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Physics, Chapter 21: Vibrations and Sound

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Vibrations and Sound

21-1 Sound

There are two aspects of sound: one is the physical aspect which involves the physics of the production, propagation, reception, and detection of sound; the other, which is the sensation of sound as perceived by the individual, depends upon physiological and psychological effects. It is not desirable to separate the two aspects of sound completely, but the main emphasis in this book must necessarily be on the physical aspect. In this chapter we shall consider mostly musical sounds. A vocabulary has been developed to describe the sensation experienced when a musical sound is heard. Such terms as the *pitch* of a sound, its *loudness*, and its *tone quality* or *timbre* are used to describe the musical sound. The physicist, on the other hand, speaks of the *frequency* of the sound, its *intensity*, and the *number* and *intensities* of the *overtones* present in a musical sound. Unfortunately, there is not a one-to-one correspondence between the terms used by the physicist and the terms used by the musician. A great deal of progress has been made in recent years as a result of tests involving thousands of persons which attempt to correlate the sensation of sound with the physical properties of sound. Some of these results will be mentioned at appropriate places in this chapter.

21-2 Frequency of a Musical Tone

A musical tone is regarded as a pleasing sound, while a noise is usually thought of as disagreeable; there are some sounds which are difficult to classify. A musical sound, for example, can be produced by a series of regular blasts of air, while a noise results when these blasts occur at irregular intervals. This can be demonstrated by means of a disk containing five concentric rings of circular holes, as shown in Figure 21-1. In the innermost ring these holes are irregularly spaced; the next four rings have circular holes which are regularly spaced. There are 40, 50, 60, and 80 holes in these rings, respectively. When this disk is rotated at uniform angular speed and a stream of air is directed at the innermost ring of holes, an unpleasant noise will be heard. But when the stream of air is directed against any of the other rings, a pleasant musical tone will be heard. When the stream of air is directed against the four outer rings from the second to the fifth in sequence, the *pitch* of the sound coming from the third ring will be higher than that from the second ring of holes; the pitch of the sound from the fifth ring will be heard as an octave higher than that from the second ring. The physicist's method of describing these tones is in terms of the *frequency* of the sound produced. For example, if the disk is rotating at the rate of 10 rps, the frequency of the sound produced by the ring with 40 holes in it is 400 vib/sec; the next ring produces 500 vib/sec; the one after that, 600 vib/sec; and the last one, 800 vib/sec. To a first approximation we can say that the pitch of a tone depends upon its frequency, the tone with the higher pitch having the higher frequency. Two tones an *octave* apart have a frequency ratio of 2:1, for example. A musician will recognize these tones, which have frequencies in the ratios 4:5:6:8, as the tones comprising a *major chord.*

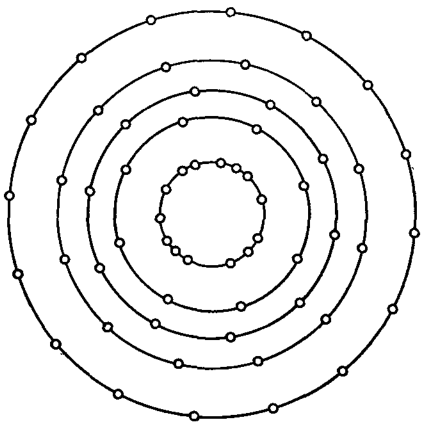

Fig. 21-1 Construction of a wheel for showing the difference between musical tones and a noise.

21-3 Resonance

An interesting phenomenon occurs when a body which is capable of vibrating at a definite frequency receives small impulses of the same frequency. These impulses set the body into vibration, with each succeeding impulse building up the amplitude of the vibration. This phenomenon, known as *resonance,* has been discussed in Section 12-7. A simple way of demonstrating resonance is to take two tuning forks having the same natural frequency and place them a short distance apart. One tuning fork is set vibrating by a hammer blow. After a short time interval, it will be found that the other tuning fork is vibrating and emits sound. The compressions and rarefactions produced in the air by the first tuning fork set the second tuning fork vibrating. Since the sound wave and the tuning fork have the same frequency, the impulses on the tuning fork are properly timed to build up its amplitude of vibration. A steady state is reached when the energy radiated by the second tuning fork is equal to the energy it receives from the first one.

Resonance can occur between any two bodies which can vibrate with the same natural frequency. An interesting example is shown in Figure 21-2, which illustrates resonance between a tuning fork and an air column. A hollow cylindrical glass tube is inserted in a jar of water. The vibrating system is the air in the hollow tube; the length of the air column can be varied by moving the tube up or down in the water. The air column ends at the surface of the water. If a tuning fork vibrating at a known frequency is held over the open end of the hollow tube and the hollow tube is raised, there will be a marked increase in loudness of the sound at some position.. At this position, the air column in the tube is set into vibration with the same frequency as the tuning fork; the two are in resonance. We may think of the process of changing the length of the air column as "tuning" it to the frequency of the wave incident upon it. This may be compared to the tuning of a radio circuit to the same frequency as the incident electromagnetic wave.

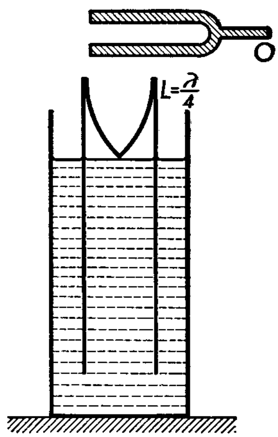

Fig. 21-2 Resonance between an air column and a tuning fork.

A tuning fork which vibrates with a frequency f emits a wave of length λ given by

$$V = f\lambda,$$

where V is the speed of sound in air. When an air column which is closed at one end is set into vibration, standing waves are produced in the air with a node at the closed end and an antinode at the other end. Since the distance between two successive nodes is half a wavelength, the distance between a node and the adjacent antinode is a quarter of a wavelength. Thus the shortest length of tube L in which the air can be in resonance with a wavelength λ is

$$L = \frac{\lambda}{4}. \tag{21-1}$$

If the tube is long enough, it will be found that resonance will occur again when the length of the tube is three quarters of a wavelength, for this length of air column will also have a node at the closed end and an antinode at the open end.

Tuning forks are often mounted on boxes whose air columns are in resonance with the sounds emitted by these forks. More energy is radiated per second from this system than from the tuning fork alone.

