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Physics, Chapter 34: Alternating Currents

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34

Alternating Currents

34-1 Resistance in A-C Circuits

If the current in a resistor varies sinusoidally, as shown in Figure 34-1, the potential difference across the terminals of the resistor will also vary sinusoidally in the same manner, *in phase* with the current, in accordance

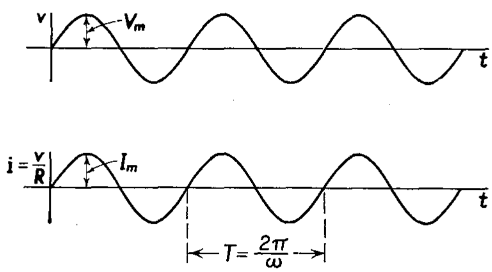

Fig. 34-1 Instantaneous values of the alternating current and voltage in a resistor.

with Ohm's law. Writing i for the instantaneous value of the current and v for the instantaneous value of the potential difference, we have

$$v = iR. \qquad (34\text{-}1)$$

The current and the potential difference reverse direction at the same time and also reach their maximum values and their minimum values simultaneously. We may describe the sinusoidal variation of the current with time by writing

$$i = I_m \sin \omega t, \qquad (34\text{-}2)$$

where I_m is the *maximum* value of the current and ω is the *angular frequency* expressed in radians per second. The angular frequency ω is related to the *frequency* f of the current through the equation

$$\omega = 2\pi f. \qquad (34\text{-}3)$$

The potential difference across the terminals of the resistor is obtained by substituting from Equation (34-2) into Equation (34-1) to obtain

$$v = I_m R \sin \omega t. \tag{34-4}$$

By analogy with Equation (34-2), we might describe the instantaneous value of the potential difference by the equation

$$v = V_m \sin \omega t, \tag{34-5}$$

where V_m represents the maximum value of the potential difference across the resistor. Comparing Equations (34-4) and (34-5), we find

$$V_m = I_m R, \tag{34-6}$$

that is, Equation (34-1) is true for the maximum values of the current and potential difference as well as every other instantaneous value.

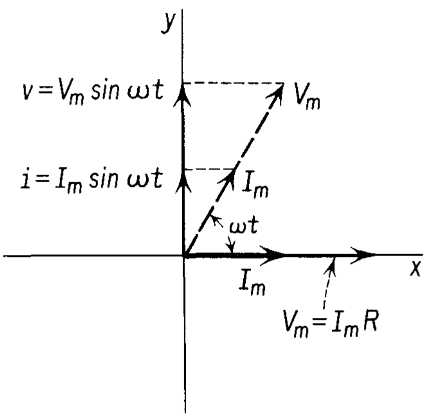

Fig. 34-2

It is convenient to think of the current and potential difference in a-c circuits in terms of the idea of the reference circle developed in our description of simple harmonic motion in Chapter 12. Let us draw a vector of length I_m along the x axis and imagine this vector to rotate in the x-y plane with an angular velocity ω, in the counterclockwise direction. The instantaneous value of the current is given by the projection of this vector onto the y axis. Thus, if the current vector is directed along the x axis at time $t = 0$, the instantaneous value of the current at that time is $i = 0$, as shown in Figure 34-2. This value is in agreement with Equation (34-2). At some subsequent time t the current vector will have rotated through an angle ωt, and the projection of the current vector onto the y axis will have a value given by

$$i = I_m \sin \omega t.$$

The potential difference between the terminals of a resistor may be represented by a vector of length V_m which is rotating with the same angular velocity. Since the current and potential difference are in phase with each other, the voltage vector is also drawn along the x axis and may be imagined to rotate along with the current vector.

One way to keep the relationships of current and voltage in mind is to think of these vectors as drawn on a transparent card which is rotated with constant angular velocity. The instantaneous voltage and current are then the projection of the voltage and current vectors onto the y axis.

