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Physics, Chapter 19: Heat Engines

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Heat Engines

19-1 Heat-Engine Cycles

In this chapter we shall consider the physical principles underlying the operations of heat engines because of the intrinsic importance of these principles and because of the part they have played in the development of fundamental physical ideas. *Heat engines* are designed and built to *convert* heat into work. In most cases the heat is obtained from the combustion of a common fuel such as coal, oil, gasoline, or natural gas. An important new source of heat that is just beginning to be used, and will be used more extensively in the future, is the *mass which is converted into energy* by means of a process called *nuclear fission*. Several power plants are now in operation which get the heat for their engines from the nuclear fission of the element uranium.

There are many different types of heat engines; we shall present brief descriptions of the operations of a few of them. In general, a heat engine utilizes a *working substance*, usually steam, or a mixture of fuel and air, or fuel and oxygen, through a series of operations known as a *cycle*. The working substance goes through a series of changes of state in this cycle, as a result of which some of the heat which has been supplied to it from a source at a high temperature is converted into work which is delivered to some external agency. Experience shows that not all of the heat supplied is converted into work; the heat which has not thus been utilized is delivered by the engine to some outside reservoir at a lower temperature.

The actual processes that occur in the operation of a heat engine are fairly complex. We can, however, simplify matters by replacing the actual heat-engine cycle by an ideal cycle which can produce the same transformations of heat and work. In such an ideal engine the working substance starts in some state designated by its pressure, its volume, and its temperature, is taken through a cycle in which its state continually changes, and then is brought back to its original state; the cycle then starts over again.

The operation of an ideal heat engine can be represented schematically by the diagram shown in Figure 19-1. A quantity of heat Q_1 is delivered to the engine during one cycle by some source of heat, and the engine performs an amount of work $\mathscr{W}$ on some outside agency and rejects an amount of heat Q_2 to another reservoir of heat. Since the substance in the engine returns to its original state at the end of the cycle of operations, it contributes no energy to this cycle. There is therefore no change in the internal energy of the working substance; that is,

$$U_f - U_i = 0.$$

From the first law of thermodynamics applied to this cycle, we get

$$Q_1 - Q_2 = \mathscr{W}. \qquad (19\text{-}1)$$

The *thermal efficiency* e of a heat engine is defined as

$$e = \frac{\text{work done during one cycle}}{\text{heat added during one cycle}},$$

or

$$e = \frac{\mathscr{W}}{Q_1}. \qquad (19\text{-}2)$$

Substituting the value for $\mathscr{W}$ from Equation (19-1), we get

$$e = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}. \qquad (19\text{-}3)$$

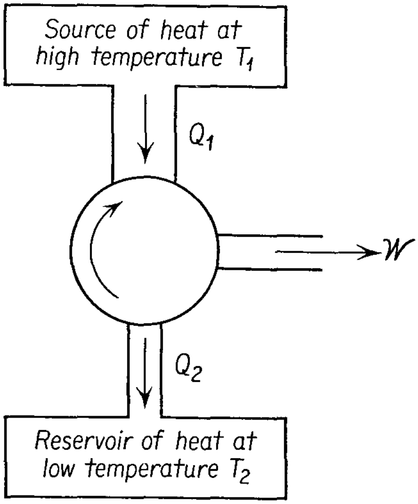

Fig. 19-1 Schematic diagram of the operation of a heat engine.

Equation (19-3) shows that the thermal efficiency of an engine is less than 100 per cent because a quantity of heat Q_2 is not transformed into work during the cycle. Experience shows that every heat engine rejects some heat during the exhaust stroke; one need merely recall the hot gases coming from the exhaust of an automobile engine or the steam exhausted by the engine of a steam locomotive. No engine has been built which takes in a quantity of heat Q_1 from some source and converts it completely into work.

19-2 The P-V Diagram

As we have already seen in Section 16-4, the work done by a gas at a pressure P in expanding through an increment of volume ΔV is given by $P\,\Delta V$. It is therefore convenient to plot the behavior of a gas on a diagram having pressure as the ordinate and volume as the abscissa when our interest is focused upon the mechanical work done on or by the gas. As the pressure or the volume of a gas is changed, the temperature may also be changed; heat may be added to the gas or taken from the gas. To describe properly the changes in the state of the gas, it would be necessary to represent the initial state of the gas as a point on a P-V-T diagram, and to represent the changes in the state of the gas as a curve in this diagram. Such a curve is called the *thermodynamic path* and represents the succession of values of

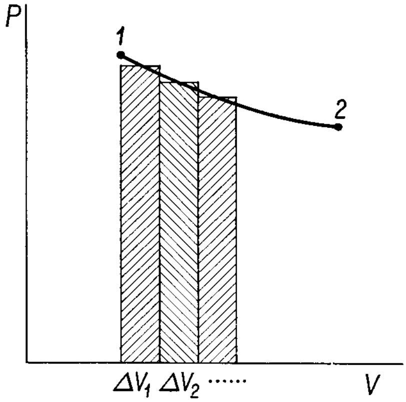

Fig. 19-2 Thermodynamic path projected onto a P-V plane.

