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Physics, Chapter 30: Magnetic Fields of Currents

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Magnetic Fields of Currents

30-1 Magnetic Field around an Electric Current

The first evidence for the existence of a magnetic field around an electric current was observed in 1820 by Hans Christian Oersted (1777–1851). He found that a wire carrying current caused a freely pivoted compass needle

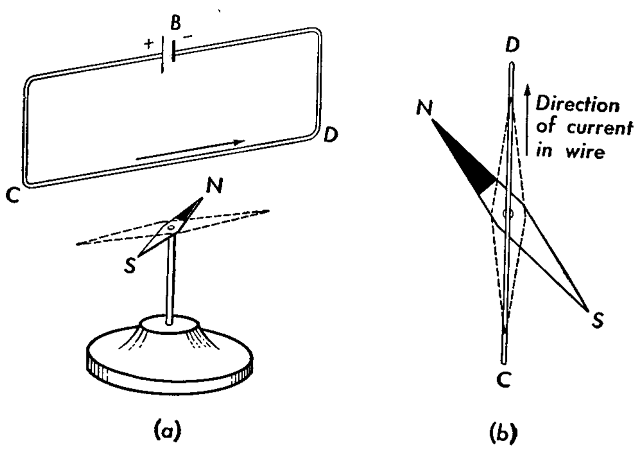

Fig. 30-1 Oersted's experiment. Compass needle is deflected toward the west when the wire CD carrying current is placed above it and the direction of the current is toward the north, from C to D.

in its vicinity to be deflected. If the current in a long straight wire is directed from C to D, as shown in Figure 30-1, a compass needle below it, whose initial orientation is shown in dotted lines, will have its north pole deflected to the left and its south pole deflected to the right. If the current in the wire is reversed and directed from D to C, then the north pole will be deflected to the right, as seen from above. In terms of the forces acting on the poles, these forces are clearly perpendicular to the direction of the current and to the line from the nearest portion of the wire to the pole itself.

The magnetic field in the neighborhood of a wire carrying current can be investigated either by exploring the region with a small compass or by using iron filings. When a wire carrying current is passed perpendicularly through a plane board and iron filings are sprinkled on the board, the

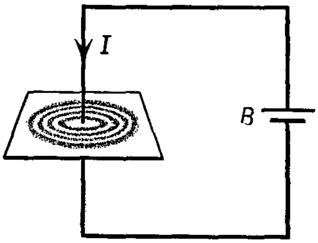

Fig. 30-2 Pattern formed by iron filings showing the circular magnetic field around a wire carrying current.

filings form a circular pattern, as shown in Figure 30-2. Thus the magnetic field generated by the wire carrying current is circular in a plane at right angles to the current. The circles are concentric, with their common center at the position of the wire. The direction of the magnetic field can be

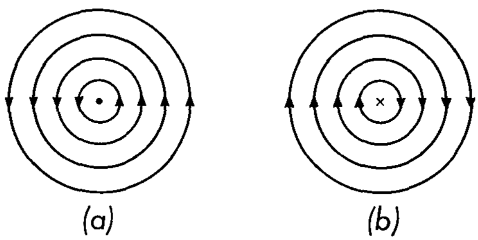

Fig. 30-3 Direction of the magnetic field around a wire (a) when the current is out of the paper; (b) when the current is into the paper. The dot represents a head-on view of an arrow while the cross represents a rear view.

determined with the aid of a small compass. If we look along the wire so that the current is coming toward us, the magnetic field is counterclockwise. If we draw a dot to represent a head-on view of an arrow and a cross to represent a rear view of an arrow, we may show the direction of the magnetic field, associated with current in a wire perpendicular to the plane of the paper when the current is coming toward the reader in Figure 30-3(a), and when the current is away from the reader in Figure 30-3(b). A small compass placed anywhere in the field will orient itself tangent to one of these circles with its north pole in the direction of the arrow.

When the wire carrying current is bent in the form of a circular loop, as shown in Figure 30-4, the direction of the magnetic field around each small portion of the wire may be determined from the above observations.

If the current in the loop is counterclockwise when viewed from the positive x axis, the direction of the magnetic field at the center of the loop is perpendicular to the plane of the loop and in the positive x direction.

