of their planning. In this same classroom debrief, Rowan went on to describe borrowing slides without looking closely at the material as acceptable for them in calculus because they viewed themselves as an expert in that subject material, whereas they could not throw together slides for the material in O&NS and feel prepared.

> I ask [Rowan] about their planning. I ask them if they usually solve those problems in the activities before class, since they mentioned that they do usually know the solutions. [Rowan] says that they don't usually solve the problems per se, but they would figure out the classification for each of the word problems, e.g., What type of division problem is this? I say, so you wouldn't write out equations or make drawings, but you would figure out the type of problem? [Rowan] says yes, they wouldn't solve with all of those things but figure out how to classify. I ask [Rowan] how that

compares to how they would plan if the activity were something like, for example, What fraction of the square is shaded? type problems. [Rowan] says that they wouldn't really solve that kind of problem before class because that is more of a math-I-know-how-to-do problem. [Rowan] says that they can just look at that and figure out, oh this is 1/3 of 1/2, this is 1/9, etc. And the problem is just done, so they aren't worried about that. They say that it's the problems that are totally for this class where you have to learn things like, expectations around the first number being number of groups and second number being size of each group (definition of multiplication) where they need to figure out what the solution is in advance, because they don't know this material very well, unlike mathematics. They actually have to think for stuff that is solely this class…I ask [Rowan] if they had any prior experiences come up for them lately. [Rowan] right away says, oh yeah. They say that today's class made them think about when they were teaching calculus last semester and they needed to borrow [faculty member]'s slides for calculus and they just totally didn't prepare for class ahead of time, but it was fine and that was material that they are an expert at, really. But with this class, no they didn't borrow slides, but it felt similar to that experience because they just threw slides together with only some vague understanding, didn't really prep for class, and were totally unprepared because the material for this class is so different. They don't know this material. (Researcher Fieldnotes, Week 12 Classroom Debrief)

Hence, for Rowan, it was more acceptable to plan lessons on the fly in calculus than in O&NS due to perceptions of their expertise. And it was not typical for Rowan to spend a lot of time working out or thinking through mathematics tasks in advance if they felt they already knew how to solve the problem. Rowan's experiences suggest that it is important to consider one's mathematics identity when preparing instructors for teaching. Might mathematicians or other mathematics instructors spend less time preparing for class if they identify more strongly with the subject material? To what extent do mathematicians or other mathematics instructors equate planning lessons to their own understanding of mathematics content? How might that impact instruction? Are mathematics instructors not just learning to lecture on the fly for the sake of time, but learning to wing-it based on their perceptions of comfort or expertise with the course material?

Moreover, Rowan's experiences surrounding lesson planning suggest that the ways in which instructors are enculturated to plan lessons matters. Having experiences in which Rowan was expected to lecture with very little time to prepare in advance, as well as having little in terms of preparation or support to plan lessons (both for O&NS and more generally) set Rowan on a path towards learning that teaching lessons "on the spot," especially if they have expertise in the material, is an acceptable sacrifice for the sake of survival. If little comes in the way of preparation and support for planning, then instructors are left with little to rely on but sociomathematical norms and the skills and mindsets developed from their prior experiences. For future research and for institutions or departments that prepare newer instructors, I suggest that mathematics instructors' beliefs around planning and the ways in which instructors learn and are enculturated to plan lessons are more critically examined. Furthermore, the development of instructional

resources for mathematics content instructors, such as lesson planning templates, may be an avenue for further exploration. Suppa (2018) and Suppa et al. (2020) recommend the use of educative curriculum materials to support newer instructors of mathematics content courses for elementary PTs. I echo Suppa and colleagues' sentiments and would encourage both the development of curricular resources to support instructors to plan lessons in deliberate (and efficient) ways *and* resources for instructional leaders to engage instructors in their learning to use curriculum materials. It is not enough for instructors to have access to a plethora of resources—they need support to learn how to use them.

Preparation and Support for Grading Matters

Rowan's experiences suggest that instructors need opportunities to learn to evaluate and provide feedback on student work. Rowan had never graded students' homework and had little prior experience evaluating or providing feedback on mathematics solutions involving explanations and drawings. Although automated grading in convened courses may have provided Rowan with more time to focus on instruction and make progress on their degree, automating these tasks also removed opportunities to learn how to grade. Without additional experiences or preparation for grading at some point earlier in their teaching career or current support for grading while teaching O&NS, grading assignments in O&NS proved difficult, and eventually "insurmountable" for Rowan.

