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Physics, Chapter 27: Direct-Current Circuits

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Direct-Current Circuits

27-1 Electromotive Force

When a charged particle traverses a closed path in an electrostatic field in space, the total work done on the particle is zero. The electric field is *conservative*. Associated with each point in the field, there is a fixed value of the electrical potential.

In a simple circuit consisting of a chemical cell and a resistor, as shown in Figure 27-1, we have seen that the current flows through the resistor from the positive terminal of the cell to the negative terminal of the cell. The positive terminal is at the higher potential, so that the current flows from the higher to the lower potential outside the cell, but inside the cell the direction of current flow is

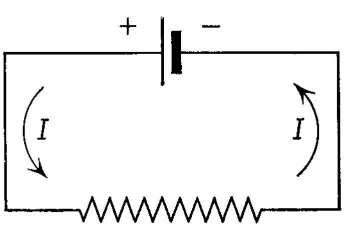

Fig. 27-1

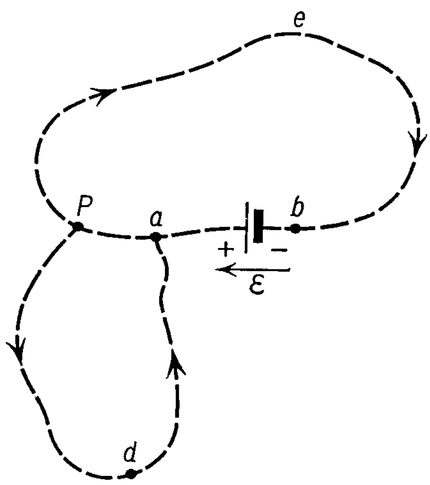

Fig. 27-2

from the lower to the higher potential. The cell must do work upon the charge in order to raise its potential.

Let us consider the effect of moving a positive probe charge through a closed path in an electric field in which such a cell is located. As long as the path of the charged particle does not pass through the cell, the work done in traversing a closed path is zero, as in the path *PdaP* in Figure 27-2. When the path traverses the cell, the particle may acquire energy as a result of the conversion of chemical energy to electrical energy, as in the path *PebaP*, or the particle may lose energy as the result of the conversion of electrical energy to chemical energy if this same path is traversed in the direction opposite to the original direction, as in the path *PabeP*. If one takes into account only the mechanical work done by the agency moving the probe charge, the cell appears to be a place where energy may be gained or lost, depending on the direction in which the passage through the cell is effected. A similar result would be obtained by passage of the probe charge through any electric generator in which the conversion of energy from some other form to electrical energy is reversible.

The concept of *electromotive force* has been introduced to describe the energy relations associated with electric circuits which incorporate chemical cells or other electric generators. If a net quantity of work $\mathscr{W}$ is done in carrying a charge q around a closed path in which no current is flowing, as in a circuit when the switch is open, the total electromotive force $\mathscr{E}$ in that path is defined as

$$\mathscr{E} = \frac{\mathscr{W}}{q}. \tag{27-1}$$

In this definition of electromotive force $\mathscr{E}$, abbreviated *emf*, it is important to specify that the charge may traverse the path with arbitrary slowness. In other words, if the work done in traversing the path depends upon the speed of the particle, we will imagine that the particle is carried around the path with near zero speed. Since the work done when a charge traverses a resistor depends upon the current, which may be related to the speed of the charged particle, the work done in very slowly moving a probe charge through a resistor will approach zero, as long as the potential difference between the ends of the resistor is zero.

From Equation (27-1) we see that the units of electromotive force are the same as the units of potential difference. In the *mks* system of units, the unit of emf is the volt. The emf is not a force but an energy per unit charge. If the potential difference across the terminals of a cell is 2 volts when no current is flowing, we say that the cell has an emf of 2 volts.

Since the work done on a probe charge q in traversing the path *PadP* is zero, the emf along this path is zero. In traversing the path *PebaP*, if the potential difference between the terminals of the cell is V, the work done by the cell on the charge is Vq, and the emf is

$$\mathscr{E} = \frac{Vq}{q} = V,$$

for the particle has more energy when it has returned to P than it had initially. Positive work has been done on the particle. In traversing the same path in the opposite direction $PabeP$, the emf is

$$\mathcal{E} = \frac{-Vq}{q} = -V,$$

for the particle has done work in traversing the cell. In other words, negative work has been done on the particle, so that its energy is less upon its return to P than it was initially. The *emf of a cell* or of a generator thus has a sense, and *is directed from the negative terminal of the cell to its positive terminal* inside the cell. When a positive charge passes through a cell in the direction of the emf, it gains in potential energy. When a positive charge passes through the cell in a direction opposite to the emf, it loses potential energy. In both cases the change in electrical potential is equal to the magnitude of the emf $\mathcal{E}$ of the cell.

27-2 Series and Parallel Connections

An electric circuit may be very simple and consist of one or two electrical devices connected to a source of power, or it may be very complex and consist of many different elements connected in a variety of ways. In