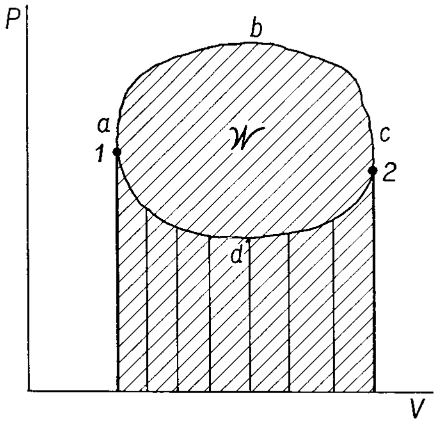

Fig. 19-3 A cycle of operations of an engine.

pressure, volume, and temperature of the gas as its state is changed. If the thermodynamic path is projected onto the P-V plane, as in Figure 19-2, the changes in the temperature of the gas are not shown on the path, but the work done in "moving" the gas along its path from condition 1 to condition 2 through a series of volume increments ΔV_1, ΔV_2, and so on, is clearly the area under the path.

Let us suppose that we alter the state of the gas from condition 1 to condition 2, in Figure 19-3, by either of the two alternate paths represented by *adc* or *abc*. Since the work done is represented by the area under the curve, the work done along the upper route is greater than the work done along the lower route. But since in either case the state of the gas, that is, the internal energy of the gas, at condition 2 at the end of each path is the same, we must assume that different quantities of heat were delivered to the gas along the two routes, to be consistent with the first law of thermodynamics.

If the gas is taken from the point *a* around the path *abcda* back to its initial condition, the device would operate as an engine, for the work done *by* the gas in expanding is greater than the work done *on* the gas when it is compressed, and the net work done by the gas $\mathscr{W}$ is represented by the area within the closed path. Since the gas at the end of the cycle is at the same pressure, volume, and temperature as it was at the beginning of the cycle, its final internal energy must be the same as its initial internal energy, and, since $U_f = U_i$, we must have

$$\mathscr{W} = Q;$$

that is, the work done by the gas in its passage around the closed path must result from the conversion of an equal quantity of heat Q which was supplied to the gas during the cycle. In the cycle of Figure 19-3 neither the changes in temperature of the gas nor the places where heat entered the gas or left the gas are shown.

The properties of a gas make it a likely substance for use in the conversion of heat energy into mechanical work. The volume changes associated with changes in the pressure or temperature of a gas are essential to the performance of mechanical work, for it is clear that any substance used as a working substance in a heat engine must be capable of changing its volume if it is to be able to do mechanical work.

If the passage around the closed cycle of Figure 19-3 had been made in the counterclockwise direction, as *adcba,* instead of in the clockwise direction, the device would operate as a refrigerator, for the work done by the gas in expanding along the path *adc* would have been less than the work required to compress the gas along the path *cba,* and the work $\mathscr{W}$, equal to the area within the closed path, would have been done on the gas. In accordance with the sign convention established in Section 15-6, work done on a system is negative work, so that the first law of thermodynamics leads to the result that a negative amount of heat has been added to the system; that is, a net quantity of heat Q equal to $\mathscr{W}$ in magnitude has been removed from the gas.

Thus any apparatus which carries a gas through a closed reversible cycle can be operated as an engine, converting some of the heat absorbed in the path *abc,* when the gas was expanding, into mechanical work, the heat not converted into mechanical work being removed from the gas when it was compressed along the path *cda.* The same apparatus operated in reverse would absorb heat during its expansion over the path *adc,* and, as the result of the mechanical work performed on the gas, would deliver the absorbed heat plus an additional quantity of heat equivalent to the work done on the gas during its compression over the path *cba.* A reversible heat engine can thus be operated as a refrigerator simply by reversing the sense of the thermodynamic path.

19-3 The Carnot Cycle

An interesting cycle from the theoretical point of view is the *Carnot cycle.* This consists of two isothermal processes and two adiabatic processes.