Oesterle (2011) similarly reported that the newer mathematics content instructors in their study found assessment challenging due, at least in part, to utilizing unfamiliar methods of evaluation, as well as ambiguity in the types of knowledge instructors should be measuring. Having prior experiences or preparation for assessing students'

understanding beyond evaluating procedural understanding on traditional mathematics exams may support newer instructors to learn to evaluate and provide feedback in myriad ways and on the myriad ways in which students can demonstrate understanding. Incorporating time to discuss grading within collaborative course structures may also support instructors in thinking through the potential benefits and limitations to various grading schemes, deciding on fair evaluations of student work and feedback that might support student growth, and overall alleviating some of the anxiety and uncertainty surrounding this work.

Cross-departmental Relationships Matter

Rowan's experiences with elementary mathematics methods (EMM) instructors suggest that cross-departmental relationships matter. Given that O&NS is designed to be a part of a collaborative STEM semester at Cardinal University with shared assignments between O&NS (a course offered in the mathematics department) and EMM, an elementary science methods course, and a technology course (all courses offered in the teaching and learning department), having a community in which instructors from all of these courses work to create a cohesive experience and support each other is important. As an instructor of the mathematics content course (O&NS), the instructors that Rowan worked more closely with were the instructors of EMM. Unfortunately, Rowan's expectations for support from EMM instructors were shattered after the joint O&NS-EMM instructor meetings unexpectedly dissolved five weeks into the semester. Because of this event, Rowan started to believe that EMM instructors did not think that O&NS instructors were worth their time.

Although the abrupt end to these joint meetings was upsetting, Rowan might have obtained more support from their corresponding EMM instructor, Willow. However, Rowan did not think being paired up with a new instructor was beneficial. Thus, Rowan's experiences additionally suggest that mindsets regarding professional relationships (e.g., desires for apprenticeship vs. egalitarian models) might also play a role in a newer instructor's decision whether to seek support from individual instructors. A more collaborative relationship between departments and a willingness to navigate their challenges alongside another newer instructor might have offered a means of mutual support for both Rowan and Willow.

Oesterle (2011) recommended strengthening the relationships between departments of mathematics and departments of teaching and learning for the potential support that such a relationship could offer newer mathematics content instructors (e.g., in learning to use physical manipulatives). I echo Oesterle's (2011) sentiment and emphasize that such a relationship could additionally support newer mathematics content instructors in learning to address complex questions from elementary PTs surrounding the intersections of elementary mathematics content, K-6 students' mathematical thinking, and K-6 schools. But I also emphasize that this relationship ought to be and has the potential to be mutually beneficial. Engaging in conversations about the mathematics elementary PTs are learning has the potential to strengthen both mathematics content and mathematics methods instructors specialized mathematics content knowledge. These conversations may further support mathematics methods instructors in enacting mathematics tasks that illustrate particular pedagogical strategies or educational theories.

Mathematicians and teacher educators need to be in healthy dialogue with each other if programs want to improve both instructors' teaching experiences and elementary PTs' learning experiences. However, more research is needed on how to establish and sustain collaborative and supportive relationships. Given that mathematics content and methods instructors may come and go, especially if they tend to be GSIs, these relationships may be difficult to create. Moreover, departments may not incentivize faculty members to construct these relationships. The problem of cross-departmental relationships is complex and requires further investigation to be supportive of everyone involved.

Communicating Expectations for Students Matters

Zopf (2010) and others have argued that teaching mathematics content courses is special, particularly for the types of knowledge involved in teaching such courses. Rowan's experiences validate the notion that teaching mathematics content courses is special in terms of the knowledge instructors need, but also offer insight into the implications of teaching such a course as a newer instructor. Specifically, the course material was not only new, "different", and "hard" for Rowan, but likely for their students as well. Just as Rowan was used to teaching courses in which students are learning mathematics content for the sake of learning mathematics (and not for teaching mathematics), Rowan's PTs were also used to learning mathematics content for the sake of learning mathematics content (as opposed to teaching it). The implication of this idea was not fully formed by Rowan until later in the semester after seeing their O&NS students' homework sets without answers to some of the questions that asked for

explanations. Rowan realized that not only did they themself need an introduction to the course and its purposes at the beginning of the semester, but so did their O&NS students.

