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Physics, Chapter 29: The Magnetic Field

Henry Semat
City College of New York

Robert Katz
University of Nebraska-Lincoln, rkatz2@unl.edu

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29

The Magnetic Field

29-1 Natural and Permanent Magnets

Natural magnets, called lodestones, have been known since ancient times. The lodestone, a magnetic oxide of iron called magnetite (Fe_3O_4), was mentioned by Thales of Miletus. By the eleventh century the magnetic compass was known to the Chinese, and in the twelfth century references to
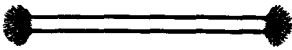

Fig. 29-1 Iron filings cling to the poles of a bar magnet.

the compass were made in Western Europe. The lodestone is capable of attracting pieces of iron and of imparting permanent magnetism to other pieces of iron so that these too could attract iron filings. If an iron bar is magnetized, as the result of being near a piece of lodestone, and is then dipped into iron filings, the filings will cling mostly to the ends of the bar, as shown in Figure 29-1. These ends are called the *poles* of the magnet.

Fig. 29-2 A simple magnetic compass needle.

The use of the magnet as a compass depends upon the fact that if a permanent magnet is suspended so that it can swing freely in a horizontal plane, the magnet will set itself in nearly a north-south direction. The end which points toward the north is called the north-seeking pole, or simply the *north pole*, while the other end is called the *south pole*. A line joining the south to the north pole is called the *axis* of the magnet. A simple magnetic compass is shown in Figure 29-2. When such a compass is mounted upon a sheet of cork and is floated in water, the cork sheet remains at rest, implying that the net force on the compass is zero.
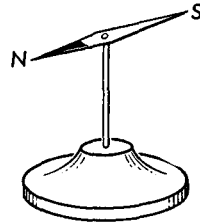
A ship's compass, a somewhat more elaborate instrument, is shown in Figure 29-3. It consists of a compass card fastened to a set of parallel magnets with their north poles facing in the same direction and mounted
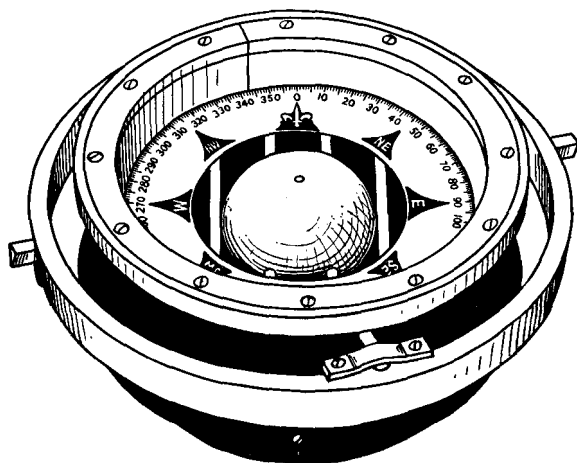

Fig. 29-3   A ship's compass. The compass card is mounted on four parallel magnets.

so that the whole assembly rotates freely in a horizontal plane. The case housing the compass is mounted in a set of bearings so that it can remain horizontal even when the ship rolls and pitches. A marker on the case housing the compass shows the direction in which the ship is headed.

29-2   Coulomb's Law of Force

Suppose that a bar magnet is suspended from its center by a string so that it can swing freely in a horizontal plane. If the north pole of a second bar magnet is brought near the north pole of the first one, there will be a force of repulsion between these poles. The suspended magnet will move so that its north pole goes away from the north pole of the approaching magnet. On the other hand, there will be a force of attraction when the north pole of one magnet approaches the south pole of the other one. In his book, *De Magnete*, William Gilbert (1540–1603) first showed that the earth could be considered as a huge spherical magnet with two poles, and that the compass needle was oriented by the forces exerted by the earth's magnetism.

About two centuries later Charles Augustin de Coulomb (1736–1806) measured the law of force between the poles of two magnets. A long thin magnet was suspended from a fine wire, and the pole of another thin magnet was brought near a like pole of the suspended magnet, as shown in Figure

29-4. Because of the force between these two poles, the suspended magnet experienced a torque which caused it to rotate about the wire as an axis, thus twisting the wire. Coulomb studied the force between the poles of two magnets as a function of the distance between them; he also studied the force between magnets of different strengths, keeping the distance between poles constant. He found that the force between two poles of two magnets varied inversely with the square of the distance between them, and varied directly with the strengths of the poles. The results of Coulomb's experiments could be summed up by stating that *the force between the two magnetic poles is proportional to the product of the strengths of the poles and inversely proportional to the square of the distance between them,* or

$$F = k_1 \frac{p_1 p_2}{r^2}, \qquad \text{(29-1)}$$

Fig. 29-4   Coulomb type torsion balance. *NS*, magnet of pole strength p_1, is suspended from elastic fiber. $N'S'$ is another magnet of pole strength p_2.

in which p_1 is the pole strength of the first magnet, p_2 is the pole strength of the second magnet, r is the distance between the poles, k_1 is a constant of proportionality, and F is the force between them. The force between the two poles is directed along the line joining the two poles; it is attractive between unlike poles and repulsive between like poles.

