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Physics, Chapter 23: The Electric Field

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The Electric Field

23-1 The Electric Field

We have previously described the gravitational field as one way of thinking about gravitational forces (Section 6-16). If an object of mass m at rest at a point P experienced a force, we could attribute that force to the presence of the gravitational field. In a similar way we may attribute the force experienced by an electric charge at rest at a point P to the presence of an *electric field* at that point. From the preceding chapter we recognize that the existence of a force on a charged particle is due to the presence of other charged particles in the vicinity, but for many purposes it is unnecessary to have a precise knowledge of the positions of these charges or their magnitudes. In the same way we have utilized a knowledge of the gravitational field intensity g to analyze the trajectories of projectiles without detailed knowledge of the mass distribution which gave rise to g.

We can use a very small body containing a small charge q as a means of exploring the electric field in any region of space. If the charge q experiences a force $\mathbf{F}$ at a given point, the *electric field intensity* $\mathbf{E}$ at this point is defined by the equation

$$\boxed{\mathbf{E} = \frac{\mathbf{F}}{q}.} \quad \textbf{(23-1)}$$

The electric field intensity at a point in an electric field is the force per uni- charge at this point. The test charge or probe charge q should be sufficiently small so that it will not change the distribution of charge that gives rise to the field to be measured. Ideally, one can take smaller and smaller test charges q, measure the force on each test charge at the given point, and take the limit of the ratio of F/q as q gets smaller as the electric field intensity at the point.

The electric field intensity is a *vector* quantity; it is the result of dividing force, a vector, by charge, a scalar. Equation (23-1) is analogous to Equation (6-18), which has been used to define the gravitational field. In using Equation (23-1) we must remember to substitute the sign of q as well as its numerical value to get the correct directional relationship between the vector quantities **E** and **F**. The direction of the electric field is opposite to the direction of the force on a negatively charged particle. The units of electric field intensity in the mks system may be expressed as *newtons per coulomb*.

As we have seen in Chapter 22, the equations relating electrical quantities take on different appearances in the cgs and mks systems of units.

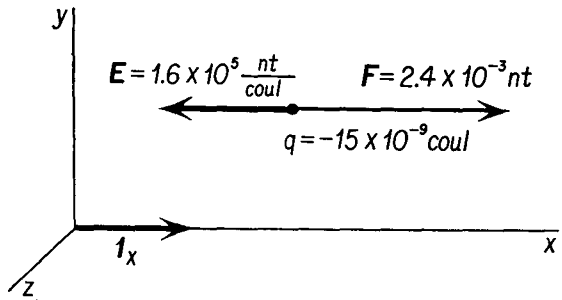

Fig. 23-1 The electric field intensity is opposite in direction to the force on a negatively charged particle.

In order to avoid complication in the body of the chapter, the equations in this and subsequent chapters will be developed using mks units. Discussions relating the two systems of units will be in smaller type to distinguish them from the principal development. The principal equations of the chapter are repeated in both systems of units in Table 23-1, and the relationships between the units of the two systems are stated in Table 23-2, at the end of the chapter.

In defining the intensity of the electric field, we must emphasize that the test body used to probe the field is *at rest*. We shall see in a subsequent chapter that a moving charged particle may experience a force *proportional to its speed*. Such a force is due to the presence of a *magnetic* field. The magnetic field does not exert a force on a charged particle at rest. The electric field may therefore be defined by the force on a *stationary* particle. Once the electric field intensity is known, it may be used to compute the force on a charged particle without regard to whether the particle is at rest or in motion. The force on a charged particle at a given point due to the electric field is determined from Equation (23-1) as

$$\mathbf{F} = \mathbf{E}q \tag{23-1a}$$

and is independent of the speed of the particle.

Illustrative Example. A small particle having a charge of -15×10^{-9} coulomb experiences a force of 2.4×10^{-3} nt in the positive x direction, as shown in Figure 23-1. Find the electric field intensity at that point.

The magnitude of the electric field intensity is

$$E = \frac{F}{q} = \frac{2.4 \times 10^{-3}\ \text{nt}}{15 \times 10^{-9}\ \text{coul}}$$

$$= 1.6 \times 10^{5}\ \frac{\text{nt}}{\text{coul}}.$$

The electric field is in the negative x direction, opposite to the direction of the force on a negatively charged particle. If we write $\mathbf{1}_x$ for a unit vector in the positive x direction, the electric field intensity may be written as

$$\mathbf{E} = -1.6 \times 10^{5} \times \mathbf{1}_x\ \frac{\text{nt}}{\text{coul}}.$$

23-2 Electric Charge as the Source of Electric Field

In the mks system of units, the vector form of Coulomb's law may be written as

$$\mathbf{F}_2 = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\,\mathbf{1}_r.$$

We may consider the charge q_2 as a probe charge used to explore the electric field. The electric field intensity $\mathbf{E}$ at the point where the charge q_2 is located may be found by dividing both sides of the above equation by q_2. Thus

$$\mathbf{E} = \frac{\mathbf{F}_2}{q_2} = \frac{q_1}{4\pi\epsilon_0 r^2}\,\mathbf{1}_r,$$

and, dropping the subscript 1 from q_1, the electric field intensity at a distance r from a point charge q, in vacuum, is given by the equation

$$\boxed{\mathbf{E} = \frac{q}{4\pi\epsilon_0 r^2}\,\mathbf{1}_r.} \qquad \textbf{(23-2)}$$

The unit vector $\mathbf{1}_r$ is directed from the charge q which generates the field to the point P where we imagine the test charge to be located, as shown in Figure 23-2.

Illustrative Example. A point charge of $-10\ \mu$coul is located at the origin. Find the electric field intensity at a point in the x-y plane whose coordinates are (3 m, 4 m).

The location of the charge and the field point P are shown in Figure 23-3.