Although any material may be used as the working substance, we shall make use of an ideal gas as the working substance. Suppose that this gas is contained in a metal cylinder with a tight-fitting piston, as shown in Figure

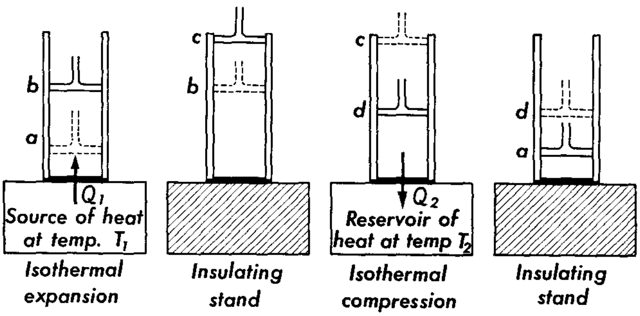

Fig. 19-4 Steps in the operation of a Carnot cycle.

19-4. Let the walls of the cylinder and the top of the piston be covered with thick layers of insulating material. Suppose we place the cylinder on a stove or other source of heat; heat will flow through the bottom of the cylinder into the gas. When equilibrium is reached, the temperature of

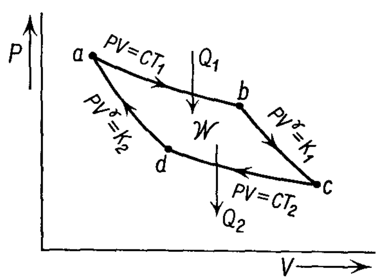

Fig. 19-5 Graphical representation of the steps in a Carnot cycle with an ideal gas as the working substance.

the gas T_1 will be the same as that of the source, and its pressure and volume will be represented by the point a in the graph of Figure 19-5. Now let the gas expand slowly so that its temperature remains T_1 while its volume increases to the point b. During this isothermal expansion, a quantity of heat Q_1 is delivered to the gas. The curve ab represents the isothermal expansion of the gas, and the area under it represents the work done by the gas during this expansion. Now imagine that the cylinder is placed on an insulated plate so that no heat can flow in or out of the cylinder. Let the gas now expand adiabatically from volume b to volume c. The amount of work done during this adiabatic expansion is the area under the curve bc. As a result of this expansion, the temperature of the gas will drop to some value T_2. Now place this cylinder on another reservoir at a temperature T_2; this reservoir may consist of a mixture of ice and water, for example. Now compress the gas isothermally to point *d*. During this process, work will be done *on* the gas equal to the area under the curve *cd*; at the same time, a quantity of heat Q_2 will be delivered to this reservoir. Now place the cylinder on the insulating stand and compress the gas *adiabatically* until it is back to the state *a*. During this adiabatic compression, work will be done *on* the gas equal to the area under the curve *da*. The cycle has now been completed.

During this Carnot cycle, a quantity of heat Q_1 was delivered to the gas at temperature T_1 and a quantity of heat Q_2 was rejected by the gas to the lower reservoir at temperature T_2, and the net work $\mathcal{W}$ was delivered to the outside. This work is equal to the area enclosed by the curves *abcda*. Since the internal energy of the gas was restored to its original value, the work $\mathcal{W}$ is given by

$$\mathcal{W} = Q_1 - Q_2, \tag{19-1}$$

and the thermal efficiency is

$$e = 1 - \frac{Q_2}{Q_1}. \tag{19-3}$$

19-4 Absolute Thermodynamic Temperature Scale

The Carnot cycle is used to define the *absolute thermodynamic scale of temperature.* The properties of the working substance do not enter into the calculation of the efficiency. The only quantities which enter into this discussion are the temperatures of the two heat sources. Let us now arbitrarily define these two temperatures by the following relationship:

$$\boxed{\frac{Q_1}{Q_2} = \frac{T_1}{T_2};} \tag{19-4}$$

that is, the ratio of these two temperatures is the ratio of the quantities of heat extracted from and delivered to these sources by an engine operating in a Carnot cycle between these two temperatures. The efficiency of the Carnot engine now becomes

$$e = 1 - \frac{T_2}{T_1}. \tag{19-5}$$

We see that its efficiency can be 100 per cent only if the temperature of the lower heat source is 0° on this scale.

We can now choose the size of the degree to suit our convenience. In the scientific scale known as the Kelvin scale of temperature, the difference between the temperature of boiling water at atmospheric pressure and the temperature of melting ice at atmospheric pressure is set equal to 100°, thus

$$T_s - T_i = 100°. \tag{19-6}$$

This makes the size of the degree on the Kelvin scale the same as that on the centigrade scale. As we shall see, the temperatures on the Kelvin scale defined by Equation (19-4) are identical with the temperatures previously introduced for the absolute gas scale of temperature. But the Kelvin scale is independent of the properties of any particular substance. On this scale, the temperature of the ice point is $T_i = 273.15°K$. From now on, we shall make no distinction between the Kelvin scale and the absolute scale. Another scale used by engineers sets $T_s - T_i = 180°$. This scale is called the absolute Fahrenheit scale of temperature. On this scale, $T_i = 491.8°$ abs F.