Following the analogy of Coulomb's law for electric charges, we may write this proportionality in the form of an equation, but we must first define either the constant of proportionality k_1 or the magnitude of the unit pole. As before, in the *cgs* system of units, the constant of proportionality k_1 is set equal to 1, when the medium is vacuum, and the subsequent equation is used to define a *unit pole*. The units based upon this equation are then called the *cgs electromagnetic units* (abbreviated emu). In the form of an equation, we have

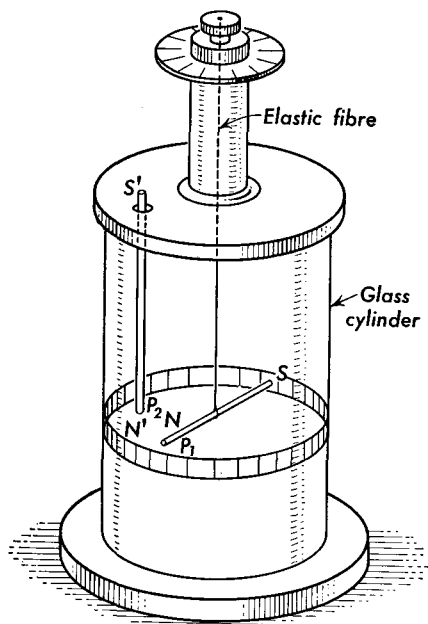cgs emu $$\mathbf{F}_2 = \frac{p_1 p_2}{r^2} \mathbf{1}_r \qquad \text{(29-1a)}$$

for the force in dynes between two poles in vacuum (or in air for which this equation is very nearly correct). In Equation (29-1a) $\mathbf{F}_2$ is the force on the pole p_2 in dynes, r is the distance between the poles in centimeters, and the unit vector $\mathbf{1}_r$ is directed from the pole p_1 to the pole p_2. The pole strength is expressed in unit poles, writing a north pole as positive and a south pole as negative. *A cgs unit pole is* implicitly *defined* in Equation (29-1a) *as one which, placed one centimeter from a like pole in vacuum, will repel it with a force of one dyne.*

In the mks system of units, the constant of proportionality is written as $\dfrac{1}{4\pi\mu_0}$, so that Coulomb's law becomes

mks units
$$\mathbf{F}_2 = \frac{1}{4\pi\mu_0} \frac{p_1 p_2}{r^2} \mathbf{1}_r.$$
(29-1b)

The constant μ_0 is assigned the value $4\pi \times 10^{-7}$, for reasons which must be deferred until a subsequent chapter. Equation (29-1b) implicitly defines an *mks unit pole* called a *weber*. In Equation (29-1b) $\mathbf{F}_2$ is the force on pole p_2 expressed in newtons, p_1 and p_2 are expressed in webers, and r is expressed in meters. *An mks unit pole of strength one weber is one which, placed one meter from a like pole in vacuum, will repel it with a force of* $10^7/(4\pi)^2$ *nt.* This is a force of $10^{12}/(4\pi)^2$ dynes. From Equation (29-1a) two cgs unit poles placed 1 m apart repel each other with a force of 10^{-4} dyne. Thus we see that

$$1 \text{ weber} = 10^8/(4\pi) \text{ cgs poles.} \tag{29-2}$$

In a form appropriate to Equation (29-1b) the value of μ_0 may be given as

$$\mu_0 = 4\pi \times 10^{-7} \frac{\text{weber}^2}{\text{nt m}^2}. \tag{29-3}$$

As the theory of magnetism is developed, other dimensions will be stated for μ_0 which are equivalent to the dimensions given above.

The properties of a magnet and of magnetic poles appear to be similar to the properties of electric charge, and yet there are extremely significant differences. For example, when a long magnetized needle is broken in two, new poles appear at the broken ends, so that each piece of the needle is observed to contain a north pole and a south pole. *An isolated magnetic pole has never been observed in nature.* Poles always occur in pairs. In calculating the forces on a magnet due to the presence of other magnets, we must always take into account the forces due to the north and the south poles of the magnets generating the force.

In a Coulomb type of experiment however, it is possible to use magnets long enough so that when two poles are placed near each other, the other poles are far enough removed so that they do not appreciably affect the result. The pole of a magnet is simply a small region of the magnet which acts as a center of force rather than the location of isolated magnetic charge. In spite of the fact that we shall find it convenient to speak of the magnetic field in terms of magnetic poles, we must always bear in mind that the smallest element of magnetism observed physically is not a magnetic pole but a magnetic dipole.

29-3   Intensity of Magnetic Field

Because of the effect that a magnet produces on magnetic materials placed anywhere in its neighborhood, we can say that a *magnetic field* exists in the neighborhood of a magnet. We can make a quantitative statement of the intensity of the magnetic field at some point in space by placing the north pole of a long, thin, needlelike magnet at this point. If the pole strength of this magnet is p, its north pole will experience a force **F**. The *magnetic field intensity* **H** at this point is defined as the force **F** divided by the pole strength p, or, in the form of an equation,

$$\mathbf{H} = \frac{\mathbf{F}}{p}. \qquad\qquad \textbf{(29-4)}$$

The *magnetic field intensity* **H** is a vector quantity, whose magnitude is given by the magnitude of the force on a unit north pole, and whose direction is the direction of the force on a north pole. Algebraically, it is conventional to represent a north pole as a positive pole and a south pole as a negative pole. In cgs emu the force is stated in dynes, the pole strength is cgs unit poles, and H is given in *oersteds*. If the intensity of the magnetic field H is known at a particular point in space, the force that would be experienced by each pole of a magnet placed at this point is given by Equation (29-4). For example, if the north pole of a long thin magnet having a pole strength of 2 cgs poles is placed at a point where the intensity is 45 oersteds, the force on it is 90 dynes.