34-2 Effective Values of Current and Voltage

One of the important effects of a current is the production of *heat* in its passage through a resistor. The heating effect is used to define the *effective* value of a given alternating current as compared to a steady *direct* current. *The effective value of an alternating current* I_{eff} *is equal to the direct current which would develop the same heat in a resistor in the same period of time.* To find the effective value of the current, we must find the heat liberated in a resistor in one complete cycle. The rate of development of heat is the instantaneous power $\mathcal{P}$ given by

$$\mathcal{P} = i^2R = I_m^2R \sin^2 \omega t,$$

and the heat $\mathcal{W}$ developed in a complete cycle of period T $(= 2\pi/\omega)$ is given by

$$\mathcal{W} = \int_0^T I_m^2R \sin^2 \omega t \, dt.$$

Now
$$\int \sin^2 x \, dx = \tfrac{1}{2}x - \tfrac{1}{4} \sin 2x,$$

so that
$$\mathcal{W} = \tfrac{1}{2}I_m^2RT.$$

The heat which would be developed by a direct current I_{eff} in the same time would be

$$\mathcal{W} = I_{\text{eff}}^2RT.$$

Equating these two values, we find

$$I_{\text{eff}}^2 = \tfrac{1}{2}I_m^2,$$

or
$$I_{\text{eff}} = \frac{I_m}{\sqrt{2}} = 0.707I_m. \tag{34-7}$$

The value of I_{eff} calculated above is the square root of the average or mean of the square of the instantaneous current. The effective value of the current is therefore called the root mean square or rms current, I_{rms}.

The effective value of the potential difference between the terminals of the resistor may be found in the same way, for the instantaneous power may be expressed in terms of the voltage as

$$\mathcal{P} = \frac{v^2}{R} = \frac{V_m^2}{R} \sin^2 \omega t.$$

Thus we may write

$$V_{\text{eff}} = \frac{V_m}{\sqrt{2}} = 0.707V_m. \tag{34-8}$$

Following convention, we shall write the symbols I and V without subscripts to designate effective values of current and potential difference in a-c circuits. It must be emphasized that the effective values given by Equations (34-7) and (34-8) are correct only when the current and voltage are varying sinusoidally. The customary a-c power supplied by utility companies is designated as 110 volt, 60 cycle. This means that the company endeavors to operate its generators so that the frequency of the alternating voltage delivered to the user is 60 cycles/sec, and that the *effective* voltage is 110 volts. The maximum or *peak* voltage available at the receptacle is therefore $V_m = 156$ volts.

In a-c circuits composed of resistors, inductors, and capacitors, the only circuit element which consumes electrical power and converts it into heat is the resistor. The power consumed in an a-c circuit in which V_R is the effective voltage across the terminals of the resistor is therefore

$$\mathcal{P} = V_R I_R, \tag{34-9a}$$

$$\mathcal{P} = I_R^2 R, \tag{34-9b}$$

$$\mathcal{P} = \frac{V_R^2}{R} \cdot \tag{34-9c}$$

In Equations (34-9) the symbol $\mathcal{P}$ is understood to represent the average power, while the symbols V_R and I_R are understood to represent the *effective* values of the current through the resistor and the voltage across the resistor.

34-3 Inductance in an A-C Circuit

We have already seen that if a potential difference v is supplied from an external source to a circuit containing inductance L and resistance R, then

$$v = Ri + L\frac{di}{dt} \cdot \tag{32-10}$$

If the resistance of the circuit is negligible, then

$$v = L\frac{di}{dt} \cdot$$

When the current in the inductor is sinusoidal,

$$i = I_m \sin \omega t,$$

and

$$\frac{di}{dt} = I_m\omega \cos \omega t.$$

To find the phase relationship between the emf induced in the inductor and the current in it, we apply the trigonometric indentity

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm \cos x.$$

Thus

$$v = LI_m\omega \sin\left(\omega t + \frac{\pi}{2}\right). \tag{34-10}$$

This equation may be written as

$$v = V_m \sin\left(\omega t + \frac{\pi}{2}\right), \tag{34-11}$$

where V_m is the maximum value of the voltage across the inductor. If we wish to relate the *maximum* value of the voltage drop across the inductor to the *maximum* value of the current in it, we find, comparing Equations (34-10) and (34-11),

$$V_m = I_m\omega L. \tag{34-12}$$

The maximum value of the current in the inductor and the maximum value of the potential difference between its terminals do not occur at the same time, for the potential difference is greatest when the rate of change