The unit vector $\mathbf{1}_r$ is shown directed from the charge q at the origin to the point P. Substituting into Equation (23-2), we find

$$\mathbf{E} = \frac{-10 \times 10^{-6}\ \text{coul}}{4\pi \times 8.85 \times 10^{-12}\,\dfrac{\text{coul}^2}{\text{nt m}^2} \times 25\ \text{m}^2} \times \mathbf{1}_r,$$

$$\mathbf{E} = -3.6 \times 10^3\,\frac{\text{nt}}{\text{coul}} \times \mathbf{1}_r.$$

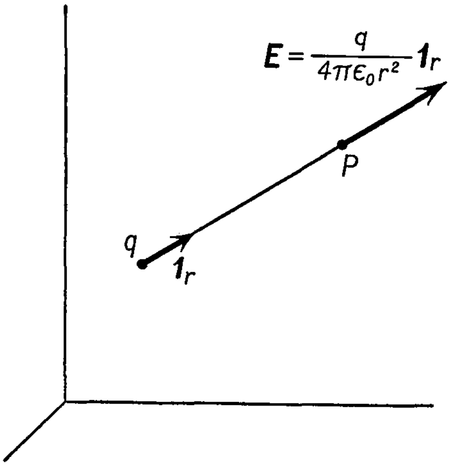

Fig. 23-2

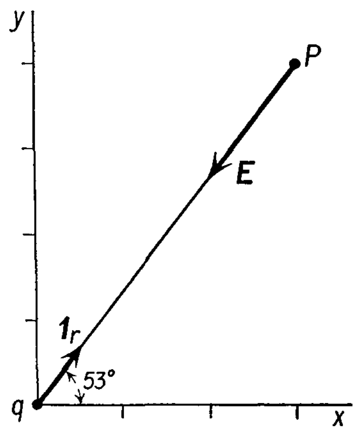

Fig. 23-3

Thus the magnitude of $\mathbf{E}$ is given by

$$E = 3.6 \times 10^3\,\frac{\text{nt}}{\text{coul}},$$

and its direction is in the direction of $-\mathbf{1}_r$, that is, toward the origin along a line making an angle of 53° with the positive x axis, as shown in the figure.

23-3 Electric Field Due to a Collection of Point Charges

A probe charge q placed at a point P in the neighborhood of any number of point charges $q_1, q_2, q_3, \cdots$ will experience a force which is the vector sum of the forces produced by the individual charges on it. Thus

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots,$$

where $\mathbf{F}$ is the resultant force on charge q and $\mathbf{F}_1$ is the force on it produced by the charge q_1, $\mathbf{F}_2$ the force produced by charge q_2, and so forth.

Since
$$\mathbf{E} = \frac{\mathbf{F}}{q} = \frac{\mathbf{F}_1}{q} + \frac{\mathbf{F}_2}{q} + \frac{\mathbf{F}_3}{q} + \cdots,$$

therefore

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \cdots. \tag{23-3}$$

Hence the electric field intensity at any point P produced by a set of point charges in its neighborhood is the vector sum of the electric field intensities produced by the individual charges at the same point.

Illustrative Example. Two point charges $q_1 = 5$ μcoul and $q_2 = -5$ μcoul are separated by a distance of 0.08 m, as shown in Figure 23-4. Find the electric field intensity (a) at point a located on the line joining the two charges at a dis-

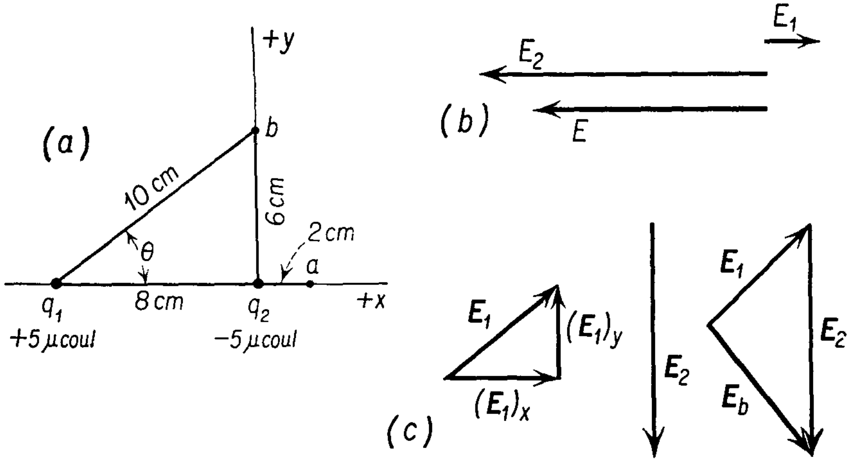

Fig. 23-4 (a) Location of charges and field points. (b) The fields E_1 due to charge q_1 and E_2 due to charge q_2 at the point a. (c) The field at b due to q_1, to q_2, and their resultant field E_b.

tance 10 cm from q_1 and 2 cm from q_2 and (b) at point b located at the vertex of a right triangle at a distance of 10 cm from q_1 and 6 cm from q_2.

(a) The electric field at a due to q_1 is

$$\mathbf{E}_1 = \frac{5 \times 10^{-6}\text{ coul}}{4\pi \times 8.85 \times 10^{-12}\dfrac{\text{coul}^2}{\text{nt m}^2} \times (0.10\text{ m})^2}\,\mathbf{1}_x$$

$$= 4.5 \times 10^6 \times \mathbf{1}_x\frac{\text{nt}}{\text{coul}}.$$

The electric field at point a due to q_2 is

$$\mathbf{E}_2 = \frac{-5 \times 10^{-6}\text{ coul}}{4\pi \times 8.85 \times 10^{-12}\dfrac{\text{coul}^2}{\text{nt m}^2} \times (0.02\text{ m})^2}\,\mathbf{1}_x$$

$$= -112.5 \times 10^6 \times \mathbf{1}_x\frac{\text{nt}}{\text{coul}}.$$

The resultant electric field is therefore

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$
$$= (4.5 - 112.5) \times 10^6 \times \mathbf{1}_x \frac{\text{nt}}{\text{coul}},$$
$$\mathbf{E} = -108 \times 10^6 \times \mathbf{1}_x \frac{\text{nt}}{\text{coul}}.$$