For the sake of definiteness, let us suppose that 1 mole of an ideal gas is carried through the Carnot cycle of Figure 19-5. When the thermodynamic path is an isotherm, such as *ab*, or *cd*, its temperature, and therefore its internal energy, remains constant. The heat Q_1 delivered to the gas in its expansion from *a* to *b* must be equal to the work $\mathcal{W}_1$ done in this expansion. From Equation (16-14) we have

$$Q_1 = \mathcal{W}_1 = RT_1 \ln \frac{V_b}{V_a}.$$

Similarly, in the isothermal compression in the path *cd*, we have

$$Q_2 = \mathcal{W}_2 = RT_2 \ln \frac{V_c}{V_d}.$$

Dividing the first of these equations by the second, we find

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \frac{\ln \frac{V_b}{V_a}}{\ln \frac{V_c}{V_d}}.$$

The equations describing the isothermal and adiabatic processes of an ideal gas in the changes of state in the Carnot cycle are $P_aV_a = P_bV_b$; $P_bV_b^\gamma = P_cV_c^\gamma$; $P_cV_c = P_dV_d$; $P_dV_d^\gamma = P_aV_a^\gamma$. Multiplying the left-hand sides of these equations together, and setting this equal to the product of the right-hand sides of the equations, we find, on factoring the product $P_aP_bP_cP_d$,

$$V_aV_b^\gamma V_cV_d^\gamma = V_bV_c^\gamma V_dV_a^\gamma,$$

or

$$(V_bV_d)^{\gamma-1} = (V_cV_a)^{\gamma-1},$$

from which

$$\frac{V_b}{V_a} = \frac{V_c}{V_d}.$$

Substituting this result into the above expression for the ratio of the heat absorbed and emitted over the isothermal portions of the cycle, we find

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2},$$

in agreement with Equation (19-4). Thus we see that the definition of temperature based upon the properties of an ideal gas is exactly equal to the definition of temperature on the Carnot cycle.

The Kelvin temperature scale represents a remarkable achievement in terms of the abstract ideas involved in the concept of temperature. Heretofore the concept of temperature was intimately bound up with the properties of matter. The very conception of the existence of an absolute zero was based upon experiments on the expansion properties of real gases at ordinary temperatures. Such a concept proved to be useless at very low temperatures where all gases liquefied. Although the practical measurement of temperature is still based upon the properties of matter, the meaning of temperature has become significantly different as the result of the absolute thermodynamic temperature scale, for now we see that temperature is intimately related to the efficiency of an ideal heat engine, the Carnot engine. We see that temperature is a measure of the *quality of heat,* for the success of any attempt to convert a given quantity of heat to mechanical energy depends upon the temperature at which that heat is available, in relation to the *ambient temperature,* the temperature of the surroundings. This, in turn, tells us a great deal about the quality of a fuel or of a combustion process. Two different fuels having the same heat of combustion do not generally produce the same amount of mechanical work. The fuel which burns at the higher temperature produces heat of higher quality, for that heat can be converted more efficiently into mechanical work.

19-5 The Second Law of Thermodynamics

It is a matter of general experience that heat always flows from a hotter to a colder substance, unless some external device is employed. So general is this observation that we could phrase a tentative hypothesis, that *heat of its own accord, will always flow from a higher to a lower temperature.* We might even extend this idea to assert the impossibility of constructing a device which, of itself, would move heat from a colder to a hotter substance. There are many devices which move heat from colder to hotter substances in everyday use, such as household refrigerators, but these are connected to the outside world through an electrical outlet. If the household refrigerator is unplugged from the power line, it no longer is able to remove heat from the freezer compartment at low temperature and deliver it to the kitchen at higher temperature.

There are many ways to formulate this second fundamental principle of thermodynamics, and they are all equivalent to each other. One form of the second law of thermodynamics is:

It is impossible to construct an engine, which, operating in a cycle, will produce no effect other than the extraction of heat from a reservoir and the performance of an equivalent amount of work.

This statement implies that every engine operating in a cycle which takes in heat from some source or reservoir must deliver some of this heat to a reservoir at a lower temperature. From the discussion of the preceding paragraph, the definition of the absolute zero is based upon the Carnot engine. The absolute zero is thus the temperature of a reservoir to which no heat will be delivered by a Carnot engine operating between some heat reservoir at a higher temperature and the reservoir at the absolute zero. But the statement of the second law of thermodynamics asserts the impossibility of constructing an engine which will eject no heat to a low-temperature reservoir. Thus we may assert that it will be impossible to achieve the absolute zero. Devices may be constructed which will come close to the absolute zero of temperature, but no device can be constructed which can achieve this temperature.