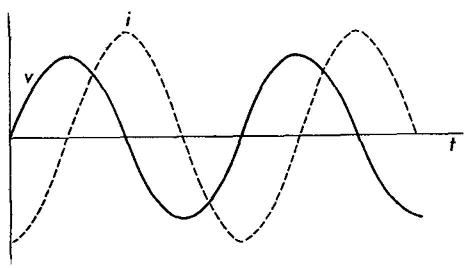

Fig. 34-3 The voltage leads the current by 90° in an inductor.

of current is a maximum; that is, when the current itself is zero. The current and voltage relationships for an inductor are drawn as a function of time in Figure 34-3. We describe these phase relationships by saying that *the voltage across an inductor leads the current through it by* 90°. The word *lead* is associated with the fact that at a time t, when the phase angle of the current is given by Equation (34-2) as ωt, the phase angle of the voltage is given by Equation (34-11) as $\omega t + \pi/2$. This statement does not imply that there is a voltage across the terminals of the inductor before any current is flowing through it, but is rather to be applied to the steady state when an alternating current has been established.

These phase relationships may be described with the aid of appropriate vectors. If the maximum value of the current is drawn as a vector in the $+x$ direction, the maximum value of the voltage across the inductor may be drawn as a vector in the $+y$ direction, as shown in Figure 34-4. If we think of these vectors as drawn onto a transparent card which is rotated in the counterclockwise direction with angular velocity ω, the projection of these vectors on the y axis at any instant of time t may then be thought of as the instantaneous values of the current and voltage. At a time t, when the phase angle of the current vector is ωt, the phase angle of the voltage vector is $\omega t + \pi/2$. The instantaneous value of the current is, from Figure 34-4,

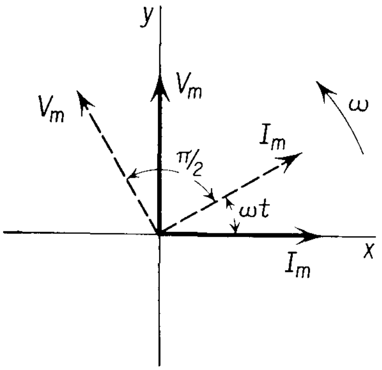

Fig. 34-4

$$i = I_m \sin \omega t, \tag{34-2}$$

while the instantaneous value of the voltage is

$$v = V_m \sin\left(\omega t + \frac{\pi}{2}\right), \tag{34-11}$$

so that, from Equation (34-12),

$$v = I_m \omega L \sin\left(\omega t + \frac{\pi}{2}\right)\cdot \tag{34-10}$$

It is customary to use the symbol X_L, called the *inductive reactance*, defined by means of the equation

$$X_L = \omega L = 2\pi f L, \tag{34-13}$$

to describe the behavior of an inductor in an a-c circuit. In these terms Equation (34-12) becomes

$$V_m = I_m X_L. \tag{34-14}$$

The inductive reactance is expressed in *ohms* when the inductance is expressed in *henrys* and the frequency f is expressed in cycles per second, or the angular frequency ω is expressed in radians per second.

34-4 Capacitance in an A-C Circuit

When a steady potential difference is applied to a capacitor, current flows only while the capacitor is being charged. There is a transient rise of the potential difference across the terminals of the capacitor which occurs in an exponential manner, similar to the transient increase in current through an inductor, described in Section 32-6. If an alternating voltage is applied to its terminals, the capacitor is charged and discharged periodically, and we say that current flows *through* the capacitor, even though no electrons actually pass through the dielectric medium separating the capacitor plates. While a steady direct current may flow through a resistor or an inductor, it is clear that the average direct current in a capacitor in a sufficiently long time interval must be zero, for as much charge must flow in one direction as in the other.