(b) The electric field at b due to charge q_1 is

$$E_1 = \frac{5 \times 10^{-6}\ \text{coul}}{4\pi \times 8.85 \times 10^{-12} \dfrac{\text{coul}^2}{\text{nt m}^2} \times (0.10\ \text{m})^2}$$
$$= 4.5 \times 10^6 \frac{\text{nt}}{\text{coul}},$$

and the x and y components of this field are

$$(E_1)_x = E_1 \times \cos\theta = 4.5 \times 10^6 \frac{\text{nt}}{\text{coul}} \times \tfrac{8}{10};$$
$$(E_1)_x = 3.6 \times 10^6 \frac{\text{nt}}{\text{coul}}.$$
$$(E_1)_y = E_1 \times \sin\theta = 4.5 \times 10^6 \frac{\text{nt}}{\text{coul}} \times \tfrac{6}{10};$$
$$(E_1)_y = 2.7 \times 10^6 \frac{\text{nt}}{\text{coul}}.$$

The electric intensity at b due to q_2 is

$$\mathbf{E}_2 = \frac{-5 \times 10^{-6}\ \text{coul}}{4\pi \times 8.85 \times 10^{-12} \dfrac{\text{coul}^2}{\text{nt m}^2} \times (0.06\ \text{m})^2} \mathbf{1}_y;$$
$$\mathbf{E}_2 = -12.5 \times 10^6 \times \mathbf{1}_y \frac{\text{nt}}{\text{coul}}.$$

The components of the resultant electric intensity are therefore given by

$$E_x = (E_1)_x = 3.6 \times 10^6 \frac{\text{nt}}{\text{coul}}.$$
$$E_y = (E_1)_y + (E_2)_y = (2.7 - 12.5) \times 10^6 \times \frac{\text{nt}}{\text{coul}};$$
$$E_y = -9.8 \times 10^6 \frac{\text{nt}}{\text{coul}}.$$

The resultant field may be expressed as the vector sum of its x and y components as

$$\mathbf{E} = (3.6 \times 10^6 \times \mathbf{1}_x - 9.8 \times 10^6 \times \mathbf{1}_y) \frac{\text{nt}}{\text{coul}}.$$

23-4 Electric Field Due to a Continuous-Charge Distribution

When the electric field is established by a continuous distribution of charge rather than by a collection of point charges, we may compute the electric field intensity by imagining the charge distribution to be cut up into small volume elements in which the entire charge of an element is considered to be concentrated at some point of the volume element, say its center. The electric intensity may then be computed by applying Equation (23-3) to this collection of charges. More generally, we apply the methods of the calculus and replace the sum by an integral in the limit of an extremely fine subdivision.

If $d\mathbf{E}$ is the contribution to the electric intensity at the point P from a volume element whose charge is dq, located at a distance r from the point P, we may write

$$\mathbf{E} = \int d\mathbf{E} = \int \frac{dq}{4\pi\epsilon_0 r^2}\,\mathbf{1}_r, \tag{23-4}$$

where $\mathbf{1}_r$ is the unit vector directed from the element of charge dq to the point P. Let us define the *charge density* ρ of a continuous-charge distribution as the quantity of charge per unit volume. The charge dq in a volume element dv is then given by

$$dq = \rho\, dv,$$

and Equation (23-4) becomes

$$\mathbf{E} = \int \frac{\rho\, dv}{4\pi\epsilon_0 r^2}\,\mathbf{1}_r. \tag{23-4a}$$

In Equations (23-4) the integration must be carried out over the entire charge distribution. These equations are useful for symbolic purposes only. In order to carry out the integration, it is necessary to replace them by equations which yield the components of the electric intensity at the point P. Thus

$$E_x = \int dE_x,$$

$$E_y = \int dE_y,$$

and so on.

Illustrative Example. Calculate the electric intensity at a point on the axis of a uniformly charged narrow ring of charge.

Let us locate the ring, of radius a, in the x-y plane. The axis of the ring is along the z coordinate axis. Let the charge of the ring be q. Its linear charge density is therefore $\dfrac{q}{2\pi a}$. The electric field intensity dE, contributed by an element of the ring to a point P located at $(0, 0, z)$, is shown in Figure 23-5. The element subtends an angle $d\theta$ and has a charge dq given by

$$dq = a\,d\theta \cdot \frac{q}{2\pi a} = \frac{q\,d\theta}{2\pi}.$$

The intensity dE is given by

$$dE = \frac{\dfrac{q\,d\theta}{2\pi}}{4\pi\epsilon_0(a^2 + z^2)}.$$

Since the electric intensity contributed by each element of the ring is in a different direction in space, we cannot integrate without first finding components along the coordinate axes. The z component of dE is given by

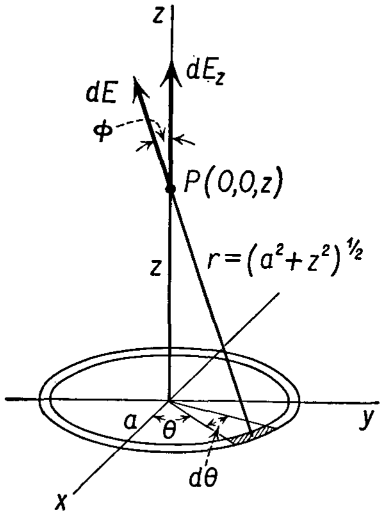

Fig. 23-5

$$dE_z = dE\cos\phi = dE\,\frac{z}{(a^2 + z^2)^{1/2}}$$

$$= \frac{1}{4\pi\epsilon_0}\,\frac{q}{2\pi}\,\frac{z}{(a^2 + z^2)^{3/2}}\,d\theta.$$

Since a, the radius of the ring, and z, the coordinate of the field point, are fixed, we may integrate over the entire region of charge by integrating $d\theta$ over the range 0 to 2π to obtain

$$E_z = \frac{q}{4\pi\epsilon_0}\,\frac{z}{(a^2 + z^2)^{3/2}}.$$

Because of the symmetry of the figure with respect to the z axis, the components of the electric intensity in a direction perpendicular to the z axis at the point P must sum to zero. For every element of the ring which contributes a component to the electric intensity parallel to the x-y plane, there is an equal element across the diameter of the ring which contributes a component in the opposite direction. Thus the electric intensity at all points on the axis of the ring is parallel to the axis of the ring and is given by the above formula.