21-4 Beats

When two bodies having slightly different natural frequencies are set into vibration, the two waves emitted by them will *interfere* with each other. At some instant the two waves will be in the same phase, and there will be a reinforcement of the waves, resulting in a wave of increased amplitude. At

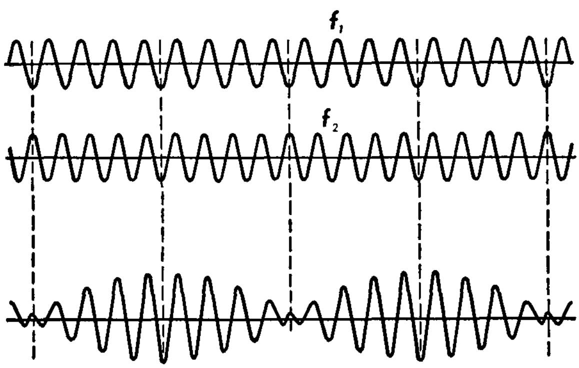

Fig. 21-3 Beats produced by the interference of two waves of slightly different frequencies.

some other instant the two waves will be completely out of phase; that is, a compression and a rarefaction will meet, resulting in a decreased amplitude producing a sound of very low intensity. The addition of two waves of slightly different frequency is illustrated in Figure 21-3. If one wave has a frequency f_1 and the other a frequency f_2, the number of times that these waves will get out of phase in a unit time can be shown to be $f_1 - f_2$; this is called the number of *beats* per unit time. If the two frequencies differ slightly, a series of beats will be heard; that is, the loudness of the sound will decrease noticeably $f_1 - f_2$ times in 1 sec. For example, if one tuning fork is emitting 256 vib/sec and another is emitting 260 vib/sec, 4 beats will be heard each second.

The phenomenon of beats is frequently used in tuning two sources of sound to the same pitch. This is a very accurate method of tuning, since the ear can perceive beats which occur only once in about 10 sec. When there are only a few beats per second, the sound produces an unpleasant effect. When the difference in frequencies is large, no beats can be distinguished in the sound produced.

21-5 Doppler Effect

In the previous discussions it was tacitly assumed that the source of sound and the observer were at rest with respect to each other. But when the source of sound is moving with respect to the observer, the pitch of the sound appears to be different from that when the two are stationary with respect to each other. There are two distinct cases to be considered: one in which the source is moving and the observer is at rest, and the other in which the observer is moving and the source is stationary. In both cases the frame of reference is fixed in the air.

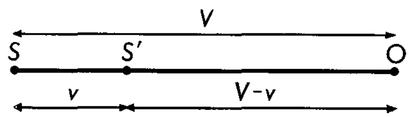

Fig. 21-4 The sound waves which are emitted in unit time by the source as it moves from S to S' with speed v toward the observer at O are contained in the distance $S'O$.

Suppose that an *observer is stationary at O*, as shown in Figure 21-4, and that the *source of sound is stationary at S*. If the source emits f vib/sec, the length of the wave λ emitted by it will be

$$\lambda = \frac{V}{f},$$

where V is the velocity of the sound. For simplicity, let us choose S to be at a distance from O equal to the distance traveled by sound in 1 sec; that is, $SO = V \times 1$ sec, where V is the speed of sound. Then when the source and the observer are both stationary, there will be f waves in the distance SO, each of length λ.

Let us now suppose that the *source is moving* with speed v *toward O*. At the end of 1 sec the source will have moved to S', where $SS' = v \times 1$ sec. During this time the source has emitted f vibrations; the first one has already reached the observer at O, and the last one has just left the source at S'. These f vibrations are therefore located in the region $S'O$, whose length is

$$S'O = (V - v) \times 1 \text{ sec}.$$

Since f waves have been emitted in this second, the length of these waves is

$$\lambda' = \frac{V - v}{f}. \qquad \textbf{(21-2)}$$

These waves travel with the velocity of sound V, and the frequency f' with which they reach the ear is therefore

$$f'\lambda' = V. \qquad \textbf{(21-3)}$$

Eliminating λ' from Equations (21-2) and (21-3) yields

$$f' = f\frac{V}{V - v}. \tag{21-4}$$

In other words, more waves will now reach the ear per second than reach it when the source is stationary. This will be interpreted as a sound of higher pitch. The change in pitch produced by the relative motion of source and observer is known as the *Doppler effect.*

The same reasoning can be applied to show that when the *source is moving away from the observer* with a velocity v, the frequency f' of the sound reaching the observer is given by

$$f' = \frac{V}{V + v}f. \tag{21-5}$$

The pitch of the sound in this case is lower than the pitch of the sound when the source is stationary.

It is instructive to analyze the Doppler effect in terms of the waves emitted by the moving source. Let us assume that the source emits spherical waves which, in Figure 21-5, are drawn as circles with successive positions of the source as centers. These successive positions are shown at time intervals equal to T, the period of the vibrations emitted by the source. In the figure, S is the present position of the source, S_1 is the position of the source at a time T earlier than S, S_2 the position at a time $2T$ earlier, and S_3 the position at a time $3T$ earlier. The wave emitted when the source was at S_1 has traveled a distance VT, where V is the speed of sound; hence this wave is represented by a circle of radius VT. Similarly, the wave emitted when the source was at S_2 is drawn as a circle of radius $2VT$, that emitted from S_3 is drawn as a circle of radius $3VT$. The source of sound is moving toward the right with a speed v less than V.

An observer in front of the moving source will receive more waves per second than if the source had been at rest. Conversely, an observer behind the moving object will receive fewer waves per second than if the source had been at rest. The observer in front of the moving source will hear a higher pitched sound than the observer behind the source. When the moving source passes the observer, he will always note a drop in the pitch of the sound.