By the end of the semester, Rowan learned the importance of communicating their expectations early on so that their students might also be ready for a new and different experience. For an instructor who is newer to teaching mathematics content courses, but also newer to teaching more generally, thinking beyond their own challenges to the challenges of their students may be difficult. Thus, Rowan's learning suggests an important implication for professional development programs and for instructional leaders looking to support newer mathematics content instructors: Instructors should not only be oriented to the ways in which teaching mathematics content courses might be new and different for instructors, but oriented to the ways in which mathematics content courses might be new and different for *students*.

Continuity Matters

Throughout this study, I often found that Rowan's prior experiences, preparation, and support illuminated many aspects of Rowan's challenges and successes. Notions of continuity offered an important lens for making sense of questions regarding how and why Rowan's challenges and successes manifested. Moreover, continuity supported me in understanding Rowan's challenges and successes more closely from their perspective. Some might only seek to critique instructors based on notions of instructor knowledge or expertise, but thinking through a lens of continuity supported me in avoiding some of the deficit-oriented mindsets towards instructors that can persist within various spaces in higher education. Rather than fixating solely on my own values and biases and constantly asking what Rowan *should* do in a particular situation, I instead asked what Rowan *would*

do or *could* reasonably be expected to do given their background, their goals for their teaching, and the conditions under which they had learned to teach and were still learning to teach.

An asset-oriented approach is not just for teaching K-12 students while PTs get left behind, and it is not just for teaching PTs while content and methods instructors get left behind. If we, as instructors of mathematics, believe that an asset-oriented approach is important for learning, then we need to extend this mindset to each and every group we aim to engage in learning. More broadly, we need to extend this mindset to people and humanity in general. Instructors are humans who need to feel seen and heard. Caring about and attending to continuity in instructors' stories offers an important means of engaging in an asset-oriented and humanizing approach to preparation and support for newer instructors.

A Resounding Theme Across Implications

Looking across these implications, there is one main thread connecting these ideas: the backgrounds, prior experiences, preparation, and support of an instructor matters for understanding many of the ways in which their challenges and successes in teaching and learning manifest. This is important for thinking about instructional practices, because when there is little preparation and support for a particular aspect of an instructor's practice, they may rely on previously developed mindsets or skills to navigate uncertainty in teaching. This is also important when thinking about developing relationships across departments because instructors with diverse backgrounds have the potential to offer each other mutually beneficial support based on their individual assets and strengths. And this is additionally important when thinking about developing

relationships with students, because they too bring prior experiences in learning mathematics that need to be understood to effectively communicate expectations and establish a productive and supportive learning environment. Ultimately, instructors bring who they are into their teaching context with aspirations for who they want to be and what they want to do, and this needs to be taken into account by education researchers, instructional leaders, and others invested in the professional development of mathematics instructors.

Summary of Suggestions for Future Research

Based on my findings and implications, I have a few recommendations for future research. First, researchers interested in investigating or developing instructors' mathematics content knowledge for teaching elementary PTs ought to consider learning about instructors' prior experiences learning elementary mathematics. In this way, researchers may be able to better understand or draw on instructors' strengths, and better understand or support instructors' orientations to teaching particular mathematics content, such as division with fractions. Second, mathematics education researchers interested in preparing newer mathematics instructors ought to investigate the culture of their respective departments. Shared experiences among individuals in mathematics communities are ripe with implicit and explicit norms that are critical to understanding instructors' behaviors, mindsets, and orientations. For researchers already interested in systemic departmental change efforts, understanding culture is essential to changing it. Third, I recommend that research pays more attention to students' assignments in mathematics content courses and their associated grading systems. Evaluating and assessing students' work and providing feedback for students does not receive as much

attention as classroom instruction but is an important way in which instructors make sense of students' learning and communicate the value of students' work to the students. Fourth, I recommend that notions of continuity be integrated into research related to mathematics instructor learning and mathematics education research more generally. Learning and growth is a long game connected to instructor identity and experience. Exploring instructor growth from a perspective of continuity may better support researchers in understanding the kinds of experiences and opportunities for reflection that develop instructors (e.g., as agentive and empathetic) in the long run.