A simple way of determining the direction of the magnetic field relative to the direction of the current is given by the *right-hand rule.* If the current is in a straight portion of the wire, then, if we imagine the thumb of the right hand to be placed along the wire and pointing in the direction of the current, the curled fingers of the right hand will point in the direction of the magnetic field. If the current flows in a circular path, then, if we imagine the fingers of the right hand curving in the direction of the current, the thumb will point in the direction of the magnetic field inside the coil, as in Figure 30-4.

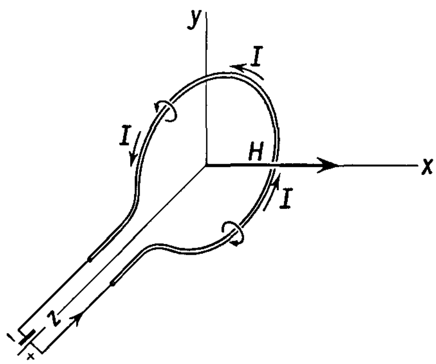

Fig. 30-4 Magnetic field produced by a current in a circular loop of wire. The magnetic field at the center is at right angles to the plane of the loop.

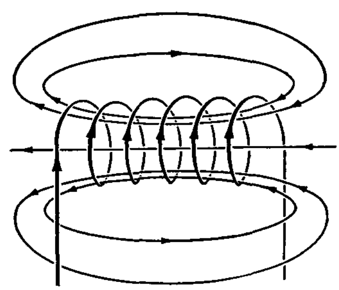

Fig. 30-5 Magnetic field due to a current in a long cylindrical coil or helix.

When, as in Figure 30-5, the wire is wound in the form of a helix which is very long in comparison with its diameter, the magnetic field outside the coil is very similar to that of a bar magnet, while the field inside the coil is uniform and parallel to the axis of the coil. At large distances the field produced by a circular loop of wire carrying current is indistinguishable from the field generated by a small magnet. According to a theory first presented by Ampère, it is possible to imagine that all magnetic effects are due to circulating currents, and to attribute the magnetic fields generated by permanent magnets to circulating currents within molecules. These currents are associated with orbital motions of electrons about some attractive center within the molecule.

30-2 Magnetic Field Generated by a Wire Carrying Current

As soon as Oersted's discovery was announced, scientists in Europe began repeating and extending these experiments. Interest in Oersted's results was aroused because this was the first time a force had been observed which was not directed along the line joining the two bodies but was perpendicular to this line. Since Newton had developed the concept of universal gravitation, both electrostatic and magnetostatic forces had been studied. All these forces obeyed an inverse square law and were directed along the line joining the bodies responsible for the forces. Biot and Savart (1820) determined the magnetic field produced by the current in a long straight wire. The experiments of Biot and Savart showed that the magnetic field intensity H at a distance a from a long straight wire carrying current I was directly proportional to the magnitude of the current and inversely proportional to the distance from the wire; that is,

$$H = k_2 \frac{2I}{a}, \tag{30-1}$$

where k_2 is a constant of proportionality. The direction of the magnetic field intensity was always normal to the wire and to the perpendicular from

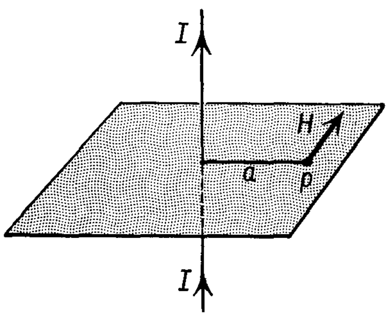

Fig. 30-6 The field at a point P due to a long straight wire is perpendicular to the plane formed by the wire and the normal a dropped from P to the wire. In this figure H is directed into the plane of the paper, as given by the right-hand rule.

the point where the field was being evaluated to the wire. The direction of the magnetic field was given by the right-hand rule, as shown in Figure 30-6.

The value of the constant k_2 depends upon the system of units used to measure H, I, and a. In the mks system of units, the current I is expressed in amperes, the distance a is expressed in meters, and the magnetic field intensity H is expressed in mks units, stated in Section 29-3 as newtons per weber. In this system of units the constant k_2 is $\frac{1}{4\pi}$, and Equation (30-1) becomes

$$H = \frac{I}{2\pi a}. \tag{30-1a}$$

In Section 29-3 we mentioned that the units of H in the mks system were commonly expressed as amperes per meter. The justification for expressing H in these terms is provided by Equation (30-1a). The units of H may be stated equivalently as newtons per weber or as amperes per meter, and the choice of units will depend upon which form is more convenient for the problem in hand.