Illustrative Example. A bar magnet 24 cm long has a pole strength of 800 cgs unit poles. Calculate the intensity of the magnetic field at point A 16 cm from the magnet, measured along the line which is the perpendicular bisector of the bar.

Referring to Figure 29-5, we note that the point A is 16 cm from the center C of the bar and 20 cm from each end, N and S. For purposes of calculation, let us imagine a long thin magnet of unit pole strength with its north pole at A,

and apply Coulomb's law to calculate the force exerted on it by each of the poles of the bar magnet. The force of repulsion on the unit north pole at A due to the north pole N is, from Equation (29-1a)

$$F = \frac{800 \times 1}{400} \text{ dynes} = 2 \text{ dynes}$$

directed away from N along NA. The force of attraction on the unit north pole at A due to the south pole S is also 2 dynes but is directed toward S along AS. Adding these two forces vectorially, we get 2.4 dynes acting parallel to the axis of the magnet as the force on a unit north pole at A. The intensity of the magnetic field at A is therefore

$$H = \frac{2.4 \text{ dynes}}{\text{cgs pole}} = 2.4 \text{ oersteds.}$$
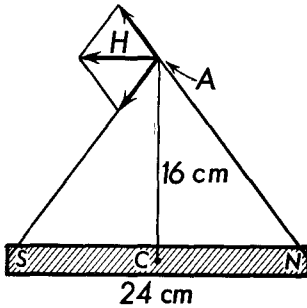

Fig. 29-5

If the north pole is taken away and if a south pole is placed at A, it will experience an equal force in the opposite direction. A magnet so small that it can be considered as though both poles are practically at A will line up parallel to the direction of the field at A, with the north pole pointing in the direction of the field. A magnetic field may therefore be mapped by small compass needles, which will orient themselves parallel to the direction of the field.

When iron filings are introduced into a magnetic field, the filings become magnetized. The magnetized filings orient themselves in the direction of the field. Figure 29-6 shows a typical arrangement of iron filings in the neighborhood of a bar magnet as seen in one plane.

As we shall see in a subsequent chapter, a magnetic field may also be generated by a wire carrying current, and, in fact, this is the most common method for the generation of a magnetic field. This has been recognized in the choice of the units of magnetic field intensity H in the mks system where H is generally stated in units of *amperes per meter*. Until we have developed the concepts appropriate to the generation of the magnetic field by current in a wire, we shall represent the magnetic field in units of *newtons per weber* in the mks system of units, a form appropriate to the definition of H in Equation (29-4). The relationship between the mks and cgs units of the magnetic field intensity H may be stated as

$$1 \frac{\text{nt}}{\text{weber}} = 4\pi \times 10^{-3} \text{ oersted.} \tag{29-5}$$

This conversion factor may be verified by recalling that
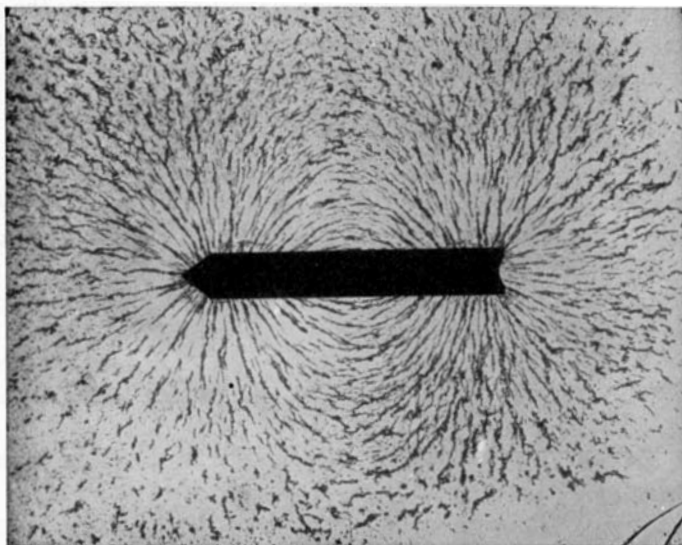
$$1 \text{ nt} = 10^5 \text{ dynes,}$$

Fig. 29-6 Pattern formed by iron filings in the neighborhood of a bar magnet.

and $\qquad$ 1 weber $= \dfrac{10^8}{4\pi}$ cgs poles.

Thus $\qquad$ $1\ \dfrac{\text{nt}}{\text{weber}} = 1\ \dfrac{\text{nt}}{\text{weber}} \times \dfrac{10^5\ \text{dynes}}{1\ \text{nt}} \times \dfrac{1\ \text{weber}}{10^8/4\pi\ \text{cgs pole}}$

$$= 4\pi \times 10^{-3}\ \dfrac{\text{dyne}}{\text{cgs pole}}$$

$$= 4\pi \times 10^{-3}\ \text{oersted.}$$

29-4   Magnetic Moment

If a bar magnet of length s and pole strength p is placed in a uniform magnetic field of intensity H, its north pole will experience a force Hp in the direction of the field, while its south pole will experience an equal force in the opposite direction, as shown in Figure 29-7(a). If the magnet is not parallel to the field, the magnet will experience a torque tending to rotate it so that its alignment is parallel to the field. The magnet will experience two forces equal in magnitude but opposite in direction, so that the net force on the magnet is zero.