In making the computation, we have followed the procedure outlined in the preceding paragraph, by integrating the components of the electric intensity vector. This must be done in integrating any vector quantity, for the reason that the process of integration is essentially the process of computing the limit of an algebraic sum.

23-5 Lines of Force

To visualize the direction and magnitude of the electric field in space, it is convenient to make use of the concept of *lines of force*, which was first introduced in Section 6-16 to represent the gravitational field. *The electric lines of force are drawn so that a tangent to the line at any point will give the direction of the electric field intensity at that point. The magnitude of the electric intensity is given by the number of lines passing perpendicularly through a unit area centered at the point,* as shown in Figure 23-6. No two lines of force may cross each other, for this would infer that the force on a unit positive charge had two directions at the point of crossing.

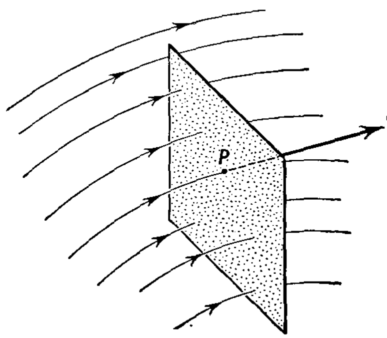

Fig. 23-6 Representation of the electric field intensity at P by the number of electric lines of force passing perpendicularly through a unit area at P.

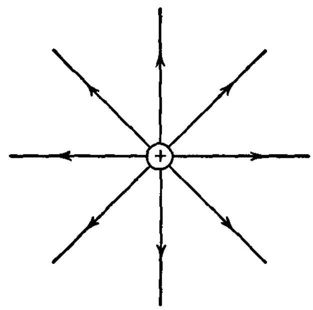

Fig. 23-7 The electric field around a small positive charge is radial and directed away from the charge.

Let us consider the appearance of the lines of force about a positive charge. Since the electric field is directed radially away from a positive charge, as shown in Figure 23-7, the lines of force are also directed away from the positive charge. The lines of force surrounding a negative charge are directed radially toward the negative charge. Thus, in vacuum, a line of force begins on positive charge and terminates on negative charge. According to Equation (23-2), the electric intensity about a charged body in vacuum is

$$E = \frac{q}{4\pi\epsilon_0 r^2}.$$

The total number of lines of force passing through a sphere of radius r concentric with q is N, the product of the electric intensity, or the number of lines per unit area, by the area of the sphere, or

$$N = 4\pi r^2 E,$$

so that

$$\boxed{N = \frac{q}{\epsilon_0}.} \qquad \textbf{(23-5)}$$

Thus the total number of lines of force radiating from a positively charged body in vacuum is given by the charge of the body divided by ϵ_0. The number of lines of force terminating on a negatively charged body is given by the same quotient.

23-6 Gauss's Theorem

The lines of force radiating from a positive charge must terminate on an equal and opposite negative charge somewhere in the universe. In dealing with an isolated charged body, as shown in Figure 23-7, we think of the lines of force as terminating on negative charges at infinity. Suppose we enclose a region of charge-free space with a sphere, or with some other simple closed surface which can be reshaped into a sphere. Let us count the total number of lines of force coming out of the surface, tallying those leaving the surface as positive, and those entering the surface as negative. Since there is no charge within the closed surface, no lines of force originate or terminate within the surface. Any line of force which enters the surface at one point must leave the surface at some other point. There can be no net lines of force leaving or entering a volume that does not enclose a charge; that is, there are as many lines entering the closed surface as there are leaving it.

If there are several charges or a charge distribution within the closed surface, each coulomb of positive charge generates $1/\epsilon_0$ lines of force, and the same number of lines of force must terminate on each coulomb of negative charge. The net number N of lines of force leaving the surface must be given by

$$N = \frac{\sum q}{\epsilon_0}. \tag{23-5a}$$

To determine the relationship between the electric intensity and the number of lines of force, let us consider an element of area of a closed surface of magnitude ΔA. If the lines of force are perpendicular to this surface element, the electric intensity E is given by the number of lines ΔN leaving the volume through this surface element, divided by the area ΔA, in accordance with the convention we have chosen for representing the electric field by lines of force. In the form of an equation

$$E = \frac{\Delta N}{\Delta A}.$$

Now suppose the surface makes some angle with the lines of force, as shown in Figure 23-8. If the outward drawn normal to the surface makes an angle θ with the lines of force, the component of the element of area which is perpendicular to the lines of force is given by $\Delta A \cos\theta$, so that

$$E = \frac{\Delta N}{\Delta A \cos\theta},$$

or

$$N = E\,\Delta A \cos\theta.$$

If we consider $\Delta\mathbf{A}$ as a vector quantity, as indicated in Section 8-2, whose magnitude is given by the area ΔA and whose direction is given by

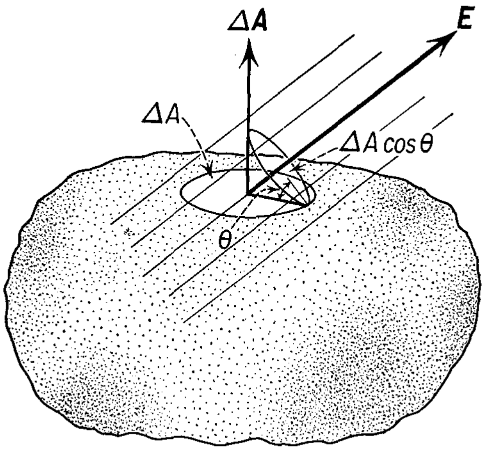

Fig. 23-8

the outward drawn normal to the surface, we can apply the definition of a scalar product given in Section 7-3, to write

$$\Delta N = \mathbf{E}\cdot\Delta\mathbf{A}.$$

To find the total number of lines of force leaving the surface, we imagine that the element of area $\Delta\mathbf{A}$ becomes very small, and in the limit we replace the symbol Δ by the symbol d, and integrate dN over the entire surface. This process may be represented symbolically as

$$N = \int dN = \int \mathbf{E}\cdot d\mathbf{A}. \tag{23-6}$$

On substituting for N its value in terms of the charge contained within the closed surface, we find

$$\boxed{\int \mathbf{E}\cdot d\mathbf{A} = \sum \frac{q}{\epsilon_0}\,.} \tag{23-7}$$

Equation (23-7) is a very important equation in electrostatics, and is known as *Gauss's theorem.* The theorem states that the *integral of the normal component of the electric field intensity over a closed surface is equal to the algebraic sum of the charge contained within that surface divided by* ϵ_0.

