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Physics, Chapter 31: Forces on Moving Charges and Currents

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Forces on Moving Charges and Currents

31-1 Force on a Charge Moving in a Magnetic Field

Ampère was the first to show that wires carrying current experienced forces when placed in magnetic fields. Later it was shown that charged particles moving in magnetic fields also experience forces. Common applications of these phenomena are the electric motor, the galvanometer, and the cathode-ray tube. Since a moving electric charge is equivalent to a current, we shall first consider the force acting on a charge q moving with velocity $\mathbf{v}$ in a magnetic field of induction $\mathbf{B}$. Experiment shows that the force $\mathbf{F}$ acting on the charge q is at right angles to the directions of both $\mathbf{v}$ and $\mathbf{B}$. This provides us with a basic distinction between the two magnetic field vectors $\mathbf{B}$ and $\mathbf{H}$.

We have already shown that the force on a magnetic pole at rest is determined by the magnetic intensity $\mathbf{H}$. We have seen that moving charges and currents produce $\mathbf{H}$. The relation between the magnetic induction $\mathbf{B}$ and the magnetic intensity $\mathbf{H}$ has been given in Equation (29-15a) as

$$\mathbf{B} = \mu\mathbf{H},$$

where μ is the permeability of the medium.

The force $\mathbf{F}$ on a charge q moving with velocity $\mathbf{v}$ in a field of induction $\mathbf{B}$ is given by

$$\mathbf{F} = k_3 q\mathbf{v}\times\mathbf{B}. \tag{31-1}$$

As shown in Figure 31-1 the force is perpendicular to the plane formed by the vectors $\mathbf{v}$ and $\mathbf{B}$ and is directed in accordance with the right-hand rule for the vector product. The constant k_3 is a constant of proportionality which is assigned the value unity in the mks system of units. Thus if the force $\mathbf{F}$ is expressed in newtons, the charge q in coulombs, the velocity $\mathbf{v}$ in meters per second, and the magnetic induction $\mathbf{B}$ in webers per square meter, Equation (31-1) may be rewritten as

$$\boxed{\mathbf{F} = q\mathbf{v}\times\mathbf{B}.} \tag{31-2}$$

We may consider Equation (31-2) as a definition of **B**. From the above equation we see that if **v** is parallel to **B**, the force on the moving charge is zero. This suggests a method of determining the direction of **B** by use of cathode-ray tubes. If a charge of 1 coul moving perpendicular to the field with a velocity of 1 m/sec experiences a force of 1 nt, then the field has an induction of 1 weber/m^2.

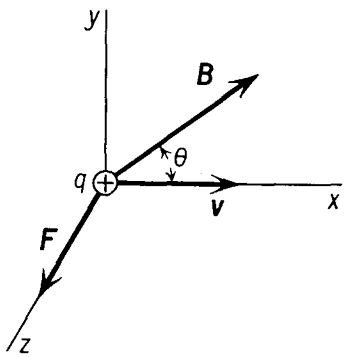

Fig. 31-1 The force **F** exerted on a charge q moving with velocity **v** in a magnetic field of induction **B**. **B** and **v** are in the x-y plane; **F** is along the z axis.

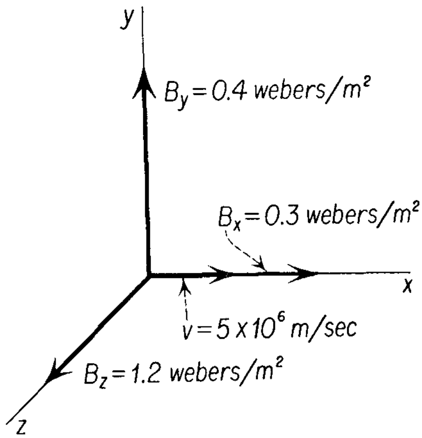

Fig. 31-2

Illustrative Example. A proton whose charge is 1.60×10^{-19} coul is moving in the x direction with a velocity of 5×10^6 m/sec in a magnetic field whose components are $B_x = 0.3$ weber/m^2, $B_y = 0.4$ weber/m^2, $B_z = 1.2$ weber/m^2. See Figure 31-2. Find the x, y, and z components of the force on the proton.

The x component of the field is parallel to the velocity; thus the force on the moving charge due to B_x is zero.

From Equation (31-2) the direction of the force on the particle due to the y component of the field is in the z direction, and since the angle between **v** and $\mathbf{B}_y$ is 90° we have

$$F_z = qvB_y = 1.60 \times 10^{-19}\text{ coul} \times 5 \times 10^6\text{ m/sec} \times 0.4\text{ weber/m}^2,$$

so that

$$F_z = 3.2 \times 10^{-13}\text{ nt}.$$

Applying the right-hand rule for the vector product, we see that the force on the particle due to the z component of the field is in the $-y$ direction. The angle between **v** and $\mathbf{B}_z$ is 90°. The y component of the force on the particle is therefore

$$F_y = -1.60 \times 10^{-19}\text{ coul} \times 5 \times 10^6\text{ m/sec} \times 1.2\text{ weber/m}^2,$$

$$F_y = -9.6 \times 10^{-13}\text{ nt}.$$

The resultant force F is of magnitude

$$F = (F_y^2 + F_z^2)^{1/2} = 9.74 \times 10^{-13}\ \text{nt}.$$

In the Gaussian system of units the constant k_3 has the value $1/c$, and we must express the units of B in gausses and of H in oersteds. As before, we must express q in statcoulombs, I in statamperes, v in centimeters per second, and F in dynes. The relationship between B and H is

$$\mathbf{B} = \kappa_m \mathbf{H}. \tag{31-3}$$

While κ_m is a pure number, we may think of it as expressed in units of gausses per oersted for convenience in calculation.