Following the terminology developed in electrostatics where we called an assemblage of two equal and opposite charges an electric dipole, we call a magnet with its two equal and opposite poles a *magnetic dipole*. The *magnetic dipole moment* **m** is a vector quantity directed from the south pole

of the magnet to its north pole and is of magnitude given by the product of the pole strength p by the separation of the poles s. In the form of an equation the magnitude of the dipole moment is given by
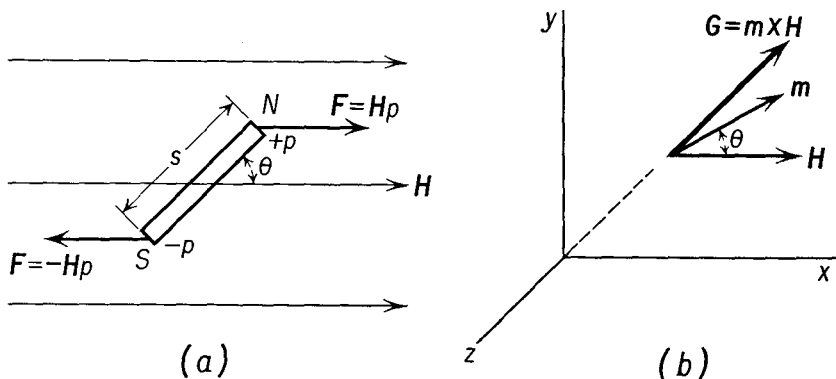
$$m = ps. \tag{29-6}$$

Fig. 29-7 (a) Forces on a bar magnet in a uniform magnetic field. The plane of the paper is the x-y plane. (b) When the magnetic moment vector **m** and the magnetic field **H** lie in the x-y plane as shown, the torque **G** on the magnet is parallel to the z axis.

If **s** is the displacement vector directed from the south pole of the magnet to its north pole, the magnetic dipole moment vector **m** is given by the equation

$$\boxed{\mathbf{m} = p\mathbf{s},} \tag{29-6a}$$

which may be compared to the corresponding definitions in electricity given in Equation (25-11). The cgs electromagnetic units of magnetic dipole moment are *cgs pole centimeters*, while the mks units of dipole moment may be expressed as *weber meters*.

Let us calculate the magnitude of the torque G about the center of the magnet due to the effect of the magnetic field **H** upon the magnetic dipole **m**, as shown in Figure 29-7(b). If the dipole-moment vector makes an angle θ with **H**, the torque is

$$G = Hps \sin \theta,$$

or, in terms of the magnetic moment m as defined in Equations (29-6), the torque is given by

$$G = Hm \sin \theta. \tag{29-7}$$

As shown in the figure, the torque tends to rotate the magnet in the clockwise direction. Following the right-hand rule, the torque vector is in the

$-z$ direction. Using the notation of the vector product developed in Section 7-4, we may write

$$\boxed{\mathbf{G} = \mathbf{m} \times \mathbf{H}.} \tag{29-7a}$$

We may apply this result to the measurement of the magnetic field. If a small bar magnet of moment of inertia I is suspended in a magnetic field H, the torque on the magnet will be given by Equation (29-7), and if the angle made by the magnet with the field direction is small, we may replace $\sin \theta$ by θ. Observing that the direction of the torque vector is opposite to the direction of the angular displacement, we obtain

$$G = -Hm\theta \tag{29-8}$$

By comparing Equation (29-8) with Equation (12-2), we see that the magnet is subjected to an elastic restoring torque which is proportional to the angular displacement θ and opposite in direction. The magnet will therefore vibrate in simple harmonic motion. The period of the motion T will be given by Equation (12-17) as

$$T = 2\pi \sqrt{\frac{I}{mH}}. \tag{29-9}$$

Thus the magnetic field intensity H may be determined by measuring the period of vibration, T of a magnet of known magnetic moment M and moment of inertia I.

Illustrative Example. A bar magnet 8 cm long has a mass of 20 gm and a pole strength of 60 cgs unit poles. Determine (a) its magnetic moment and (b) its period when placed in a field whose intensity is 100 oersteds.

(a) The magnetic moment of the bar magnet is, from Equation (29-6),

$$m = 60 \text{ poles} \times 8 \text{ cm} = 480 \text{ pole cm.}$$

To determine the period it is necessary to find the moment of inertia. The moment of inertia of a uniform bar about an axis through its center of gravity is given by the expression

$$I = \tfrac{1}{12}ML^2,$$

where M is the mass of the bar and L is its length. Therefore

$$I = \tfrac{1}{12} \times 20 \text{ gm} \times 64 \text{ cm}^2 = 106.7 \text{ gm cm}^2.$$

(b) Substituting these values in Equation (29-9), we find

$$T = 2\pi \sqrt{\frac{106.7}{480 \times 100}} \text{ sec} = 0.296 \text{ sec.}$$

