1994

Reconciling Two Rate Level Indications: A Chain Rule Approach

Cheng-Sheng Peter Wu

Aetna life and Casualty

Follow this and additional works at: http://digitalcommons.unl.edu/joap

Part of the Accounting Commons, Business Administration, Management, and Operations Commons, Corporate Finance Commons, Finance and Financial Management Commons, Insurance Commons, and the Management Sciences and Quantitative Methods Commons

http://digitalcommons.unl.edu/joap/155

This Article is brought to you for free and open access by the Finance Department at DigitalCommons@University of Nebraska - Lincoln. It has been accepted for inclusion in Journal of Actuarial Practice 1993-2006 by an authorized administrator of DigitalCommons@University of Nebraska - Lincoln.
Reconciling Two Rate Level Indications: A Chain Rule Approach

Cheng-Sheng Peter Wu*

Abstract

The problem considered is that of reconciling two rate level indications that are based on several common factors, but have been made at different review periods. A popular approach to this problem is the so-called sequential replacement method, which calculates the impact of each individual factor. Unfortunately, this method has a serious deficiency: the estimated impact of a factor depends upon the order of the replacement. To counteract this defect, a new approach, called the chain rule approach, is developed. Using this approach, an explicit formula is given for calculating the impact and the marginal impact of each factor.

Key words and phrases: sequential replacement approach, factor impact, marginal factor impact

1 Introduction

Pricing (property/casualty) actuaries often have to deal with situations where two rate level indications have been produced at different rate review periods. If the two indications differ, then underwriters, marketing personnel, and regulators usually want to know the reasons

*Cheng-Sheng Peter Wu, A.C.A.S. (1993), is a senior actuarial associate at Aetna Life and Casualty, Hartford, CT. He received his masters degrees in chemical engineering and in statistics from the Pennsylvania State University. Mr. Wu has several papers published (or in press) in the areas of automotive engineering, tribology (lubrication engineering) and statistics.

Mr. Wu's address is: Personal Auto SBU-TS24, Aetna Life & Casualty, 151 Farmington Ave., Hartford CT 06156, USA.

He thanks the anonymous referees and the editor for their detailed comments and suggestions.
behind the difference. This is particularly true when rate level indica-
tions increase 20 percent, 30 percent, or even 50 percent in one year for
volatile lines such as workers' compensation. Such significant increases
may result from various factors, including a high trend, deteriorating
experience, or a change in the loss development pattern.

In order to explain the difference between two rate level indications,
the pricing actuary may need to estimate the individual impact of each
rating factor on the change. Because the rate level indication function
is usually a nonlinear function of the underlying rating factors, the por-
tion of the overall change due to any given factor depends on the values
of other factors.

In Section 2 we describe the approach now in use, the so-called se-
quential replacement approach. A better method, called the chain rule
approach is introduced in Section 3.

2 The Sequential Replacement Approach

2.1 The Definition

One method that some actuaries use to reconcile two rate level indi-
cations is the sequential replacement approach. The sequential replace-
ment approach starts with the prior review indication and replaces the
prior review rating factors sequentially (one by one) with the current re-
view factors. The method then concludes that the impact of any factor
is the change in the indication when that particular factor is replaced
in the indication calculation. In other words, suppose that there are
m factors and at time t, for (t = 0, 1, 2,...), the ith factor is
notated by \( X_{t,i} \) and the vector of the m factors at time t is denoted by
\( X_t = (X_{t,1}, X_{t,2}, \ldots, X_{t,m}) \). The indication at time t is \( I_t \), given by

\[
I_t = f(X_t)
\]

where \( f \) is a real valued function of the m factors. The change in the
indication is \( \Delta I_t = I_{t+1} - I_t \). How do we calculate the change in indica-
tion due to a particular factor? According to the sequential replacement
method, the change in indication between times t and t + 1 for factor i
is \( \Delta I_t(i) \) where

\[
\Delta I_t(i) = f(X_{t+1,1}, X_{t+1,2}, \ldots, X_{t+1,i-1}, X_{t+1,i}, X_{t,i+1}, \ldots, X_{t,m}) - f(X_{t+1,1}, X_{t+1,2}, \ldots, X_{t+1,i-1}, X_{t,i}, X_{t,i+1}, \ldots, X_{t,m}).
\]

The factors can be labeled in any order, provided the order is maintained throughout the analysis.
An obvious problem with using equation (2) to measure the impact of a specific factor is that the size of the estimated impact depends on how the factors are labeled and the order in which they are replaced in equation (2). The following example will illustrate this problem.