Many would-be inventors have sought to construct perpetual-motion machines in violation of both the first and the second laws of thermodynamics. Perpetual-motion machines, intended to deliver more energy than they receive, are examples of attempts to violate the first law. An inventor who succeeded in circumventing the second law, that is, design a perpetual-motion machine of the second kind, would be able, for example, to drive a ship across the ocean by extracting heat from the ocean's water and converting it to mechanical energy. He could operate electrical generating stations by extracting heat from the earth, or he could propel an airplane by extracting heat from the air. If our inventor could have a reservoir at the absolute zero at his disposal, he might appear to be able to achieve his goal, for his engine would expel no heat to his reservoir, so that there should be little difficulty about keeping the reservoir at the absolute zero once it had been cooled to that temperature. Unfortunately, the reservoir at absolute zero is unattainable.

19-6 The Refrigerator

In principle, a refrigerator may be thought of as a heat engine operated in reverse. As shown schematically in Figure 19-6, heat Q_2 is taken from some source or sources at a low temperature, work $\mathscr{W}$ is done on the engine by means of some outside agency such as an electric motor, and a quantity of heat Q_1 is delivered to a source at a higher temperature. The source from which heat is extracted is usually the food in the refrigerator. The source which received the heat Q_1 is usually the air surrounding the refrigerator. In the electrically operated refrigerator, the electric motor runs a compressor which consists essentially of a cylinder, a piston, and two valves

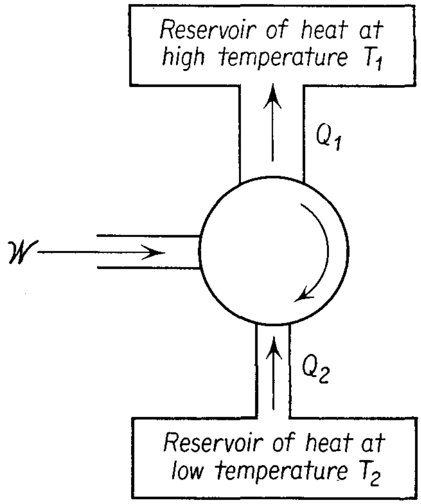

Fig. 19-6 Schematic diagram of the operation of a refrigerator.

just like the cylinder of a steam engine. The working substance used is called the refrigerant and may be ammonia, sulphur dioxide, or any other substance whose boiling point is fairly low. The refrigerant is taken through a cycle of operations which is described below, and at the end of this cycle its internal energy remains unchanged. For the first law of thermodynamics as applied to the refrigerator, we can write

$$Q_1 - Q_2 = \mathscr{W},$$

or

$$Q_1 = Q_2 + \mathscr{W}; \tag{19-7}$$

that is, the heat delivered to the air in the kitchen is greater than that taken from the food.

A typical cycle of operations for the refrigerant is as follows: suppose we start with the refrigerant, say ammonia, as a liquid, at high pressure and at room temperature, and allow some of it to pass through a valve or throttle into a region of lower pressure (see Figure 19-7). This process is called a throttling process (see Section 17-5). During this process the temperature also drops, and some of the ammonia is vaporized. This mixture is now led into the evaporation chamber in which the remaining liquid is vaporized at this low temperature and pressure. It is during this process of vaporization that heat is extracted from the food and water in the refrigerator and is used to vaporize the ammonia. The ammonia vapor is now taken into the compressor and is compressed adiabatically to a high pressure and a temperature slightly above room temperature. This compressed fluid is then sent through pipes which are cooled by the circulating air around them. It is during this process that the heat Q_1 is given out by the refrigerant and the refrigerant is brought back to its initial state.