The relationship between gausses and webers per square meter may be found by calculating, from Equation (31-1), the magnetic induction in gausses required to exert a force of 1 nt on a charge of 1 coul moving perpendicularly to the magnetic field with a velocity of 1 m/sec. By definition, this field is 1 weber/m². The known numerical values are

$$\begin{aligned} F &= 1\ \text{nt} = 10^5\ \text{dynes}, \\ q &= 1\ \text{coul} = 3 \times 10^9\ \text{stcoul}, \\ v &= 1\ \text{m/sec} = 10^2\ \text{cm/sec}, \\ c &= 3 \times 10^{10}\ \text{cm/sec}, \\ \theta &= 90^\circ. \end{aligned}$$

From Equation (31-1) expressed in cgs units, we have

$$F = q\frac{v}{c}B\sin\theta,$$

$$10^5\ \text{dynes} = \frac{3 \times 10^9\ \text{stcoul} \times 10^2\ \text{cm/sec}}{3 \times 10^{10}\ \text{cm/sec}} \times B,$$

$$B = 10^4\ \text{gausses}.$$

Thus a magnetic field whose induction is 10^4 gausses will produce the same effect on a moving charge as a field whose induction is 1 weber/m². In the form of an equation we may write

$$1\ \text{weber/m}^2 = 10^4\ \text{gausses}. \tag{31-4}$$

31-2 Path of a Charged Particle in a Magnetic Field

When a charged particle is projected into a uniform magnetic field, the component of the field parallel to the velocity exerts no force on the particle. It is convenient to resolve the velocity of the particle into components parallel and perpendicular to the field, and to consider separately the motion of the particle perpendicular to the field.

Let us consider a charged particle that is moving with velocity **v** perpendicular to a magnetic field of induction **B**. The particle will experience a force Bqv constant in magnitude and directed perpendicularly to the velocity of the particle. Hence the particle will move in a circular path with uniform speed v. The force exerted by the magnetic field on the charged particle is the centripetal force which produces an acceleration $a = v^2/r$, where r is the radius of the circular path. Applying Newton's second law to this case, we get

$$F = \frac{mv^2}{r} = Bqv,$$

from which

$$r = \frac{mv}{Bq}. \qquad (31\text{-}5)$$

If the motion of the charged particle is not transverse to the field, the symbol v in Equation (31-5) must be interpreted as the component of the velocity which is perpendicular to the magnetic field. The motion of the

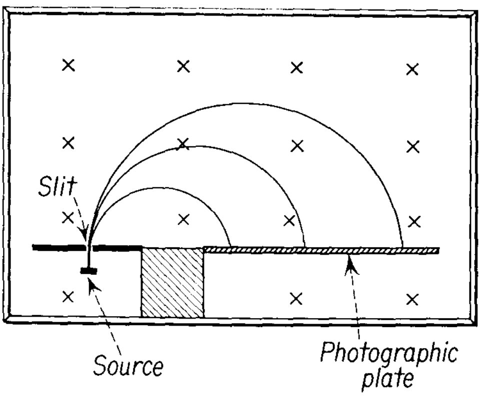

Fig. 31-3 Beta-ray spectrometer.

particle will then consist of a motion of translation parallel to the field and, at the same time, a motion in a circular orbit in a plane perpendicular to the field. The effect of these two simultaneous movements is to produce a corkscrewlike motion, or a helical motion whose axis is parallel to the magnetic field.

Note that the force on the particle moving in the magnetic field is always perpendicular to its velocity. The magnetic field therefore *does no work* on the charged particle, but only deflects it. This property of a uniform magnetic field may be utilized to measure the velocities of charged particles. In a *beta-ray spectrometer,* used to measure the velocities of electrons emitted in radioactive disintegrations, electrons or beta rays are emitted into a uniform magnetic field of known magnetic induction B in a direction perpendicular to the field. The particles are deflected into circular orbits and strike a photographic film after completing a semicircle, as shown in Figure 31-3. The diameter of the orbit can be measured from the photographic film; the momentum *mv* of the electrons can then be determined, and the energy of the emitted particles may also be calculated.