29-5   Atomic Magnetic Moments

When a small magnet is placed in a uniform magnetic field, it experiences a torque. When the magnet is placed in a *nonuniform* field, the field acting at the south pole of the magnet may be given by $\mathbf{H}_s$, while that acting at the north pole of the magnet may be given by $\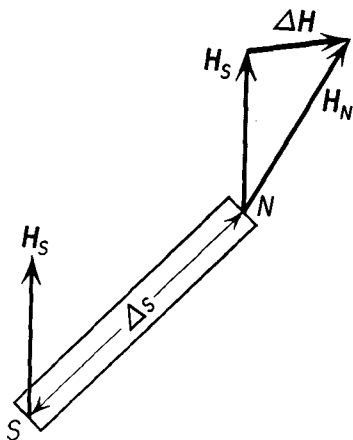mathbf{H}_n = \mathbf{H}_s + \Delta\mathbf{H}$, as shown in Figure 29-8, where $\Delta\mathbf{H}$ is the vector difference between the fields at these two poles. The net force on the magnet is due to the change in the field $p\,\Delta\mathbf{H}$, for if p is the pole strength of the magnet, the force on the magnet is

$$\mathbf{F} = p\mathbf{H}_n - p\mathbf{H}_s = p\,\Delta\mathbf{H}.$$

Let us suppose that the magnet is physically small, of length Δs. We multiply and divide the right-hand side of the equation above by the quantity Δs to find

$$\mathbf{F} = p\,\Delta s\,\frac{\Delta\mathbf{H}}{\Delta s}.$$

Fig. 29-8

The product $p\,\Delta s$ is the magnetic moment m of the magnet, while the quantity $\Delta\mathbf{H}/\Delta s$ is the rate of change of the magnetic field directed from the south to the north pole. Thus we have, in the limit of extremely small magnets,

$$\mathbf{F} = m\,\frac{\Delta\mathbf{H}}{\Delta s} = m\,\frac{d\mathbf{H}}{ds}, \tag{29-10}$$

where the direction in which we take the derivative of the magnetic field intensity is the direction of the magnetic moment.

A method of measuring atomic magnetic moments devised by Stern and Gerlach (1921) was to generate a beam of atoms, to send them through a nonuniform or inhomogeneous magnetic field of known inhomogeneity, and to observe the deflection of the beam. In the Stern-Gerlach experiment a beam of silver atoms was obtained by heating silver to a high temperature in an oven in vacuum. A narrow beam of silver atoms coming from an oven O, Figure 29-9, after passing through defining slits S_1 and S_2, was allowed to pass through an inhomogeneous magnetic field and strike a plate P. The inhomogeneous field was produced by an electromagnet with specially designed pole pieces. One pole piece was in the form of a knife-edge, while the other pole piece had a channel cut in it parallel to the

knife-edge. From the pattern on the photographic plate, shown in Figure 29-10, and a knowledge of the speed of the particles, the magnetic moment
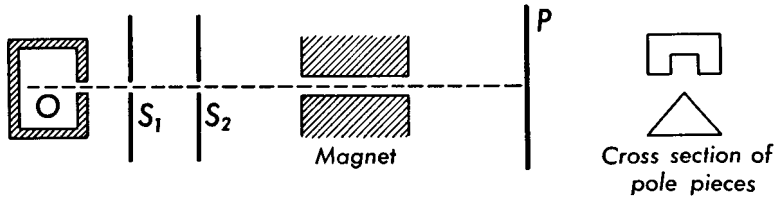

Fig. 29-9   Arrangement of apparatus in Stern–Gerlach experiment.

of silver could be calculated. The magnetic dipole moment of silver, m_{Ag}, is now known to be

$$m_{Ag} = 0.93 \times 10^{-20} \text{ emu.}$$

It may be remarked parenthetically that the magnetic moment of the silver atom, as measured in the Stern-Gerlach experiment, is actually the magnetic moment of the valence electron of silver. A glance at Table 5 of Appendix A shows that silver, in the normal state, has a single electron,

Fig. 29-10 Type of pattern made by a beam of silver atoms (a) without magnetic field on; (b) with magnetic field on.

(a)   (b)
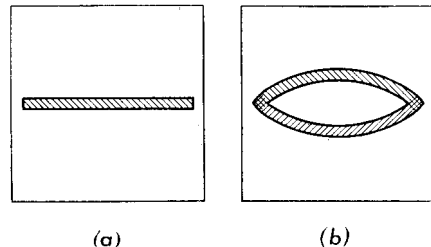
an s electron outside of a closed, stable configuration of the other 46 electrons. The sum of the magnetic moments of these 46 electrons is zero. The magnetic moment of an electron is ascribed to its intrinsic spin. One interesting result of the above experiment was that the silver atom was deflected in only one of two possible directions, as indicated by the splitting of the line into two lines. The interpretation of this is that a spinning electron, when in a magnetic field, cannot have any arbitrary direction in space. The spin of an electron may be considered as a vector quantity, since there is an angular momentum associated with it directed along the axis of spin. The magnetic moment is also a vector quantity directed along this axis. When an electron enters a magnetic field, its axis can take only one of two possible orientations, either parallel to the direction of the

magnetic field or antiparallel, that is, oppositely directed, depending upon the direction of its spin.   This limitation of the possible orientations of the axis of spin of an electron is called *space quantization.*

Measurements of the magnetic moment, by the methods of Stern and Gerlach and by other methods developed subsequently, have been made of atoms, molecules, and subatomic particles such as electrons, protons, neutrons, and many nuclei.   These measurements are an important part of our knowledge of the properties of matter.