The frequency of the sound f' received by the observer in front of the source is given by Equation (21-4) and can be derived very simply by referring to Figure 21-5. The distance between successive wave fronts which reach this observer is $(V - v)T$ and is therefore the length of the wave λ' perceived by the observer, that is,

$$\lambda' = (V - v)T,$$

but, for the source,

$$T = \frac{1}{f}\cdot$$

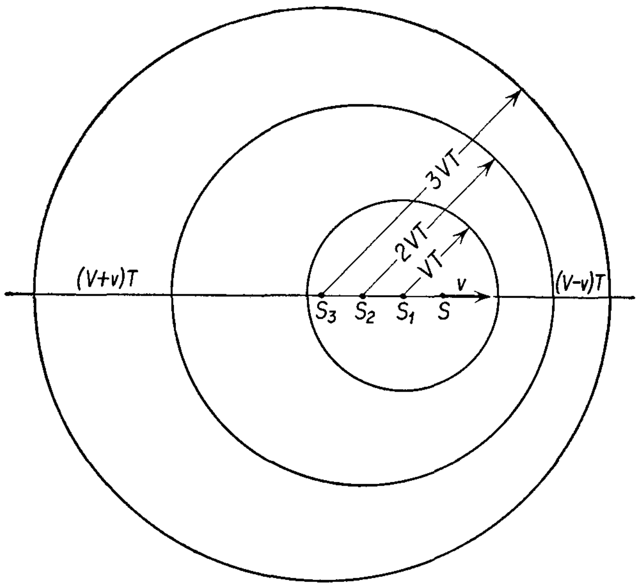

Fig. 21-5 Waves emitted by source moving to the right with speed v.

Therefore

$$\lambda' = \frac{V - v}{f},$$

and since these waves travel with speed V in the air, the frequency f' of these waves is given by

$$f'\lambda' = V, \tag{21-3}$$

so that

$$f' = f\frac{V}{V - v} \tag{21-4}$$

follows immediately.

Equation (21-5) can be derived in a similar manner.

Figure 21-5 can be used to determine the pitch of the sound heard by a stationary observer who is not in the line of motion of the source. The distance between wave fronts increases from the smallest value $(V - v)T$

for an observer in front of the source to its largest value $(V + v)T$ for an observer behind the source. To an observer at right angles to the line of motion, the distance between wave fronts is simply VT or λ; that is, it is exactly the same as if the source were stationary.

When the source is stationary and the *observer* is *moving* toward the source with a velocity v, the pitch is higher than that heard when the observer is stationary, but the actual value of the new frequency is slightly different from that given by Equation (21-4). The wavelength of the sound in air remains unchanged, but as the observer moves toward the source, he receives more waves per second than he receives when standing still. If he moves toward the source with a velocity v, he will receive v/λ additional waves per second, or a total of

$$f' = \frac{V}{\lambda} + \frac{v}{\lambda},$$

or

$$f' = \frac{V + v}{\lambda}.$$

Now since

$$f\lambda = V,$$

we get

$$\boxed{f' = \frac{V + v}{V} f,} \tag{21-6}$$

which gives the new frequency of the sound heard by the observer.

In a similar manner, if the observer is moving away from the source of sound, it can be shown that the frequency f' of the sound heard by the observer is given by

$$\boxed{f' = \frac{V - v}{V} f,} \tag{21-7}$$

which is a sound of lower pitch than that heard by the observer when stationary.

It must be emphasized that, in any one of these cases, the observer hears only one tone; he does not hear a change in pitch. Only when the motion is changed can he hear a change in pitch. Such a change in pitch can be observed when a train which is sounding its whistle passes an observer; the observer will hear a drop in pitch as the train passes him.

An interesting combination of the Doppler effect and the phenomenon of beats can be produced by moving a tuning fork rapidly toward a wall. A stationary observer will receive two sounds, one directly from the tuning fork and one reflected from the wall. The apparent source of sound of the reflected wave is the image of the tuning fork formed by the wall acting as a plane mirror. While the tuning fork is moving away from the observer, its image is moving toward him. The direct wave from the tuning fork will have a lower pitch than the wave coming from its image, and the observer will hear beats.

Illustrative Example. The siren of a fire truck is emitting a tone whose frequency is 1,200 vib/sec. The fire truck is traveling with a speed of 60 mi/hr. A man in the street notices a drop in pitch as the truck passes him. Determine the change in frequency of the tone heard by this observer.

While the fire truck was moving toward the observer at a speed of 88 ft/sec, he heard a tone whose frequency was higher than 1,200 vib/sec. This frequency f_1' can be determined from Equation (21-4),

$$f_1' = f\frac{V}{V - v} = 1{,}200 \times \frac{1{,}100}{1{,}100 - 88}\,\frac{\text{vib}}{\text{sec}},$$

$$f_1' = 1{,}304\,\frac{\text{vib}}{\text{sec}}\cdot$$

As the fire truck passed the observer, it moved away from him with a speed of 88 ft/sec, and the tone he heard had a frequency lower than 1,200 vib/sec. This frequency can be determined from Equation (21-5),

$$f_2' = f\frac{V}{V + v} = 1{,}200\,\frac{1{,}100}{1{,}100 + 88}\,\frac{\text{vib}}{\text{sec}},$$

$$f_2' = 1{,}111\,\frac{\text{vib}}{\text{sec}}\cdot$$

Hence the drop in pitch of the tone heard by the observer was due to a change in frequency of

$$f_1' - f_2' = 193\,\frac{\text{vib}}{\text{sec}}\cdot$$

21-6 Velocity of Source Greater than Velocity of Sound

When a body such as a projectile, a jet plane, or a rocket moves with a velocity v greater than the velocity of sound V in the medium, it sets up a compressional wave, as shown in Figure 21-6. The wave front, sometimes called *a shock wave*, is a cone, with the moving body at its apex S. The cone inside which the sound waves travel can be constructed by drawing spherical waves which originated at various positions of the source during its motion. In Figure 21-6 S is the present position of the source, S_1 its position at a time t earlier, S_2 its position at a time $2t$ earlier, and S_3 its position at a time $3t$ earlier, where t is an arbitrary time interval; let us call it one unit of time. With S_1 as a center we draw a circle of radius $V \times 1$, with S_2 as a center we draw a circle of radius $V \times 2$, and so forth. These circles represent the present positions of the compressions which started from S_1, S_2, and S_3. The wave front is the tangent to these circles. In a three-dimensional diagram this wave front would be a cone whose elements were tangent to spheres

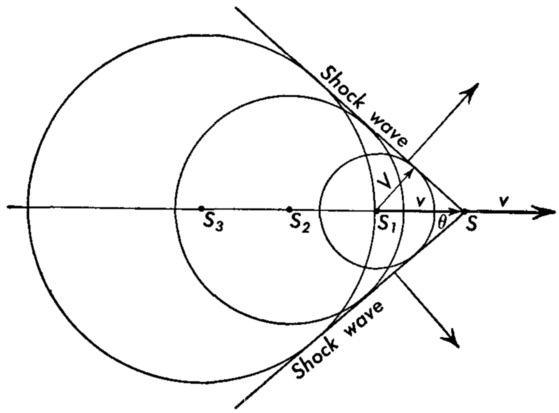

Fig. 21-6 Waves emitted by source moving with speed $v = 1.5\ V$.

of radii V, $2V$, and $3V$, respectively. In the unit of time that the wave progresses a distance V, the source moves a distance v, as shown in the figure. The angle θ that an element of the cone makes with the direction of motion of the source is

$$\boxed{\sin \theta = \frac{V}{v},} \tag{21-8}$$

since V is at right angles to the element of the cone and v is the hypotenuse of the right triangle.