The deflection of charged particles in magnetic fields is today widely utilized in scientific and technical apparatus such as beta-ray spectrometers,

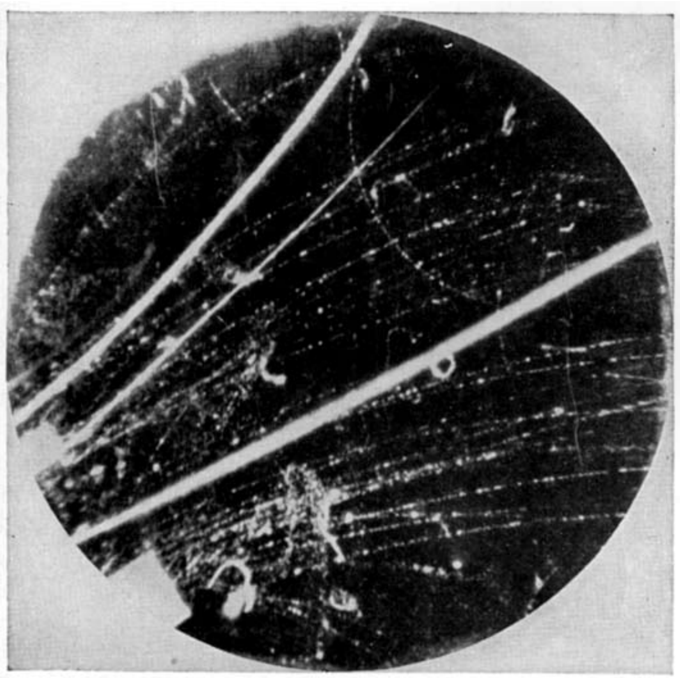

Fig. 31-4 Cloud chamber photograph showing curved paths of particles moving at right angles to a magnetic field. The magnetic field is directed into the paper and the particles originate from a source at the left. The three heavy tracks are those of protons; the numerous light tracks curved in the opposite direction are those of electrons. (Photograph by H. R. Crane.)

mass spectrometers, cathode-ray tubes (as used in television), the cyclotron (accelerates charged particles for nuclear research), the magnetron (a vacuum tube for the production of microwaves used in radar), and many others. The deflection of charged particles in a magnetic field may be seen in the accompanying photograph of tracks in a Wilson cloud chamber, shown in Figure 31-4. In this photograph the magnetic field is perpendicular to the paper and is directed into the plane of the paper.

31-3 Force on a Charged Particle in an Electromagnetic Field

We have already seen that a charged particle in an electric field of intensity **E** experiences a force $q\mathbf{E}$. This force does not depend upon the velocity of the particle. When a charged particle moves with a velocity **v**, it will experience a force given by the equation

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}). \tag{31-6}$$

Equation (31-6) is simply a combination of results already stated in Equations (23-1a) and (31-2). We may think of Equation (31-6) as the defining equation for the electric field intensity **E** and the magnetic induction **B**. To determine these fields in a region of space, we may imagine that a probe charge q is directed through the region. The force on the particle is determined by observing the deflection of the particle, and the electric and magnetic fields are determined by calculation from Equation (31-6). In a similar way we may imagine that Equation (30-7) is the defining equation for the magnetic intensity **H** and the electric displacement **D**, through the force on a moving pole.

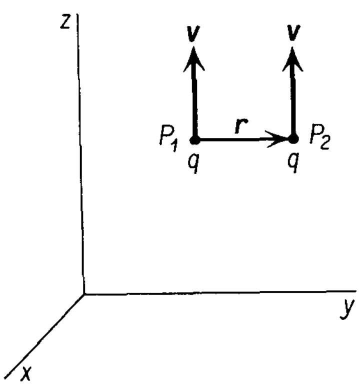

Fig. 31-5

In our initial discussion of Coulomb's law of force between magnetic poles, in Section 29-2, we stated that the constant μ_0 was to be assigned to a value of $4\pi \times 10^{-7}$ for reasons which had to be deferred to a later chapter. At this point we wish to justify the choice of a numerical value for μ_0. To do this we will calculate the ratio of the magnetic force to the electric force between two moving charged particles.

Let us consider the force between two equal charges q moving with the same velocity **v** in a direction parallel to the z axis, as shown in Figure 31-5. The instantaneous positions of the charges are P_1 and P_2, as shown in the figure. The particles are fixed in position relative to each other. The electric force on the charge at P_2 due to the presence of an equal charge at P_1 is $\mathbf{F}_e$, whose magnitude is

$$F_e = \frac{q^2}{4\pi\epsilon_0 r^2}\cdot$$

The magnetic force on the charge at P_2 may be obtained by first computing the magnetic field intensity H generated by the motion of the charge at P_1

with respect to the coordinate frame from Equation (30-6),

$$\mathbf{H} = \frac{q\mathbf{v} \times \mathbf{1}_r}{4\pi r^2}.$$

The relationship between $\mathbf{B}$ and $\mathbf{H}$ in vacuum is given by $\mathbf{B} = \mu_0\mathbf{H}$, so that

$$\mathbf{B} = \frac{\mu_0 q\mathbf{v} \times \mathbf{1}_r}{4\pi r^2}.$$

Since the velocity $\mathbf{v}$ is in the z direction and the unit vector $\mathbf{1}_r$ is in the y direction, from P_1 to P_2, the magnetic induction $\mathbf{B}$ is in the $-x$ direction. The magnitude of the magnetic force F_m on the charged particle at P_2 may be obtained from