29-6   Terrestrial Magnetism

We have already shown that a bar magnet, when suspended so that it can swing freely about a vertical axis, will ultimately come to rest in an approximately north-south position.   The use of a compass is based upon this
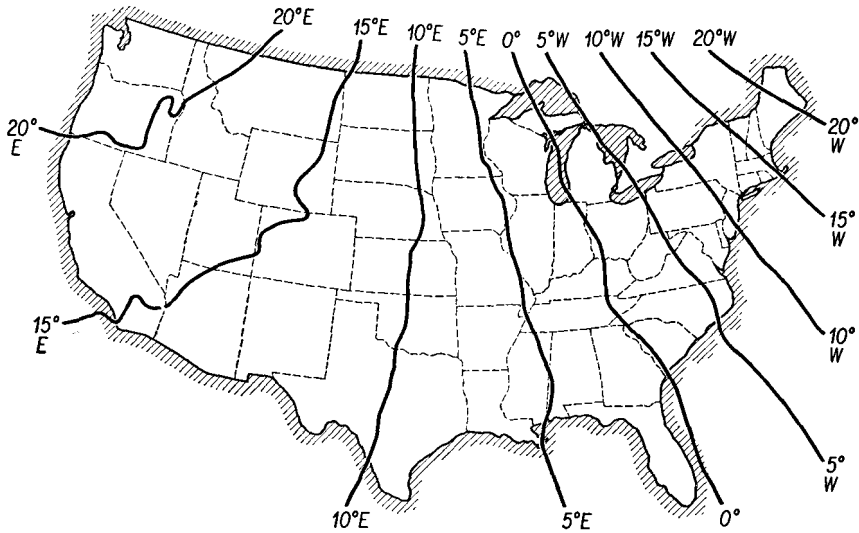

Fig. 29-11   Chart showing isogonic lines for the United States, 1955.   From the 1955 edition of chart 3077, of the U.S. Coast and Geodetic Survey, showing distribution of magnetic declination throughout the United States.   (Courtesy of Coast and Geodetic Survey.)

observation.   It is important, however, to know how much the magnetic north-south direction differs from the geographic or true north-south direction.   Various surveys have been made to determine this difference for various points of the earth's surface.   This difference is termed the *declination* of the magnetic from the geographic north-south direction.   It differs from point to point on the earth's surface and also is found to vary in a

complex manner with time. Charts are plotted for any region of the earth's surface with lines showing equal declinations, or *isogonic* lines, marked on them. One such chart is shown in Figure 29-11. In New York the declination is about 11°W, in Detroit it happens to be 3°W, while in San Francisco it is about 18°E.

The declination gives the direction of the horizontal component only of the earth's magnetic field at any one point. The fact that there is a vertical component to the earth's magnetic field can be shown by the following simple experiment. A long unmagnetized steel needle is mounted so that it can pivot about a horizontal axis through its center of gravity. It is balanced in the horizontal position and pointed in the direction in which a compass needle would point. The needle is now magnetized by means of a strong magnet and then allowed to swing freely. It will be found that the magnetized needle no longer balances in the horizontal position but dips with its north pole downward. The angle of dip varies from place to place over the earth. At New York the angle of dip is 72° with the horizontal, near the equator it is almost 0°, and at the magnetic pole in the Northern Hemisphere it is 90°. In this manner the earth's magnetic poles are located.

Fig. 29-12   Angle of dip.

When the dip needle is properly used as described above, its direction is that of the earth's magnetic field at the given place. The horizontal component H_H of the intensity of the earth's magnetic field can easily be determined by means of an oscillating horizontal magnet, as described above. By means of a simple vector diagram, illustrated in Figure 29-12, the total magnetic field intensity H and its vertical component H_V can be computed.

The earth behaves as though it contained a very large magnet with its south pole in the Northern Hemisphere located at and below Boothia Peninsula in northern Canada, and the other pole in the Southern Hemisphere near Ross Sea in Antarctica. The origin of the earth's magnetic field is at present unknown. The hypothesis that seems most reasonable is that the earth's magnetic field is produced by electric currents within the earth, probably within its core. If the above hypothesis is accepted, one must explain the source of these electric currents.

29-7   A Molecular Theory of Magnets

An insight into the origin of the magnetic properties of magnetic substances can be obtained from the following simple experiment. First magnetize a long steel needle or steel strip; then break the magnet in half. By dipping
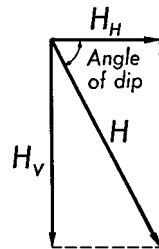
these parts in iron filings, it will be found, as sketched in Figure 29-13, that each half is itself a magnet. By testing the polarity of each part, it will be found that opposite poles appear on the faces where the break was made. If one of these parts is again broken in half, we will again have two magnets.

Fig. 29-13 A bar magnet broken in half forms two bar magnets.

This process can be repeated until the magnets become too small to handle. Each part broken from the original magnet will always be found to be magnetized. This suggests that a magnet is made up of very tiny magnets, perhaps of molecular size.