If the charge on one plate of the capacitor at any instant is q, the potential difference between the plates of the capacitor at that instant is v, given by

$$v = \frac{q}{C}.$$

The charge on the capacitor plate is equal to the integral of the current over the time during which the charge was flowing into the capacitor, so that

$$Cv = \int i\,dt.$$

If the current is sinusoidal,

$$i = I_m \sin \omega t, \tag{34-2}$$

and we have

$$Cv = \int I_m \sin \omega t\,dt,$$

$$Cv = \frac{-I_m}{\omega} \cos \omega t,$$

$$v = \frac{I_m}{\omega C} \sin\left(\omega t - \frac{\pi}{2}\right). \tag{34-15}$$

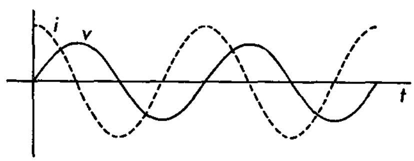

Fig. 34-5 The current in a capacitor leads the voltage across the capacitor by 90°, or the voltage lags by 90°.

In the above development, the constant of integration, representing the initial charge on the capacitor, has been set equal to zero. The potential difference between the plates of the capacitor may be expressed as

$$v = V_m \sin\left(\omega t - \frac{\pi}{2}\right), \tag{34-16}$$

where

$$V_m = \frac{I_m}{\omega C}. \tag{34-17}$$

Thus we see that the potential difference between the plates of the capacitor lags behind the current by 90°, as shown in Figure 34-5. If the maximum value of the current is drawn as a vector in the $+x$ direction, the maximum voltage across the capacitor may be represented as a vector drawn in the

$-y$ direction, as shown in Figure 34-6. Once again the instantaneous values of the current and voltage are to be found by examining the projection of these two vectors on the y axis, and imagined to rotate in the counterclockwise direction with angular velocity ω.

It is customary to use the symbol X_c, called the *capacitive reactance*, defined by means of the equation

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}, \tag{34-18}$$

to describe the behavior of a capacitor in an a-c circuit. In these terms Equation (34-17) becomes

$$V_m = I_m X_c. \tag{34-19}$$

When alternating current flows through an ideal capacitor, no electrical energy is consumed by the capacitor. Instead, the capacitor stores energy in the electric field between its plates while being charged, and returns that energy to the circuit when it is discharging.

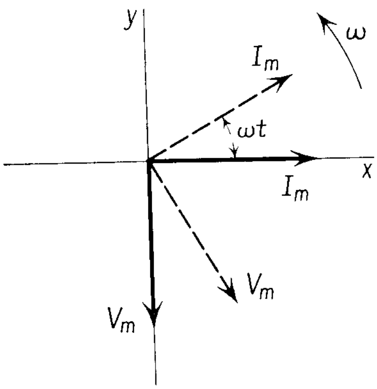

Fig. 34-6

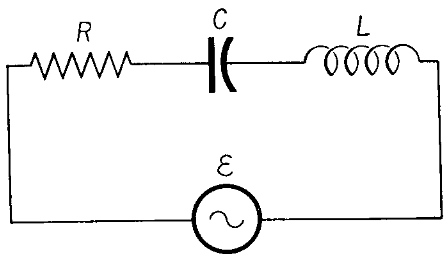

Fig. 34-7

In our initial discussion of capacitance in Chapter 25, we saw that one application of capacitors was to store electrical energy. By far the largest application of capacitors is as a circuit element in a-c circuits. One important use of capacitors is associated with the fact that direct current cannot flow through a capacitor, but that alternating current does. The capacitor may be used to block the flow of direct current.

34-5 Series Circuits

Let us suppose that an alternating current is flowing in a series circuit made up of a resistor R, an inductor L, and a capacitor C, and that these are connected to an a-c generator, as shown in Figure 34-7. At any instant of time the current must be the same in all parts of the circuit. If the current varies sinusoidally with time, as given by

$$i = I_m \sin \omega t, \tag{34-2}$$

the potential difference across the circuit elements will be

$$v_R = I_m R \sin \omega t, \tag{34-4}$$

$$v_L = I_m \omega L \sin\left(\omega t + \frac{\pi}{2}\right), \tag{34-10}$$

$$v_C = \frac{I_m}{\omega C} \sin\left(\omega t - \frac{\pi}{2}\right); \tag{34-15}$$

and the value of the emf across the terminals of the generator at any instant will be the sum of these potential differences at the same instant of time, according to Kirchhoff's second law (Section 27-6). To find the maximum value of the emf from these equations, we should have to find the instant t at which the sum of these equations is a maximum, and then add the separate potential differences at that time. The problem may be greatly simplified by use of the *vector diagram.*