$$\mathbf{F}_m = q\mathbf{v} \times \mathbf{B},$$

$$F_m = qvB \sin\theta.$$

At P_2 the magnetic induction $\mathbf{B}$ is in the $-x$ direction, and the velocity of the particle is in the z direction. Since the two vectors $\mathbf{v}$ and $\mathbf{B}$ are perpendicular to each other, the factor $\sin\theta$ in the vector product is equal to 1, and the magnitude of the magnetic force is equal to

$$F_m = \frac{\mu_0}{4\pi}\frac{q^2v^2}{r^2}.$$

The ratio of the magnetic force to the electric force must be a pure number, without dimensions. We obtain

mks units $$\frac{F_m}{F_e} = \mu_0\epsilon_0 v^2. \tag{31-7a}$$

Thus the quantity $\mu_0\epsilon_0$ must have the dimensions of $1/v^2$; that is, its dimensions must be $(\text{sec}/\text{m})^2$.

When the same calculation is carried out in Gaussian units, we find that

Gaussian units $$\frac{F_m}{F_e} = \frac{v^2}{c^2}. \tag{31-7b}$$

The ratio of the magnetic force to the electric force must be the same in whatever system of units the calculation is carried out. In Gaussian units both the velocity of the particle and the velocity of light are expressed in centimeters per second, while in mks units the velocity of the particle is expressed in meters per second. In order to equate the results of the two calculations, let us now express all velocities in meters per second. This will not alter the ratio in Equation (31-7b). We obtain

$$\mu_0\epsilon_0 v^2 = \frac{v^2}{c^2},$$

or

mks units
$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}, \tag{31-8}$$

where c is the velocity of light in meters per second.

Our procedure in developing the mks system of electrical units was first to define the size of the coulomb from the statcoulomb. This determined the choice of the constant ϵ_0. Once a choice of ϵ_0 had been made, there could be no freedom of choice of the constant μ_0, for the magnitude of that constant was determined by the connection between electric and magnetic forces, as developed in Equation (31-8).

31-4 Force on a Wire Carrying Current in a Magnetic Field

We have already seen in Section 30-7 that a charge dq moving with a velocity **v** could be considered equivalent to a current I in an element of wire **ds** through the relationship

$$\mathbf{v}\, dq = I\, d\mathbf{s}. \tag{31-9}$$

From Equation (31-2) the force on a moving charge element dq is given by

$$d\mathbf{F} = dq\, \mathbf{v} \times \mathbf{B},$$

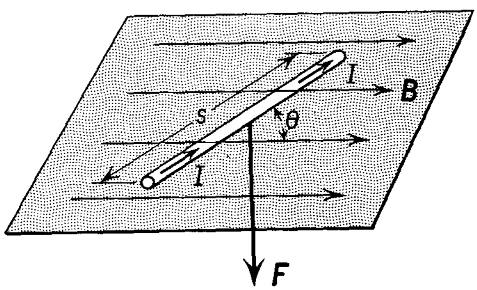

Fig. 31-6

and, substituting from Equation (31-9), we find the force $d\mathbf{F}$ on a current element $I\, d\mathbf{s}$ to be

$$d\mathbf{F} = I\, d\mathbf{s} \times \mathbf{B}. \tag{31-10}$$

This force is perpendicular to the plane formed by the current element and the direction of the field. For the special case of a long straight conductor of length s in a uniform field of magnetic induction B, the magnitude of the force F on the wire is given by

$$F = BIs \sin \theta, \tag{31-11}$$

where θ is the angle between the direction of the current in the wire and the direction of B, as shown in Figure 31-6. The force is perpendicular to the plane formed by s and B, and its direction may be found by the right-hand rule for the vector product of Equation (31-10). The force on a wire carrying current in a magnetic field is the basis for the operation of electric motors and galvanometers.

Illustrative Example. A wire 5 m long lies along the x axis. Find the components of the force on the wire when it carries a current of 2 amp in the positive x direction if it is placed in a uniform magnetic field for which

$$B_x = 0.3 \text{ weber/m}^2,$$

$$B_y = 0.4 \text{ weber/m}^2,$$

$$B_z = 1.2 \text{ webers/m}^2.$$

Since the wire lies along the x axis, the force on the wire due to B_x is zero. The angle made by the wire with the y axis and with the z axis is 90°. The force exerted on the wire due to the y component of the field is in the $+z$ direction. Calling this force F_z, we have

$$\begin{aligned} F_z &= B_y I s \\ &= 0.4 \text{ weber/m}^2 \times 2 \text{ amp} \times 5 \text{ m} \\ &= 4.0 \text{ nt.} \end{aligned}$$

The force exerted on the wire due to the z component of the field is in the $-y$ direction. Calling this force F_y, we have

$$\begin{aligned} F_y &= -B_z I s \\ &= -1.2 \text{ weber/m}^2 \times 2 \text{ amp} \times 5 \text{ m} \\ &= -12 \text{ nt.} \end{aligned}$$

31-5 Torque on a Coil Carrying Current

Let us assume that a rectangular coil of sides a and b is carrying current I in a region of space in which there is a uniform field of magnetic induction B. If the plane of the coil is perpendicular to the direction of B, as shown in Figure 31-7 where B is directed into the paper, symmetrical elements of the coil at g and h at the top and bottom of the coil experience equal and opposite forces. Since the forces on symmetrical elements of the wire are directed along the same straight line, the resultant force on the coil and the resultant torque on the coil due to these elements are zero. Similarly, the elements at c and d contribute a net force of zero and a net torque of zero.