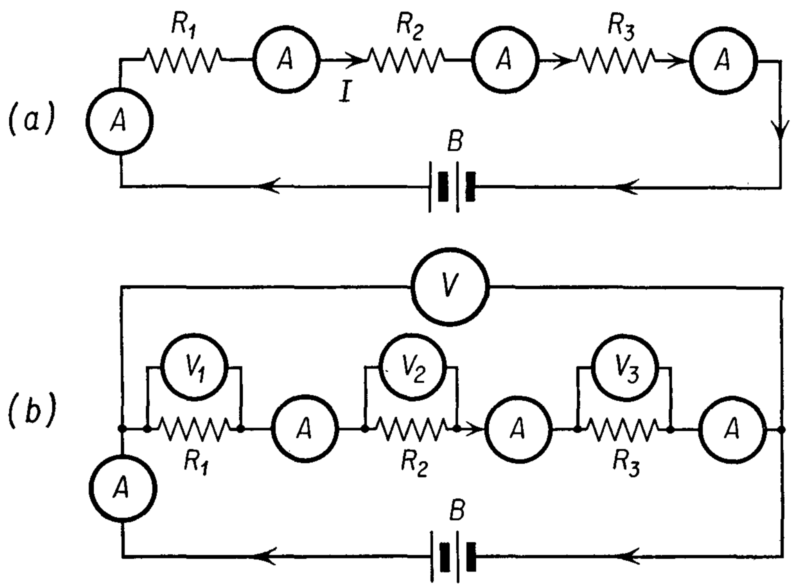

Fig. 27-3 Resistors connected in series. (a) The current is the same in each resistor. (b) The difference of potential across all the resistors in series is equal to the sum of the differences of potential across each of the resistors.

practical applications it is important to be able to determine the *equivalent resistance* of any circuit or any section of the circuit in terms of the resistances of the individual elements of a circuit. Suppose we have a circuit consisting of a battery, three resistors, and four ammeters, connected as shown in Figure 27-3(a). This is called a *series* circuit, and the various elements are said to be connected in *series*. In a series circuit *the current is the same in all parts of the circuit.* The same current flows in each of the resistors, each of the ammeters, and the battery.

Another type of connection is shown in Figure 27-4 in which the three resistors are connected in *parallel.* Since the difference of potential between two points, such as C and D, can have only one value, the difference of potential across each resistor is the same. A voltmeter connected across C and D will give the difference of potential across each resistor and, in this case, will also give the difference of potential across the terminals of the battery. The voltmeter is always connected in parallel with that portion of the circuit whose voltage is to be measured, while the ammeter is always connected in series in that portion of the circuit in which the current is to be measured.

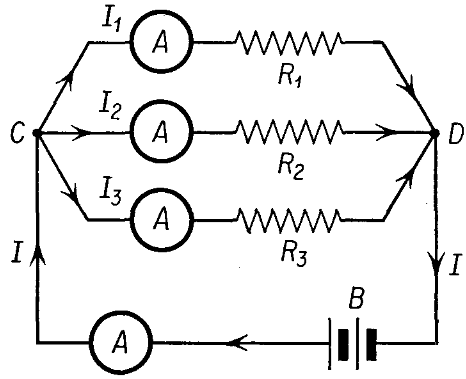

Fig. 27-4 Resistors in parallel. The difference of potential is the same across each resistor.

In Figure 27-3(b), if three voltmeters are connected across the three resistors in series, R_1, R_2, and R_3, and a fourth voltmeter is connected across all three resistors, it will be observed that the difference of potential V across all three resistors in series is equal to the sum of the differences of potential across each of the resistors, or

$$V = V_1 + V_2 + V_3. \tag{27-2}$$

Applying Ohm's law to each resistor and remembering that the current I is the same in each one, we get

$$V_1 = IR_1, \qquad V_2 = IR_2, \qquad \text{and} \qquad V_3 = IR_3,$$

which yields, upon substitution in Equation (27-2),

$$V = IR_1 + IR_2 + IR_3. \tag{27-3}$$

The equivalent resistance R of this circuit is one which would have the same current I flowing in it when the same potential difference V is applied to it; that is,

$$V = IR. \tag{27-4}$$

Equating these two values of V, we get

$$R = R_1 + R_2 + R_3. \tag{27-5}$$

The equivalent resistance of a group of resistors connected in series is the sum of their individual resistances.

If ammeters are inserted into the circuit containing three resistors in parallel, as shown in Figure 27-4, it will be observed that the current I which leaves the battery divides at C in such a way that it is equal to the sum of the currents in the individual resistors, or

$$I = I_1 + I_2 + I_3, \tag{27-6}$$

where I_1 is the current in R_1, I_2 the current in R_2 and I_3 the current in R_3. These currents recombine at D and flow back to the battery. Applying Ohm's law to each resistor and remembering that the potential difference V is the same across each one, we get

$$V = I_1R_1, \qquad V = I_2R_2, \qquad \text{and} \qquad V = I_3R_3.$$

We may therefore write Equation (27-6) as

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3};$$

and if R is the equivalent resistance of these three parallel resistors, then

$$I = \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3},$$

from which

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}; \tag{27-7}$$

this is the relationship between the equivalent resistance R and the individual resistances of three resistors in parallel. The reciprocal of the resistance is called the *conductance* of the resistor. Equation (27-7) may be read as follows; the equivalent conductance of a parallel circuit is the sum of the conductances of the individual resistors connected in parallel. Equation (27-7) also shows that the equivalent resistance of a parallel combination is less than the resistance of any one of the resistors.

Illustrative Example. Find the potential difference between the terminals of the 12-ohm resistor of Figure 27-5(a) if the battery has an emf of 36 volts and has no internal resistance.

Let us first determine the equivalent resistance of the three resistors in the circuit. The equivalent resistance of the parallel combination of the 6- and

12-ohm resistors may be found as

$$\frac{1}{R} = \frac{1}{6 \text{ ohms}} + \frac{1}{12 \text{ ohms}},$$

or

$$R = 4 \text{ ohms}.$$

We may replace the parallel combination by a single 4-ohm resistor, as shown in Figure 27-5(b). This series combination of two resistors has an effective resistance of 6 ohms. Thus the three resistors of Figure 27-5(a) may be replaced by a single 6-ohm resistor, as shown in Figure 27-5(c).