If we draw a vector representing the maximum value of the current I_m along the $+x$ axis, the vectors representing the maximum values of the potential differences across the terminals of the various circuit elements are shown in Figure 34-8. We imagine the diagram to be drawn on a transparent card which is rotating in the counterclockwise direction with angular velocity ω. The instantaneous value of the current and of the voltage across each element at a given time t is to be determined by finding the projection of the appropriate vector onto the y axis. Thus the instantaneous emf of the generator is given by adding the y components of the three voltage vectors.

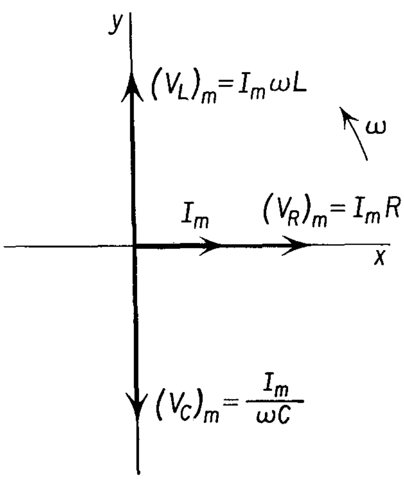

Fig. 34-8

The concepts of vector addition tell us that the sum of the y components of three vectors is equal to the y component of their resultant. Thus we may add the three voltages vectorially and draw a new diagram, shown in Figure 34-9, in which the maximum value of the current I_m is one vector, represented along the x axis, and the vector sum of the separate voltages, represented as V_m, is the other. The vector V_m is clearly equal to the maximum value of the potential difference across the three circuit elements and is therefore the maximum value of the emf of the generator. The angle ϕ is the angle by which the voltage vector leads the current vector

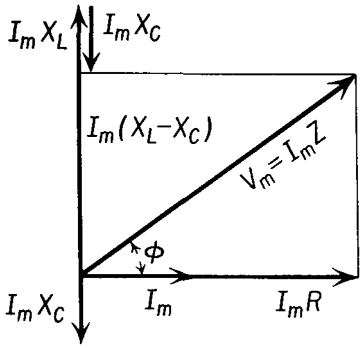

Fig. 34-9 Vector diagram of the voltages and currents in a series circuit containing resistance, inductance, and capacitance.

in the figure and is therefore the phase difference between voltage and current.

From the figure we find that

$$\mathcal{E}_m = V_m = \{[I_m(X_L - X_C)]^2 + (I_m R)^2\}^{1/2}, \tag{34-20}$$

$$\mathcal{E}_m = V_m = I_m\,[(X_L - X_C)^2 + R^2]^{1/2}. \tag{34-21}$$

The quantity in brackets in Equation (34-21) is called the *impedance Z* of the series circuit. Thus

$$Z = [(X_L - X_C)^2 + R^2]^{1/2}. \tag{34-22}$$

The maximum value of the voltage across the terminals of an a-c circuit may be expressed in terms of the impedance as

$$V_m = I_m Z. \tag{34-23}$$

This equation may be taken as the definition of the impedance of any a-c circuit.

The instantaneous value of the voltage may be seen from Figure 34-9 to be

$$v = V_m \sin(\omega t + \phi), \tag{34-24}$$

where the phase angle ϕ is given by

$$\phi = \arc\tan \frac{X_L - X_C}{R}\,. \tag{34-25}$$

The power delivered to the circuit is consumed only in the resistor. The power consumed is therefore the product of the *effective* voltage across the resistor by the *effective* current, according to Section 34-2:

$$\mathcal{P} = V_R I. \qquad \textbf{(34-9a)}$$

The effective values of voltage and current used in Equation (34-9a) are equal to the maximum values of these quantities divided by the square root of 2. Thus

$$\mathcal{P} = \frac{(V_R)_m I_m}{2}.$$

From Figure 34-9 the maximum voltage across the resistor is related to the maximum voltage across the entire circuit through the cosine of the phase angle