In one form of cgs units called the *Gaussian* system of units, toward which we have been building our cgs unit system, the units of electrical quantities such as charge, current, electric field intensity, electric displacement, potential, and resistance are based upon the statcoulomb as the unit of charge, while units of magnetic quantities, the oersted and the gauss, are based upon the cgs unit pole. In this system the constant k_2 is $1/c$, where c is the velocity of light in centimeters/per second, given approximately as $c = 3 \times 10^{10}$ cm/sec. To simplify the presentation of the material of this chapter, all equations will be presented in the mks system of units.

Traditionally, Equation (30-1) has been used to define a new unit of current called the *electromagnetic unit of current*, by setting k_2 equal to 1. This unit is called the emu of current, or the *abampere*. Electromagnetic units of current will rarely be used in this book.

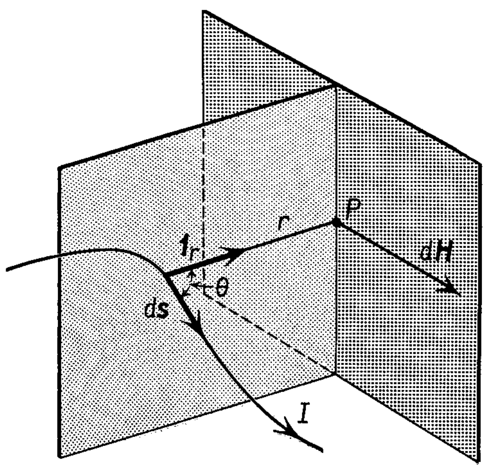

Fig. 30-7 The magnetic field intensity $d\mathbf{H}$ at any point P a distance r from an element $d\mathbf{s}$ of wire carrying current I is perpendicular to the plane of $d\mathbf{s}$ and $\mathbf{1}_r$, where $\mathbf{1}_r$ is the unit vector directed from $d\mathbf{s}$ to P. The element $d\mathbf{H}$ is shown coming out of the plane of the paper when $d\mathbf{s}$ and $\mathbf{1}_r$ lie in the plane of the paper.

30-3 Magnetic Field Generated by Current Element

The results of the experiments of Biot and Savart on the magnetic field around a wire carrying current form the basis of a fundamental physical law for the determination of magnetic fields produced by currents. If we consider any small element of wire of length ds in which there is a current I (see Figure 30-7), its contribution $d\mathbf{H}$ to the magnetic field intensity at a point P located a distance r from it, is given by

$$\boxed{d\mathbf{H} = \frac{I\, d\mathbf{s} \times \mathbf{1}_r}{4\pi r^2}.} \tag{30-2}$$

The unit vector $\mathbf{1}_r$ is directed from the current element to the field point P. The vector $d\mathbf{s}$ is tangent to the direction of the current. The direction of $d\mathbf{H}$ is perpendicular to both $d\mathbf{s}$ and $\mathbf{1}_r$. Equation (30-2) is sometimes called the law of Biot and Savart, and also *Ampère's law.*

The total magnetic field intensity $\mathbf{H}$ at a point P due to a given current distribution is found by summing vectorially (that is, integrating) the contributions $d\mathbf{H}$ from all the elements $d\mathbf{s}$ of the wire. For most geometrical arrangements of the conductor, this summation is beyond the reach of the calculus and must be carried out by numerical methods of integration or by use of an electronic computer. We shall consider only two cases, the flat circular coil and the long straight wire.

30-4 Magnetic Field of a Flat Circular Coil

Let us use Ampère's law to calculate the magnetic field intensity at the center of a single circular loop of wire carrying a current I. As shown in

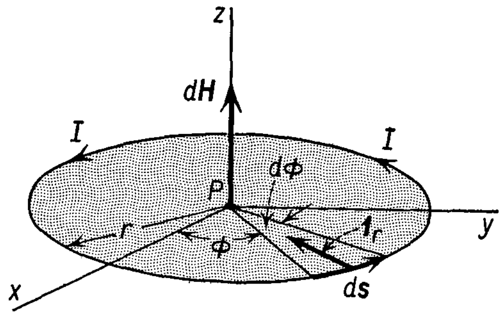

Fig. 30-8

Figure 30-8, each element of the loop of wire makes a right angle with the line drawn from that element to the field point P at the center of the circle. The direction of the magnetic field $d\mathbf{H}$ due to each element of the loop $d\mathbf{s}$ is in the positive z direction when the current is in the counterclockwise direction, as shown in the figure. Since the contribution to the magnetic field from every element of current is in the same direction, the problem of obtaining the vector sum of the contributions to the total magnetic field intensity is reduced to a scalar sum, and we may set up the problem in the form of an integral.