2.2 Example 1: The Sequential Replacement Approach

Assume that the following generic loss ratio formula\(^2\) is used to calculate a rate indication:

\[
I = \frac{X \times C + (1 - C) \times B}{ELR} - 1 \tag{3}
\]

where \(I\) is the indication; \(X\) is the insurer's ultimate, on-level, and trended experience; \(C\) is the credibility; \(B\) is the experience applied to the complement of credibility; and \(ELR\) is the permissible or expected loss ratio.

Further, assume that the rating factors and indications underlying the prior and current reviews are as follows.

<table>
<thead>
<tr>
<th>Review Data</th>
<th>(X)</th>
<th>(C)</th>
<th>(B)</th>
<th>(ELR)</th>
<th>(I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prior</td>
<td>0.7000</td>
<td>0.8000</td>
<td>0.4000</td>
<td>0.6000</td>
<td>0.0667</td>
</tr>
<tr>
<td>Current</td>
<td>1.0000</td>
<td>0.7000</td>
<td>0.6000</td>
<td>0.6500</td>
<td>0.3538</td>
</tr>
</tbody>
</table>

The increase in the indication from the prior to current review is \(\Delta I = 0.3538 - 0.0667 = 0.2871\), which is not unusual. The sequential replacement approach may proceed as follows: let \(\Delta I(X)\) be the impact due to the insurer's experience, \(X\). From equation (3), it follows that

\[
\Delta I(X) = \frac{1.0 \times 0.8 + (1.0 - 0.8) \times 0.4}{0.6} - \frac{0.7 \times 0.8 + (1.0 - 0.8) \times 0.4}{0.6} = 0.4.
\]

Let \(\Delta I(C)\) be the impact due to the credibility, \(C\). Then,

\[
\Delta I(C) = \frac{1.0 \times 0.7 + (1.0 - 0.7) \times 0.4}{0.6}.
\]

\(^2\)To keep this example simple, many of the rating factors, such as the on-level factor, trend, and loss development factor, are not considered in equation (3). The impacts of these factors will be discussed in detail in the next section.
Let $\Delta I(B)$ be the impact due to the experience applied to the complement of credibility, $B$.

$$
\Delta I(B) = \frac{1.0 \times 0.7 + (1.0 - 0.7) \times 0.6}{0.6} - \frac{1.0 \times 0.7 + (1.0 - 0.7) \times 0.4}{0.6}
$$

Let $\Delta I(ELR)$ be the impact due to the expected loss ratio, $ELR$. Then,

$$
\Delta I(ELR) = \frac{1.0 \times 0.7 + (1.0 - 0.7) \times 0.6}{0.65} - \frac{1.0 \times 0.7 + (1.0 - 0.7) \times 0.6}{0.6}
$$

In the above calculations, the order of replacement is $X$ first, then $C$, then $B$, and finally $ELR$. If this order of replacement changes, however, the impact of each factor may change. For example, when the order of replacement is $ELR$ first, then $B$, then $C$, and finally $X$, we get

$$
\Delta I(ELR) = -0.0821, \quad \Delta I(B) = 0.0615
$$

On the other hand, when the order of replacement is $B$ first, then $X$, then $ELR$ and finally $C$, we get

$$
\Delta I(B) = 0.0667, \quad \Delta I(X) = -0.1179, \quad \Delta I(ELR) = -0.0821, \quad \Delta I(C) = -0.0615.
$$

Given this problem with the sequential replacement approach, a new method is needed to compute the impact of each factor that is independent of the order of the computations. The chain rule approach described below solves this problem.
3 The Chain Rule Approach

