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Energy Efficiency and Goodput Analysis in Two-Way Wireless Relay Networks

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Abstract—[1] **In this paper, we study two-way relay networks (TWRNs) in which two source nodes exchange their information via a relay node indirectly in Rayleigh fading channels. Both Amplify-and-Forward (AF) and Decode-and-Forward (DF) techniques have been analyzed in the TWRN employing a Markov chain model through which the network operation is described and investigated in depth. Automatic Repeat-reQuest (ARQ) retransmission has been applied to guarantee the successful packet delivery. The bit energy consumption and goodput expressions have been derived as functions of transmission rate in a given AF or DF TWRN. Numerical results are used to identify the optimal transmission rates of which the bit energy consumption is minimized or the goodput is maximized. The network performances are compared in terms of energy and transmission efficiency in AF and DF modes.**

I. INTRODUCTION

Recently, there has been much interest on two-way relay networks (TWRNs) in which two source nodes T_1 and T_2 without a direct link communicate with each other via a relay node. The architecture of TWRNs makes it possible to better exploit the channel multiplexing of uplink and downlink wireless medium [1]. The source nodes initially send their data to the relay node. The received data is combined employing a certain method according to the Amplify-and-Forward (AF) or the Decode-and-Forward (DF) mode and gets broadcasted from the relay back to both source nodes. With the application of network coding and channel estimation techniques [2], T_1 and T_2 can perform self-interference cancelation and remove their own transmitted codewords from the received signal. Four time slots needed in a traditional one-way transmission for the forward and backward channels to accomplish one-round information exchange between T_1 and T_2 via the relay node can be reduced to two in TWRNs by comparison.

In a realistic multi-user wireless network, e.g. IS-856 system [3] which has more relaxed delay requirements, the transmission power is fixed while the rate can be adapted according to the channel conditions. Moreover, Automatic Repeat reQuest (ARQ) techniques have been applied to improve the transmission reliability above the physical layer [4]. Prior works [5], [6] also show there is a compromise between transmission rate R and ARQ such that the network average successful throughput, i.e., the goodput, can be maximized at an optimal rate R^*. In addition to the goodput analysis, we in this paper are interested in energy-efficient operation. In such cases, the energy consumption due to retransmission should also be taken into account to evaluate the energy efficiency with respect to R. Hence, we investigate the joint optimization of R (at the physical layer) and the number of ARQ retransmissions (at the data-link layer) by adopting a cross-layer framework in TWRNs.

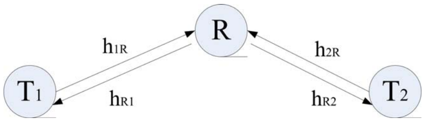

Fig. 1. Two-Way Relay Network

The remainder of this paper is organized as follows. Section II introduces the TWRN model, channel assumptions as well as its general working mechanism. In Sections III and IV, we investigate the Markov chain model under both AF and DF modes to derive the analytical expressions for the bit energy consumption and goodput. In Section V, the numerical results are shown to compare the system performance in AF and DF modes. Section VI provides the conclusion.

II. SYSTEM FORMULATION AND CHANNEL ASSUMPTIONS

In Figure 1, we depict a 3-node TWRN where source nodes T_1 and T_2 can only exchange information via the relay node. Codewords x_1 and x_2 from T_1 and T_2, respectively, have equal length and unit energy. All nodes are working in half-duplex mode and the channel fading coefficients between T_1 and the relay, and T_2 and the relay are modeled as complex Gaussian random variables with distributions $h_{1r} \sim \mathcal{CN}(0, \sigma_1^2)$ and $h_{2r} \sim \mathcal{CN}(0, \sigma_2^2)$. Without loss of generality, we also assume channel reciprocity such that h_{r1} and h_{r2} have identical distributions as h_{1r} and h_{2r}, respectively. Odd and even time slots have equal length, which is the time to transmit one codeword, and are dedicated for uplink and downlink data transmissions, respectively. ACK and NACK control packets are assumed to be always successfully received and the trivial processing time is ignored. Additive Gaussian noise at the receiver terminals is modeled as $n \sim \mathcal{CN}(0, \sigma_n^2)$.