From Equation (30-2) we have

$$d\mathbf{H} = \frac{I\, d\mathbf{s} \times \mathbf{1}_r}{4\pi r^2}.$$

The factor $d\mathbf{s} \times \mathbf{1}_r$ is equal in magnitude to the product of the magnitude ds by the magnitude of $\mathbf{1}_r$ (which is unity), by the sine of the angle between them. Since the angle between the two vectors is 90°, the magnitude of this vector product is simply the magnitude ds. The direction of this vector product is in the $+z$ direction, so that we may represent it as

$$d\mathbf{s} \times \mathbf{1}_r = ds\, \mathbf{1}_z,$$

where $\mathbf{1}_z$ is a unit vector in the positive z direction. Thus we have

$$d\mathbf{H} = \frac{I\, ds}{4\pi r^2}\, \mathbf{1}_z,$$

where the magnitude of $d\mathbf{H}$ is the factor multiplying $\mathbf{1}_z$.

In order to carry out the integration, we must represent the variables of the problem in terms of some simple parameter. Let us measure the angular position of an element of current by the angle ϕ, from the x axis, positive in the counterclockwise direction. The angle subtended by the element of length ds is $d\phi$ such that

$$ds = r\, d\phi.$$

Thus we have

$$dH = \frac{Ir\, d\phi}{4\pi r^2},$$

and, integrating around the circumference of the circle by letting ϕ go from 0 to 2π, we find

$$H = \int_0^{2\pi} \frac{I\, d\phi}{4\pi r}$$

$$= \frac{I}{2r}.$$

The complete representation of the magnetic field intensity at the center of a single circular loop of wire of radius r carrying a current I in the counterclockwise direction is given by

$$\mathbf{H} = \frac{I}{2r}\, \mathbf{1}_z. \tag{30-3}$$

In Equation (30-3) the appropriate choice of units to represent H is amperes per meter when I is in amperes and r is expressed in meters.

In the event that we have a coil of N turns of wire wound as a flat circular coil rather than a single circular loop, each turn of the coil contributes a magnetic field at the center of the coil, as given by Equation (30-3). Since these field contributions are all in the same direction, the total magnetic field is N times that given by Equation (30-3). A circular coil of wire of N turns carrying current I in the counterclockwise direction, when viewed from the positive z axis, as in Figure 30-8, has at its center a magnetic field $\mathbf{H}$ given by

$$\mathbf{H} = \frac{NI}{2r}\mathbf{1}_z. \tag{30-4}$$

One of the early forms of current-measuring instruments was the *tangent galvanometer*, which consisted of a single circular loop of wire with a short compass needle placed at its center, as shown in Figure 30-9. When a current I was sent through the loop, it set up a magnetic field of intensity

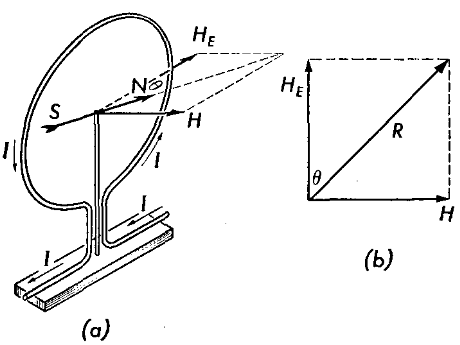

Fig. 30-9 (a) Tangent galvanometer. The plane of the circular coil is parallel to the earth's magnetic field. (b) The two magnetic fields which act on the N-pole of the small compass needle. H_E is the horizontal component of the earth's magnetic field; H is the intensity of the magnetic field produced by the current. R is the resultant of these two field intensities. $H = H_E \tan\theta$.

H at its center, producing a force Hp on each pole of the magnet. In the figure the force on the north pole is shown directed to the right, while that on the south pole is directed to the left. These two forces produced a torque causing the compass needle to rotate out of the plane of the coil. The horizontal component of the earth's magnetic field produced an opposite torque on the compass needle. The compass needle came to rest at some angle θ with the plane of the loop, at which angle the two opposing torques were equal in magnitude. A simple calculation shows that the current in the loop is proportional to the tangent of the angle θ, hence the name tangent galvanometer. This was the instrument refined by Lord Kelvin as the detector for use in the first successful operation of the Atlantic cable.