3.1 Definition

Again, let $x_t = (x_1, x_2, \ldots, x_m)$ denote a vector of the $m$ factors used in determining the prior rate level indication at time $t$, and $f(x_t)$ be the rate level indication function. Consider what happens when there are infinitesimal changes in the rating factors. The total differential of the indication function can be calculated by the chain rule of differentiation (Edwards, 1973, Chapter 2):

$$df(x_t) = \sum_{i=1}^{m} \frac{\partial f(x_t)}{\partial x_{t,i}} dx_{t,i}. \tag{4}$$

Let $x_{t+1}$ be the current vector of rates. Then for small changes, however, equation (4) can be approximated by

$$\Delta f(x_t) = f(x_t + \Delta x_t) - f(x_t) \approx \sum_{i=1}^{m} \frac{\partial f(x_t)}{\partial x_{t,i}} \Delta x_{t,i}, \tag{5}$$

where $\Delta x_t = x_{t+1} - x_t = (\Delta x_{t,1}, \Delta x_{t,2}, \ldots, \Delta x_{t,m})$. From equation (5), the individual impact of factor $i$ may be approximated by $\left[\frac{\partial f(x_t)}{\partial x_{t,i}}\right] \times dx_{t,i}$. Its marginal impact is approximated by $\frac{\partial f(x_t)}{\partial x_{t,i}}$. Note that this approach is not affected by the order of the estimation sequence.

In the real world, however, the chain rule approach has a serious limitation. For many real world applications, the changes in $x_t$ are not necessarily small so equation (5) cannot be used. To cope with a significant change in $x_t$, a multivariate Taylor series expansion can be used. Recall the multivariate Taylor series expansion:

$$\Delta f(x_t) = \sum_{i=1}^{m} \frac{\partial f(x_t)}{\partial x_{t,i}} \Delta x_{t,i} + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{\partial^2 f(x_t)}{\partial x_{t,i} \partial x_{t,j}} \Delta x_{t,i} \Delta x_{t,j} + \cdots. \tag{6}$$

The chain rule can be approximated by the first order Taylor series expansion given in equation (5). This is only an approximation, however. But because we know the (exact) value of $\Delta f(x_t)$ we can make this approximation exact.

Now, by the mean value theorem, there is at least one point, $\mathbf{x}$, given by

$$\mathbf{x} = x_t + \tau \Delta x_t \quad \text{with} \quad [0 \leq \tau \leq 1] \tag{6}$$

$^3$ $f(x_t)$ is assumed to be at least twice differentiable in each of its parameters.
for which the first order Taylor series approximation is exact; see for
example, Edwards (1973, Chapter 2). The theorem, however, does not
indicate where \( x \) is. One obvious choice is to use the mid-point between
\( x_t \) and \( x_{t+1} \) (i.e., at \( \tau = 0.5 \)) to evaluate the partial derivatives. As we
will see, there is a better choice.

Consider the equation (as a function of \( \tau \)):

\[
H(\tau) = \Delta f(x_t) - \sum_{i=1}^{m} \left. \frac{\partial f(x_t)}{\partial x_{t,i}} \right|_{x_t=x_t^*} \times \Delta x_{t,i}
\]  

(7)

where \( x \) is given in equation (6). Let \( \tau^* \) be the smallest value of \( \tau \) for
which \( H(\tau) = 0 \), and let \( x_t^* \) be defined as

\[
x_t^* = x_t + \tau^* \Delta x_t.
\]  

(8)

The mean value theorem only guarantees the existence of \( \tau^* \). We can
determine \( \tau^* \) by first plotting \( H(\tau) \) for \( \tau = k/100, k = 1, 2, \ldots , 100 \) and
observing the number and approximate location of the roots of \( H(\tau) \).
Then \( \tau^* \) can be obtained more accurately using well-known numeri­
cal root-finding methods such as the bisection method or the secant
method. (See, for example, Burden and Faires, 1985, Chapter 2.) In
most practical situations, we expect \( \tau^* \) to be close to 0.5, i.e., \( \tau^* \approx 0.5 \).

The marginal impact and the impact of factor \( i \) can be defined as
follows.