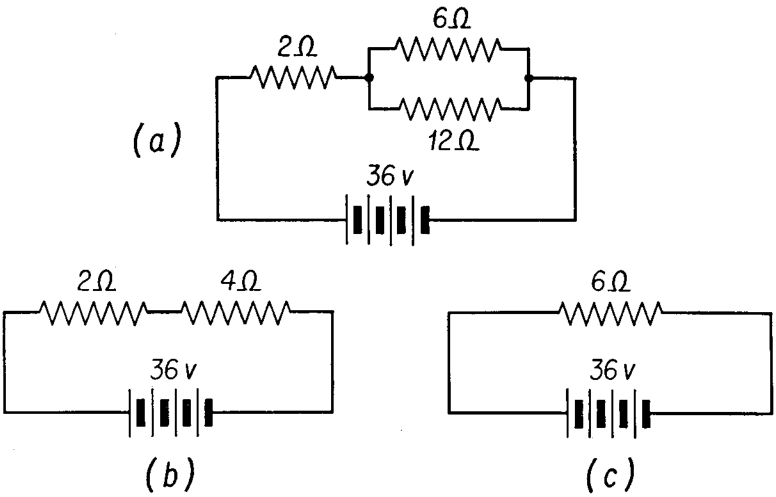

Fig. 27-5

From Ohm's law the current in the circuit of Figure 27-5(c) is

$$I = \frac{V}{R} = \frac{36 \text{ volts}}{6 \text{ ohms}}$$

$$= 6 \text{ amp}.$$

Thus a current of 6 amp flows from the battery in each of the equivalent circuits of Figure 27-5. In Figure 27-5(b) the potential difference between the terminals of the 4-ohm resistor is therefore equal to 24 volts. Replacing the 4-ohm resistor by the original parallel combination, as in Figure 27-5(a), the potential difference between the terminals of the 12-ohm resistor is 24 volts.

27-3 Terminal Voltage

In practical electrical generators the passage of current through the generator is accompanied by the evolution of heat within the generator. To describe the evolution of heat in a quantitative way, we say that practical generators have an emf and an *internal resistance* as well. It is customary to represent the internal resistance of a practical cell by a resistor r which is in series with a resistanceless cell of emf $\mathcal{E}$. The potential difference as measured by a voltmeter connected across the terminals of the cell ab (Figure 27-6) will depend upon the current I flowing through the cell.

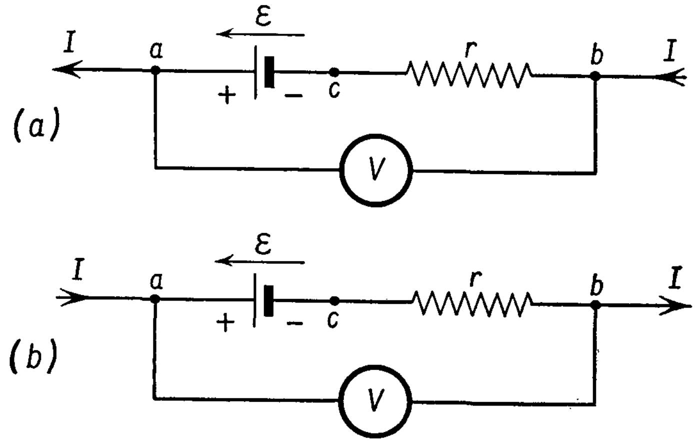

Fig. 27-6 Terminal voltage and emf of (a) cell discharging and (b) cell being charged.

Let us suppose that the current through the cell is in the direction of the emf, as shown in Figure 27-6(a). The current through the internal resistance r is in the direction bc. According to Ohm's law, the point c must be at a lower potential than the point b by an amount given by

$$V = Ir.$$

If we carry a unit positive probe charge from the point b to the point c and then to the point a, the potential falls by Ir in passing from b to c and rises by $\mathcal{E}$ on passing from c to a. The potential difference between b and a is given by

$$V_a = V_b - Ir + \mathcal{E}, \quad \textbf{(27-8)}$$

so that

$$V_a - V_b = \mathcal{E} - Ir. \quad \textbf{(27-9)}$$

Thus the difference of potential between the terminals of the cell, also called the *terminal voltage*, is less than the emf of the cell when the current is in the direction of the emf; this is the case when the cell is discharging.

When the direction of the current is opposite to that of the emf, as in Figure 27-6(b), we note that the point c must be at a higher potential than the point b by an amount Ir, so that the potential difference between points

b and a is given by

$$V_a - V_b = \mathcal{E} + Ir. \tag{27-10}$$

When a battery is being charged, the current is in a direction opposite to its emf, and its terminal voltage is greater than $\mathcal{E}$.

As a dry cell or a storage battery deteriorates with use, its emf remains substantially constant, but its internal resistance increases. The cell must be tested under conditions in which it is required to supply currents appropriate to its use, in order to determine whether the cell is "dead." When the terminal voltage is appreciably less than the emf, the battery may no longer be suitable for its intended application.

27-4 Voltmeters and Ammeters

The voltmeters and ammeters used in electrical measurements are constructed from a basic meter movement which will be described in a subsequent chapter. For our present purposes we need only know that this basic meter is a device which measures current, called a *galvanometer*. Galvanometers are usually constructed so as to have scale deflections proportional to the current passing through them. The galvanometer has an internal resistance.

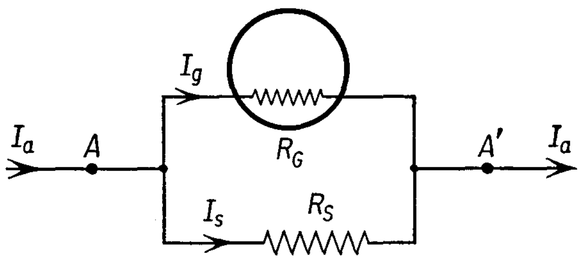

Fig. 27-7 Ammeter.