There are two more key assumptions: 1) channel codes support communication at the instantaneous channel capacity

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levels, and outages, which occur if transmission rate exceeds the instantaneous channel capacity, lead to packet errors and are perfectly detected at the receivers; 2) depending on whether packets are successfully received or not, ACK or NACK control frames are sent and received with no errors. Based on above network formulations, we can further discuss the TWRN working procedure according to the current network states under AF and DF relay schemes, and find out the inherent impact of the transmission rate R on network performances.

III. AMPLIFY-AND-FORWARD TWRN

A. Network Model

The TWRN in AF mode can be visualized as two bi-directional cascade channels where in the odd time slots, T_1 and T_2 send individual codewords simultaneously to the relay and the signals are actually superimposed in the wireless medium. The relay will then amplify the received signals proportional to the average received power and broadcast the combined signals back to T_1 and T_2 in the even time slots.

According to Fig.1, the received signals at the relay in odd time slots is

$$y_r = \sqrt{P_1}h_{1r}x_1 + \sqrt{P_2}h_{2r}x_2 + n, \tag{1}$$

where P_1 and P_2 are the transmit power of T_1 and T_2 respectively. The relay will forward y_r with a scaling factor β which is

$$\beta = \frac{\sqrt{P_r}}{\sqrt{P_1\sigma_1^2 + P_2\sigma_2^2 + \sigma_n^2}}. \tag{2}$$

where P_r is the relay's transmit power.

Here, we normalize the variance of the channel between T_1 and T_2 as $\sigma^2 = 1$ and by using a normalized distance factor [7] $k = \frac{d_{T_1,Relay}}{d_{T_1,T_2}} \in (0,1)$ while assuming that the variances of the other links are proportional to $d^{-\alpha}$ where α is the path loss coefficient, we then have $\sigma_1^2 = k^{-\alpha}\sigma^2$ and $\sigma_2^2 = (1-k)^{-\alpha}\sigma^2$. At the end of even time slots, the received signals at T_1 and T_2 can be written as

$$\begin{aligned} y_1 &= \sqrt{P_1}\beta h_{r1}h_{1r}x_1 + \sqrt{P_2}\beta h_{r1}h_{2r}x_2 + \beta h_{r1}n + n_1 \\ y_2 &= \sqrt{P_1}\beta h_{r2}h_{1r}x_1 + \sqrt{P_2}\beta h_{r2}h_{2r}x_2 + \beta h_{r2}n + n_2, \end{aligned} \tag{3}$$

where n_1, n_2, and n are i.i.d Gaussian noise components. Assuming the instantaneous channel state information is perfectly known at T_1 and T_2, the self-interference part can be removed from y_1 and y_2 and the signals that will be used for decoding can be represented by

$$\begin{aligned} \widehat{y}_1 &= \sqrt{P_2}\beta h_{r1}h_{2r}x_2 + \beta h_{r1}n + n_1 \\ \widehat{y}_2 &= \sqrt{P_1}\beta h_{r2}h_{1r}x_1 + \beta h_{r2}n + n_2. \end{aligned} \tag{4}$$

The cascade channel instantaneous rate from T_1 to T_2 and from T_2 to T_1 are hence represented by

$$\begin{aligned} R_{12} &= \log\left(1 + \frac{|\beta h_{r2}h_{1r}|^2 P_1}{(1+|\beta h_{r2}|^2)\sigma_n^2}\right) \\ R_{21} &= \log\left(1 + \frac{|\beta h_{r1}h_{2r}|^2 P_2}{(1+|\beta h_{r1}|^2)\sigma_n^2}\right). \end{aligned} \tag{5}$$

To describe the network mechanism more accurately, we need to formulate the protocol of TWRN in AF mode as follows:

1) Each transmission round contains two consecutive time slots. In the odd slot, source nodes T_1 and T_2 both transmit codewords to the relay with transmission rate R bits/sec/Hz, and the relay in the following even slot broadcasts βy_r back to source nodes.
2) At the end of one transmission round, T_1 and T_2 perform self-interference cancelation (SIC) to subtract their own weighted messages, and decode $\widehat{y}_1$ and $\widehat{y}_2$, respectively. If the decoding fails, an outage event will be declared on that cascade link.
3) The outage event on the cascade link $T_1 - T_2$ (or $T_2 - T_1$) is defined as the probability of the event $R_{12} < R$ (or $R_{21} < R$). ACK or NACK packets would be sent back to the relay based on successful transmission or outage. The relay will also notify T_1 and T_2 whether a new codeword or an old codeword should be (re)transmitted in the next odd time slot with the control packet information.

The network state transition diagram of the AF TWRN can be modeled as a Markov chain as shown in Fig. 2, where the probability on each path denotes the probability of the transition between two states. p_{12} and p_{21} are defined as the outage probabilities on the cascade $T_1 - T_2$ and $T_2 - T_1$ links and are given by

$$p_{12} = \Pr(R_{12} < R) = \Pr\left(\frac{|h_{r2}h_{1r}|^2}{\frac{1}{\beta^2} + |h_{r2}^2|} < \frac{(2^R-1)\sigma_n^2}{P_1}\right) \tag{6}$$

$$p_{21} = \Pr(R_{21} < R) = \Pr\left(\frac{|h_{r1}h_{2r}|^2}{\frac{1}{\beta^2} + |h_{r1}^2|} < \frac{(2^R-1)\sigma_n^2}{P_2}\right). \tag{7}$$

(6) and (7) can be determined using the cumulative distribution function of the random variable X [8]

$$F_X(x) = 1 - \frac{1}{\mu_2}\int_0^\infty e^{-\frac{x(a+z)}{\mu_1 z} - \frac{z}{\mu_2}}dz \tag{8}$$

where

$$X = \frac{Y_1Y_2}{a + Y_2}, \tag{9}$$

and Y_1 and Y_2 are independent exponential random variables with mean μ_1 and μ_2, and a is a constant. In this context, we know

$$\begin{cases} \mu_1 = E[|h_{1r}|^2], \quad \mu_2 = E[|h_{r2}|^2], \quad for \quad p_{12} \\ \mu_1 = E[|h_{2r}|^2], \quad \mu_2 = E[|h_{r1}|^2], \quad for \quad p_{21} \\ \alpha = \frac{1}{\beta^2} \end{cases} \tag{10}$$

With the given network parameters, p_{12} and p_{21} can be derived accordingly.

B. Goodput Analysis

In Fig. 3, it is explicitly seen at the beginning of each odd time slot that both T_1 and T_2 transmit to the relay such that at the beginning of each even time slot, the relay is always in

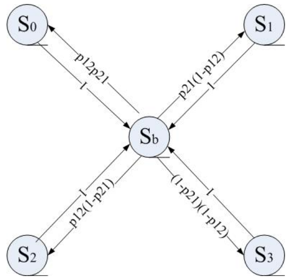

Fig. 2. Markov Chain of TWRN in AF

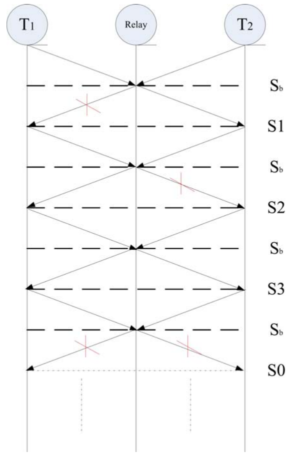

Fig. 3. State Transition of TWRN in AF

the ready-for-broadcasting state S_b. The relay will amplify and forward the signals to T_1 and T_2 in the even time slot and the state S_b will consequently transit to S_0, S_1, S_2, or S_3 with certain probabilities. Moving to S_0 means both transmissions to T_1 and T_2 have failed and moving to S_3 means both transmissions have succeeded. While moving to S_1 or S_2 means only the transmission to T_2 or T_1 is successful respectively. We first determine the following four equations according to the state transitions in the Markov chain to derive the probability of each state:

$$\begin{cases} p(S_0) = p(S_b)p_{12}p_{21} \\ p(S_1) = p(S_b)p_{21}(1-p_{12}) \\ p(S_2) = p(S_b)p_{12}(1-p_{21}) \\ p(S_3) = p(S_b)(1-p_{21})(1-p_{12}) \\ p(S_0)+p(S_1)+p(S_2)+p(S_3)+p(S_b) = 1. \end{cases} \tag{11}$$