Gauss's theorem is of great usefulness in computing the electric field intensity E of symmetric charge distributions, in which it is possible to have some knowledge about the symmetry of E itself from observing the

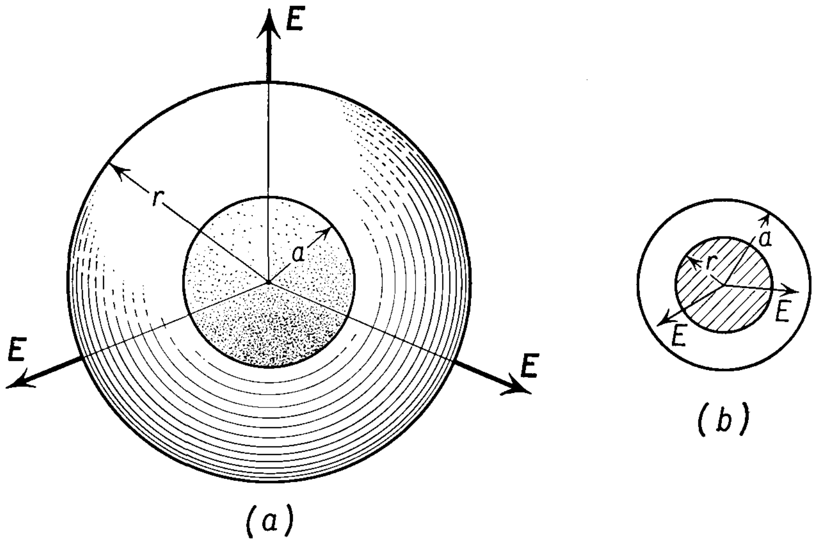

Fig. 23-9 Gaussian sphere of radius r concentric with charged sphere of radius a. (a) Gaussian sphere outside the charged sphere; (b) Gaussian sphere inside the charged sphere.

symmetry of the charge distribution. Let us consider the electric intensity associated with a sphere having a charge q distributed uniformly throughout its volume. To compute the electric intensity by subdividing the sphere into small volume elements and integrating, in the manner indicated in Section 23-4, is a tedious job. It is far simpler to observe that the value of E must be the same on all points of a second spherical surface, concentric with the first, because of the symmetry of the charge distribution. Let us draw such a surface of radius r, called a *Gaussian surface,* as shown in Figure 23-9. The radius of the uniformly charged sphere is a.

Since the direction of **E** is everywhere radial, the angle made by **E** with the normal to the surface of the Gaussian sphere is everywhere 0°. The quantity $\mathbf{E}\cdot d\mathbf{A}$ in Equation (23-7) reduces to the product of the magnitudes of these two quantities, and we have

$$\int E\, dA = \frac{q}{\epsilon_0}.$$

Since E is everywhere constant over the surface of the Gaussian sphere, the quantity E may be taken outside the integral. The integral then represents the surface area of the Gaussian sphere, $4\pi r^2$. Thus we have

$$4\pi r^2 E = \frac{q}{\epsilon_0},$$

which leads to

$$E = \frac{q}{4\pi\epsilon_0 r^2} \tag{23-8}$$

for the magnitude of the electric field, whose direction is specified by the fact that it is everywhere radial. Notice that *the electric intensity everywhere outside a uniformly charged sphere is as though the charge were all concentrated at the center of the sphere.* Since the law of universal gravitation is of the same form as Coulomb's law, the above result is also valid for the gravitational field about a uniform spherical mass distribution. It was for this reason that we could treat the earth as though its mass were concentrated at its center. The same result obviously applies to a uniformly charged spherical shell and, in fact, to any distribution of charge having spherical symmetry. Thus any distribution of charge having spherical symmetry generates an electric field outside the charge distribution which is as though the entire charge were concentrated at the center of the sphere.

If we wish to find the electric intensity inside the charge distribution we draw a Gaussian sphere of radius $r < a$ concentric with the charged sphere, and again we observe that the electric field intensity must be radially directed and must be of equal magnitude at all points of the Gaussian sphere, from considerations of symmetry. Applying Gauss's theorem, we find

$$4\pi r^2 E = \frac{1}{\epsilon_0}\left(\frac{4}{3}\pi r^3 \rho\right),$$

where ρ is the density of charge within the sphere and is given by

$$\rho = \frac{q}{\frac{4}{3}\pi a^3} = \frac{3q}{4\pi a^3}.$$

Thus we have

$$E = \frac{\rho r}{3\epsilon_0} = \frac{qr}{4\pi a^3 \epsilon_0}. \tag{23-9}$$

Comparing the results of Equations (23-8) and (23-9), we see that both equations lead to the same result at the surface of the charged sphere where

$r = a$, namely

$$E = \frac{q}{4\pi\epsilon_0 a^2}. \tag{23-10}$$

The results of Equations (23-8) and (23-9) have been plotted in Figure 23-10. The electric intensity has a maximum value at the surface of the charged sphere and diminishes to zero at the center of the sphere and at infinity. These results are of some interest in practical problems, as, for example, in the electrical effects associated with the flight of an airplane

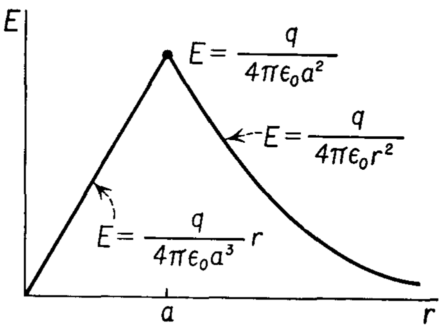

Fig. 23-10 Electric field intensity of a uniformly charged sphere of radius a as a function of the distance r from the center of the sphere.

through a thundercloud. These results are also of interest in atomic physics, where the electric intensity at the position of an outer-valence electron is made up of the field contributed by the central nucleus, which follows Coulomb's law, and the field of the inner electrons, which may be approximated by a uniformly charged sphere. The field experienced by the outer electron no longer follows an inverse square law when this electron penetrates the charged cloud, and this, in turn, serves to determine some important properties of atoms.