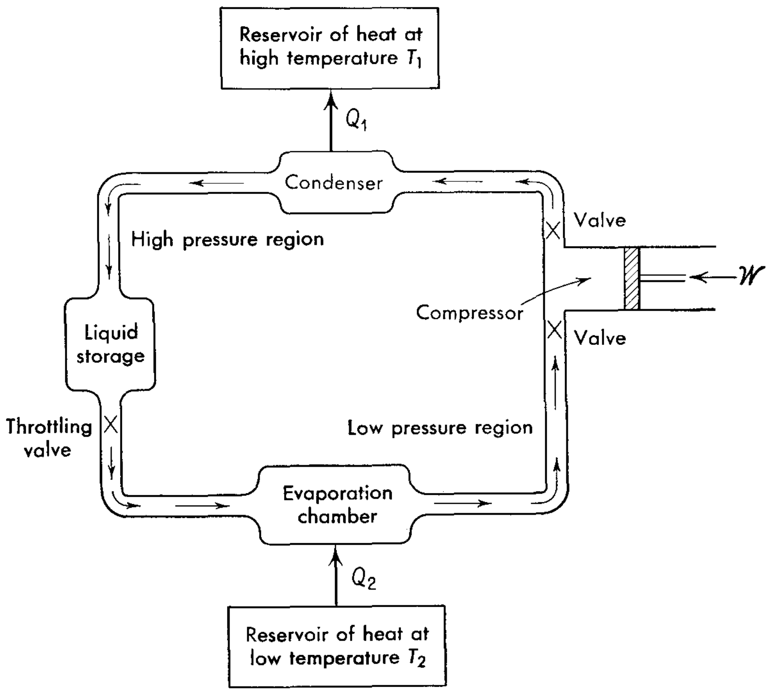

Fig. 19-7 Schematic diagram outlining the processes which occur in a refrigerator using ammonia as the refrigerant.

What is desired in a refrigerator is the extraction of an amount of heat Q_2 from the cold source with the performance of as little work $\mathscr{W}$ as possible. Instead of talking about the efficiency of a refrigerator, engineers use the term *coefficient of performance* of a refrigerator, defined as

$$K = \frac{Q_2}{\mathscr{W}} \tag{19-8}$$

where K is the coefficient of performance. In most practical refrigerators K has the value of 5 or 6. The smaller the amount of work needed to extract a given amount of heat, the greater is the coefficient of performance. For example, if 1,000 cal of heat are extracted from the food in the refrigerator and the motor which operates the compressor performs an amount of work equivalent to 200 cal, then the coefficient of performance of this refrigerator is 5. The heat Q_1 delivered to the air in the kitchen is 1,200 cal.

19-7 Practical Heat Engines

The Carnot engine and the Carnot refrigerator are imaginary devices. No one has yet built an operating Carnot engine, and it is extremely unlikely that such a device will ever be built. Yet the Carnot engine is of very great practical value, for the imaginary experiments we have conducted with the Carnot engine have enabled us to establish the meaning of temperature more clearly than before and to set limits on the efficiency of real engines, without regard for practical problems such as minimizing friction or achieving perfect fit between piston and cylinder. In fact, we may assert that *no reversible engine operating between two given heat reservoirs at different temperatures can be more efficient than a Carnot engine*, for this would constitute a violation of the second law of thermodynamics. Let us suppose that there were an engine more efficient than the Carnot engine. We shall imagine that the second engine is operated in reverse, as a refrigerator between the two temperature reservoirs, and is driven by the Carnot engine, operating between the same two reservoirs. The two devices together would constitute a single self-acting device which would pump heat from the reservoir at low temperature to the reservoir at high temperature and would produce no other effect, which is in clear violation of the second law.

The steam engine, the gasoline engine, the diesel engine, the turbine, the jet engine, and the rocket engine are all primarily heat engines. Some of these are external-combustion engines, in which the fuel is burned in a combustion chamber and the heat is transferred to the engine proper, while others are internal-combustion engines in which the combustion of the fuel takes place within the engine itself. In all of these engines, it is the heat liberated in combustion rather than any explosion of the fuel which is converted to mechanical work. For this reason the efficiency of all these engines is fundamentally limited by the efficiency of a Carnot engine. Modern engine designers, seeking increased efficiency, are limited by the properties of materials at high temperatures, for only by operation at high temperatures can the efficiency of present-day engines be significantly improved. Much of the effort of metallurgical and ceramic research is concentrated in the study and development of materials suited to the construction of more efficient engines.

19-8 A Heat Pump

The analysis of the refrigeration cycle shows that, by the performance of a certain amount of work $\mathcal{W}$, a quantity of heat Q_2 is taken from a reservoir at a low temperature T_2, and a larger quantity of heat Q_1 is delivered to a reservoir at a higher temperature T_1. Lord Kelvin, in 1852, suggested that this is just what is desired in the operation of a *heat pump*. It took about 75 years for the first practical heat pump to be put into operation; they are coming into more common use now for heating homes in the winter and cooling them in the summer. A schematic diagram showing the operation