**Definition 1** Given a vector of \( m \) factors \( x_t = (x_1, x_2, \ldots , x_m) \) and the
rate level indication function \( f(x_t) \), The marginal impact of factor \( i \), for
\( i = 1, 2, \ldots , m \), is \( \text{MIF}(i) \) where

\[
\text{MIF}(i) = \left. \frac{\partial f(x_t)}{\partial x_{t,i}} \right|_{x_t=x_t^*} 
\]  

(9)

The impact of factor \( i \) can be defined as follows:

**Definition 2** The impact of factor \( i \), for \( i = 1, 2, \ldots , m \), is \( \Delta I(i) \) where

\[
\Delta I(i) = \left. \frac{\partial f(x_t)}{\partial x_{t,i}} \right|_{x_t=x_t^*} \times \Delta x_{t,i}
\]  

(10)

\[
= \text{MIF}(i) \times \Delta x_{t,i}.
\]

3.2 Example 1 (Continued): The Chain Rule Approach

Let \( x_t = (X, C, B, ELR) \). Recall equation (3),

\[
I = f(x_t) = \frac{X \times C + (1 - C) \times B}{ELR} - 1.
\]
Clearly the partial derivatives are:
\[
\begin{align*}
\frac{\partial I}{\partial X} &= \frac{C}{ELR} \\
\frac{\partial I}{\partial C} &= \frac{(X - B)}{ELR} \\
\frac{\partial I}{\partial B} &= \frac{(1 - C)}{ELR} \\
\frac{\partial I}{\partial ELR} &= -\left(\frac{X \times C + (1 - C) \times B}{ELR^2}\right).
\end{align*}
\]

Given the data in Table 1, we have \(x_t = (0.7, 0.8, 0.4, 0.6)\) and \(\Delta x_t = (0.3, -0.1, 0.2, 0.05)\). Using equation (7), we have \(\tau^* = 0.4900\). Notice that, as expected, \(\tau^*\) is close to 0.5. From equation (8), \(x_t^* = x_t + 0.4900 \Delta x_t = (0.8470, 0.7510, 0.4980, 0.6245)\). Equations (9) and (10) now can be used to obtain the marginal impact and the impact of each factor. For example, the marginal impact of the factor \(ELR\) is
\[
MIF(ELR) = -\frac{0.8470 \times 0.7510 + (1 - 0.7510) \times 0.4980}{(0.6245)^2} = -1.9490.
\]

The impact of factor \(ELR\) is \(-1.9490 \times 0.05 = -0.0975\). Table 2 shows the marginal impact and the impact of each factor in this example.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Impact of Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor</td>
<td>MIF</td>
</tr>
<tr>
<td>(X)</td>
<td>1.2026</td>
</tr>
<tr>
<td>(C)</td>
<td>0.5588</td>
</tr>
<tr>
<td>(B)</td>
<td>0.3987</td>
</tr>
<tr>
<td>(ELR)</td>
<td>-1.9490</td>
</tr>
<tr>
<td>Total</td>
<td>0.2871</td>
</tr>
</tbody>
</table>

4 Example 2: Workers' Compensation Rating

4.1 The Problem

Next, an example from workers' compensation is considered. The example shows how to adjust the indication formula in order to consider the impacts of rating factors for trend, loss development, and any intervening rate changes.
Assume that we are proceeding on a state rate review in which the rate is stipulated by the Rating Bureau in that state. The insurer is free to use flexible rating tools, however, such as rate deviation, dividends, or schedule rating, to compete in the state.

Suppose the following information is given:

- The prior review uses the experience of 1990 accident year ending 12/31/90 evaluated as of 3/31/1992 (15 month maturity);
- The current review uses the experience of 1991 accident year ending 12/31/91 evaluated as of 3/31/1993 (15 month maturity);
- The Bureau's loss ratio is applied to the complement of credibility, and the prior and current reviews also use the 1991 and 1992 accident year experience, respectively, evaluated as of the same maturity date for the insurer's loss ratio;
- The target average effective date for the prior review is 7/1/1993;
- The target average effective date for the current review is 7/1/1994;
- An exponential trend with a 6 percent annual trend amount is used in the prior review for both insurer's and Bureau's loss ratios;
- An exponential trend with a 10 percent annual trend amount is used in the current review for both insurer's and Bureau's loss ratios; and
- There is a rate change of 15 percent between the two review periods.