23-7 Conductors

By the term *electrostatics* we mean the study of the properties of electric charge *at rest.* Within the framework of electrostatics, there can be no electric field within a conductor, for by the word *conductor* we imply that electric charge is free to move. If there were an electric field within a conductor, there would be a force on the free electrons of that conductor; these electrons would be accelerated; hence they would not be at rest. Thus, simply as a matter of the consistency of our definitions, we must conclude that, for distributions of charge which are in static equilibrium, the electric intensity within a conductor is zero. In making such a statement we are speaking of an idealized conductor, for any material object is made up of nuclei and electrons which are held together by electric forces. There are intense electric fields on a submicroscopic level of examination. In the present approximation of a conductor, we imagine the conductor to be made up of a completely homogeneous material which may be subdivided into infinitesimal parts without altering its properties. This approximation has been found experimentally valid as long as the smallest subdivision we permit ourselves to examine is one which contains hundreds of atoms. For practical purposes this is still a very small volume element.

Since the electric field within a conductor is zero, we know from Gauss's theorem that the electric charge within any portion of a conductor must be zero. There must be an equal quantity of positive and negative charge within any subvolume of the conductor. Hence, *all the charge on a charged conductor must reside on its surface.* If a hole is made within the body of a conductor, the electric intensity within that hole must be zero.

Much electrical equipment is built so that all the working parts are contained within a metallic box, generally built of sheet copper or aluminum, and called an *electrostatic shield.* Electric charges outside that box cannot produce any electric field within the box, and consequently cannot affect the operation of equipment within the box. The box therefore provides a shield against outside electrical disturbances. This effect may be demonstrated by placing an electroscope in the vicinity of an electrostatic generator or a highly charged rod. If the rod is positively charged, some negative charge is attracted to the ball of the electroscope, with the result that the leaves become positively charged and repel each other, as shown in Figure 23-11(a). If a metallic cap is placed over the electroscope, the electroscope is shielded, and no matter how great the charge on the rod,

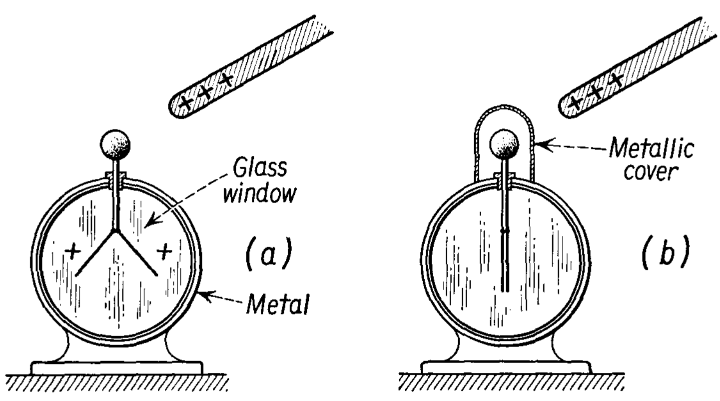

Fig. 23-11 (a) Leaves of an electroscope diverge when a charged rod is brought near it. (b) When an electroscope is shielded by a metallic cover, the charged rod does not affect the electroscope.

the leaves remain vertical, as shown in Figure 23-11(b). The metallic braid woven over the insulation of wires used in many electronic circuits is another illustration of the practical use of electrostatic shielding.

23-8 Field Outside the Surface of a Conductor

The electric field intensity immediately outside the surface of a conductor must be directed perpendicularly to the surface of the conductor. To understand this, we observe that if the electric field were oblique to the surface, it could be resolved into components parallel and perpendicular to the surface. Once again, our argument is based upon the definition of electrostatics. If there were a component of the electric intensity parallel to the conducting surface, the free electrons on the surface of the conductor would be accelerated and would no longer be at rest.

Let us suppose that there is an electric field in the vicinity of a conducting surface. Some of the lines of force associated with that field will go toward the conducting surface and terminate abruptly on it, perpendicular to the surface; other lines of force will originate from the surface and leave it perpendicularly. There are no lines within the conductor. We have already seen that lines of force begin or end on electric charges. Thus, if there is an electric field normal to the surface of the conductor, the con-

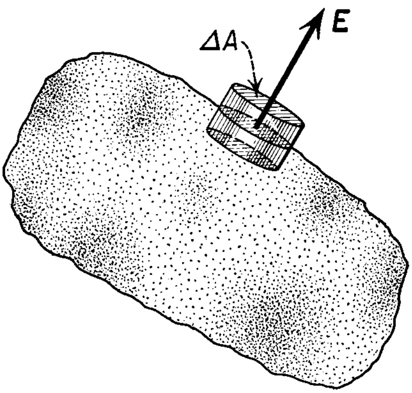

Fig. 23-12

ductor must be charged. We may apply Gauss's theorem to evaluate the relationship between the electric field at the surface of a conductor and the surface charge on that conductor.

Let us suppose that the electric charge on the surface of the conductor is of charge density σ units of positive charge per unit area. The electric field intensity normal to the surface of the conductor is E. To find the relationship between σ and E, we make use of a Gaussian surface in the shape of a *pillbox*, with cylindrical walls normal to the conducting surface, and plane top and bottom faces parallel to the conducting surface. One face of the pillbox is imagined to be within the conducting material, while the other face is just outside the conducting surface, as shown in Figure 23-12. If the area of the face of the pillbox is ΔA, the charge contained within the pillbox is $\sigma\,\Delta A$.