Illustrative Example. A circular coil of 30 closely wound turns of 15 cm average radius carries a current of 2 amp. Determine the magnitude and direction of the magnetic field at the center of this coil.

Assuming that the turns are so closely wound that the current in each one contributes the same amount to the intensity of the magnetic field at the center of the coil, the magnetic field intensity is of magnitude

$$H = \frac{NI}{2r}.$$

Substituting numerical values, we have

$$H = \frac{30 \text{ turns} \times 2 \text{ amp}}{2 \times 0.15 \text{ m}} = 200 \frac{\text{amp}}{\text{m}}.$$

Note that the *turn* is a dimensionless quantity. The direction of the magnetic field is perpendicular to the plane of the coil, as given by the right-hand rule.

30-5 Magnetic Field of a Long Straight Wire

The experimental result of Biot and Savart given in Equation (30-1) may be calculated from Equation (30-2) by integrating the contributions to the field from each element of the wire. The field point P is located a distance a from the wire, as shown in Figure 30-10. Let us measure the distance s along the wire from the foot of the perpendicular from P to the wire.

In terms of the angle θ between the element ds and the unit vector $\mathbf{1}_r$ from the element ds to the field point P, the distance r may be represented as

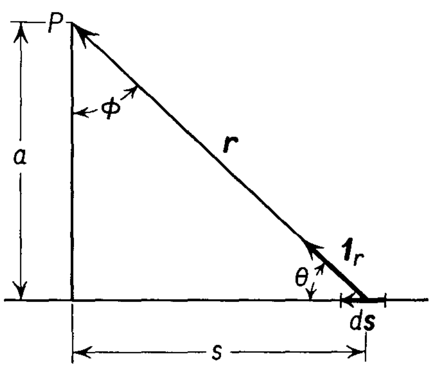

Fig. 30-10

$$r = \frac{a}{\sin\theta} = a \csc\theta.$$

To express the length of the element ds in terms of the distance a and the angle θ, we first note that the length s is given by

$$s = a \cot\theta,$$

so that, taking differentials to obtain the magnitude of ds,

$$ds = -a \csc^2\theta \, d\theta.$$

We note that contribution to the magnetic field from each element of wire is in the same direction at P; that is, it is directed out of the plane of the paper, toward the reader. If we call this direction the direction of z, then

$$d\mathbf{H} = + \frac{I\,ds \sin\theta}{4\pi r^2}\mathbf{1}_z.$$

Substituting appropriate expressions into the above equation, we find

$$dH = -\frac{Ia \csc^2\theta\, d\theta \sin\theta}{4\pi a^2 \csc^2\theta}$$

$$= -\frac{I \sin\theta\, d\theta}{4\pi a}.$$

Since all contributions to the magnetic field at P are in the same direction, we may find the magnetic field intensity at P by integrating between the limits of $\theta = \pi$ to $\theta = 0$, corresponding to limits on s of $s = -\infty$ to $s = +\infty$. Thus we have

$$H = \int_\pi^0 -\frac{I}{4\pi a}\sin\theta\, d\theta,$$

$$H = \frac{I}{2\pi a},$$

as previously stated in Equation (30-1a).

30-6 Field of a Solenoid and a Toroid

A solenoid is made by winding wire on a cylindrical form; the length s of the cylinder is generally much larger than its radius r. The adjacent turns of the wire are generally close together, as shown in Figure 30-11. The field inside the solenoid may be calculated from Ampère's law. The field is uniform over the entire region of the solenoid except near the ends, as

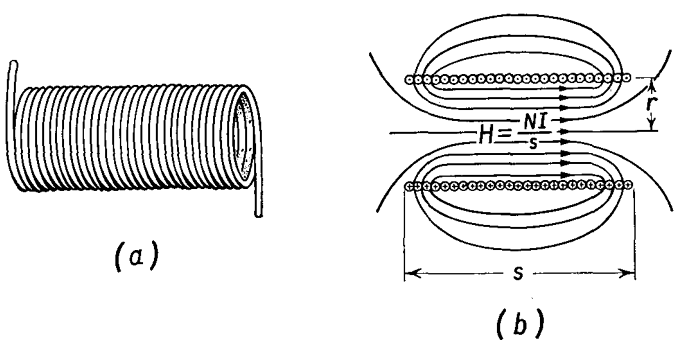

Fig. 30-11 (a) Windings of a solenoid. (b) Magnetic field intensity inside a solenoid. Current is coming out of the wires on top and going into the wires on bottom.

shown in the figure. The field intensity within a solenoid will be shown in Section 33-6 to be

$$H = \frac{NI}{s}, \tag{30-5}$$

where N is the number of turns of the solenoid, s is its length, and I is the current in it.