Let us suppose that we have a galvanometer which is so constructed that its internal resistance is R_g and that it is deflected to a full-scale reading whenever the current passing through it is I_g. If we wish to have an ammeter whose full-scale reading is I_a, we may construct such a meter from the galvanometer by connecting a resistor R_s across the terminals of the galvanometer. Such a resistor is called a galvanometer *shunt*. The shunt resistor is often connected inside the case of the galvanometer; the external connections to the instrument are shown in Figure 27-7 as AA'.

Suppose we wish the galvanometer to indicate a full-scale deflection when a current I_a flows through the connections AA'. Since the construction of the galvanometer is such that it reads full-scale deflection when the current through it is I_g, the current flowing through the shunt resistor I_s must be given by

$$I_a = I_g + I_s.$$

Since the galvanometer is in parallel with the shunt resistor, the potential difference across the galvanometer must be the same as the potential difference across the shunt resistor. Thus

$$I_g R_g = I_s R_s.$$

The value of the required shunt resistor may be determined by eliminating the current I_s from these two equations. We have

$$I_a = I_g\left(1 + \frac{R_g}{R_s}\right), \tag{27-11}$$

so that a knowledge of the resistance of the galvanometer, the current through the galvanometer for full-scale deflection, and the desired full-scale ammeter reading suffice to determine the required shunt resistance.

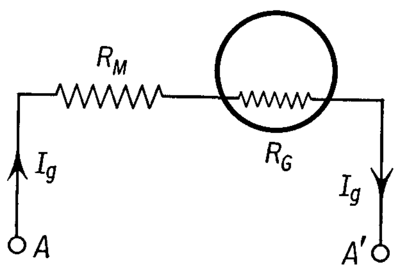

Fig. 27-8 Voltmeter.

It is impossible to make an ammeter whose full-scale deflection requires less current than is required by the galvanometer from which it was constructed. An ammeter reading large currents requires a shunt resistor of smaller resistance than does one reading small currents. Since an ammeter is connected in series with the circuit in which the current is being measured, an ammeter with very low resistance is desired in order that it may have a minimum influence upon the current in the circuit.

The same galvanometer may be used as a voltmeter by connecting a *multiplier* resistor in series with it, as shown in Figure 27-8. The terminals of the voltmeter AA' are to be connected across (in parallel with) two points in a circuit whose potential difference we wish to measure. The galvanometer reads full-scale deflection when a current I_g passes through it. The same current also passes through the multiplier resistor R_m. From Ohm's law the potential difference V, between the points A and A', is given by

$$V = I_g(R_m + R_g). \tag{27-12}$$

Thus we may construct a voltmeter with any desired full-scale reading V by inserting a resistor R_m in series with a galvanometer, if the internal resistance R_g of the galvanometer and the current I_g required for full-scale deflection are known.

A good voltmeter should be constructed from a galvanometer of high sensitivity. The voltmeter is connected in parallel with the circuit component whose voltage is being measured. In order that the circuit be least disturbed by the presence of the voltmeter, the current through the voltmeter should be small. Thus it is important that the resistance of the voltmeter be high.

It is possible to construct multipurpose meters from a single galvanometer. A number of appropriate resistors are mounted in the case with the galvanometer, and a switching arrangement is provided which connects selected resistors in series with the galvanometer for voltage measurement, while other resistors are connected in parallel with the galvanometer for use of the instrument as an ammeter.

Illustrative Example. It is desired to convert a galvanometer, that has an internal resistance of 5 ohms and gives a full-scale deflection for a current of 10 μamp, into a voltmeter whose full-scale deflection corresponds to 300 volts. What value of multiplier resistance should be used?

Substituting numerical values into Equation (27-12), we have

$$300 = 10 \times 10^{-6}(R_m + 5),$$

$$R_m = 2{,}995 \text{ ohms}.$$

To convert the same galvanometer to an ammeter with a full-scale reading of 100 μamp, a shunt resistor is used whose resistance is given by Equation (27-11).

$$100 \times 10^{-6} = 10 \times 10^{-6}\left(1 + \frac{5}{R_s}\right)$$

$$R_s = \tfrac{5}{9} \text{ ohm}.$$

27-5 The Potentiometer

When a voltmeter is used to measure a potential difference, some error is always introduced because of the fact that the voltmeter draws current from the circuit to which it is connected. When a voltmeter is used to measure the potential difference between the terminals of a cell, the measured voltage is always less than the emf because of the voltage drop across the internal resistance of the cell. In many applications it is important to be able to measure the potential difference between the two terminals without drawing current to the measuring device. In such cases a *potentiometer* is used.

The schematic circuit of a potentiometer is shown in Figure 27-9. At the heart of the potentiometer is a *slide-wire* resistor made of a uniform wire ac, provided with a sliding contact, shown in the figure at point b. The resistance of the portion of the wire from the fixed contact at a to the sliding contact at b is proportional to the length of the wire between these two points. A constant current I is maintained in the slide wire by means of the battery B, so that the potential difference between a and b is proportional to the length of the wire between these two points. In this way any potential difference from zero to the terminal voltage of the battery can be obtained between the points a and b simply by moving the sliding contact.