The electric intensity within the conductor is zero. Furthermore, since the electric field is normal to the surface, no lines of force pass through the cylindrical walls of the pillbox. In applying Gauss's theorem in the form of Equation (23-7), the only contribution to the integral of the normal component of E is obtained from the face of the pillbox outside the conductor. Thus we have

$$E\,\Delta A = \frac{\sigma\,\Delta A}{\epsilon_0},$$

and

$$E = \frac{\sigma}{\epsilon_0}. \tag{23-11}$$

If we represent a unit vector normal to the surface of the conductor and directed outward from the conductor as $\mathbf{1}_n$, the electric intensity at the surface of the conductor is related to the surface density of charge by the equation

$$\mathbf{E} = \frac{\sigma}{\epsilon_0}\mathbf{1}_n. \tag{23-11a}$$

Thus, if the charge on the surface of the conductor is positive, $\mathbf{E}$ is parallel to $\mathbf{1}_n$; hence $\mathbf{E}$ is outwardly directed. If the charge on the surface is negative, $\mathbf{E}$ is opposite in direction to $\mathbf{1}_n$ and is inwardly directed.

It is often desired to measure the electric field at the surface of a conductor, as, for example, at the belly of an airplane in flight or at the surface of the earth. One means of doing this is to measure the charge on a unit area of the surface of the conductor and to apply Equation (23-11) to determine the electric field intensity from the surface density of charge. A small section of the conducting surface of the airplane may be insulated from the remainder of the surface and may be periodically removed from the skin of the airplane, brought within the fuselage, and connected to an electroscope to measure its charge. More practically, if an electrically isolated segment of the airplane's skin is alternately covered and uncovered by a rotating conducting plate which is electrically connected to the skin, as shown in Figure 23-13, the isolated segment may be thought of as being first on the surface of the airplane, then within the conducting shell, and so on. It becomes electrically charged when it is on the surface and is discharged through a high resistance leading to the skin of the airplane when it is covered by the rotating conductor. In such an arrangement the charge which flows to and from the isolated segment through the resistor may be electronically amplified, and the electric field may be readily determined. From such measurements it is known that the average fair-weather electric field intensity at the surface of the earth is 100 nt/coul. The earth carries

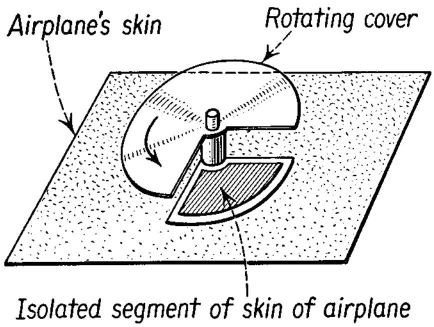

Fig. 23-13 Schematic illustration of electric field meter.

a negative charge whose total value is of the order of 500,000 coul, and whose surface density is about 0.0009 coul/km^2. The electric field intensity at the belly of an airplane in flight through a thunderstorm can be as much as 340,000 nt/coul just prior to a lightning strike.

23-9 Dielectric Strength

Let us suppose that a gas is placed in an electric field, and that one of the molecules of the gas has become ionized (or charged), say as a result of a collision with another rapidly moving molecule. The charged ion is accelerated by the electric field. The force acting upon the ion is the product of the electric field intensity E by the charge q of the ion. Suppose further that the charged particle moves an average distance l before making a collision with another molecule; l is called the *mean free path.* The work done on the particle by the electric field is given by the product Eql. This is the energy acquired by the charged particle in the interval between collisions. If the energy delivered to the ion by the field is sufficiently great so that the ion can disrupt a molecule with which it collides, at least two additional ions result from the collision process, each of which may be again accelerated by the field and may make subsequent collisions.

In this way a large number of ions may be created, and the gas then becomes electrically conducting. The electric field at which a gas becomes conducting is called the *dielectric strength* of the gas. Clearly, if there are no gas molecules present, as in vacuum, the dielectric strength is infinite, for there are no molecules present to become ionized. When only a very few molecules are present, as within a vacuum tube, an ionized particle may reach the electrodes of the tube without making a collision with another gas molecule, so that the total charge transferred is small. Again, when the pressure is quite high, the mean free path of the molecules is reduced, so that an ion cannot move far enough in the field to acquire the energy required to disrupt another molecule. The dielectric strength of the gas is high, for a large electric field intensity is required to generate secondary ionization.

The dielectric strength of air at atmospheric pressure is about 3×10^6 nt/coul. When this is exceeded, the air becomes conducting; corona dis-

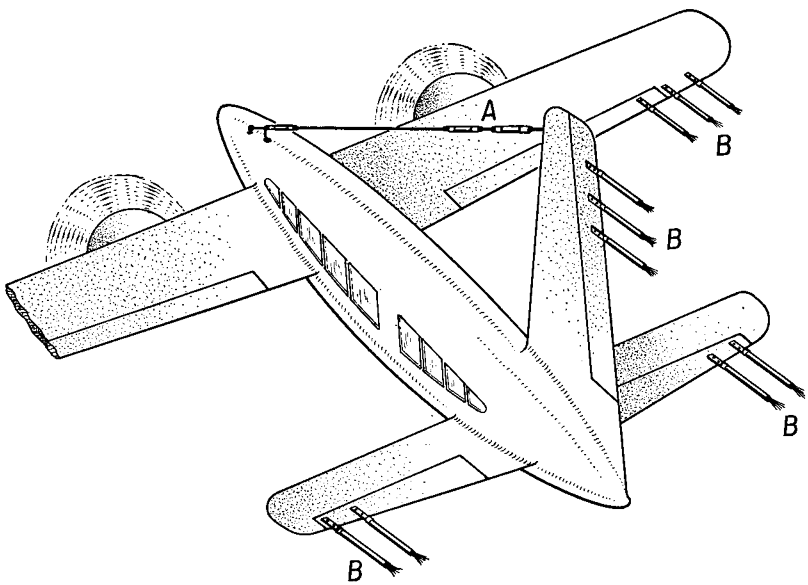

Fig. 23-14 Antistatic antennae A and static dischargers B on an airplane.

charge may be observed as a bluish glow in the region of an intense electric field, for the recombining ions give off some of their energy as light, and the sharp smell of ozone may be distinguished.