$$(V_R)_m = V_m \cos \phi,$$

so that the power consumed is

$$\mathcal{P} = \frac{V_m I_m \cos \phi}{2}.$$

In terms of the effective voltage across the circuit and the effective current through the circuit

$$\mathcal{P} = VI \cos \phi. \qquad \textbf{(34-26)}$$

The factor $\cos \phi$ is called the *power factor.*

It is customary to analyze a-c circuits in terms of effective current and voltage rather than maximum current and voltage. Since the effective values are related to the maximum values by a constant factor, the square root of 2, a vector diagram may be drawn to relate effective values just like Figure 34-9, without altering any of the phase relationships between voltage and current.

In general, the voltage and current indicated by a-c measuring instruments are effective values rather than maximum values.

In terms of the effective values we have for each element:

$$V_R = IR, \qquad \textbf{(34-27a)}$$

$$V_L = IX_L, \qquad \textbf{(34-27b)}$$

$$V_C = IX_C; \qquad \textbf{(34-27c)}$$

and for the entire circuit,

$$V = IZ. \qquad \textbf{(34-27d)}$$

Illustrative Example. A series a-c circuit consists of a capacitor of 8 μfd, an inductor of 600 millihenrys, and a resistor of 48 ohms resistance. Determine (a) the current in the circuit, (b) the voltage across each element in the circuit, (c) the phase angle, and (d) the power supplied to the circuit when the terminal voltage of the 60-cycle generator is 220 volts.

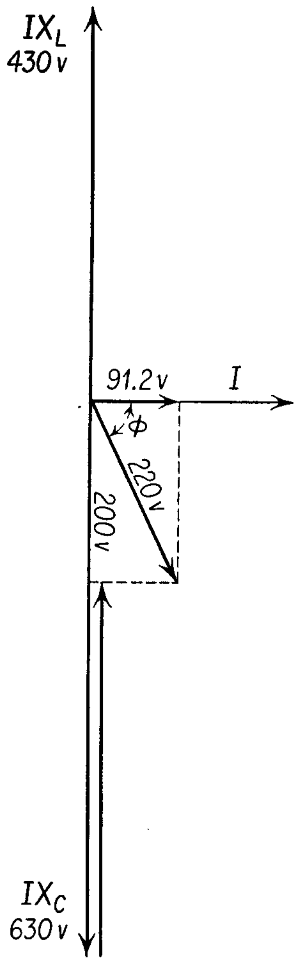

Fig. 34-10

The capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 60 \times 8 \times 10^{-6}} \text{ ohms}$$
$$= 332 \text{ ohms.}$$

The inductive reactance is

$$X_L = 2\pi fL = 2\pi \times 60 \times 0.60 \text{ ohms} = 226 \text{ ohms.}$$

The total impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{(48)^2 + (226 - 332)^2} \text{ ohms} = 116 \text{ ohms.}$$

(a) The current in the circuit is

$$I = \frac{V}{Z} = \frac{220}{116} = 1.90 \text{ amp.}$$

(b) Constructing the vector diagram as shown in Figure 34-10, with the current I along the x axis, we find that the voltage across the resistor is

$$V_R = IR = 1.90 \times 48 \text{ volts} = 91.2 \text{ volts}$$

in phase with the current and therefore drawn along the x axis; the voltage across the capacitance is

$$V_C = IX_C = 1.9 \times 332 \text{ volts} = 630 \text{ volts}$$

in the negative y direction, since the curent leads the voltage by 90°; the voltage across the inductance is

$$V_L = IX_L = 1.9 \times 226 \text{ volts} = 430 \text{ volts}$$

in the positive y direction, since the current lags behind the voltage by 90°.

The impressed voltage V is the vector sum of these individual voltages and is given by

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(91.2)^2 + (630 - 430)^2} = 220 \text{ volts.}$$

This is a convenient way of checking the calculations.

(c) The phase angle is determined from the equation

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{226 - 332}{48},$$

so that

$$\tan\phi = \frac{-106}{48} = -2.21,$$

and

$$\phi = -65°40';$$

hence the current leads the voltage by 65°40′.