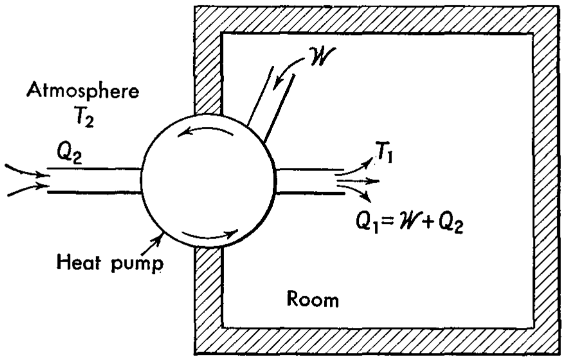

Fig. 19-8 Schematic diagram of the operation of a heat pump which takes a quantity of heat Q_2 from the atmosphere at a low temperature T_2 and pumps a quantity of heat Q_1 into the room at a higher temperature T_1 when an amount of work $\mathscr{W}$ is done on it.

of a heat pump is sketched in Figure 19-8. This heat pump will take heat Q_2 from the atmosphere and pump heat Q_1 into the room at the higher temperature T_1 with the performance of work $\mathscr{W}$ on the compressor of the heat pump. The quantity of heat Q_1 that is delivered to the room is given by

$$Q_1 = \mathscr{W} + Q_2,$$

although the energy that is paid for on the electric bill is represented by the work $\mathscr{W}$ which is done by the electric motor in operating the compressor.

By reversing the flow of the refrigerant in the heat pump, the temperature of the room can be kept below that of the atmosphere, a very desirable feature on hot summer days. In this case a quantity of heat Q_2 will be taken from the room at a low temperature T_2, a quantity of work $\mathscr{W}$ will be done on the compressor to accomplish this task, and a quantity of heat $Q_1 = \mathscr{W} + Q_2$ will be delivered to the outside.

19-9 Entropy

We have seen that in a Carnot cycle the quotient of the quantity of heat Q absorbed from or given out to a reservoir divided by the temperature T of the reservoir is a constant, for

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}. \tag{19-4}$$

If we regard Q as an algebraic quantity which is positive for heat absorbed by a body and negative for heat given out by a body, we may write

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = 0,$$

or

$$\sum \frac{Q_i}{T_i} = 0.$$

If we write ΔQ for the (algebraic) quantity of heat transferred to the engine at the temperature T over a small increment in the thermodynamic path, we have

$$\sum \frac{\Delta Q}{T} = 0. \tag{19-9}$$

The quantity

$$\Delta S = \frac{\Delta Q}{T} \tag{19-10}$$

is called the change in *entropy* ΔS. Thus the entropy change in the working substance in a cycle of operation of a Carnot engine is zero.

Let us consider the change in entropy of the universe associated with the flow of a quantity of heat Q from a reservoir at high temperature T_1 to a reservoir at low temperature T_2. Since heat has flowed out of the high-temperature reservoir, we call the heat change of the reservoir negative, and the entropy change of the high-temperature reservoir ΔS_1 is

$$\Delta S_1 = -\frac{\Delta Q}{T_1},$$

while the change in entropy of the low-temperature reservoir ΔS_2 is

$$\Delta S_2 = +\frac{\Delta Q}{T_2}.$$

The total change in entropy of the universe is

$$\Delta S = \Delta S_1 + \Delta S_2 = \frac{\Delta Q}{T_2} - \frac{\Delta Q}{T_1},$$

and, since T_1 is greater than T_2,

$$\Delta S > 0.$$

Similarly, we may consider the change in entropy of the universe when work $\Delta \mathscr{W}$ is converted to heat by friction against the surface of a reservoir at temperature T. The work is converted to heat $\Delta Q = \Delta \mathscr{W}$ (in appropriate units), and the entropy change of the reservoir is

$$\Delta S = \frac{\Delta Q}{T} \cdot$$

Since no heat has passed into or out of the agent doing the work, ΔS represents the entropy change of the universe, and again ΔS is a positive quantity.

Experience shows that in every thermodynamic process the change of entropy of the universe is equal to or greater than zero, and, in fact, natural thermodynamic processes move in such a direction as to increase the entropy of the universe. This may be written mathematically as

$$\boxed{\Delta S \geqq 0.} \tag{19-11}$$

Further analysis of the concept of entropy of a substance indicates that the entropy is a measure of the statistical probability for the arrangement of molecules of the substance in a particular state. A state of perfect order might be represented as one of zero entropy. In this sense the tendency for the entropy of the universe to increase might be described as a tendency for the universe to proceed from a highly organized to a completely disorganized state, a tendency for the universe to run down. In this very general way the concept of entropy has been used to explain the tendency of isolated organic matter to decompose, and to explain that life processes which seem to increase the degree of organization of nature can only take place if an even greater disorganization is taking place elsewhere in the universe, as in the sun. If a quantity of heat ΔQ is transferred by radiation from the sun at temperature T_s to a solar furnace at temperature T_f, the change in entropy of the sun is $-\dfrac{\Delta Q}{T_s}$, while the change in entropy of the furnace is $\dfrac{\Delta Q}{T_f}$. The change in entropy of the universe in the transfer of heat energy must be greater than or equal to zero, according to Equation (19-11).