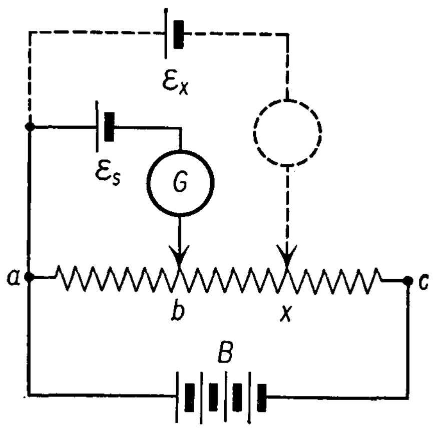

Fig. 27-9 A potentiometer.

In order to use the potentiometer, it is necessary first to calibrate it with the aid of a *standard cell* such as the Weston normal cell whose emf has been previously determined. The positive terminal of the standard cell is connected to the same point a as the positive terminal of the battery, while the negative terminal of the standard cell is connected to the point b through a sensitive galvanometer. The sliding contact b is moved along the slide wire until there is no current through the galvanometer, indicating that there is no potential difference across the terminals of the galvanometer. Thus the emf of the standard cell is equal to the potential difference V_{ab} along the slide wire between these points. If the resistance of the slide wire between the point a and the point b is R_{ab} we have,

$$\mathcal{E}_s = V_{ab} = IR_{ab}.$$

To determine the emf of an unknown cell $\mathcal{E}_x$, the standard cell is replaced by the unknown cell, and the sliding contact is shifted to a new position x to achieve a balance. We have

$$\mathcal{E}_x = V_{ax} = IR_{ax}.$$

Dividing the second of these equations by the first, we obtain

$$\mathcal{E}_x = \mathcal{E}_s\left(\frac{R_{ax}}{R_{ab}}\right).$$

If l_b is the length of the slide wire from a to b and l_x is the length from a to x, and since the resistance of a uniform wire is proportional to its length, we have

$$\frac{R_{ax}}{R_{ab}} = \frac{l_x}{l_b},$$

so that

$$\mathcal{E}_x = \mathcal{E}_s\left(\frac{l_x}{l_b}\right). \tag{27-13}$$

The measurement of the emf of a cell is thus reduced to the measurement of two lengths along a uniform slide wire. Since the potentiometer uses a galvanometer to determine the *null condition,* the condition of no current flow, an extremely sensitive instrument may be used. Furthermore, we have no need to compound the errors of measurement by first measuring the current from the battery or the resistance of the slide wire. Any potential differences may be measured relative to a standard cell with an accuracy limited only by the uniformity of the slide wire and the sensitivity of the galvanometer.

The potentiometer may be used to calibrate voltmeters and ammeters, to measure potentials in an electrolytic plotting tank, to measure the thermal emf in a thermocouple for the measurement of temperature, and, in fact, wherever an accurate determination of potential difference is required.

27-6 Kirchhoff's Laws

Complicated circuits made up of resistors and sources of emf often cannot be readily resolved into series and parallel combinations of resistors. The procedures for solving such networks were first stated by Gustav Robert Kirchhoff (1824–1887) and are known as Kirchhoff's laws. In order to state concisely these rules for the solution of circuit problems, we shall first define two terms, a *junction* and a *loop.* A *junction,* or branch point, is a point where three or more conductors, or branches, are electrically connected. A *loop* is any closed path in an electrical circuit.

The concept of conservation of charge leads to the first of Kirchhoff's laws, which states that *the algebraic sum of the currents flowing into a junction is zero.*

$$\sum I = 0.$$

In the above equation, representing Kirchhoff's first law, current flowing into a junction is generally regarded as positive current, while current flowing out of the junction is regarded as negative. The current flowing into any junction must equal the current flowing out of that junction. If this were not true, we would have to imagine that charge could be created or destroyed at a junction, in contradiction of the principle of conservation of electric charge.

The second of Kirchhoff's laws may be stated as *the algebraic sum of the potential differences around a closed loop is equal to zero.* Since the potential differences between the terminals of the circuit elements are composed of emf's, and IR drops across resistors, we may write Kirchhoff's second law symbolically as

$$\sum \mathcal{E} + \sum IR = 0.$$

In effect, this rule states that the potential of a point in an electric circuit is a fixed quantity. If we start at any point in the circuit and imagine that we transport a probe charge around a closed loop, the algebraic sum of the potential changes associated with passing from one junction to the next in passing around the loop is equal to zero.

To apply Kirchhoff's laws to a network, we first imagine the network to be broken up into loops, as shown in Figure 27-10, in which circuit elements have been represented as rectangles. We choose these loops to be as simple as possible, so that each circuit element is contained in at least one loop. Some circuit elements may be part of more than one loop. We assign arbitrarily a direction to the current in each loop. We indicate this direction by an arrow, and assign to the current an algebraic magnitude represented by the symbol I_1, I_2, and so on. In drawing these currents as continuous through a junction, we have automatically fulfilled Kirchhoff's first law. In the event that we have assigned the wrong direction to any of these loop currents, the numerical solution for that current will yield a negative answer.

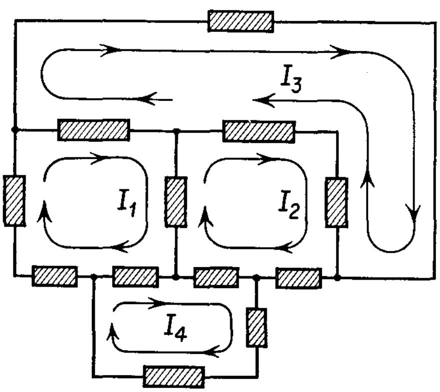

Fig. 27-10 Current loops.