The following loss ratio formula is used in this example to calculate the rate level indication:

\[ I = \frac{T \times F}{ELR} \times (X \times D \times L \times C + (1 - C) \times B) - 1 \]  

where \( I \) is the rate level indication; \( X \) is the insurer's on-leveled but untrended and undeveloped loss ratio; \( D \) is the loss development factor; \( L \) is the loss adjustment expense factor; \( C \) is the credibility; \( B \) is the untrended Bureau loss ratio; \( T \) is the trend factor; \( F \) is the flexible rating factor (such as rate deviation and schedule rating); and \( ELR \) is the expected loss ratio.

Table 3 lists all the values assumed for these rating factors in the two reviews and the resulting prior and current review indications.
Table 3

Review Data for Example 2

<table>
<thead>
<tr>
<th>Factors</th>
<th>Prior $(x_t)$</th>
<th>Current $(x_{t+1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>0.4200</td>
<td>0.4400</td>
</tr>
<tr>
<td>$D$</td>
<td>1.3500</td>
<td>1.3750</td>
</tr>
<tr>
<td>$L$</td>
<td>1.1500</td>
<td>1.1480</td>
</tr>
<tr>
<td>$C$</td>
<td>0.8500</td>
<td>0.9000</td>
</tr>
<tr>
<td>$B$</td>
<td>0.6300</td>
<td>0.6500</td>
</tr>
<tr>
<td>$T$</td>
<td>1.1910</td>
<td>1.3310</td>
</tr>
<tr>
<td>$F$</td>
<td>1.0200</td>
<td>1.0100</td>
</tr>
<tr>
<td>$ELR$</td>
<td>0.7200</td>
<td>0.7050</td>
</tr>
<tr>
<td>$I$</td>
<td>0.0946</td>
<td>0.3159</td>
</tr>
</tbody>
</table>

Both reviews use an exponential trend, but with different annual trend amounts: 6 percent for the prior review and 10 percent for the current review. The trending period for both reviews is the same, three years: from 7/1/90 to 7/1/93 for the prior review and from 7/1/91 to 7/1/94 for the current review. Thus, the trend factor in the prior review was $(1.06)^3 = 1.1910$, while in the current review it is $(1.10)^3 = 1.331$. In addition, the overall indication change is

$$\Delta I = 0.3159 - 0.0946 = 0.2213.$$  

Before applying the chain rule approach, several adjustments must be made to the indication formula given in equation (11). Adjustments are made to the following factors: rate on-level, trend, and loss development. This is because these rating factors must be compared at the same point in time between the two reviews. Thus, adjustments are necessary if there have been any rate changes between reviews, different trends are selected, or if the experience is evaluated on different maturity dates.

One rating factor not considered in this example is the benefit changes between reviews. Similar to the rate change, the insurer's loss ratio and the Bureau's loss ratio reflect all benefit changes through the reviews. Therefore, the adjustment for benefit change will impact the formula in essentially the same way as the adjustment for rate change, as discussed below.
4.2 Adjustment for Rate Change

The insurer's loss ratios and the Bureau's loss ratios listed in Table 3 reflect all rate changes through each review. Because there was a 15 percent rate change between the two review periods, the loss ratios are inconsistent. One way to adjust for the rate change is to recalculate the loss ratios in the current review without the 15 percent rate change and add one more rating factor, $R$, for the rate change to equation (11). Let $X'$ be the insurer's adjusted loss ratio and $B'$ be the Bureau's adjusted loss ratio. Table 4 shows their values.

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Adjustment for Rate Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factors</td>
<td>Prior</td>
</tr>
<tr>
<td>$X'$</td>
<td>0.4200</td>
</tr>
<tr>
<td>$B'$</td>
<td>0.6300</td>
</tr>
<tr>
<td>$R$</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Note: $0.4400 \times 1.1500 = 0.5060$ and $0.6500 \times 1.1500 = 0.7475$

4.3 Adjustment for Trend

The impact of trend on the rate indication can be split into two parts: the impact due to the trend amount and the impact due to the trend date. In this example, the annual trend amounts are different between the two reviews: 6 percent in the prior review and 10 percent in the current review. Also, the impact of the trend date must be evaluated separately because a more recent review will trend the on-level experience into a later effective date, which is one year later in this example. The trend date impact represents the increase in costs from the prior target average effective date to the current target average effective date.