A toroid is made by winding wire on a ring or doughnut-shaped form called a *torus*. The field of the toroid is entirely confined to the space within the toroid. A toroid may be generated by bending a long straight solenoid

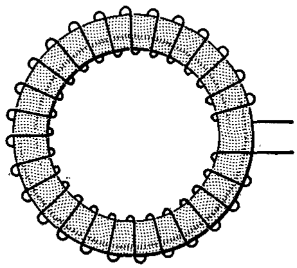

Fig. 30-12 Toroid.

in the form of a ring. The magnetic field intensity within the toroid is given by Equation (30-5), where s now represents the mean circumference of the ring, shown in Figure 30-12.

The solenoid and the toroid are frequently used as standard means of achieving known, uniform magnetic fields.

30-7 Equivalence of a Moving Charge and a Current

Our original definition of current was based on the flow of charges through a surface in a given time interval; it remains to be shown that a set of moving charges will produce the same magnetic effect as a current in a wire. This was first shown experimentally by H. A. Rowland in 1876. He used an ebonite disk having metallic sectors distributed near the rim of the disk. The metallic sectors were charged electrically, and the disk was set into rapid rotation. A magnetic needle suspended near the disk was deflected by the magnetic field set up by the moving charges. The direction of the deflection was the same as that which would have been produced by the current in a circular loop of wire coinciding with the rim of the disk. When the direction of rotation was reversed, the deflection of the magnetic needle was also reversed. More recently (1929) R. C. Tolman set a charged cylinder oscillating about its axis and observed that this produced an alternating magnetic field, the same as that produced in the neighborhood of an alternating current. These experiments show that, as far as the magnetic effect is concerned, a moving charge and a current in a wire are equivalent.

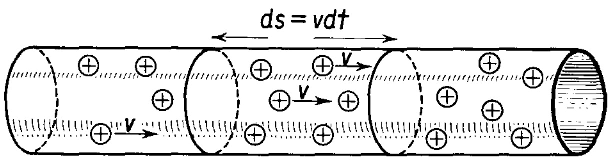

Fig. 30-13

Figure 30-13 shows a tube in which a current I consists of the motion of positive charges with uniform velocity v. From the definition of current

$$I = \frac{dq}{dt},$$

where dq is the quantity of charge which passes through a complete cross section of the tube in a time dt. The charge will traverse a length ds of the tube, where

$$ds = v\,dt.$$

Eliminating dt from these two equations yields

$$I\,ds = v\,dq.$$

In other words, a charge dq moving with velocity v may be considered to be equivalent to a current element of length ds carrying a current I.

We may find the magnetic field H generated by a moving charge q by substituting the above results into Equation (30-2). We find

$$\boxed{\mathbf{H} = \frac{q\mathbf{v} \times \mathbf{1}_r}{4\pi r^2}.} \tag{30-6}$$

30-8 Coulomb's Law and Ampère's Law

Let us consider the magnetic field generated by a charged particle mounted on an airplane. If we calculate the magnetic field generated by the moving charge from Equation (30-6), we note that, to an observer on the ground, the charge is moving with the velocity v of the airplane, but, to an observer on the airplane, the charge is moving with zero velocity. Thus an observer on the surface of the earth is able to detect a magnetic field due to the motion of the charged particle on the airplane, but an observer on the airplane cannot detect a magnetic field due to the charge on the airplane, for that charge has no motion with respect to him. By definition, the magnetic field intensity H in a particular reference frame is the force on a unit north pole at rest in that frame. If our formulation is to be consistent, we must expect that the force exerted by a charge upon a magnetic pole is the same when these are in relative motion, regardless