To each loop we apply Kirchhoff's second law; starting at a particular point in the circuit, we imagine that we carry a unit positive probe charge from junction to junction. The probe charge may be carried around the loop in any direction, without regard to the direction of the loop current. When the probe charge is carried through a source of emf in the direction of the emf, passing from the negative to the positive terminal, the potential of the probe charge is increased by the magnitude of the emf. When the probe charge is carried through the source of emf from the positive to the negative terminal, its potential is decreased in amount by the magnitude of the emf. When the probe charge passes through a resistor in the direction of the current, the potential of the probe charge is decreased, for the direction of the current in a resistor is from high to low potential. If the probe charge is passed through the resistor in a direction opposite to the direction of the current, the potential of the probe charge is increased. Because of the manner in which we have indicated the loop currents, some elements will have current contributions from more than one loop current. Since the net current through the resistor determines the potential difference between its terminals, we may treat the potential change due to each loop current as an independent contribution to the potential change of the probe charge.

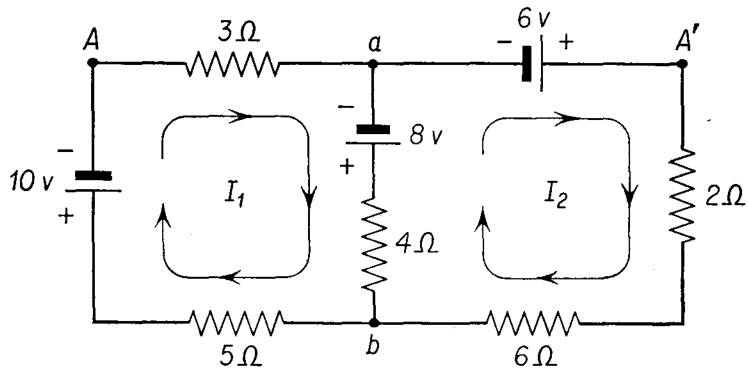

Fig. 27-11

Illustrative Example. Find the potential difference between the points a and b in the circuit of Figure 27-11.

We imagine the circuit to be broken into two closed loops, as shown in the figure, with the 8-volt cell and the 4-ohm resistor common to both loops, and represent the currents in the two loops by I_1 and I_2 which have been drawn arbitrarily in the clockwise direction. We imagine that we have a unit positive probe charge at the point A, and we carry the probe charge around the first loop in the direction of the current in this loop. We shall represent an increase in the potential of the probe charge as positive and a decrease of the potential of the probe charge as negative. The algebraic sum of the potential changes of the probe charge in passing around the loop must be equal to zero.

On passing through the 3-ohm resistor in the direction of the current I_1, the potential of the probe charge falls. The change in potential of the probe charge is $-3I_1$. The probe charge next passes from the negative to the positive terminal of the 8-volt cell, so that its potential change is $+8$ volts. There are two contributions to the potential change of the probe charge in passing through the 4-ohm resistor. Since it passes through the resistor in the direction of the current I_1, the potential of the probe charge decreases by $4I_1$. At the same time the probe charge passes through the resistor in a direction opposite to I_2, so that its potential increases $4I_2$. The net change in potential of the probe charge is

$$-4I_1 + 4I_2.$$

Next the probe charge passes through the 5-ohm resistor where its potential changes by $-5I_1$. Finally it returns to the point A by passing through the 10-volt cell from the positive to the negative terminal, making a change in potential of -10 volts. Adding all these changes algebraically and setting the sum equal to zero, we find

$$-3I_1 + 8 - 4I_1 + 4I_2 - 5I_1 - 10 = 0,$$

from which

$$-12I_1 + 4I_2 - 2 = 0$$

for the equation resulting from the application of the loop condition to the first loop.

Starting with the point A' and traversing the second loop in the counterclockwise direction, opposite to the direction of the current I_2, we find

$$-6 + 8 - 4I_1 + 4I_2 + 6I_2 + 2I_2 = 0,$$

yielding

$$-4I_1 + 12I_2 + 2 = 0$$

as the equation resulting from the second loop.

We may solve these equations simultaneously to find

$$I_1 = I_2 = -\tfrac{1}{4}\text{ amp.}$$

The currents in both loops are therefore opposite to the directions indicated in Figure 27-11.

Having found the currents in the circuit, the remainder of the problem is quite straightforward. Since I_1 and I_2 are equal in magnitude and opposite in direction between a and b, there is no current between the junctions a and b. To find the potential difference between these two points, we imagine the unit probe charge to be at position a. At this point its potential is V_a. To move the probe charge to position b it must pass through the 8-volt cell, its potential being increased by the emf of the cell, and through the 4-ohm resistor. Since there is no current through the resistor, there is no potential change on passing through it. The probe charge is then at the potential of point b, V_b. In the form of an equation

$$V_a + 8\text{ volts} = V_b;$$

$$V_b - V_a = 8\text{ volts.}$$

27-7 Back EMF of a Motor

As we have seen in Section 27-1, the concept of electromotive force is a useful one in connection with the operation of generators of electricity. We may extend this concept so that it is applicable to motors as well, through the idea of a *back emf*. It is possible to construct a d-c motor so that the direction of rotation of the motor depends upon the direction of the current through it. Unlike chemical cells or generators, which are polarized so that they convert some other form of energy to electrical energy when the charge passes in one direction and convert electrical energy back to that other form when the charge passes in the reverse direction, a motor always converts electrical energy to mechanical energy.

It is possible to describe this property of the motor electrically in terms of an emf, called a back emf, which is always opposite in direction to the current. The electrical power $\mathcal{P}$ converted by the motor to mechanical power is

$$\mathcal{P} = \mathcal{E}I,$$

where I is the current delivered to the motor, and $\mathcal{E}$ is its back emf. In general, a motor may be described by its back emf $\mathcal{E}$ and its internal resistance R when it is part of an electrical circuit.