The cone of sound moves through the medium with the speed v of the source. Outside the cone, no sound can be heard.

When an airplane wing moves through the air, it produces a change in the pressure, or a pressure pulse, which travels through the air with the speed of sound. If the speed of the airplane is less than the speed of sound, the pressure pulse travels ahead of the wing and, in effect, sets up the flow pattern of the air ahead of it (see Figure 9-9). When the speed of the airplane is greater than the speed of sound through the air, the wing meets the air head on, producing a shock wave which travels across the wing. This shock wave increases the drag on the wing and also sets up great stresses in it. In aircraft engineering the ratio of the speed v of a plane to the speed V of sound in air through which it is traveling is called the *Mach number*.

Thus the Mach number chosen for Figure 21-6 is 1.5. The bow wave, or the wake from a speedboat, is a similar phenomenon.

21-7 Intensity and Loudness

The *intensity* of a wave at any point in space is defined as *the amount of energy passing perpendicularly through a unit area at this point in unit time.* The intensity can be expressed in ergs per cm^2 per sec or in watts/cm^2. The intensity of the sound received from any source depends upon the rate at which the source emits energy, upon the distance of the observer from the source, and upon the reflections which the waves undergo from the walls, ceiling, floor, and objects in the room. If the size of the source is small in comparison with its distance from the observer, and if no reflection or absorption takes place, the intensity of the sound at any place will vary inversely as the square of its distance from the source, but this is rarely the case with sound waves. In terms of the sound wave which reaches the observer, it can be shown that *the intensity depends upon the square of the amplitude of vibration of the particles in the wave and upon the square of its frequency.*

The *loudness* of a sound is a sensation experienced by the observer, and although loudness is related to the intensity of the sound, the relationship between the two is not a simple one. Waves in air may be detected by the normal human ear if their frequencies lie between about 20 cycles/sec and 20,000 cycles/sec and if their intensities are within a certain range; the range of intensities audible to the ear also depends on the frequency of the wave. Those waves which can be heard are called *sound waves.* Figure 21-7 shows the range of frequencies and their intensities which are perceived as sound by the normal human ear; the intensity of the wave is plotted along the y axis, while the frequency of the wave is plotted along the x axis. Because of the wide range of intensities, these are plotted not on a uniform scale but on a logarithmic scale. One scale shows the intensities in watts/cm^2. Another scale shows the intensities in terms of the pressure changes in the wave which strikes the eardrum; since the pressure in a wave varies sinusoidally, the *effective* or *root mean square* (abbreviated rms) values of the pressure changes are used. The lower curve represents the *threshold of audibility.* A point on this curve represents the smallest intensity of a sound of given frequency which is just audible to the average ear. The ear is most sensitive to sounds of about 3,000 cycles/sec. At a certain intensity, known as the *threshold of feeling,* the sound is not heard but is felt by the ear as a tickling sensation. Above the threshold of feeling the intensity may be so great as to be painful. The region between the two curves represents the range of hearing. The range of intensities to which the ear is sensitive is about a millionfold. Because of this large range of intensities, a *logarithmic scale* has been adopted for expressing the *level of intensities of sound,* taking the zero level at about the limit of audibility of sound. The intensity level B of a sound is defined as

$$B = 10 \log \frac{I}{I_0}, \tag{21-9}$$

where I is the intensity of the sound and I_0 is the zero level of intensity which is taken arbitrarily to be equal to 10^{-16} watt/cm^2 or 10^{-12} watt/m^2.

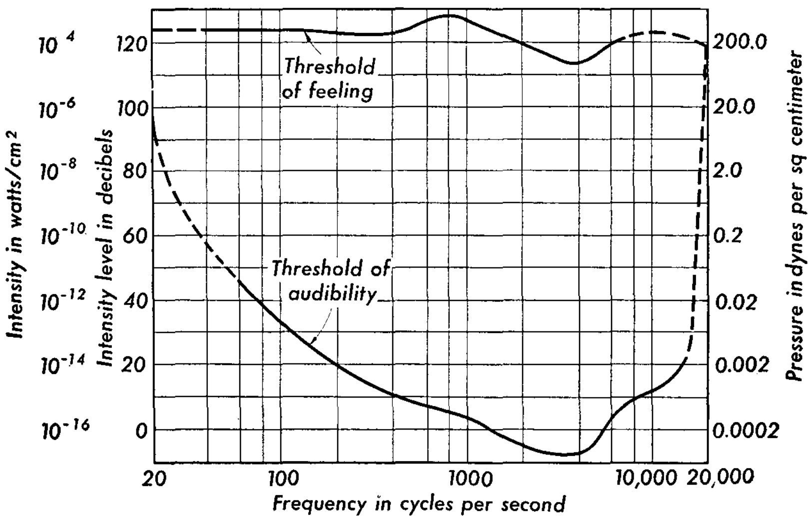

Fig. 21-7 Range of frequencies and their intensities which are perceived by the human ear. (After H. Fletcher, *Reviews of Modern Physics*, January, 1940.)

The intensity level B is expressed in *decibels* (db). Thus if a sound has an intensity $I = 10^{14}$ watt/cm^2, its intensity level is

$$B = 10 \log \frac{10^{-14}}{10^{-16}} \text{ db},$$

or

$$B = 10 \log 100 \text{ db},$$

from which

$$B = 20 \text{ db}.$$

Sound levels have been measured at various places under a variety of conditions. For example, inside some noisy subway cars the sound level is about 100 db, while the threshold of feeling (or pain) is about 120 db; the sound level of a whisper is about 15 db.

The shape of Figure 21-7 is of interest in connection with the high-fidelity reproduction of music. It is common experience that a radio or phonograph sounds better when played at high volume, for at low volume a significant range of frequencies is reproduced below the threshold of audibility. It is unreasonable to expect reproduced sound to have the fullness of orchestral music unless the intensity of sound in the home is equal to that of the concert hall. In modern high-fidelity phonographs an attempt has been made to compensate for the response of the ear by introducing a contoured volume control which decreases the intensity at both high and low frequencies at a lesser rate than the middle frequencies, as the volume control is turned down.