After solving the set of equations in (11), we obtain $p(S_b) = \frac{1}{2}$. We know from the inherent characteristics of the AF TWRN that the data exchange only happens in the broadcasting phases with successful packet delivery. Therefore, the system goodput is defined similarly as in [1] by

$$\begin{aligned} &\eta_{AF} \\ &= p(S_b)R(2(1-p_{12})(1-p_{21}) + p_{12}(1-p_{21}) + p_{21}(1-p_{12})) \\ &= \frac{R(2-p_{12}-p_{21})}{2}. \end{aligned} \tag{12}$$

(12) indicates through the terms p_{12} and p_{21} that a higher transmission rate R will result in higher packet error rates (outage), leading to more ARQ retransmissions which equivalently reduce the data rate. Intuitively, a balance between transmission rate and the number of ARQ retransmissions needs to be found such that the goodput is maximized.

C. Average Bit Energy Consumption

Energy efficiency has always been a major concern in wireless networks. Recently, power or energy efficiency in wireless one-way relay networks have been extensively studied. In [9], the average bit energy consumption E_b is minimized by determining the optimal number of bits per symbol, i.e., the constellation size, in a specific modulation format. Similarly as in one-way relay channels, the outage probabilities in TWRNs are functions of the transmission rate R. For instance, there could be an increased number of outage events on the cascade channels when codewords are transmitted at a high rate. In such a case, more retransmissions and higher energy expenditure are needed to accomplish the reliable packet delivery. Therefore, we are interested in a possible realization of TWRN operation, which can provide a well-balanced performance on both the goodput η and the required energy. We evaluate this by formulating the average bit energy consumption E_b required for successfully exchanging one information bit between T_1 and T_2.

We then evaluate E_b by considering long-term transmissions on TWRN. Regardless of the previous state, whenever the relay is in state S_b and is broadcasting, the resulting state would be any of the other four states previously described. Assume there are K rounds of two-way transmission, each of which consists of a pair of consecutive time slots and each codeword has L bits. Therefore, with $\sum_{i=0}^{3} K_i = K$, where K_i is the number of transmission rounds corresponding to state S_i, the average bit energy consumption could be derived as the ratio of total energy consumption over total bits successfully exchanged:

$$\begin{aligned} E_b &= \lim_{K\to\infty} \frac{K(P_1+P_2+P_r)\frac{L}{R}}{K_3 2L + K_1 L + K_2 L} \\ &= \lim_{K\to\infty} \frac{(P_1+P_2+P_r)\frac{L}{R}}{\frac{K_3}{K}2L + \frac{K_1}{K}L + \frac{K_2}{K}L} \\ &= \frac{P_1+P_2+P_r}{(2-p_{12}-p_{21})R}. \end{aligned} \tag{13}$$

IV. DECODE-AND-FORWARD TWRN

A. Network Model

The DF TWRN differs from the AF TWRN in that there is a crucial intermediate decoding procedure at the the relay when it has received the codeword from the uplink transmission.