When the orientation of the coil is changed so that the normal to the plane of the coil makes an angle ϕ with the direction of B, as shown in Figure 31-8, the symmetrical elements g and h at the top and bottom of the coil still experience collinear forces in the $-z$ and $+z$ direction, respectively, which are equal and opposite, and therefore contribute zero net force and torque. The forces on elements at c and d are still equal and opposite, directed parallel to the x axis in the figure, but they are no longer in the same straight line; they are now separated by a distance $a \sin \phi$ [Figure 31-8(a)].

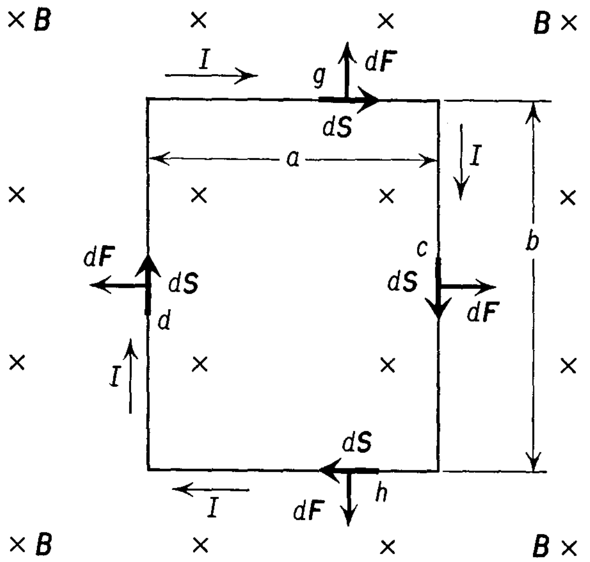

Fig. 31-7

The force on each side b of the coil is given by Equation (31-11) as

$$F = Bib$$

for the angle θ between the vertical wires, and the direction of B is 90°. The resultant force on the coil is zero, for the forces on each side of the coil are in opposite directions, but the torque G on the coil is now

$$G = BIba \sin \phi.$$

Since the area A of the coil is given by

$$A = ba,$$

we may express the torque on the coil as

$$G = BIA \sin \phi. \tag{31-12}$$

If the coil is closely wound, having N turns, a torque equal to that given by Equation (31-12) is contributed by each turn so that we have

$$G = NBIA \sin \phi. \tag{31-13}$$

Although Equation (31-13) was derived for rectangular loops or coils of wire, it may be seen that coils of any shape, whose conductors lie in a single plane, may be approximated by a collection of rectangular coils laid side by side, as shown in Figure 31-9. Thus Equation (31-13) gives the torque on a coil of any shape when placed in a uniform magnetic field.

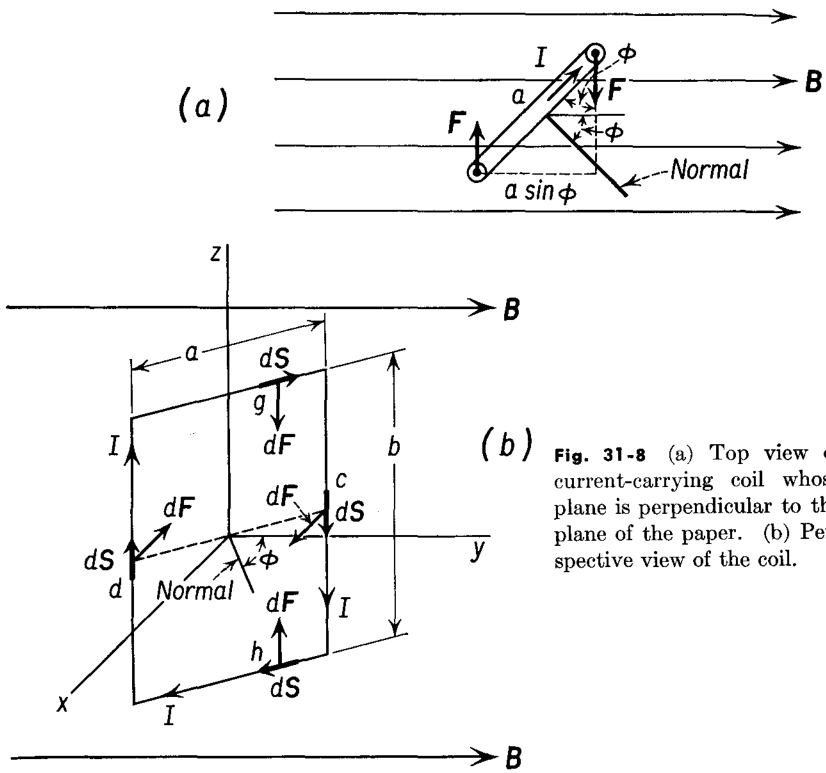

Fig. 31-8 (a) Top view of current-carrying coil whose plane is perpendicular to the plane of the paper. (b) Perspective view of the coil.