21-8 The Ear

Figure 21-8 is a diagram showing the essentials of the structure of the human ear. Sound waves enter the ear through the auditory canal and strike the eardrum. The pressure variations of the sound wave are trans-

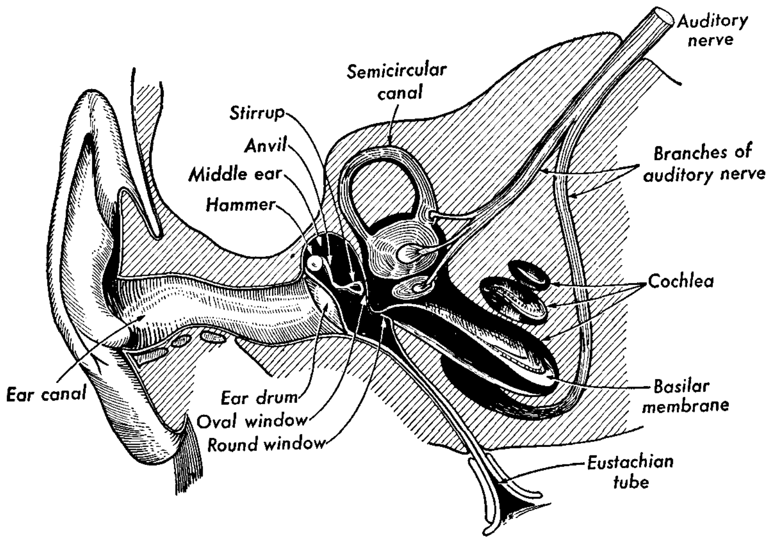

Fig. 21-8 Semidiagrammatic section of the right ear.

mitted from the eardrum by a system of three small bones, the hammer, anvil, and stirrup, to the oval window of the inner ear. The latter is filled with a liquid. The cochlea, a spiral-shaped part of the inner ear, contains the *basilar membrane* which runs along its entire length and divides it into two sections. The nerve endings of the auditory nerve are attached to one edge of the membrane. The entire length of the basilar membrane is about

30 mm (about 1.2 in.), and there are about 30,000 nerve endings attached to it. The vibrations which are transmitted to the liquid of the inner ear set the basilar membrane into vibrations, different tones affecting different sections of the membrane. These vibrations stimulate the nerve endings attached to it, and these transmit the signal along the auditory nerve to the brain.

The sense of balance is associated with the semicircular canal, and it is interesting that there are three of these natural "carpenter" levels in each ear, approximately at right angles to each other. In this way nature has apparently recognized the independence of the three rectangular components of a force, or the three mutually perpendicular directions of space.

Another aspect of the design of the ear which is of interest is the length of the ear canal. If we consider the ear canal as a pipe, closed at one end, we find that its resonant wavelength is given by Equation (21-1) as

$$\lambda = 4L. \tag{21-1}$$

The length of the ear canal is about 3.3 cm. Taking the velocity of sound as approximately 330 m/sec, we find the resonant frequency of the ear canal as

$$\begin{aligned} f &= \frac{V}{\lambda} \\ &= \frac{3.3 \times 10^4 \text{ cm/sec}}{4 \times 3.3 \text{ cm}} \\ &= 2{,}500 \text{ cycles/sec,} \end{aligned}$$

in good agreement with Figure 21-7.

If we assume that the hearing mechanism of most animals is similar to that of man, that the length of the ear canal is roughly proportional to the size of the animal, and that animals speak at frequencies appropriate to their organs of hearing, we may infer that large animals will speak in bass voices while smaller animals will utter sounds of higher frequencies.

21-9 Quality of a Musical Sound

When two tones of the same pitch and same loudness are produced by two different musical instruments, such as a violin and a clarinet, the sensations produced by them are decidedly different. We recognize this difference because of the difference in *quality* or *timbre* between these two musical sounds. One of the main reasons for this difference in quality is that each sound produced by an instrument is not a tone of a single frequency but is a complex sound consisting of several different frequencies. Another reason for the difference in quality is the manner in which the human ear responds to tones of different frequencies and different loudness. In this section we shall consider only the effect produced by the complexity of the sound on the quality of a tone emitted by vibrating bodies.

As shown previously, the vibrations in a body can be analyzed in terms of stationary waves which are set up in it by the interference of two waves traveling in opposite directions. The *fundamental* mode of vibration of a body corresponds to the longest wave, or wave of lowest frequency, which

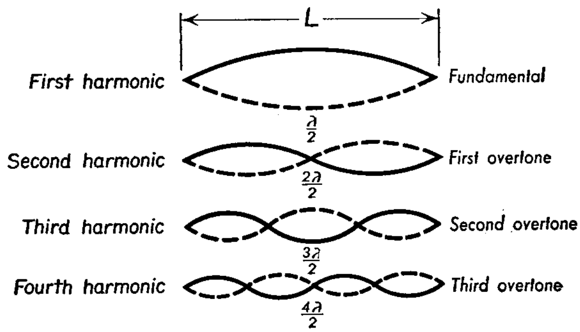

Fig. 21-9 Modes of vibration of a string which is fastened at both ends.

can be set up in the body. For example, in the case of a string which is fastened at both ends, the fundamental mode of vibration is one in which the string vibrates as a whole, as shown in Figure 21-9. Since the two ends are fastened, the standing wave must have nodes at these points, while the antinode is midway between them. The length of the string L is thus half the wavelength $\lambda/2$ of the transverse wave traveling in the string. The pitch of the sound emitted by the string when vibrating as a whole is called the *fundamental tone* of the string. The string can also vibrate in two parts, with a node in the center; the frequency of this sound is twice that of the fundamental tone. When vibrating in this manner, the pitch of the tone will be an octave higher than the fundamental. Other modes of vibration of the string are shown in the figure. When a string is set into vibration by plucking it or bowing it, several of these modes of vibration will be set up simultaneously; in addition to the fundamental tone, some of the higher pitched tones or *overtones* will be emitted by the string. *The quality of the tone will depend upon the number of overtones produced and their relative intensities.*

The same thing is true of other vibrating bodies; the quality of the sound depends upon the number and relative intensities of the overtones produced. Although, in the case of string instruments and wind instruments, the frequencies of the overtones are whole multiples of the frequency of the fundamental tone, this is not generally true of other musical instruments such as bells, chimes, and drums.