The overall trend impact can be broken into the trend amount impact and the trend duration impact as follows: The average accident date (7/1/90) of the experience period in the prior review is used as the point in time to compare the trend impact between the two reviews. First the insurer's loss ratio and Bureau's loss ratio in the current review are detrended backward from 7/1/91 to 7/1/90 using the 10 percent trend amount. Next the trend amount impact for both reviews is defined from 7/1/90 to 7/1/93, which is $1.10^3 = 1.331$ for the current review and $1.06^3 = 1.191$ for the prior review. The difference between these two numbers is due to the different trend amounts used. Because the experience in the current review is trended one year beyond
the prior review (from 7/1/93 to 7/1/94) the trend date impact for the current review is defined as 1.10, while the trend date impact for the prior review is assumed to be 1.0. The trend date impact reflects the loss cost inflation from the prior target date to the current target date.

Following the previous adjustment for the rate change in Table 4, we further adjust the indication formula for the trend impact as follows: let $TA$ be the trend amount factor and $TD$ be the trend date factor, then

<table>
<thead>
<tr>
<th>Factors</th>
<th>Prior</th>
<th>Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X''$</td>
<td>0.420</td>
<td>0.4600</td>
</tr>
<tr>
<td>$B''$</td>
<td>0.630</td>
<td>0.6795</td>
</tr>
<tr>
<td>$TA$</td>
<td>1.191</td>
<td>1.3310</td>
</tr>
<tr>
<td>$TD$</td>
<td>1.000</td>
<td>1.1000</td>
</tr>
</tbody>
</table>

*Note: $0.5060/1.10 = 0.4600$, and $0.7475/1.10 = 0.6795$."

4.4 Adjustment for Loss Development Factor

In addition to the adjustments for rate change and trend, we need to ensure that the loss data in the prior and current reviews are evaluated as of the same maturity date. That is, $X''$ and $D$ must represent the experience and development factor of the same maturity between the two reviews. If not, an adjustment must be made to one of the reviews so that the two reviews are consistent.

For example, assume that prior review data are 12 months matured, while current review data are 15 months matured. We can make an adjustment to the prior review by dividing the prior 12-to-ultimate factor into a 12-to-15 factor and a 15-ultimate factor. Then the prior experience is combined with the 12-to-15 factor. By doing so, the loss experience and development factors between the two reviews become comparable. In this workers' compensation example, however, the insurer's loss ratio and the Bureau's loss ratio between the two reviews are developed from the same maturity date to ultimate; thus, there is no need for this adjustment.

4.5 Application of the Chain Rule Approach

At this point, we have finished all the necessary adjustments, and we are ready to adjust equation (11) to reflect all of the adjustments
made thus far.

\[ I = \left[ \frac{TA \times TD \times F}{R \times ELR} \right. \times \left. (X' \times D \times L \times C + (1 - C) \times B'') \right] - 1. \quad (12) \]

Table 6 summarizes the prior data and the current (adjusted) data needed for equation (12). It directly gives us \( x_t \) and \( \Delta x_t \). From the equations for the partial derivatives, we can calculate \( \tau^* \) and hence \( x_t^* \):

\[
\begin{align*}
  x_t &= (0.42, 1.35, 1.15, 0.85, 0.63, 1.191, 1.0, 1.02, 1.0, 0.72) \\
  \Delta x_t &= (0.04, 0.025, -0.002, 0.05, 0.0495, 0.14, 0.1, -0.01, \\
                  & \quad 0.15, -0.015) \\
  \tau^* &= 0.50376 \\
  x_t^* &= (0.4402, 1.3626, 1.1490, 0.8752, 0.6550, 1.2615, 1.0504, \\
                & \quad 1.0150, 1.0756, 0.7124).
\end{align*}
\]