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APPENDIX A: DEMOGRAPHIC SURVEY QUESTIONS

1. Preferred Name
2. Preferred Pronouns (e.g., she/her, they/them, she/they)
3. List all of the undergraduate or graduate-level mathematics or education courses you have taught (or assisted teaching) followed by the semester and year you taught the course and institution in which you taught the course. (e.g., College Algebra (Fall 2020, Spring 2021); Discrete Math for Teachers (TA; Summer 2019))
4. Do you have any K-12 teaching related experiences?
5. If you do have K-12 teaching related experiences, list all of your K-12 teaching related experiences here, being sure to include the grade-level and courses/topics taught, along with the semester or year they were taught, and the school in which they were taught, as applicable. (e.g., student teaching 9^{th} grade Algebra (2020-2021) at City High School)

APPENDIX B: AUTOBIOGRAPHY PROMPTS

Mathematics Learner Autobiography Prompt

The purpose of this assignment is to reflect on the experiences *learning* mathematics that helped to shape the mathematics teacher that you are today. As you write about your mathematical experiences please use specific examples (i.e. places, people, activities) and be honest about both what you did and how you felt. This autobiography is intended to span your earliest mathematical experiences through to the present … it is your story. Be sure to include the following: earliest memories of mathematics, elementary school mathematics experiences, middle school mathematics experiences, high school mathematics experiences, college mathematics experiences, graduate-level mathematics experiences, and most recent mathematical experiences. Now, the final touch … add a title to this paper that you believe captures your story. Be creative. (adapted from McCulloch et al., 2010).

Mathematics Teaching Autobiography Prompt

The purpose of this assignment is to reflect on the experiences *teaching* mathematics that helped to shape the mathematics teacher that you are today. As you write about your mathematics teaching experiences please use specific examples (i.e. places, people, activities) and be honest about both what you did and how you felt. This autobiography is intended to span your earliest experiences teaching mathematics through to the present … it is your story. Be sure to include memories of your first mathematics teaching experiences, memories from a variety of mathematics courses you have taught, and most recent mathematics teaching experiences. Now, the final touch … add a title to

this paper that you believe captures your story. Be creative. (adapted from McCulloch et al., 2010).

APPENDIX C: PREPARATION INTERVIEW QUESTIONS AND ACTIVITY

1. Describe for me your preparation or training for teaching from the beginning up until this point. [Probe for details about the time, place, settings, and people involved in these experiences.]
2. The following is a list of possible activities you might have engaged in prior to teaching [O&NS] and possible areas you might have developed expertise in as part of your preparation for teaching. [Give participant a copy of the preparation interview table.] Note that each of these areas are not necessarily independent from each other. These areas are listed because they are discussed in research literature around preparing instructors of mathematics and may assist you in thinking about your own preparation. I'll give you a moment to read through this list. You can feel free to voice your thoughts aloud, but let me know when you are ready for my questions about it.
    a. Which of these activities or areas would you say your preparation engaged you in or covered? Please star/mark these in green.
    b. Has your preparation engaged you in any activities or covered any areas that you feel are not captured in this list? If so, let us add those activities or areas to the list. Please star/mark these in green after you write them in.
    c. Which areas do you feel you have been well prepared for? Describe an experience that highlights this. [Probe for details about the time, place, settings, and people involved in these experiences.] Please highlight these in yellow.

   d. Which areas covered by your preparation do you feel you have not been well prepared for? Why do you feel this way? Describe an experience that highlights why this is important to you. [Probe for details about the time, place, settings, and people involved in these experiences.] Please star/mark these in red.
   e. What activities or areas on this list (or not) do you wish you (or others) engaged in (more) or were covered (more) by your preparation? Why do you feel this way? Describe an experience that highlights why this is important to you (or others). [Probe for details about the time, place, settings, and people involved in these experiences.] Please highlight these in pink.