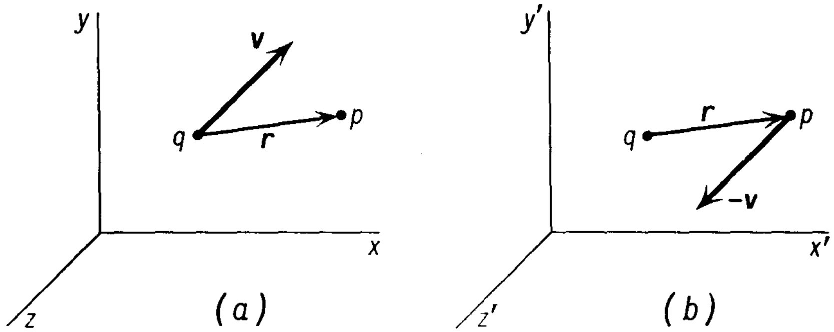

Fig. 30-14 (a) System of axes in which the magnetic pole is at rest. (b) System of axes in which the charge q is at rest.

of whether the pole is considered to be at rest or the charge is considered to be at rest.

In the coordinate frame in which the pole of strength p is at rest, as shown in Figure 30-14(a), the charge q is moving with velocity $\mathbf{v}$, and the magnetic field $\mathbf{H}$ at the position of the pole, due to the moving charge, is given by Equation (30-6) as

$$\mathbf{H} = q\frac{\mathbf{v}\times\mathbf{1}_r}{4\pi r^2}\cdot$$

The force on the magnetic pole at this point is therefore

$$\mathbf{F} = \mathbf{H}p = qp\frac{\mathbf{v}\times\mathbf{1}_r}{4\pi r^2}\cdot$$

If we consider the frame of reference to be fixed with respect to the charge q, as shown in Figure 30-14(b), we must consider the charge to be at rest and the pole to be moving with a velocity $\mathbf{v}' = -\mathbf{v}$. Substituting this into the above equation, and rearranging the position of q in the equation we find

$$\mathbf{F} = p(-\mathbf{v}')\times\frac{q\,\mathbf{1}_r}{4\pi r^2}$$

for the force exerted on the moving pole by the fixed charge. From the illustrative example of Section 25-8, we note that the terms grouped to the right of the cross in the above equation represent precisely the electric displacement **D** generated by the charge q at the position of the pole p, for

$$\mathbf{D} = \frac{q\,\mathbf{1}_r}{4\pi r^2}.$$

Thus the force on a pole p, moving with velocity **v** (dropping the prime) in a coordinate frame in which the electric displacement is **D**, is given by

$$\mathbf{F} = -p(\mathbf{v}\times\mathbf{D}).$$

Thus the force experienced by a moving magnetic pole consists of two parts. One part is due to the magnetic field **H**; this force on the pole is independent of its state of rest or motion. The second part depends upon the motion of the poles; this force is due to the velocity of the pole and to the electric displacement **D**. In the form of an equation, the force on a magnetic pole, moving with velocity **v** in a coordinate frame in which the magnetic field intensity is **H** and the electric displacement is **D**, is given by

$$\mathbf{F} = p(\mathbf{H} - \mathbf{v}\times\mathbf{D}). \tag{30-7}$$

Another interpretation of Equation (30-7) is that the force exerted on a magnetic pole by a charged particle may be calculated in one of two ways: (a) for the case in which the pole is considered to be at rest and the charge in motion, we can use Ampère's law; (b) for the case in which the charge is considered to be at rest and the pole is in motion, we can use Coulomb's law, utilizing the concept of electric displacement. In Rowland's experiment a moving charge was seen to deflect a compass needle. From Equation (30-7) we see that we must expect a compass needle to be deflected when the needle is moving in a region of space in which there is an electric

TABLE 30-1 PRINCIPAL EQUATIONS IN MKS AND GAUSSIAN UNITS

Equation	MKS	Gaussian	
(30-2)	$d\mathbf{H} = \dfrac{I\,d\mathbf{s}\times\mathbf{1}_r}{4\pi r^2}$	$d\mathbf{H} = \dfrac{I\,d\mathbf{s}\times\mathbf{1}_r}{r^2 c}$	Current element
(30-1a)	$H = \dfrac{I}{2\pi a}$	$H = \dfrac{2I}{ac}$	Straight wire
(30-5)	$H = \dfrac{NI}{s}$	$H = \dfrac{4\pi NI}{sc}$	Solenoid or toroid
(30-6)	$\mathbf{H} = \dfrac{q\mathbf{v}\times\mathbf{1}_r}{4\pi r^2}$	$\mathbf{H} = \dfrac{q\mathbf{v}\times\mathbf{1}_r}{r^2 c}$	Moving charge
(30-7)	$\mathbf{F} = p(\mathbf{H} - \mathbf{v}\times\mathbf{D})$	$\mathbf{F} = p\left(\mathbf{H} - \dfrac{\mathbf{v}}{c}\times\mathbf{D}\right)$	Pole

field, for example, if a compass needle moves between the plates of a charged capacitor.