<table>
<thead>
<tr>
<th>Factors</th>
<th>Prior</th>
<th>Current</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( X' )</td>
<td>( x_t )</td>
<td>( x_{t+1} )</td>
</tr>
<tr>
<td></td>
<td>0.4200</td>
<td>0.4600</td>
<td>0.0400</td>
</tr>
<tr>
<td></td>
<td>1.3500</td>
<td>1.3750</td>
<td>0.0250</td>
</tr>
<tr>
<td></td>
<td>1.1500</td>
<td>1.1480</td>
<td>-0.0020</td>
</tr>
<tr>
<td></td>
<td>0.8500</td>
<td>0.9000</td>
<td>0.0500</td>
</tr>
<tr>
<td></td>
<td>0.6300</td>
<td>0.6795</td>
<td>0.0495</td>
</tr>
<tr>
<td></td>
<td>1.1910</td>
<td>1.3310</td>
<td>0.1400</td>
</tr>
<tr>
<td></td>
<td>1.0000</td>
<td>1.1000</td>
<td>0.1000</td>
</tr>
<tr>
<td></td>
<td>1.0200</td>
<td>1.0100</td>
<td>-0.0100</td>
</tr>
<tr>
<td></td>
<td>1.0000</td>
<td>1.1500</td>
<td>0.1500</td>
</tr>
<tr>
<td></td>
<td>0.7200</td>
<td>0.7050</td>
<td>-0.0150</td>
</tr>
<tr>
<td></td>
<td>0.0946</td>
<td>0.3159</td>
<td>0.2213</td>
</tr>
</tbody>
</table>
Table 7
Chain Rule Results

<table>
<thead>
<tr>
<th>Factors</th>
<th>MIF</th>
<th>Impact</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X'')</td>
<td>2.4049</td>
<td>0.0962</td>
</tr>
<tr>
<td>(D)</td>
<td>0.7768</td>
<td>0.0194</td>
</tr>
<tr>
<td>(L)</td>
<td>0.9212</td>
<td>-0.0018</td>
</tr>
<tr>
<td>(C)</td>
<td>0.0599</td>
<td>0.0030</td>
</tr>
<tr>
<td>(B'')</td>
<td>0.2191</td>
<td>0.0108</td>
</tr>
<tr>
<td>(TA)</td>
<td>0.9528</td>
<td>0.1334</td>
</tr>
<tr>
<td>(TD)</td>
<td>1.1443</td>
<td>0.1144</td>
</tr>
<tr>
<td>(F)</td>
<td>1.1843</td>
<td>-0.0118</td>
</tr>
<tr>
<td>(R)</td>
<td>-1.1175</td>
<td>-0.1676</td>
</tr>
<tr>
<td>(ELR)</td>
<td>-1.6871</td>
<td>0.0253</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>0.2213</td>
</tr>
</tbody>
</table>

5 Summary

The chain rule approach has been introduced in this paper to reconcile two rate level indications that have been made at different rate review periods. This approach individually estimates the impact of each rating factor on the overall indication change. Unlike the sequential replacement approach, the chain rule approach does not depend on a sequence of estimation. This paper further indicates evaluating partial derivatives at the mid-point between the prior and current reviews provide a close approximation to the overall indication change. A workers' compensation example is given to show how to adjust the rate level indication formula for trend, loss development, and any rate and benefit changes between two reviews.

Although the main body of the discussion focuses on the loss ratio method, the developed chain rule approach can be applied equally to the pure premium method, such as the pure premium formula noted by McClenahan (1990, Chapter 2):

\[
RT = \frac{PP + FE}{1 - VE}
\]

where \(RT\) is the indicated rate per unit of exposure; \(PP\) is the trended and developed pure premium per unit of exposure; \(FE\) is the fixed expense per unit of exposure; and \(VE\) is the variable expense per unit of exposure.
While the loss ratio method develops the indicated percent change in the rate, the pure premium method develops the indicated rate. The \( PP \) term in the above formula can be subdivided into loss development and trend factors. The subsequent procedure to estimate the impact of each factor on the change in the indicated rate remains the same as described earlier in this paper.

References