On this theory of magnetism, magnetic substances are assumed to contain little magnets of approximately molecular size. When the bar of steel is unmagnetized, these molecular magnets are oriented at random, as sketched in Figure 29-14(a), so that the bar shows no polarity and has no net magnetic moment. But when the bar of steel is brought into a magnetic field, its molecular magnets rotate until they line up with the field, as shown in Figure 29-14(b); the bar thus acquires magnetization, magnetic poles, and a magnetic moment. When the steel bar is removed from the magnetic field, most of these little magnets retain their orientation so that the steel bar remains permanently magnetized, although its magnetic mo-

Fig. 29-14 (a) Molecular magnets in an unmagnetized steel bar have random orientations. (b) When the steel bar is magnetized, the molecular magnets line up parallel to the external field, producing poles at the ends of the bar.

ment may not be as big as it was when in the magnetic field. Other substances, such as soft iron, lose most of their magnetism when taken out of the magnetic field.
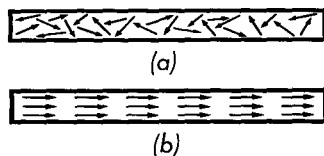
Other evidence in support of this molecular theory of magnetism may be cited. If a magnetized steel bar is dropped, hammered, or otherwise jarred, it loses some of its magnetism. If it is heated to a temperature beyond about 800°C, it loses its magnetic properties, probably because of the fact that the increased vibratory motion of the molecules destroys the orientation of the molecular magnets. This molecular theory is not, of course, the complete explanation. One would like to know the origin of these elementary magnets and why some substances are magnetic while others apparently are not magnetic. We shall consider this topic again after we have shown the relationship between electricity and magnetism.

29-8   A Theory of Magnetism

From Coulomb's law it is possible to develop a theory of magnetism based upon the magnetic dipole as the basic element of magnetism, and upon the auxiliary concept of the magnetic pole. Such a theory of magnetism is quite analogous to the theory of electrostatics developed in earlier chapters. In this theory the *magnetic field intensity* **H** is analogous to the electric field intensity **E**. The contribution to the magnetic field intensity at a point due to a magnetic pole of strength p can be written in an equation analogous to Equation (23-2) as

$$\mathbf{H} = k_1 \frac{p}{r^2} \mathbf{1}_r, \tag{29-11}$$

where, in the cgs system of units,

cgs units $k_1 = 1$

in vacuum, and **H** is expressed in oersteds when p is expressed in cgs unit poles and r is expressed in centimeters. In the mks system of units,

mks units $k_1 = \dfrac{1}{4\pi\mu_0}$ $\tag{29-12}$

in vacuum, and **H** is expressed in newtons per weber when p is expressed in webers, r is expressed in meters, and

$$\mu_0 = 4\pi \times 10^{-7} \frac{\text{weber}^2}{\text{nt m}^2}.$$

In many problems associated with the magnetic fields due to permanent magnets, it is helpful to define a magnetic potential similar to the electric potential defined in electrostatics. One essential difference between electrostatics and magnetostatics lies in the fact that there are no *conductors* of magnetism. Another difference which we have already emphasized is that the isolated magnetic pole is a useful mental construct rather than an experimentally observed entity.

To carry the analogy further, we utilize the concept of *magnetic lines of force*, developed as in Section 23-5 for the electric field. There are 4π lines of force emanating from a cgs unit north pole, and $1/\mu_0$ lines of force emanating from an mks unit north pole of pole strength 1 weber, as in Equations (23-5). Since the source of the magnetic field is the magnetic dipole, every volume element must contain as much positive magnetism as negative magnetism, so that the net number of magnetic lines of force emerging from any Gaussian surface is zero.

In dealing with the magnetic field in the material medium, we define the *magnetization* **M** as the *magnetic moment per unit volume*, just as, in a dielectric, we spoke of the polarization **P** as the electric moment per unit volume. The magnetic medium differs from the dielectric medium in that a permanent state of magnetization is commonly found in magnetic materials, while the polarization of most dielectric materials is only found in the presence of an electric field.

It is possible to produce a state of permanent electric polarization in some waxes by cooling them slowly in an electric field. These permanently polarized waxes are called *electrets*, and generate an electric field similar to the magnetic field generated by magnets. When electrets are left alone for a time, electric charges, in the form of atmospheric ions, collect on their surfaces in such a way as to neutralize the electric field generated by the electret. The electret may be revived by scraping away its surface, removing the collected ions. The fact that permanent magnets are not diminished in strength by exposure to the atmosphere may be cited as evidence that free magnetic poles do not exist in the atmosphere.

In addition to the permanent magnetization, we describe the induced magnetization in a magnetic material by the magnetic susceptibility χ_m (chi sub m) so that the induced magnetization could be written as

$$\mathbf{M} = \chi_m \mathbf{H}, \tag{29-13}$$

analogous to Equation (25-12).

Just as it was convenient to describe the electric field in the material medium by means of the electric displacement **D**, so it is convenient to describe the magnetic field in terms of the magnetic induction **B**, defined by equations analogous to Equation (25-24) as

$$\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M}. \tag{29-14a}$$

In the mks system of units, the magnetization **M**, the magnetic moment per unit volume, is expressed in webers per square meter, and the product $\mu_0\mathbf{H}$ is similarly expressed in webers per square meter. Thus the units of magnetic induction **B** are expressed in webers per square meter. We may describe the relationship between **B** and **H** in a linear medium through the equation

$$\mathbf{B} = \mu\mathbf{H}, \tag{29-15a}$$

where

$$\mu = \kappa_m \mu_0. \tag{29-16}$$

In these equations μ is called the *permeability* of the medium, and κ_m is called the relative permeability of the medium. The value of κ_m for vacuum is unity.