Another effect which helps to determine the quality of a musical tone is its transient character. A note sounded by a musical instrument is rarely sustained at constant volume for more than a moment or two. In a percussion instrument such as the piano, the intensity of the note diminishes rapidly after the note is struck. It is interesting to listen to the recording of a single piano note played backward. With modern tape-recording techniques it is possible to play a simple tune on the piano, to snip out each note, and then to paste them together so that each note is played backward but in the correct order. Each note increases in intensity and then abruptly ends. The effect is quite unrecognizable as a piano piece.

It is perhaps the transient nature of the typical musical instrument's sound which makes many electronic musical instruments seem rather unsatisfactory and lifeless, for, while an electronic instrument may generate the correct frequencies and even the correct distribution of harmonics, it is difficult to simulate the correct transient response as well.

21-10 Vibrations of Strings

The frequencies of the various modes of vibration of a string are in the ratios of whole numbers; such vibrations form a *harmonic series* with the fundamental vibration as the first harmonic, the overtone of twice this frequency as the second harmonic, and so forth. Any wavelength λ and its associated frequency f of a wave in a vibrating string are related by the equation

$$V = f\lambda,$$

where V is the speed of the transverse wave in the string. This speed is given by

$$V = \sqrt{\frac{S}{m}}, \tag{20-10}$$

where m is the linear density of the string, that is, its mass per unit length, and S is the tension in the string. Now the length of the wave in the fundamental mode of vibration is twice the length L of the string, that is,

$$\lambda = 2L; \tag{21-10}$$

therefore the frequency f_1 of the fundamental or first harmonic is

$$f_1 = \frac{V}{2L}, \tag{21-11}$$

from which

$$f_1 = \frac{1}{2L}\sqrt{\frac{S}{m}}. \tag{21-12}$$

The frequency of vibration and hence the pitch of a string can be varied by changing its length or its tension. Increasing the tension four times will double the frequency of vibration or produce a tone an octave higher. Decreasing the length of the string—for example, by pressing the string against a board with the finger—will increase the frequency or pitch of the tone emitted. Two strings of the same length and under the same tension, but of different linear densities, will have different frequencies for their fundamental tones.

The number of overtones set up in a string can be controlled to some extent by the method of bowing or plucking the string. For example, if the string is plucked in the center, those modes of vibration will be most intense which have antinodes at this point; the overtones will consist of the odd harmonics. If the string is plucked at a distance of about one seventh of its length from one end, the seventh harmonic will be absent, but most of the even harmonics will be present; the quality of this tone will be noticeably different from that heard when the string is plucked at the center, even though in each case the pitch heard is that of the fundamental tone.

The presence of overtones in a vibrating string can be shown very easily by plucking the string at one quarter of its length from one end and then placing a finger lightly on the center of the string; this will stop the fundamental mode of vibration, but the second harmonic can continue to vibrate, since it has a node at the center. When the finger is placed at the center, the fundamental tone will no longer be heard, but its octave will be heard, showing that the second harmonic is present.

Another method for demonstrating the presence of overtones in a vibrating string is illustrated in Figure 21-10. A steel wire is stretched between two posts A and B on a board, and these two points are connected to the primary coil of a step-up transformer. The secondary coil of this transformer is connected to an amplifier and a loud-speaker. Several U-shaped magnets are placed on the board so that the wire can vibrate freely between the poles of these magnets. Suppose that these magnets are so placed that their north poles are all on one side of the wire and their south poles are on the opposite side, as shown in Figure 21-10(a). When the string is plucked near the center, an induced electromotive force (emf) will be set up in the wire. This will be amplified, and the fundamental tone will be heard coming from the speaker.

To show the presence of the second harmonic, we use only two magnets, each placed at a distance of about one quarter of the length of the string from the ends A and B, but with opposite poles on the same side, as shown in Figure 21-10(b). The magnets are placed in this position because the string vibrates in two parts, with the portions of the string on either side of the central node moving in opposite directions. If the string is now plucked at a point near one of the magnets, a tone twice the frequency of the fundamental will be heard. If, while the string is still vibrating, one of the magnets is reversed so that like poles are on the same side, the fundamental tone will be heard. If the magnet is again reversed, the octave will be heard. This clearly demonstrates that both the fundamental tone and the first overtone are present at the same time.

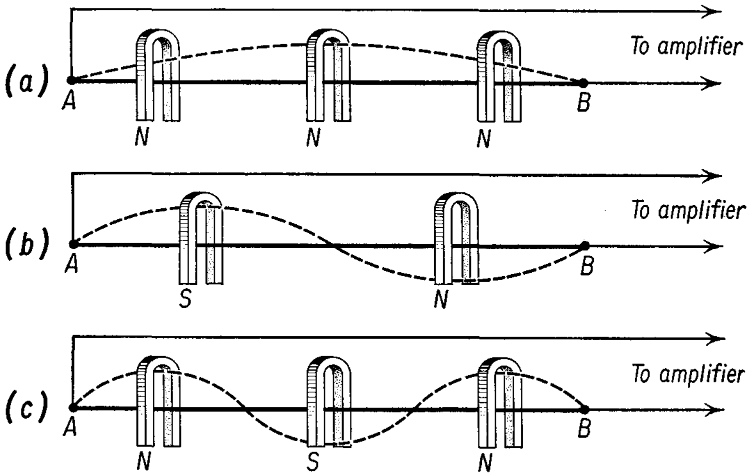

Fig. 21-10 Method for demonstrating the presence of overtones in a vibrating wire. The ends of the string *AB* are connected to the primary of a transformer whose secondary coil is connected to an amplifier and loud-speaker. *N* and *S* represent those poles of the magnets which are in front of the wire.

To show the presence of the second overtone, or third harmonic, three magnets are used, as shown in Figure 21-10(c). If the string is plucked at a point near one of the end magnets, the third harmonic will be heard clearly. If, while the string is still vibrating, the center magnet is reversed, the fundamental tone will be heard, thus showing that the first and third harmonics are both present. This method can be extended to show the presence of several of the other harmonics.