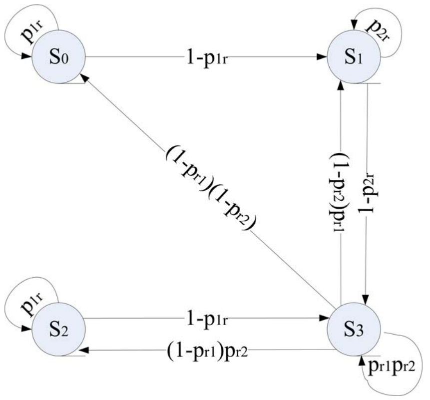

Fig. 4. Markov Chain of TWRN in DF

If both source nodes are allowed to send codewords to the relay simultaneously in the uplink transmission, the decoding at the relay has to deal with the multiple access problem in a realistic application, with successive interference cancelation techniques. To reduce the hardware complexity and increase the feasibility of implementation, we hereby adopt the DF TWRN mode from [1] where the relay performs sequential Decode-and-Forward. The outage probabilities on $T_1 - Relay$, $T_2 - Relay$, $Relay - T_1$, $Relay - T_2$ links are denoted as p_{1r}, p_{2r}, p_{r1} and p_{r2}. The protocol for sequential DF TWRN is described as follows:

1) In the initial state S_0, the relay's buffer is empty and the relay first polls on T_1 until it receives codeword x_1 successfully with probability $1 - p_{1r}$. Then, the state moves to S_1 which means the relay holds x_1 in the buffer. Otherwise, the state remains as S_0 with probability p_{1r}.
2) If the relay already has x_1, it starts polling T_2. The state S_1 either changes to S_3 with probability $1 - p_{2r}$ upon successfully receiving x_2, or stays in S_1 with p_{2r}.
3) When the relay has both x_1 and x_2, it generates a new codeword $y_n = \sqrt{\frac{P_r}{2}}x_1 + \sqrt{\frac{P_r}{2}}x_2$ according to a Gaussian codebook of 2^{2LR} with equal power allocation. Then at rate R, it broadcasts to T_1 and T_2, which will perform SIC to decode x_2 and x_1, respectively. Accordingly, the state will transit to S_0, S_1, S_2 or S_3 with the corresponding probabilities $(1 - p_{r1})(1 - p_{r2})$, $(1 - p_{r2})p_{r1}$, $(1 - p_{r1})p_{r2}$ or $p_{r1}p_{r2}$ respectively.
4) At the beginning of next transmission round, the relay will decide to poll a new codeword from T_1 (or T_2) based on the previous state being S_0,S_2 (or S_1) or just retransmits the previous y_n if the previous state was S_3.

The network state transition diagram of the DF TWRN can be modeled as in Fig. 4 with detailed probabilities on each path. Since the relay receives and decodes x_1 and x_2 at different time slots, the received signals at the relay from uplink transmissions can be represented as

$$\begin{aligned} y_{1r} &= \sqrt{P_1}h_{1r}x_1 + n_1 \\ y_{2r} &= \sqrt{P_2}h_{2r}x_2 + n_2. \end{aligned} \tag{14}$$

Similar to (4), the signals that will be used for decoding at T_1 and T_2 after SIC has been performed can be written as

$$\begin{aligned} x_{r1} &= \sqrt{\frac{P_r}{2}}h_{r1}x_2 + n_1 \\ x_{r2} &= \sqrt{\frac{P_r}{2}}h_{r2}x_1 + n_2. \end{aligned} \tag{15}$$

B. Goodput Analysis

Similarly as in the discussion of the goodput of AF TWRN in Section III, the data exchange only occurs upon the successful signal receptions at T_1 and T_2 at the end of the broadcasting time slot. Hence, initially, it is necessary to calculate the probability of being in State 3, i.e., $p(S_3)$.

We start from calculating the outage probabilities on the forward and backward channels as

$$\begin{aligned} p_{1r} &= 1 - e^{\frac{-(2^R-1)\sigma_n^2}{\mu_1 P_1}} \\ p_{2r} &= 1 - e^{\frac{-(2^R-1)\sigma_n^2}{\mu_2 P_2}} \\ p_{r1} &= 1 - e^{\frac{-(2^R-1)2\sigma_n^2}{\mu_1 P_r}} \\ p_{r2} &= 1 - e^{\frac{-(2^R-1)2\sigma_n^2}{\mu_2 P_r}}, \end{aligned} \tag{16}$$

where $\mu_1 = \sigma_1^2$ and $\mu_2 = \sigma_2^2$. The probabilities of buffer states can be solved by noting the following relations from Fig. 4:

$$\begin{cases} p(S_0) = p(S_0)p_{1r} + p(S_3)(1 - p_{r1})(1 - p_{r2}) \\ p(S_1) = p(S_0)(1 - p_{1r}) + p(S_1)p_{2r} + p(S_3)(1 - p_{r2})p_{r1} \\ p(S_2) = p(S_2)p_{1r} + p(S_3)(1 - p_{r1})p_{r2} \\ p(S_3) = p(S_3)p_{r1}p_{r2} + p(S_1)(1 - p_{2r}) + p(S_2)(1 - p_{1r}). \end{cases} \tag{17}$$