In the cgs system of units Equation (29-14a) may be re-expressed as

cgs units
$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}. \tag{29-14b}$$

The units of magnetic induction **B** are expressed in *gausses*. The terminology for this system of units is not so well worked out as in the case of mks units, so that each quantity in Equation (29-14b) is expressed in different units. Thus **H** is expressed in oersteds and **M** is expressed in cgs poles per square centimeter. The linear relationship between **B** and **H**, for a linear medium, is then expressed as

cgs units $$\mathbf{B} = \kappa_m \mathbf{H}.$$ (29-15b)

This neat analogy between the electric field and the magnetic field is very useful, but it neglects the most important of all magnetic effects, the generation of a magnetic field by an electric current. It was discovered by Hans Christian Oersted in 1820 that an electric current in a wire caused a compass needle to be deflected. Subsequent investigations have shown that the magnetic field exerts a force on a moving charge, and that a changing magnetic field induces a current in a wire. The theory outlined above must be supplemented to include these effects.

In contrast to the case of the electric field in which the electric displacement **D** plays a minor role, important in simplifying calculations but not at the foundation of electrostatics, the magnetic induction **B** is of great significance, for, as we shall see, **B** describes the effects of the field on a moving charge, while **H** describes the effects of the field upon a pole. From a fundamental physical viewpoint the pole is a property of bulk matter and is not a fundamentally significant magnetic parameter. Since isolated poles do not exist in nature, we are forced to relegate the magnetic field intensity **H** to the role of an auxiliary quantity and to think of the magnetic induction **B** as the fundamental magnetic-field vector. The sources of the magnetic field are today understood to be electric currents and the magnetic dipole moments of the fundamental physical particles, while the probe by means of which we are enabled to examine the magnetic field is either a moving charged particle or an elementary magnetic dipole.

Problems

29-1. A bar magnet having a pole strength of 400 cgs poles and a length of 5 cm is placed so that its south pole is at the origin and its north pole is located at the point whose coordinates are (5 cm, 0). Find the magnitude and direction of the force exerted by this magnet on the north pole of a long, slender, needlelike magnet whose pole strength is 20 cgs poles, when that pole is placed at a point whose coordinates are (a) (0, 10 cm), (b) (-10 cm, 0), and (c) (5 cm, 3 cm).

29-2. A bar magnet having a pole strength of 10^{-5} weber is placed along the y axis so that its south pole is at the origin and its north pole is located at $y = 0.1$ m. Find the magnitude and direction of the force exerted by this magnet on the north pole of a needlelike magnet of pole strength 10^{-6} weber when that north pole is located at a point whose coordinates are (a) (0, -0.2 m), (b) (0.24 m, 0), and (c) (0.03 m, 0.05 m).

29-3. Two identical bar magnets are placed side by side with their axes parallel and separated by a distance of 10 cm. Find the magnitude and direction of the force between them when like poles face each other. The pole strength of each magnet is 300 cgs poles, and the length of each magnet is 20 cm.

29-4. Repeat the calculation of Problem 29-3 in the case that unlike poles face each other and the pole strength of the magnets is 10^{-5} weber. Perform your calculations in mks units.

29-5. What is the intensity of the magnetic field at a point 20 cm from the north pole of a bar magnet, measured along the axis of the magnet, if the magnet is 30 cm long and has a pole strength of 400 cgs poles?

29-6. What is the magnetic field intensity generated by the bar magnet at each of the points indicated in Problem 29-1?

29-7. What is the magnetic field intensity generated by the bar magnet at each of the points indicated in Problem 29-2?

29-8. A bar magnet 4 cm long, having a pole strength of 12 cgs poles, is placed in a uniform magnetic field with the axis of the magnet perpendicular to the direction of the magnetic field. The intensity of the magnetic field is 45 oersteds. Determine the torque acting on this magnet.

29-9. A small compass needle oscillates with a period of 2.5 sec when placed at a point where the horizontal component of the earth's magnetic field is 0.2 oersted. Determine the magnetic moment of the compass needle if its moment of inertia about the axis of rotation is 24 gm cm^2.

29-10. A bar magnet 6 cm long is suspended by a wire passing through its center. The magnet is placed in a uniform field with the axis of the magnet perpendicular to the direction of the field. The torque, as measured by the twist of the wire, is 1,500 dynes cm. Determine (a) the magnetic moment of the magnet and (b) the pole strength. The intensity of the field is 50 oersteds.

29-11. A small bar magnet has a period of oscillation of 2 sec when pivoted in a uniform magnetic field of 36 nt/weber. When it is used to measure the intensity of a second magnetic field, the period is found to be 0.40 sec. Determine the intensity of the second magnetic field.

29-12. A uniformly magnetized rod has a pole strength of 10 webers. The magnet has a cross-sectional area of 1 cm^2 and a length of 10 cm. (a) What is the magnetic moment of the magnet? (b) What is the magnetization of the magnet?

29-13. By analogy with the electric field in a material medium (Sections 25-6 to 25-8), write a formula for the magnetic field intensity H due to a magnetic pole of pole strength p when that pole is immersed in a medium of relative permeability κ_m (a) in cgs units and (b) in mks units.

29-14. From the result of Problem 29-13, state Coulomb's law of force between two poles immersed in a medium of relative permeability κ_m (a) in mks units and (b) in cgs units.