The operation of a moving-coil galvanometer, or *D'Arsonval galvanometer*, is based upon the torque on a coil in a magnetic field. The essentials of such a galvanometer are a strong permanent magnet shaped so that the two poles face each other across a gap in which the magnetic induction is fairly uniform, a rectangular coil of many turns of fine wire, and a fine spring attached to the coil. This spring may be in the form of a spirallike fine watch spring, in which case the coil is mounted on jeweled bearings, or the spring may be in the form of a long wire, one end of which is rigidly attached at the top of the frame housing the coil and the other end is attached to the coil and is part of the conductor leading current into the coil. As seen from Equation (31-13), the torque on the coil is proportional to the current through the coil. At a particular value of the current, the coil is deflected until the restoring torque of the spring is equal to the torque generated by the magnetic field. The deflection may be indicated by a pointer moving over a scale or by means of a beam of light reflected from

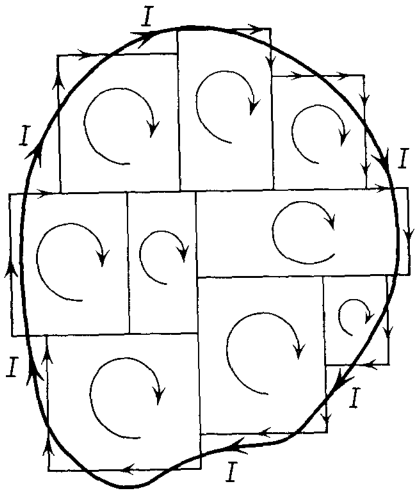

Fig. 31-9 A plane coil of arbitrary shape may be approximated by a set of rectangular coils. Note that the currents on all interior wires cancel out.

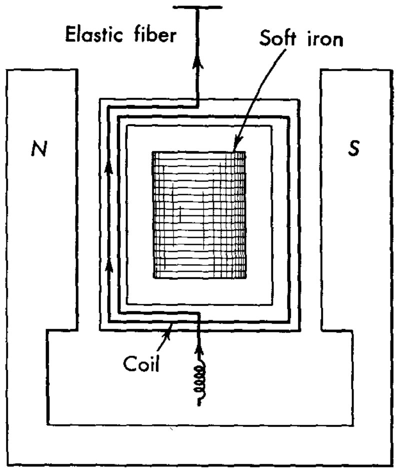

Fig. 31-10 Essentials of a D'Arsonval type of galvanometer. The soft iron core is introduced to shape the magnetic field so that it is radial, thus obtaining a galvanometer in which the deflection is proportional to the current through the coil.

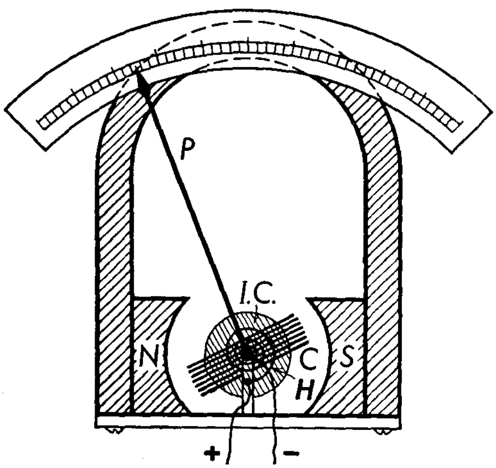

Fig. 31-11 Essentials of a moving coil galvanometer. C is the moving coil, N and S are the poles of a permanent magnet, $I.C.$ is the soft iron core, P is the pointer, and H is the hairspring.

a mirror which turns with the coil. Schematic diagrams of the essentials of a D'Arsonval galvanometer, and a typical panel-type moving-coil galvanometer are shown in Figures 31-10 and 31-11. In practice the pole pieces are curved so that the magnetic field is nearly radial. This produces a galvanometer in which the deflection is nearly proportional to the angular displacement.

31-6 Force between Two Parallel Conductors

When two parallel conductors carry current, there is a force between them which is repulsive when the currents are in opposite directions and attractive when the currents are in the same direction. The force is due to the interaction between the current in one wire and the magnetic field at that wire due to current in the other wire. We may calculate the force between the two conductors by utilizing the results of the preceding chapter. If the current in the wire shown as 1 in Figure 31-12 is I_1, the magnetic field intensity at the position of wire 2, due to the current I_1 in wire 1, is