It will be noted that the voltage across a capacitor or an inductor in an a-c series circuit may be much greater than the voltage across the terminals of the circuit.

(d) The power supplied by the generator is

$$
\begin{aligned}
P &= VI \cos \phi \\
&= 220 \times 1.9 \times \cos 65°40' \\
&= 173 \text{ watts.}
\end{aligned}
$$

34-6 Resonance in an A-C Series Circuit

A case of very great interest is one in which the current and the voltage are in phase in an a-c series circuit containing resistance, inductance, and capacitance. This can be established by adjusting the values of C and L so that

$$X_L = X_C, \tag{34-28}$$

in which case the phase angle ϕ will be zero, since

$$\tan \phi = \frac{X_L - X_C}{R}. \tag{34-25}$$

Putting in the values for X_L and X_C in Equation (34-28), we get

$$2\pi f L = \frac{1}{2\pi f C},$$

from which

$$f = \frac{1}{2\pi\sqrt{LC}}. \tag{34-29}$$

When this condition is fulfilled, the current in the circuit will be a maximum, since the impedance is a minimum, and will be given simply by

$$I = \frac{V}{R},$$

since for this case

$$Z = R.$$

A circuit for which Equation (34-29) holds is said to be in *resonance* at the frequency f. When the frequency of the a-c supply is that given by Equation (34-29), there is a maximum transfer of energy from the generator to the circuit, since the phase angle ϕ is zero and the power factor is 1. For example, if, in the illustrative example of the previous section, sufficient inductance is added to the circuit either by inserting more iron in the inductance coils or by adding additional inductance coils so that the circuit is in resonance at 60 cycles/sec then the current in the circuit would be increased to its maximum value given by

$$I = \frac{V}{R} = \frac{220}{48}\text{ amp} = 4.58\text{ amp.}$$

To determine the new value of the inductance, we can solve Equation (34-29) for L and get

$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 \times 3{,}600 \times 8 \times 10^{-6}}\text{ henry}$$
$$= 0.88\text{ henry} = 880\text{ millihenrys.}$$

34-7 The Transformer

The electric energy which is transmitted from the generating station to the consumer is transmitted during a time interval t, and, in designing a transmission system, it is the power or the rate at which the energy is transmitted that is of importance. If the terminal voltage of a d-c generator is V, the power delivered to the transmission line is

$$\mathcal{P} = VI,$$

where I is the current in the line. If the transmission line has a resistance R, then the rate at which heat is developed in the line is I^2R, and hence the power $\mathcal{P}$ delivered to the consumer is

$$\mathcal{P} = VI - I^2R. \tag{34-30}$$

A greater amount of power can be transmitted to the consumer by reducing the resistance of the power line, that is, by using wires of larger diameters, or else by transmitting the power at smaller currents. The latter method means stepping up the voltage at the generating station.

It has been found difficult to build d-c generators which will develop emf's greater than about 3,000 volts. Hence to transmit direct current at higher voltages, it would be necessary to connect several generators in series. This practice is not commonly followed in this country. Another difficulty is that, for safe handling, the voltage at the consumer's end of the line must be comparatively low—not more than a few hundred volts— and no efficient methods have been developed for stepping down the voltage of a d-c line. For a-c generating stations, however, the problem is entirely different. With the aid of a device known as a *transformer*, it is possible to step up the voltage at the transmission line to any desired value, and then to use another transformer at the consumer's end of the line to step down the voltage to a safe, usable value, the power having meanwhile been transmitted at a high voltage and low current. Modern transmission lines are operated at voltages as high as 250,000 volts.

A transformer consists of two coils near each other. In most transformers these coils are wound on closed iron cores such as that shown in Figure 34-11(a). The conventional diagram of an iron-core transformer is shown in Figure 34-11(b). For special uses, particularly in some radio circuits, transformers are made without iron cores; these are usually called air-core transformers.

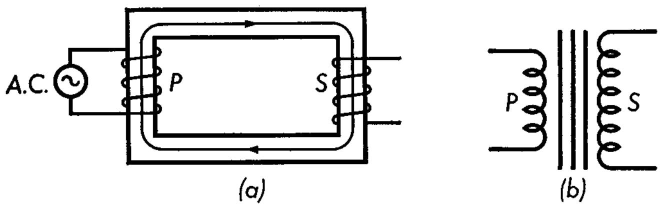

Fig. 34-11 Iron-core transformer.