Solving the equations in (17) with given outage probabilities, we can obtain the following results for the buffer states:

$$\begin{cases} p(S_0) &= \frac{(1-p_{2r})(1-p_{r1})(1-p_{r2})}{D} \\ p(S_1) &= \frac{(1-p_{1r})(1-p_{r2})}{D} \\ p(S_2) &= \frac{(1-p_{2r})(1-p_{r1})p_{r2}}{D} \\ p(S_3) &= \frac{(1-p_{1r})(1-p_{2r})}{D}, \end{cases} \tag{18}$$

where the polynomial in the denominators is denoted by

$$D = 3 - 2p_{1r} - 2p_{2r} - p_{r1} - p_{r2} + p_{r1}p_{2r} + p_{1r}p_{r2} + p_{1r}p_{2r}. \tag{19}$$

Therefore, the system goodput in the DF mode can be derived as

$$\begin{aligned} &\eta_{DF} \\ &= p(S_3)R\left(2(1 - p_{r1})(1 - p_{r2}) + p_{r1}(1 - p_{r2}) + p_{r2}(1 - p_{r1})\right) \\ &= \frac{R(2 - p_{r1} - p_{r2})(1 - p_{1r})(1 - p_{2r})}{3 - 2p_{1r} - 2p_{2r} - p_{r1} - p_{r2} + p_{r1}p_{2r} + p_{1r}p_{r2} + p_{1r}p_{2r}}. \end{aligned} \tag{20}$$

C. Average Bit Energy Consumption

E_b in the DF TWRN is more complicated to calculate than in the AF TWRN where each transmission round has fixed power as can be seen in (13). Hence, in the DF scenario, we have to separate the energy expenditure into two parts, energy consumption in the first stage and energy consumption in the second stage. The first stage denotes the state transition from any of 4 previous states to state S_3, where the relay holds two codewords x_1 and x_2 in its buffer and is ready to broadcast. The second stage is that the relay broadcasts its newly generated codeword and the state transits back to any of the four states again.

Considering the relay's buffer is to be loaded with both codewords x_1 and x_2 from any of the previous states on the first stage, the energy consumption conditioned on the previous state S_0, S_1, S_2, or S_3 on this particular transition will be

$$\begin{aligned} E_{S_0} &= \frac{P_1 L}{(1-p_{1r})R} + \frac{P_2 L}{(1-p_{2r})R} \\ E_{S_1} &= \frac{P_2 L}{(1-p_{2r})R} \\ E_{S_2} &= \frac{P_1 L}{(1-p_{1r})R} \\ E_{S_3} &= 0. \end{aligned} \tag{21}$$

On the second stage, the energy consumption for broadcasting is always $\frac{P_r L}{R}$, so the average bit energy consumption for one information bit successfully exchanged on the DF TWRN can be computed as

$$\begin{aligned} E_b &= \frac{\sum_{i=0}^{3}(E_{S_i} + \frac{P_r L}{R})p(S_i)}{(2(1-p_{r1})(1-p_{r2}) + (1-p_{r1})p_{r2} + (1-p_{r2})p_{r1})L} \\ &= \frac{E_{S_0}p(S_0) + E_{S_1}p(S_1) + E_{S_2}p(S_2) + \frac{P_r L}{R}}{(2-p_{r1}-p_{r2})L}. \end{aligned} \tag{22}$$

whenever the state probabilities and outage probabilities are known.