When an airplane becomes electrically charged in flight through precipitation, or when a charge is induced on the surface of an airplane when it flies near a thundercloud, corona discharge may take place from the propellers, from the wingtips, and from the radio antennae. The discharge from the antennae is particularly serious, for the erratic nature of the discharge generates radio noise called *precipitation static;* this often drowns out the signal from a radio station used by the pilot for communication and navigation purposes. Static dischargers, shown in Figure 23-14, have been placed upon the wingtips of many airplanes so that the electric charge accumulating on the airplane may be discharged noiselessly to the air. At the same time the wire radio antennae of the airplane have been insulated with an insulating material of high dielectric strength, and specially designed antenna fittings have been used to minimize the electric field intensity at the terminations of the wires so that the corona discharge does not take place from the antenna wire itself or from its associated fittings.

The dielectric strength of insulating materials is a technically important property of insulators. Their values range from about 10^6 to about 10^8 nt/coul. It is interesting that many solid insulating materials have a dielectric strength which is not appreciably greater than air. Solid insulating material in electrical apparatus is used generally as a spacer, to keep conductors from making contact with each other rather than to improve upon the insulating properties of atmospheric air.

TABLE 23-1 PRINCIPAL EQUATIONS IN THE MKS AND CGS UNITS

Equation	MKS	CGS	
(23-1)	$\mathbf{E} = \mathbf{F}/q$	Same as mks	Electric field
(23-2)	$\mathbf{E} = \frac{q}{4\pi\epsilon_0 r^2}\mathbf{1}_r$	$\mathbf{E} = \frac{q}{r^2}\mathbf{1}_r$	Field of a point charge
(23-5)	$N = \frac{q}{\epsilon_0}$	$N = 4\pi q$	Lines of force
(23-7)	$\int \mathbf{E}\cdot d\mathbf{A} = \sum \frac{q}{\epsilon_0}$	$\int \mathbf{E}\cdot d\mathbf{A} = 4\pi q$	Gauss's theorem
(23-11a)	$\mathbf{E} = \frac{\sigma}{\epsilon_0}\mathbf{1}_n$	$\mathbf{E} = 4\pi\sigma\mathbf{1}_n$	Field outside a conductor

TABLE 23-2 CONVERSION FACTORS RELATING MKS AND CGS UNITS

Quantity	Symbol	MKS Unit	CGS Unit
Charge	q	1 coul	$= 3 \times 10^9$ stcoul (esu)
Electric intensity	E	$1\ \frac{\text{nt}}{\text{coul}}$	$= \frac{1}{3 \times 10^4}\ \frac{\text{dyne}}{\text{stcoul}}$ (esu)
Force	F	1 nt	$= 10^5$ dyne

$$\text{Permittivity of free space: } \epsilon_0 = 8.85 \times 10^{-12}\,\frac{\text{coul}^2}{\text{nt m}^2},$$

$$\epsilon_0 = \frac{1}{36\pi \times 10^9}\,\frac{\text{coul}^2}{\text{nt m}^2}.$$

Problems

23-1. What is the intensity of the electric field at a distance of 20 cm from a small sphere charged to +1,600 stcoul?

23-2. What is the intensity of the electric field at a distance of 40 cm from a small body charged with −7,200 stcoul?

23-3. An electric charge of +15 stcoul is placed 25 cm from one of −40 stcoul. Determine the magnitude and the direction of the electric field intensity at the mid-point of a line joining them.

23-4. In Problem 23-3 determine the electric field intensity at a point in the first quadrant 20 cm from the +15-stcoul charge and 15 cm from the −40-stcoul charge. The line connecting the two charges is along the x axis, and the 15-stcoul charge is at the origin. State your answer in terms of x and y components of the electric field intensity.

23-5. Two equal charges each of +2,500 stcoul are placed 24 cm apart, along the y axis. Determine the electric field intensity at a point in the right-hand half plane 15 cm from each charge. State your answer in terms of the unit vectors $\mathbf{1}_x$ and $\mathbf{1}_y$.

23-6. The electric field intensity at a point P near a charge of 144 stcoul is 9 dynes/stcoul. Where must a charge of +324 stcoul be placed to reduce the field intensity at P to zero? Give the position of P relative to each charge.

23-7. Two small charged bodies are placed 25 cm apart along the x axis. One has a charge of +600 μcoul and the other has a charge of −1,800 μcoul. Find the electric field intensity at a point 60 cm from the positive charge and 65 cm from the negative charge. State your answer in terms of unit vectors along the x and y axes.

23-8. A small charge of +12 stcoul is placed in a uniform electric field of 300 dynes/stcoul. Determine the force on this charge.

23-9. A body having a mass of 0.01 gm and a charge of 1 μcoul is placed in a uniform electric field. What must be the magnitude and direction of the electric field intensity if the body is to remain at rest under the influence of the electric and gravitational fields.

23-10. How many lines of force emanate from a charge of +5 μcoul (a) in the mks system of units? (b) In the cgs system of units?

23-11. A metallic sphere is charged to 10 μcoul, the charge being uniformly distributed over the surface of the sphere. If the sphere is 1 m in diameter, find the electric field intensity (a) at the surface of the sphere and (b) at the center of the sphere.

23-12. A large metal sphere contains a surface charge of 2 μcoul/m^2. What is the electric field intensity at the surface of the sphere?

23-13. A thin spherical shell of charge of radius 1 m has a total charge of 1 coul. (a) What is the electric field intensity at the center of the shell? (b) What is the electric field intensity at a point 50 cm from the center of the shell? (c) What is the electric field intensity 2 m from the center of the shell?

23-14. Derive a formula for the electric field intensity at distance r from the center of a long uniformly charged cylinder of charge of radius a, of charge density ρ per unit volume (a) when r is less than a and (b) when r is greater than a. Check your results for these two cases by comparing them when r is equal to a. Use a Gaussian surface composed of a cylinder of radius r concentric with the charged cylinder and apply the consequences of cylindrical symmetry to Gauss's theorem.