The amount of energy transferred directly from the vibrating string to the air is very small. To permit a greater transfer of energy, the strings are mounted on various types of solid boards, called *sounding boards*, usually made of metal or wood. These sounding boards are set into *forced* vibration by the vibrating string; the vibrations of these boards set larger quantities of air into motion, thus producing a more intense sound. In some string instruments of the violin type, there are air columns in the instrument which also vibrate. All of these vibrating systems make their contribution to the quality of the sound emitted by the instrument.

Illustrative Example. One of the steel strings of a piano is 50 cm long and has a linear density of 0.60 gm/cm. When struck with the hammer, it emits a tone whose fundamental frequency is 520 vib/sec. Determine (a) the tension in the string and (b) the frequencies of the first and second overtones of this string.

The tension in the string can be determined from Equation (21-12). Solving this equation for S, we get

$$S = 4L^2f^2m.$$

Substituting the numerical values, we get

$$S = 4 \times 2{,}500 \times 270{,}000 \times 0.60 \text{ dyne}$$
$$= 16.2 \times 10^8 \text{ dynes.}$$

The first overtone of a string is its second harmonic, and the second overtone is its third harmonic, therefore their frequencies are

$$f_2 = 1{,}040 \text{ vib/sec,}$$

and

$$f_3 = 1{,}560 \text{ vib/sec.}$$

21-11 Vibrating Air Columns

Wind instruments such as the clarinet, the trumpet, and the pipe organ all have vibrating air columns to reinforce some of the sounds produced by the source of sound. In this discussion we shall consider only cylindrical pipes such as those commonly used in pipe organs. There are two general classes of such pipes, the *open* pipe, that is, a pipe open at both ends; and a *closed* pipe, that is, a pipe closed at one end; the end containing the source of vibrations is always considered an open end. The vibrations can be produced in one of several ways, such as blowing air against a reed and setting it vibrating, or blowing a thin sheet of air against a thin lip at one end and setting the air into vibration. Whatever the method of setting up the vibrations, the column of air in the pipe will reinforce those modes of vibration corresponding to the standing waves which can be set up in this column.

Figure 21-11(a) shows several modes of vibration which can be set up in a closed organ pipe. The method for determining these modes of vibration depends upon the fact that only those vibrations can exist in the air column which have a node at the closed end and an antinode at the open end. It can be seen that the fundamental mode of vibration corresponds to a wavelength λ which is four times the length L of the air column. The first overtone possible in this case is one whose wavelength λ is four thirds of the length L of the air column. The frequency of the first overtone is therefore three times the frequency of the fundamental tone. The frequency and the wavelength are related by the usual equation

$$V = f\lambda,$$

where V is the speed of sound in air.

An examination of all other possible modes of vibration of an air column in a closed pipe shows that all the overtones are odd harmonics; their frequencies are in the ratios of 1:3:5:7:... and so on.

A pipe open at both ends must have antinodes at these ends. The fundamental mode of vibration of the air in an open pipe is shown in Figure 21-11(b); its wavelength λ is twice the length of the pipe. The first overtone has a wavelength λ which is equal to the length L of the pipe; its frequency is therefore twice that of the fundamental. An examination of the other possible modes of vibration shows that all the harmonics may be set up in this air column.

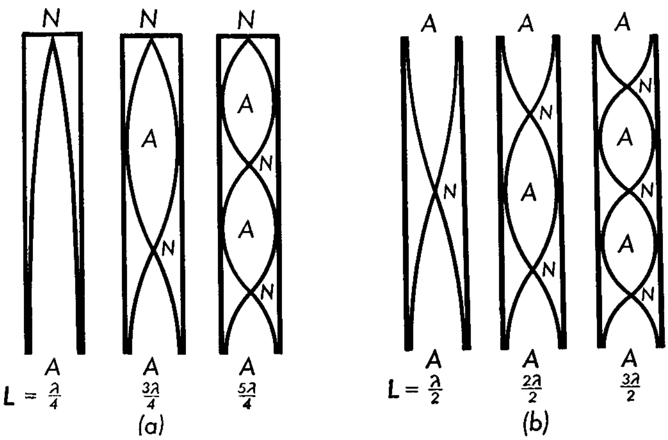

Fig. 21-11 (a) Modes of vibration of an air column closed at one end. (b) Modes of vibration of an air column open at both ends. The nodes are at the positions marked N, and the antinodes are at the positions marked A.

In practice, small corrections need to be applied to the actual length of a pipe to yield an effective length. We shall ignore these corrections in this and subsequent discussions.

A simple comparison of the open and closed pipes will show that if two such pipes of the same length are emitting their fundamental tones, the open pipe will emit a tone an octave higher than the closed pipe.

The pitch of a sound produced by an air column can be varied without varying the length of the air column. For example, when the air is blown harder against the reed, one of the overtones produced in the air column may have a greater intensity than the fundamental, and the pitch of the sound will correspond to this overtone. A bugle, for example, has a fixed length, but different tones can be obtained from it by changing the tension of the lips and the manner of blowing the air through them.

21-12 Acoustical Problems

From our discussion of the natural frequency of organ pipes and of the phenomenon of resonance, we see that any open pipe has a natural frequency of vibration, and that such a pipe will be excited to large amplitudes only when it is excited at its natural frequency. In many engineering problems it is necessary to maintain a free flow of air, while at the same time it is desired to prevent the passage of sound. The air-intake section of an aircraft wind tunnel is a typical illustration. If the propellers which drive the air of the wind tunnel are maintained at a fixed rotational speed, it is possible to admit air to the wind tunnel through a nest of tubes whose dimensions are chosen so that their natural frequencies are far from the frequency of the sound generated by the propellers. Such a nest of tubes constitutes an *acoustic filter* which discriminates against sound waves of certain frequencies. Other examples of sound filters in common use are the mufflers of automobiles and guns. The familiar effect produced by shouting into a barrel is an example of acoustic filtering.

Let us consider the effect of the size of a room on the sound produced within the room by a musical instrument. As a first approximation we may consider the room to be broken up into an imaginary nest of tubes whose length is the longest dimension of the room. Since these tubes are closed at both ends, the resonant wavelength of each tube will be

$$\lambda = 2L,$$

where L is the length of the room. A typical living room about 22 ft long will have a resonant frequency of about 50 cycles/sec. It should not be expected that a recorded organ recital will be reproduced with the same tonal balance in such a living room as it was originally performed in a cathedral.