V. NUMERICAL RESULTS AND COMPARISONS

In this section, we present the numerical results to evaluate the system performance of TWRN in both AF and DF modes. The network configurations are assumed to be as follows: Relay is located in the middle between T_1 and T_2 which means $k = 0.5$. The power spectrum density of the Gaussian white noise is $\sigma_n^2 = 10^{-10}$ and the channel bandwidth is set to $B = 10^6$ Hz. Path loss coefficient is $\alpha = 3.12$ [10]. We also assume the same transmit power for both source nodes and the relay, which is $P_1 = P_2 = P_r = P$ and define the SNR by $\gamma = \frac{P}{\sigma_n^2}$.

Firstly, we are particularly interested in how the goodput varies as a function of the transmission rate R at specific SNR values. In Fig. 5, η_{AF} and η_{DF} are plotted as functions of R, with solid and dashed lines corresponding to AF and DF modes, respectively. On each curve with a given specific γ value, it is immediately seen that the goodput first increases within low R range and then begins to drop once the rate is increased beyond the optimal R^* which maximizes the goodput η. Additionally, at low values of the rate R, the AF TWRN has higher goodput η_{AF}, while beyond a certain rate R, DF starts to outperform, regardless of the SNR γ value.

To better illustrate the goodput performance in AF and DF modes, we look into transmission efficiency by defining a normalized rate $\frac{\eta}{R}$ and plotting it in Fig. 6. The normalized rate is always decreasing when R increases at all SNR's in both AF and DF. In other words, increasing outage probabilities due to increasing R has eventually resulted in more ARQ retransmissions. Specifically in the high SNR scenario, the normalized rate levels off between 0.6 and 0.7 in AF and 0.9 and 1 in DF, which means the transmission efficiency doesn't change too much within this rate range. In addition, the AF mode seems to have higher normalized rate in low rate range.

In Fig. 7, we analyze the energy efficiency. We notice that the difference of the average bit energy consumptions between two modes is insignificant up until $R = 1$, but DF stills has a better energy efficiency with a lower E_b regardless of γ. However when compared with the corresponding points ($R < 1$) in Fig. 6, it is shown that even though DF can achieve a slightly lower E_b than AF, it also suffers a lower transmission efficiency in the metric of lower normalized rate. Above $R = 1$ in low SNR scenario of $\gamma = 0\ dB$, DF predominates with both higher normalized rate and lower E_b until R approaches about 6 bits/sec/Hz. Similar results can be observed on the high SNR scenarios also. In Fig. 8, we study the impact of SNR on the normalized rate in both AF and DF. Basically the normalized rate is increasing as SNR increases at all transmission rates. At low SNR e.g. $\gamma = -5\ dB$, DF performs better than AF. As SNR approaches $\gamma = 20\ dB$, the normalized rate gets close to 1 (or 0.7) in AF (or DF) mode. Consequently, one way to improve on the transmission efficiency is to increase SNR in the TWRN. Considering overall impacts of rate R and SNR, we can always find an scheme for the TWRN to achieve optimality in respect to the goodput η, the average bit energy consumption E_b or the transmission efficiency.

VI. CONCLUSION

In this paper, we have studied the two-way relay networks working in Amplify-and-Forward and Decode-and-Forward modes. In each mode, we set up a Markov chain model to analyze the state transition in details. ARQ transmission is employed to guarantee the successful packet delivery at the end and mathematical expressions for the goodput and bit energy consumption have been derived. Several interesting results are observed from simulation results: 1) the transmission rate R can be optimized to achieve a maximal goodput in both AF and DF modes; 2) generally the transmission efficiency is higher in AF within a certain R range, while the DF can achieve a slightly higher energy efficiency instead; 3) increasing SNR will always increase the normalized rate regardless of R. Hence, it's possible the network performance be optimized in a balanced manner to maintain a relatively high goodput as well as a low E_b.