3. Describe for me your preparation or training for teaching [O&NS]. [Probe for details about the time, place, settings, and people involved in these experiences.]
4. [Give participant a second copy of the preparation interview table.] Let's fill in those areas and activities that you added to the first copy of the table to this second copy.
   a. What activities or areas (on this list or not) do you think might be especially important for preparation to teach [O&NS]? Why do you think these activities or areas might be important?
   b. What areas do you feel you have been well prepared for with respect to teaching [O&NS]? Describe an experience that highlights this. [Probe for details about the time, place, settings, and people involved in these experiences.] Please highlight these in yellow.

c. What areas covered by your preparation do you feel you have not been well prepared for with respect to teaching [O&NS]? Why do you feel this way? Describe an experience that highlights why this is important to you. [Probe for details about the time, place, settings, and people involved in these experiences.] Please star/mark these in red.

d. What activities or areas (on this list or not) do you (or others) wish you engaged in (more) or were covered (more) with respect to teaching [O&NS]? Why do you feel this way? Describe an experience that highlights why this is important to you. [Probe for details about the time, place, settings, and people involved in these experiences.] Please highlight these in pink.

5. [Follow-up questions to ask if time permits.]

    a. What made you want to teach [O&NS]?

    b. What did you think about [O&NS] before you started teaching it a few weeks ago? Who did you talk to and what kinds of things did they say?

    c. Which of the courses that you've taught have made the biggest impact on you and why?

    d. Which course or prior experience has impacted what you're doing for [O&NS] the most right now and why?

    e. To what extent has TA-ing for the two-week summer mathematics content course impacted you as an instructor?

    f. What were you doing Summer [Year 2], Summer [Year 3], and Summer [Year 4]?

g. What were you doing Spring [Year 4]?

h. What made you decide to teach Intermediate Algebra over College Algebra?

i. What made you decide to keep teaching Intermediate Algebra for another full year?

j. What made you decide to lecture Calculus I in Fall [Year 5]?

k. Were you in 11th grade when you were student teaching/tutoring? What all did that entail and how did that come about?

l. When did you get a list of recommended class activities for this week (week two) from [Robin] and was it just a list of activities or were there other notes in that list?

m. Do you have access to a teacher's edition of the textbook for [O&NS]? If so, when did you acquire it?

n. How are you planning on grading homework and other assignments for the course?

Table C1

Possible Activities and Areas Related to Preparation to Teach

Activities	Areas
Structured conversations about teaching	Constructing a syllabus
Opportunities to reflect on your teaching (guided by instructional mentor(s))	Creating assignments
Opportunities to observe others' teaching (followed by debriefing/discussion)	Planning instruction
Opportunities to be observed teaching	Identifying lesson or course

(followed by debriefing/discussion)	objectives/goals
Opportunities to teach courses	Selecting tasks and participation structures
Opportunities to co-teach or assist teaching a course (taught by instructional mentor(s))	Anticipating student thinking, questions, and actions
	Eliciting and responding to student thinking, questions, and actions
	Leading/facilitating a discussion
	Motivating students to learn mathematics
	Evaluating/assessing student work
	Providing feedback on student work
	Eliciting and responding to student feedback (e.g., mid-semester or end of course evaluations)
	Communicating with students
	Adapting and modifying the course
	Modeling instructional practice (with meta-commentary)
	Familiarity with professional research literature related to learning and teaching mathematics
	Familiarity with common core (and state) mathematical practices and standards

APPENDIX D: SUPPORT INTERVIEW QUESTIONS AND ACTIVITY

1. First, describe for me all the ways in which you are supported to teach [O&NS]. This includes describing the different types of support you have access to or use, as well as how you feel about each of them.
2. Similar to our interview last time, the following is a list of possible supports for teaching [O&NS]. [Give participant a copy of the support interview table] Note that each of these supports are not necessarily independent from each other. These supports are listed because they are discussed in research literature around supporting mathematics content instructors and may assist you in thinking about your own support for teaching. I'll give you a moment to read through this list. You can feel free to voice your thoughts aloud, but let me know when you are ready for my questions about it.
    a. Which of these supports correspond to the supports you described having access to or using? Please star/mark these in green.
    b. Are any of the supports you described not covered by this list? If so, let us add those to the list. Please star/mark these in green after you write them in.
    c. Which of these supports do you think are especially important for teaching [O&NS]? Why do you feel this way? Describe an experience that highlights this. [Probe for details about the time, place, settings, and people involved in these experiences.] Please highlight these in yellow.
    d. Are there any supports not on this list that you think might be important for teaching [O&NS]? If so, let us add those to the list. Why do you feel

this way? Describe an experience that highlights why this is important to you (or others). [Probe for details about the time, place, settings, and people involved in these experiences.] Please highlight these in yellow.