Hence
$$\Delta S = \frac{\Delta Q}{T_f} - \frac{\Delta Q}{T_s} \geqq 0.$$

Thus the temperature of the furnace cannot exceed the temperature of the sun.

Problems

19-1. (a) Determine the thermal efficiency of a Carnot engine which operates between the temperatures of 100°C and 0°C. If 1,000 cal of heat are supplied to it, (b) how much work is done, and (c) how much heat is rejected to the low-temperature reservoir?

19-2. (a) Determine the thermal efficiency of a Carnot engine which operates between two reservoirs whose temperatures are 300°C and 0°C. If 1,000 cal of heat are supplied to it, (b) how much work is done, and (c) how much heat is rejected to the low-temperature reservoir?

19-3. Superheated steam at a temperature of 520°F is supplied to a steam engine which exhausts the steam to a condenser kept at a temperature of 60°F. Determine the maximum thermal efficiency of this steam engine.

19-4. Steam at a temperature of 227°C is supplied to a steam turbine which exhausts it to a condenser kept at a temperature of 13°C. Determine the maximum thermal efficiency of this steam turbine.

19-5. The coefficient of performance of a refrigerator is 4. It takes 3,000 cal out of a quantity of food. (a) How much work is done by the electric motor which operates this refrigerator? (b) How much heat is supplied to the surrounding air?

19-6. A tray containing 2,500 gm of water at 20°C is placed in a refrigerator whose coefficient of performance is 6. The water is changed to ice at −10°C. Determine (a) how much heat is removed from the water in converting it to ice, (b) the work done by the electric motor which operates the refrigerator, and (c) the amount of heat supplied to the surrounding air.

19-7. A two-stage turbine operates at an initial temperature of 327°C. Steam at the upper temperature is passed through the first stage and is expelled to a second stage at 227°C. The steam undergoes a further expansion in the second stage and is expelled to the atmosphere at a temperature of 127°C. Find the maximum theoretical efficiency of the turbine.

19-8. In a two-stage engine a quantity of heat Q_1 is absorbed at a temperature T_1, work $\mathscr{W}_1$ is done, and heat Q_2 is expelled at a lower temperature T_2. The second stage takes the heat expelled by the first, does work $\mathscr{W}_2$, and expels a quantity of heat Q_3 at a lower temperature T_3. Show that the efficiency of the combination is given by $(T_1 - T_3)/T_1$.

19-9. A Carnot refrigerator removes 100 cal of heat from a low-temperature reservoir at −23°C and expels 150 cal to a high-temperature reservoir. What is the temperature of that reservoir?

19-10. A refrigerator delivers heat into a room at 27°C at a rate of 1,200 watts. (a) What is the rate of extracting heat, in watts, from a low-temperature reservoir at −23°C? (b) What is the rating of the motor (in horsepower) required to operate this refrigerator?

19-11. A real engine absorbs 1,000 cal from a high-temperature reservoir at 127°C and expels 800 cal to a low-temperature reservoir at 27°C in each cycle. What is the efficiency of this engine?

19-12. One end of a copper rod is in thermal contact with a heat reservoir at a temperature of 100°C, and the other end of the rod is in contact with a heat reservoir at a temperature of 0°C. Determine the change in entropy (a) of the hot reservoir, (b) of the cold reservoir, (c) of the copper rod, and (d) of the universe when 1,500 cal of heat are transmitted through the rod.

19-13. Referring to Problem 19-1, determine the change in entropy (a) of the hot reservoir, (b) of the cold reservoir, and (c) of the universe. (d) What is the change in entropy of the working substance in a cycle of operation?

19-14. Referring to Problem 19-2, determine the change in entropy (a) of the hot reservoir, (b) of the cold reservoir, and (c) of the universe. (d) What is the change in entropy of the working substance in a cycle of operation?

19-15. A Carnot engine using helium as a working substance operates between temperatures of 27°C and 127°C. The engine does 0.1 joule of work in each cycle. Referring to Figure 19-5, determine the change in the entropy of the working substance (a) in the isothermal expansion *ab*, (b) in the adiabatic expansion *bc*, (c) in the isothermal compression *cd*, (d) in the adiabatic compression *da*, and (e) in the entire cycle.

19-16. A blunt drill is driven against a block of hardened steel by a $\frac{1}{4}$ horsepower motor. Assuming that all of the energy delivered to the motor is converted into heat, find the change in entropy of the universe in 3 minutes, assuming that the block remains at a constant temperature of 27°C.