$$H = \frac{I_1}{2\pi a}, \qquad \text{(30-1a)}$$

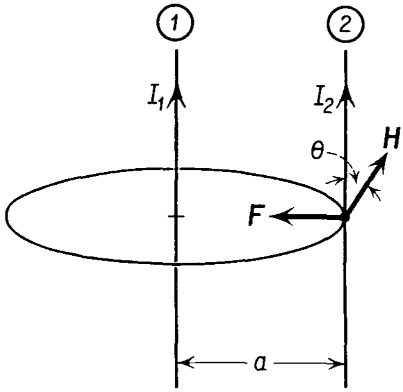

Fig. 31-12

where a is the distance between the wires. The magnetic induction B is related to the magnetic field intensity H in vacuum or in air through the equation

$$B = \mu_0 H;$$

thus the magnetic induction due to the current in wire 1 at the position of wire 2 is

$$B = \frac{\mu_0 I_1}{2\pi a}.$$

The direction of **B** is parallel to the direction of **H** and is shown in the figure as directed into the plane of the paper, perpendicular to the wire 2. We may find the force on a length s of wire 2 by application of Equation (31-11). We note that the angle θ between the direction of the current in wire 2 and the direction of **B** at wire 2 is 90°. Thus we have

$$\begin{aligned} F &= BsI_2 \\ &= \frac{\mu_0 I_1 I_2 s}{2\pi a}. \end{aligned}$$

The force per unit length of wire is given by

$$\frac{F}{s} = \frac{\mu_0 I_1 I_2}{2\pi a}. \tag{31-14}$$

From application of the rule for the vector product, as given in Equation (31-10) for the direction of the force, the force on wire 2 may be seen to be toward wire 1. An equal but opposite force is exerted on wire 2, as may be seen from an application of the same procedure to wire 2. When the currents are in opposite directions, the force is of the same magnitude but is repulsive rather than attractive.

From Equation (31-14) we note that the units of μ_0 may be expressed as nt/amp^2 as well as weber/nt m^2 or weber/amp m. The units in which μ_0 is expressed is a matter of convenience. Thus we may write

$$\mu_0 = 4\pi \times 10^{-7}\ \text{nt/amp}^2. \tag{31-15}$$

It is possible through Equation (31-14) to establish a definition of the ampere in terms of the mechanical quantities of force and distance and the constant μ_0. Thus an ampere may be defined as that current flowing in two infinitely long parallel wires separated by a distance of 1 m in air which causes a force of 2×10^{-7} nt per meter of length between them. In a device called the current balance, the force between two coils of wire connected in series and separated by a distance of a few centimeters may be used to make an absolute determination of current.

When there is an alternating current in a coil, the current is first in one direction, then in the other, following a sine or cosine function of time. The currents in adjacent turns are in the same direction, so that there is an attractive force between adjacent turns which varies from some maximum value to zero and back again. This periodic force may cause the wires of the coil to vibrate and produce an audible hum; such a noise is sometimes heard in a transformer of a radio set.

Illustrative Example. Find the force between two parallel conductors in a power distributing station. The conductors are 10 m long, carry a current of 150 amp each, and are separated by a distance of 1 cm.

From Equation (31-14)

$$F = \frac{\mu_0 I_1 I_2 s}{2a}.$$

Substituting numerical values, we have

$$F = \frac{4\pi \times 10^{-7}\ \text{nt/amp}^2 \times 150\ \text{amp} \times 150\ \text{amp} \times 10\ \text{m}}{2 \times 0.01\ \text{m}},$$

$$F = 4.5\ \text{nt}.$$

TABLE 31-1 PRINCIPAL EQUATIONS IN MKS AND GAUSSIAN UNITS

Equation	MKS	Gaussian	
(31-2)	$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$	$\mathbf{F} = q \dfrac{\mathbf{v} \times \mathbf{B}}{c}$	Moving charge
(31-3)	$\mathbf{B} = \kappa_m \mu_0 \mathbf{H}$	$\mathbf{B} = \kappa_m \mathbf{H}$	
(31-5)	$r = \dfrac{mv}{Bq}$	$r = \dfrac{mvc}{Bq}$	Circular path
(31-6)	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$	$\mathbf{F} = q\left(\mathbf{E} + \dfrac{\mathbf{v} \times \mathbf{B}}{c}\right)$	Charge
(31-10)	$d\mathbf{F} = I\, d\mathbf{s} \times \mathbf{B}$	$d\mathbf{F} = \dfrac{I}{c}\, d\mathbf{s} \times \mathbf{B}$	Element of wire carrying current
(31-13)	$G = NBIA \sin \phi$	$G = \dfrac{NBIA \sin \phi}{c}$	Torque on coil
(31-14)	$\dfrac{F}{s} = \dfrac{\mu_0 I_1 I_2}{2\pi a}$	$\dfrac{F}{s} = \dfrac{2 I_1 I_2}{a c^2}$	Parallel wires