e. You may have already covered this previously, but just in case, which supports have been most useful for you? Why do you feel this way? What makes these supports useful? Describe an experience that highlights this. [Probe for details about the time, place, settings, and people involved in these experiences.] Please highlight these in pink.

f. Is there anything about the nature of your current support for teaching [O&NS] that you would change? Why would you change these supports? In what ways would you change them? Describe an experience that highlights why this is important to you. [Probe for details about the time, place, settings, and people involved in these experiences.] Please star/mark these in red.

g. Are there any supports for teaching [O&NS] that you wish you had? Why do you feel this way? Describe an experience that highlights why this is important to you. [Probe for details about the time, place, settings, and people involved in these experiences.] Please star/mark these in blue.

3. [Follow-up question to ask if time permits and if not addressed above] [I know you've already touched on this, but in case you have more to say,] How are meetings with the other [O&NS] instructors going?

    a. What's going well? What makes you say this?

    b. What could be better? What makes you say this?

4. [Follow-up question to ask if time permits and if not addressed above] [I know you've already touched on this, but in case you have more to say,] How are the all-instructor meetings (joint O&NS-EMM instructor meetings) going?
    a. What's going well? What makes you say this?
    b. What could be better? What makes you say this?
5. [Follow-up question to ask if time permits and if not addressed above] [I know you've already touched on this, but in case you have more to say,] How are meetings with your corresponding EMM instructor going?
    a. What's going well? What makes you say this?
    b. What could be better? What makes you say this?

Table D1

Possible Support for Teaching O&NS

Activities	Resources
Mentoring	Syllabi or course objectives/learning goals[7]
Seminars related to mathematics learning and teaching	Lesson planning materials[8]
Meetings with other O&NS instructors	Assessment materials[9]
Meetings with EMM instructor(s)	O&NS textbook
Meetings with other instructors or instructional	Online databases or websites (e.g.,

[7] These materials might be from a previous semester, collectively developed for the current semester, or "handed" to you for the current semester.

[8] Above footnote applies here.

[9] And here.

leaders	wiki pages, nctm.org, Google)
Planning groups	Other curriculum materials or resources[10]
Opportunities to observe others' teaching (followed by debriefing/discussion)	Colleagues whom you can observe teaching (and with whom you can debrief/discuss their teaching)
Opportunities to be observed teaching (followed by debriefing/discussion)	Colleagues who can observe your teaching (and debrief/discuss your teaching)
Opportunities to reflect on your teaching and/or anticipate future challenges (with guidance)	Colleagues you go to for (and/or to discuss) syllabi or course objectives/learning goals
Opportunities to discuss own concerns related to teaching	Colleagues you go to for (and/or to discuss) lesson planning materials
Opportunities to rehearse teaching	Colleagues you go to for (and/or to discuss) assessment materials
	Colleagues you go to for (and/or to discuss) the O&NS textbook, online databases, websites, or other curriculum materials or resources
	Colleagues or communities you go to for advice or to talk about teaching

[10] Also here.

APPENDIX E: REFLECTIONS ON LEARNING INTERVIEW QUESTIONS

Summarizing Overall Learning

1. Last time I saw you, I asked if you could think of a metaphor that describes your experiences learning to teach [O&NS]. You described a caterpillar turning into a butterfly, was unsatisfied with this metaphor, and then started to describe classes in Dungeon and Dragons. What metaphor do you think you would use today and why?
2. Reflecting on the entire semester, what do you think are the most important things you learned as a newer instructor of [O&NS]?
3. What prior experiences do you think impacted your learning to teach [O&NS] the most? Why do you feel this way? Describe an experience that highlights this.
    a. I noticed at various points throughout the semester that you made reference to your mom. In what ways did your mom come up for you with respect to teaching [O&NS] and to what extent have those things made an impact on you and your experiences teaching [O&NS]? In what ways do you think you might have benefited (or not) from having a mom as a former schoolteacher—that other instructors who don't have this kind of personal connection might not?
4. What preparation or support do you think impacted your learning to teach [O&NS] the most? Why do you feel this way? Describe an experience that highlights this.