Illustrative Example. (a) What is the back emf of an electric motor which has an internal resistance $r = 5$ ohms and draws a current $I = 2$ amp when connected to a source whose terminal voltage V is 110 volts? (b) What is the efficiency of the motor?

(a) The schematic circuit is shown in Figure 27-12 where the motor has been represented by a generator, whose emf $\mathcal{E}$ is directed opposite to the direction of the current, in series with a resistor representing the internal resistance of the motor. If we imagine that the probe charge is initially at the point A and that we carry the probe charge in a counterclockwise direction around the loop, we find, from Kirchhoff's second law,

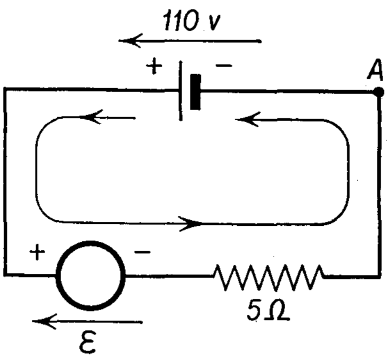

Fig. 27-12

$$V - \mathcal{E} - rI = 0.$$

Substituting numerical values, we get

$$+\,110 \text{ volts} - \mathcal{E} - 5 \text{ ohm} \times 2 \text{ amp} = 0,$$

$$\mathcal{E} = 100 \text{ volts}.$$

(b) To find the efficiency of the motor, we observe that the energy $\mathcal{W}_i$ delivered to the motor in time t is

$$\mathcal{W}_i = 110 \text{ volts} \times 2 \text{ amp} \times t = 220 \text{ watts} \times t.$$

The energy converted by the motor to mechanical energy is given by the product of the back emf by the charge which has passed through the motor in time t so that the work done by the motor $\mathcal{W}_0$ is given by

$$\mathcal{W}_0 = 100 \text{ volts} \times 2 \text{ amp} \times t \text{ sec} = 200 \text{ watts} \times t.$$

The efficiency e of the motor is

$$e = \frac{\mathscr{W}_0}{\mathscr{W}_i} = \frac{200 \text{ watts} \times t}{220 \text{ watts} \times t},$$

$$e = 0.91.$$

Problems

27-1. Two resistors of 30 and 45 ohms resistance, respectively, are connected in parallel, and the combination is connected to a 120-volt source. Determine (a) the effective resistance of this combination and (b) the current through each resistor.

27-2. Three coils of 20, 30, and 50 ohms resistance, respectively, made of uniform wire, are connected in parallel, and the group is then connected to a 110-volt source. Find (a) the resistance of the combination and (b) the current through each resistance.

27-3. Three resistors having resistances of 15, 25, and 50 ohms, respectively, are connected in series, and a difference of potential of 120 volts is maintained across the combination. (a) What is the current in each resistor? (b) What is the voltage across each resistor? (c) How much power is supplied to this combination?

27-4. In the circuit sketched in Figure 27-13, the current in the 10-ohm coil is 4.5 amp. (a) Calculate the value of the resistance R. (b) Determine the amount of heat developed in 1 min in the 10-ohm coil.

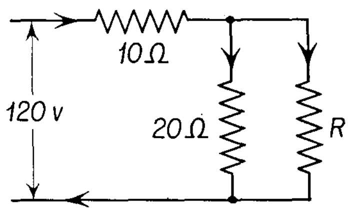

Fig. 27-13

27-5. Three resistors having resistances of 4, 8, and 12 ohms, respectively, are connected in series. A storage battery maintains a difference of potential of 12 volts across the combination. How much power is delivered to each resistor?

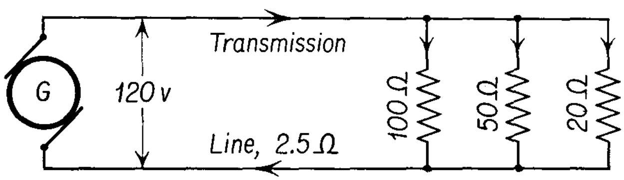

Fig. 27-14

27-6. A lamp, an electric heater, and an electric iron are connected in parallel, as shown in Figure 27-14. Their resistances, when hot, are 100, 50, and 20 ohms, respectively. If the generator produces a voltage across its terminals of 120 volts, and if the transmission line has a resistance of 2.5 ohms, find (a) the current supplied by the generator, (b) the voltage across the terminals of the lamp, (c) the current in the heater, and (d) the power consumed by the heater.

27-7. A generator which maintains a constant terminal voltage of 120 volts supplies a current of 16 amp to a group of 10 identical lamps in parallel. The line connecting the generator to the lamps has a resistance of 0.5 ohm. (a) What is the voltage at the lamps? (b) What is the resistance of each lamp?

27-8. Figure 27-15 is a diagram of a part of an electric circuit. If the current in the 6-ohm resistor is 3 amp, find the following quantities: (a) the reading of the voltmeter connected between C and D; (b) the current in the 8-ohm resistor; (c) the reading of the ammeter placed between B and C; (d) the potential difference between points A and B; and (e) the current in the 20-ohm resistor.

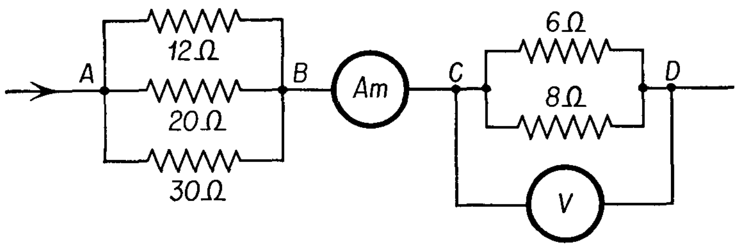

Fig. 27-15

27-9. Two lamps, each rated at 40 watts and 120 volts, are used as resistors in a circuit. If the two lamps are connected in series, (a) what is their combined resistance and (b) how much power is dissipated by these lamps when the voltage across the two is 120 volts?