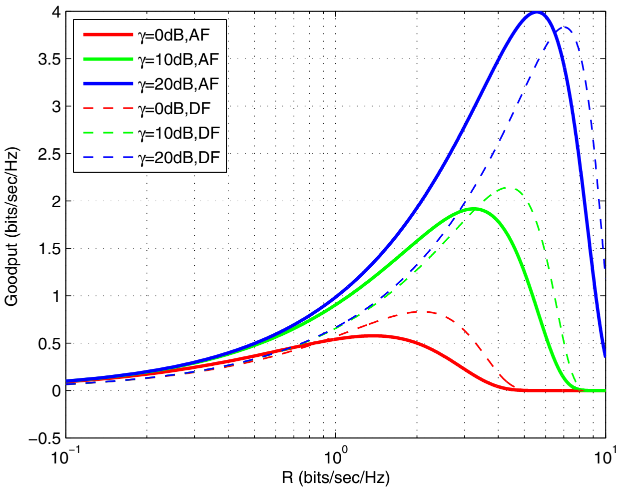

Fig. 5. Goodput Vs. Transmission Rate in AF and DF TWRN

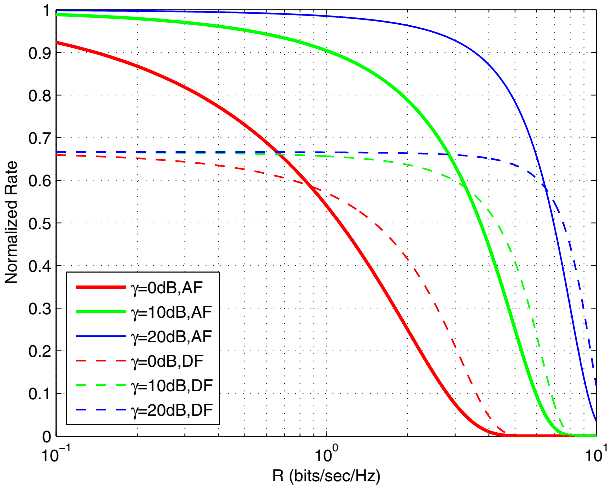

Fig. 6. Normalized Transmission Rate Vs. Transmission Rate

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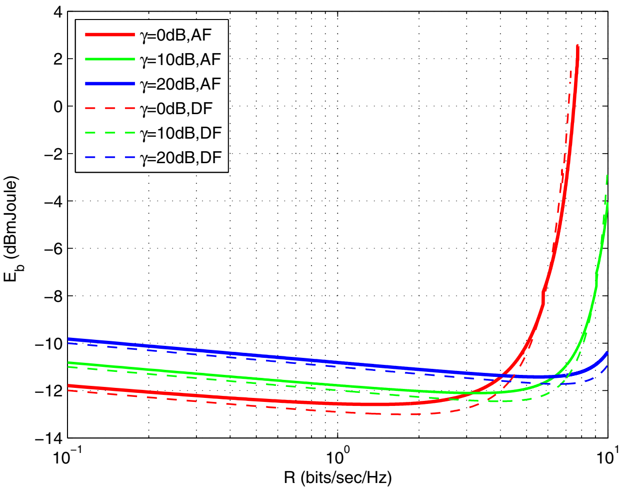

Fig. 7. Average Bit Energy Vs. Transmission Rate

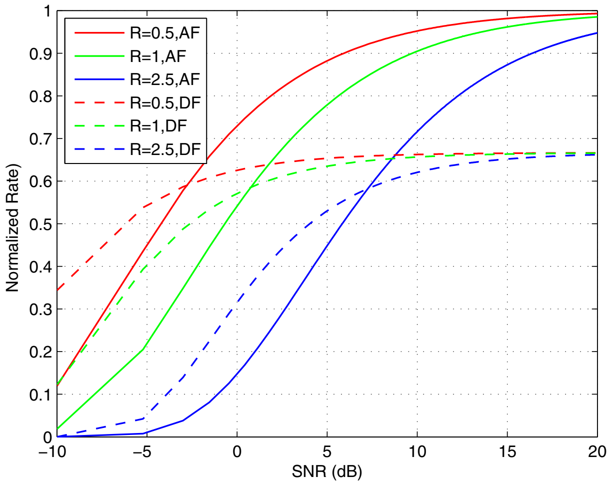

Fig. 8. Normalized Transmission Rate Vs. SNR

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