TABLE 31-2 CONVERSION FACTORS RELATING MKS AND GAUSSIAN UNITS

Quantity	Symbol	MKS Unit	Gaussian Unit	
Magnetic intensity	H	1 amp/m = 1 nt/weber	$= 4\pi \times 10^{-3}$ oersted	(emu)
Magnetic induction	B	1 weber/m^2	$= 10^4$ gausses	(emu)
Electric intensity	E	1 nt/coul = 1 volt/m	$= \dfrac{10^{-4}}{3} \dfrac{\text{statvolt}}{\text{cm}}$	(esu)
			$= \dfrac{10^{-4}}{3} \dfrac{\text{dyne}}{\text{stcoul}}$	(esu)
Electric displacement	D	1 coul/m^2	$= 3 \times 10^5$ stcoul/cm^2	(esu)
Charge	q	1 coul	$= 3 \times 10^9$ stcoul	(esu)
Current	I	1 amp	$= 3 \times 10^9$ statamperes	(esu)

Problems

31-1. A proton of charge $+4.8 \times 10^{-10}$ stcoul moves with a speed of 10^9 cm/sec at right angles to the direction of a magnetic field of 300 gausses. Find the force on the proton due to its motion with respect to the magnetic field.

31-2. The mass of a proton is 1.67×10^{-24} gm. What will be the radius of the circle in which the proton of Problem 31-1 moves?

31-3. In a beta-ray spectrometer an electron moving perpendicularly to a magnetic field of magnetic induction 150 gausses moves in a circle of 15 cm diameter. Find the momentum of the electron.

31-4. A proton moves with a velocity of 10^4 m/sec in the $+x$ direction in a magnetic field whose components are $B_x = 0.03$ weber/m^2, $B_y = 0.04$ weber/m^2, $B_z = 14$ webers/m^2. Find the components of the force on the proton.

31-5. A particle of charge 10 μcoul has a velocity whose components are $v_x =$ 3 m/sec, $v_y = 4$ m/sec. The particle moves in a magnetic field whose components are $B_x = 0.5$ weber/m^2, $B_y = 1.2$ webers/m^2, and $B_z = 1.5$ webers/m^2. Find the components of the force on the particle.

31-6. The magnetic induction between the poles of a large electromagnet is 1,000 gausses in a direction from east to west. The field extends vertically for a distance of 5 cm. A wire is suspended vertically in the field, and a current of 15 amp is sent through it. Determine the magnitude and direction of the force on the wire.

31-7. A wire parallel to the y axis carries a current of 10 amp. The wire is 0.5 m long. The magnetic induction in the vicinity of the wire has an x component of 0.3 weber/m^2, a y component of -1.2 webers/m^2, and a z component of 0.5 weber/m^2. Find the components of the force on the wire.

31-8. A current of 8 amp is directed downward in a vertical wire 80 cm long which is suspended in a place where the intensity of the earth's magnetic field has a horizontal component of 0.25 oersted and a vertical component of 0.75 oersted. Determine the magnitude and the direction of the force on the wire.

31-9. A long uniform solenoid is wound with 1,000 turns/m of length and carries a current of 10 amp. An electron, whose velocity component parallel to the axis of the solenoid is 10^5 m/sec and whose velocity perpendicular to the axis of the solenoid is 10^6 m/sec, is ejected from a radioactive source at the center of the solenoid. The path described by the electron is a helix. (a) Find the radius of the cross section of this helix and (b) find the pitch (number of turns per meter) of the helix. The charge of an electron is -4.8×10^{-10} stcoul, and the mass of an electron is 9.10×10^{-28} gm.

31-10. A rectangular coil of 150 turns of fine wire is suspended in a magnetic field with its plane parallel to the direction of the field. The dimensions of the coil are 2 cm by 4 cm. When a current of 0.5 amp is sent through the coil, a torque of 18,000 dynes cm acts on it. Determine the magnetic induction of this field.

31-11. A straight wire 12 cm long is placed in a magnetic field of 350 gausses directed at right angles to the length of the wire. A delicate spring balance attached to the wire measures a force of 160 dynes when a current is sent through the wire. Determine the current in the wire.

31-12. A proton whose mass is 1.65×10^{-27} kg travels in a circular path of 0.35 m radius in a magnetic field of 1.2 webers/m^2. Determine (a) the velocity of the proton and (b) its period of revolution.

31-13. A rectangular galvanometer coil has 500 turns wound on a frame 2 cm by 4 cm. It hangs between the poles of a magnet where the flux density is 0.080 weber/m^2. Determine the torque which acts on this coil when the current in it is 3×10^{-8} amp.

31-14. Derive Equation (31-7b).

31-15. Two electrons are moving down the axis of a television tube with a velocity of 10^7 m/sec. The electrons are separated by a distance of 0.01 cm. (a) What is the electric force between them? (b) What is the magnetic force between them? (c) What is the resultant force between them?

31-16. A circular coil of wire of radius 10 cm and containing 50 turns is placed in a magnetic field in which B is 1.5 webers/m^2. The current in the coil is 3 amp. (a) What is the torque on the coil when its plane is parallel to the field? (b) When the normal to the plane of the coil makes an angle of 30° with the field?

31-17. One end of a flexible copper wire is fastened to a post while the other goes over a fixed pulley and has a weight W hanging from it. The portion of the wire from A to the pulley is horizontal and is in a uniform magnetic field of flux density B in the vertical direction. A current I is sent through the wire; determine the radius of the circular arc into which the wire is bent.