5. Reflecting on the entire semester, what do you think were the most significant successes you experienced as a newer instructor of [O&NS]? Why do you feel this way? Describe an experience that highlights this.
    a. To what or whom do you most attribute these successes and why?
    b. What prior experiences do you think impacted these successes the most and why?
    c. What or who do you think prepared or supported you to experience these successes the most and why?
6. Reflecting on the entire semester, what do you think were the most significant challenges you experienced as a newer instructor of [O&NS]? Why do you feel this way? Describe an experience that highlights this.
    a. To what extent have you been able to manage or overcome these challenges?
    b. What prior experiences do you think impacted your management of these challenges the most and why?
    c. What or who do you think prepared or support you to manage these challenges the most and why?

Imagining the Future/Capturing Change

7. What kinds of successes do you anticipate future newer instructors of [O&NS] might experience and why?
8. What kinds of challenges do you anticipate future newer instructors of [O&NS] might experience and why?

9. What kinds of prior experiences do you imagine would be most helpful for managing the challenges you experienced or anticipate that future newer instructors of [O&NS] might experience? Why do you feel this way?
10. What kinds of preparation or support do you imagine would be most helpful for managing the challenges you experienced or anticipate that future newer instructors of [O&NS] might experience? Why do you feel this way?
11. In the preparation interview, you used yellow highlighting to indicate the activities or areas that you felt well prepared for with respect to teaching [O&NS]. You used red to indicate the activities or areas that you did not feel well prepared for with respect to teaching [O&NS]. And you used blue hearts to indicate the activities or areas you wish that you or others engaged in more or were covered more in preparation for teaching [O&NS]. Here is the document you marked up during the interview. Is there anything that you would change about what you said then to reflect how you feel at this time? Please mark changes using purple. Is there anything you said back then that especially resonates with you now?
12. In the support interview, you used green to identify the supports you had access to or used. You used yellow highlighting to indicate the supports that you thought were especially important for teaching [O&NS]. You used pink highlighting to additionally indicate the supports that you thought had been most useful to you. You used red to indicate supports whose nature you would change in some way. And you used blue to indicate supports for teaching [O&NS] that you wished you had. Is there anything that you would change about what you said then to reflect

how you feel at this time? Please mark changes using purple. Is there anything you said back then that especially resonates with you now?

13. Overall, what do you think are the most important things for future newer instructors of [O&NS] to learn and why?
    a. In particular, what advice would you give to a newer [O&NS] instructor who traversed a similar path as you—they taught Intermediate Algebra, taught Intermediate Algebra again as a course convenor, and taught Calculus lecture and recitations?

APPENDIX F: WEEKLY REFLECTION JOURNAL PROMPTS

1. What successes have you been experiencing related to [O&NS]? Be sure to mention any people, resources, or prior experiences that might have supported you or prepared you for these successes, if applicable.
2. What challenges have you been experiencing related to [O&NS]? Be sure to mention any people, resources, or prior experiences that might have supported you or prepared you to manage these challenges, if applicable.
3. What have you been learning recently related to [O&NS]? For instance, you might describe what you have been learning as it relates to course content, students, or teaching.
4. What, if any, successes do you anticipate experiencing? Why do you anticipate these successes?
5. What, if any, challenges do you anticipate experiencing? Why do you anticipate experiencing these challenges?
6. Is there anything you wish you were more prepared for related to [O&NS]? If yes, what do you wish you were more prepared for and why?
7. Are there any supports (these might include human resources, physical or electronic resources, etc.) that you wish you had related to [O&NS]? If yes, what supports do you wish you had and why do you wish you had these?
8. What, if any, prior mathematics teaching or mathematics learning experiences came up for you this week as you prepared for, taught, or reflected on [O&NS] and why?
9. Is there anything else you would like to share about how [O&NS] went this week?