27-10. A battery has an emf of 6 volts and an internal resistance of 3 ohms. What will be the current delivered to a 100-ohm resistor connected to the terminals of the battery?

27-11. When a generator is delivering a current of 50 amp, its terminal voltage is 110 volts. When the same generator delivers a current of 100 amp, its terminal voltage is 100 volts. What are the values of (a) the emf and (b) the internal resistance of this generator?

27-12. When a storage battery is delivering a current of 20 amp to a load, its terminal voltage is 5.9 volts. When the same battery is being charged by a generator at a current of 5 amp, its terminal voltage is 6.05 volts. What are the values of (a) the internal resistance and (b) the emf of the battery?

27-13. A generator with an emf of 120 volts and an internal resistance of 5 ohms delivers a current of 3 amp to a motor whose internal resistance is 2 ohms. (a) What is the back emf of the motor? (b) What is the mechanical power delivered by the motor? (c) What is the efficiency of the motor? (d) What is the over-all efficiency of the system?

27-14. An ammeter reads a full-scale deflection of 1 amp and has a shunt resistance of 0.1 ohm. The galvanometer from which it is constructed gives a full-scale deflection when a current of 100 μamp passes through it. What is the resistance of the galvanometer?

27-15. It is desired to construct a multimeter with ammeter ranges of 0.01 amp, 0.1 amp, and 1.0 amp and with voltmeter ranges of 0.1 volt, 1.0 volt, and 10 volts from a galvanometer having a full-scale deflection at a current of 100

μamp and an internal resistance of 50 ohms. Find the value of the shunt or multiplier resistance in each case.

27-16. A voltmeter having a resistance of 100,000 ohms and a full-scale deflection of 1 volt is connected across a 50,000 ohm-resistor. The voltmeter indicates a potential difference of 0.75 volt. What is the potential difference between the terminals of the resistor when the voltmeter is disconnected?

27-17. An ammeter which has a resistance of 50 ohms and a full-scale deflection of 0.1 milliampere (abbreviated ma) is connected in series with a circuit consisting of a cell and a 200-ohm resistor. The meter reads 0.08 ma. What is the current in the circuit when the milliammeter is removed?

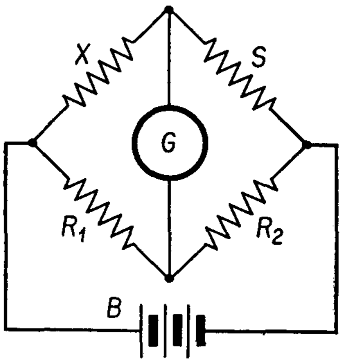

Fig. 27-16

27-18. In the Wheatstone bridge of Figure 27-16, the standard resistor S is 100 ohms, while the resistor R_1 is 150 ohms and R_2 is 50 ohms at balance. (a) What is the value of the unknown resistor X? (b) Find the current supplied by the battery whose emf is 6 volts and whose internal resistance is 3 ohms.

27-19. A battery has an internal resistance of R ohms. How large a resistor should be connected across its terminals in order that the greatest amount of heat is generated in the resistor?

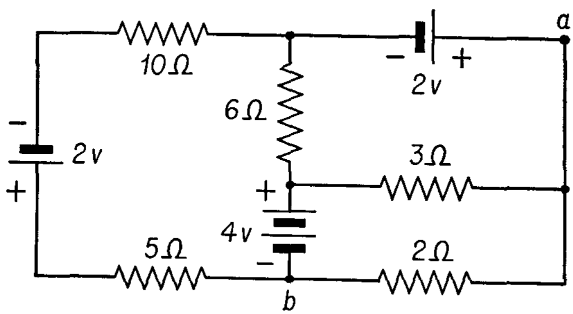

Fig. 27-17

27-20. Three 6-volt storage batteries are connected in parallel to a resistor of 100 ohms. The internal resistances of the batteries are 1 ohm, 3 ohms, and

10 ohms, respectively. (a) Find the heat generated in the 100-ohm resistor. (b) Find the current delivered by each of the storage batteries.

27-21. In the circuit of Figure 27-17, find the current through the 6-ohm resistor and the difference in potential $V_b - V_a$ between the points a and b.

27-22. A d-c generator having an emf of 110 volts and an internal resistance of 5 ohms is used to charge a 60-volt bank of storage batteries, whose internal

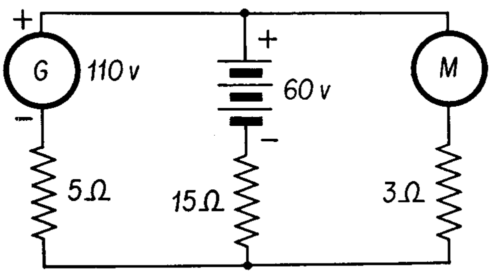

Fig. 27-18

resistance is 15 ohms, and to drive a motor whose back emf is 85 volts and whose internal resistance is 3 ohms, as shown in Figure 27-18. (a) Find the current supplied by the generator. (b) Find the charging current delivered to the batteries. (c) Find the mechanical power delivered by the motor.

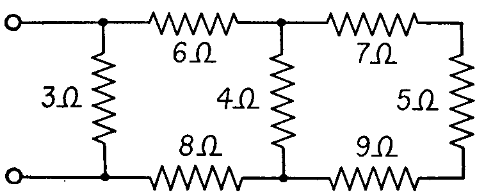

Fig. 27-19

27-23. Find the single resistance equivalent to the network of Figure 27-19.