TABLE 30-2 CONVERSION FACTORS RELATING MKS AND GAUSSIAN UNITS

Quantity	Symbol	MKS Unit	Gaussian Unit	
Pole	p	1 weber	$= \frac{10^8}{4\pi}$ unit pole	(emu)
Magnetic field intensity	H	$1 \frac{\text{nt}}{\text{weber}} = 1 \frac{\text{amp}}{\text{m}}$	$= 4\pi \times 10^{-3}$ oersted	(emu)
Charge	q	1 coul	$= 3 \times 10^9$ stcoul	(esu)
Current	I	1 amp	$= 3 \times 10^9$ statamperes	(esu)
Displacement	D	1 coul/m^2	$= 3 \times 10^5$ stcoul/cm^2	(esu)

Problems

30-1. Determine (a) in mks units and (b) in Gaussian units the intensity of the magnetic field at the center of a circular coil of 6 cm radius when it carries a current of 15 amp.

30-2. Determine (a) in mks units and (b) in Gaussian units the magnetic field intensity at the center of a closely wound circular coil of 75 turns, whose average radius is 9 cm, when the coil carries a current of 4 amp.

30-3. A circular coil of 20 cm radius consists of a single turn of wire and has a small compass needle suspended at its center so that it can swing freely about a vertical axis. The plane of the coil is set parallel to the earth's magnetic field. If the horizontal component of the earth's magnetic field is 0.2 oersted, determine the angle through which the compass needle is deflected when a current of 3 amp is sent through the coil.

30-4. Determine (a) in mks units and (b) in Gaussian units the magnetic field intensity at a distance of 0.25 m from a long straight wire in which there is a current of 32 amp.

30-5. A small compass needle, whose magnetic moment is 40 cgs poles cm and whose moment of inertia is 6.0 gm cm^2, is placed at a point 15 cm from a long straight wire. When a current is sent through the wire, the needle oscillates with a period of 7.2 sec. Determine (a) the magnetic field intensity at the position of the compass and (b) the current in the wire.

30-6. A solenoid 0.75 m long and 0.08 m in diameter is wound with 400 turns of wire. Determine the magnetic field intensity inside this solenoid when it carries a current of 6.5 amp.

30-7. A long straight wire carries a current of 25 amp. The north pole of a long bar magnet is placed 5 cm from this wire. If the pole strength of this magnet is 40 cgs poles, determine the direction and magnitude of the force on this north pole.

30-8. A long straight wire carries a current of 20 amp. The wire is parallel to the z axis, and the current flows in the positive z direction. In addition to the magnetic field generated by the current in the wire, there is an external magnetic field in the positive z direction of intensity 5 amp/m. Find the resultant magnetic field (a) at a point in the x-y plane whose coordinates are (5 m, 0) and (b) at a point in the x-y plane whose coordinates are (0, 10 m).

30-9. Derive an equation, in Gaussian units, for the magnetic field intensity at a point in the neighborhood of a long straight wire carrying current.

30-10. Derive an equation, in Gaussian units, for the magnetic field intensity at the center of a circular loop of wire.

30-11. Find the magnetic field intensity within a toroid of mean radius 10 cm wound with 100 turns of wire when the current in the toroid is 4 amp. The circular cross section of the toroid has a diameter of 1 cm.

30-12. Two long straight wires are 18 cm apart and carry currents of 36 amp each. Determine the magnetic field intensity at a point midway between them (a) when these currents are in the same direction and (b) when the currents are in opposite directions. Express your answers in both Gaussian and mks units.

30-13. In the Oersted experiment a long straight wire placed 4 cm above a compass needle carries a current of 28 amp directed to the north. Determine the torque on the compass needle if its magnetic moment is 75 cgs pole cm.

30-14. Find the force on a magnetic pole of 10^{-4} weber moving in the $+x$ direction with a speed of 10^3 m/sec between the plates of a parallel-plate capacitor in vacuum. The plates of the capacitor are 1 cm apart and are charged to a potential difference of 500 volts. The electric field between the plates of the capacitor is in the $+y$ direction.