Another aspect of auditorium acoustics which is of great importance is called the *reverberation time,* which is arbitrarily defined as the time required for the sound intensity to fall to one millionth of its original intensity. The persistence of sound in a room depends upon the absorption of sound at the walls at each reflection. In a room covered with heavy draperies, the reverberation time may be too short, and the sound seems dull. In general, the desirable reverberation time ranges from about 1 sec for a large room to about 2 sec for a large auditorium. An empty, untreated auditorium may have a reverberation time as long as 6 or 7 sec. If the reverberation time is too long, sounds reaching the listener from a speaker by a direct path and by reflected paths are of comparable intensity, and a meaningless garble results.

21-13 Ultrasonics and Supersonics

The term *supersonic* is often used to designate speeds greater than the velocity of sound. The so-called "sound barrier" to aircraft flight has nothing basically to do with sound but rather is associated with the velocity of propagation of a disturbance in air, which we call the velocity of sound. When an aircraft moves through air at speeds higher than the velocity of sound, shock waves, which are regions of considerable turbulence, are developed. The flow is no longer streamline, and control surfaces on the airplane no longer respond in a normal fashion.

The term *ultrasonic* is used to describe sound waves of frequencies greater than the human ear can hear, generally greater than about 20,000 cycles/sec. Such frequencies may be produced by a vibrating quartz crystal plate of appropriate size and shape. The plate is excited electrically through the *piezoelectric effect*, in which a dimensional change in the quartz is produced by electrical excitation. Ultrasonic frequencies may also be generated by the vibrations of a rod of magnetic material which is excited magnetically by *magnetostriction*, in which a change of dimension is produced magnetically.

Sound waves of ultrasonic frequencies have been found to disperse colloids and to produce various biological effects, such as the destruction of microorganisms, and have been used to sterilize and homogenize milk. Because of their short wavelength, such waves tend to move more nearly in a straight-line path and may cast sound shadows and be reflected from small obstacles. Thus ultrasonic waves have been used to detect flaws of suitable dimensions in opaque objects. Sound is reflected from cracks and other discontinuities, and their presence may be readily detected.

By listening for the echo, bats determine the presence of obstacles and insects by the reflection of ultrasonic sound which they emit. In order to obtain good reflections, the wavelength of the sound must not be appreciably greater than the dimensions of the obstacle. Bats feed upon beetles and moths and can sense the presence of ropes strung in a cave. They do not detect gnats and smaller insects and will fly into fine wires strung across a cave, for the dimensions of these objects are appreciably smaller than the wavelength of the ultrasonic wave the bat emits. The highest frequency a bat can utter is about 100,000 cycles per second, corresponding to a wavelength of about 0.13 in.

Problems

[NOTE: The speed of sound in air at room temperature may be taken as 1,100 ft/sec or as 330 m/sec.]

21-1. A siren wheel has 20 uniformly spaced holes near its rim and is rotated by means of a stream of air. (a) What is the frequency of the sound emitted when its speed is 44 rps? (b) What is the wavelength of the sound wave in air?

21-2. An air column closed at one end is in resonance with a sound wave whose frequency is 128 vib/sec. Determine the length of this air column.

21-3. What is the lowest-frequency note that can be produced by an organ pipe 12 ft. long which is closed at one end?

21-4. A metal tube 4 ft long has a piston placed in it near one end; the position of this piston is adjustable. A vibrating tuning fork whose frequency of vibration is 440 vib/sec is held near the open end of the tube. At what distances from the open end must the piston be placed to produce resonance?

21-5. A siren on a fire-engine truck emits a sound whose frequency is 1,000 vib/sec. What will be the frequency of the tone heard by a spectator (a) when the truck is moving toward him with a speed of 45 mi/hr and (b) when it is moving away from him with a speed of 45 mi/hr?

21-6. A fire siren in a village is emitting a sound whose frequency is 880 vib/sec. What is the frequency of the tone heard by the firemen approaching this source at a speed of 60 mi/hr?

21-7. A loud-speaker is hung from a cord attached to the ceiling of a lecture room and is connected by flexible wire to a 1,000-cycle oscillator. While emitting this note, the loud-speaker is swung toward the front wall with a speed of 3 ft/sec. (a) What is the frequency of the tone coming from the loud-speaker as heard by a student sitting in the rear of the room? (b) What is the frequency of the tone coming from the image of this loud-speaker as heard by this student? (c) How many beats per second will this student hear?

21-8. A string 100 cm long has a linear density of 0.04 gm/cm. When vibrating transversely with a node at each end, it has a frequency of 200 vib/sec. Determine (a) the speed of the wave in the string and (b) the tension in the string.

21-9. Two tuning forks are vibrating simultaneously, one with a frequency of 512 vib/sec and the other with a frequency of 516 vib/sec. How many beats are produced?

21-10. Two tuning forks A and B are observed to produce beats at the rate of 5 per second. Fork A has a frequency of 440 vib/sec. If fork B is loaded with a bit of putty, the number of beats increases to 8 per second. What is the frequency of vibration of fork B when it is not loaded?

21-11. An open organ pipe sounds the note A whose frequency is 440 vib/sec. (a) What is the length of the air column? (b) What are the frequencies of the first and second overtones produced by this air column?

21-12. A closed organ pipe sounds the note A whose frequency is 440 vib/sec. (a) What is the length of the air column? (b) What are the frequencies of the first and second overtones produced by this air column?

21-13. A steel wire 80 cm long is fastened at the ends. The mass of the wire is 2.5 gm. When plucked, it emits a tone whose fundamental frequency is 520 vib/sec. (a) Determine the tension in the wire. (b) What are the frequencies of the first and second overtones of this string?

21-14. A steel wire 3 ft long is rigidly fastened to two posts. The wire weighs 0.02 lb. When bowed, it emits a tone whose fundamental frequency is 110 vib/sec. (a) Determine the tension in the wire. (b) What are the frequencies of the first and second overtones produced by this string?

21-15. A bullet is fired with a velocity of 2,700 ft/sec. Determine (a) its Mach number and (b) the angle that the shock wave makes with the line of motion of the bullet.

21-16. A siren mounted on a car emits a note whose frequency is 500 vib/sec. (a) Determine the frequency of the sound heard by a stationary observer when the car approaches him with a speed of 70 ft/sec. (b) Determine the frequency heard by an observer moving toward the car with a speed of 70 ft/sec while the car remains stationary.

21-17. Assuming that the ear can detect sounds whose frequencies range from 20 vib/sec to 20,000 vib/sec, determine the range of wavelengths that the ear can detect.