Suppose that the primary coil P of an iron-core transformer is connected to an a-c source and that the effective voltage across its terminals is $\mathcal{E}$. Let us suppose initially that the terminals of the secondary coil are left open. The current that now flows through the primary coil sets up a magnetic field in the iron core. Because of the high permeability of the iron, practically the entire magnetic flux is inside the iron core. Since the current is alternating, the magnetic flux in the iron core is also alternating. This alternating magnetic flux induces an emf in each turn of the secondary coil, and hence the induced emf $\mathcal{E}_S$ in the secondary coil is proportional to the number of turns of wire n_S in the secondary coil. Since the same magnetic flux goes through each turn of the primary coil, an emf will be induced in each turn of the primary coil so that the total self-induced emf in the primary coil $\mathcal{E}_P$ will be proportional to the number of turns n_P in the primary coil. Since the magnetic flux is changing at the same rate inside each coil, we can write that

$$\frac{\mathcal{E}_S}{\mathcal{E}_P} = \frac{n_S}{n_P}. \tag{34-31}$$

Equation (34-31) holds for both the instantaneous values of the emf's and for their effective values.

In a well-designed transformer, $\mathcal{E}_P$ will differ very slightly from the impressed voltage $\mathcal{E}$. Thus, to a very close approximation, Equation (34-31) may be written as

$$\frac{\mathcal{E}_S}{\mathcal{E}} = \frac{n_S}{n_P}. \tag{34-32}$$

If the number of turns n_S in the secondary coil is greater than the number of turns n_P in the primary coil, the transformer is called a *step-up* transformer;

if the reverse is the case, it is a *step-down* transformer. For example, if the secondary coil has 1,000 times as many turns as the primary coil, the emf $\mathcal{E}_S$ induced in the secondary will be 1,000 times the voltage impressed across the primary.

When a load is connected to the terminals of the secondary coil, a current will flow in the secondary circuit, and power will be supplied by it. This power must, of course, come from the source of power connected to the primary coil. This transfer of power takes place through the interactions of the magnetic fields because of the current in the primary coil and that in the secondary coil. In well-designed transformers the efficiency is as high as 98 or 99 per cent. Neglecting the slight loss of power in heating the coils and the iron core, we find that the power drawn from the secondary coil must equal the power supplied to the primary coil; that is,

$$\mathcal{E} i_P = \mathcal{E}_S i_S, \tag{34-33}$$

where the symbols refer to the instantaneous values of the voltage and current in the primary and secondary coils, respectively. Or

$$\frac{\mathcal{E}}{\mathcal{E}_S} = \frac{i_S}{i_P}. \tag{34-34}$$

Since Equation (34-34) holds at any instant, it also holds for the maximum values and hence for the effective values, so that we can write

$$\frac{\mathcal{E}}{\mathcal{E}_S} = \frac{I_S}{I_P}, \tag{34-35}$$

which, combined with Equation (34-32), yields

$$\frac{I_S}{I_P} = \frac{n_P}{n_S}. \tag{34-36}$$

Thus the effective values of the currents in the primary and secondary circuits are in the inverse ratio of the numbers of turns in the two coils.

If we rewrite Equation (34-36) as

$$n_S I_S = n_P I_P, \tag{34-37}$$

we note that, if the current in the secondary is increased, as is the case when the load on the secondary is increased, the current in the primary is also increased.

In a step-up transformer the emf induced in the secondary is large but the current I_S is small, while the voltage across the primary is small and the current I_P through it is large. Both step-up and step-down transformers are used in transmitting power. A simplified version of a transmission system is shown in Figure 34-12. At the powerhouse, the a-c generator develops electric power at, say, 120 volts; the terminals of this generator are connected to the terminals of the primary coil of a step-up transformer T_1 which steps up the voltage to 12,000 volts at the terminals of the secondary coil. The two wires of the transmission line, which may be several miles long, connect the terminals of this secondary coil to the primary of a step-

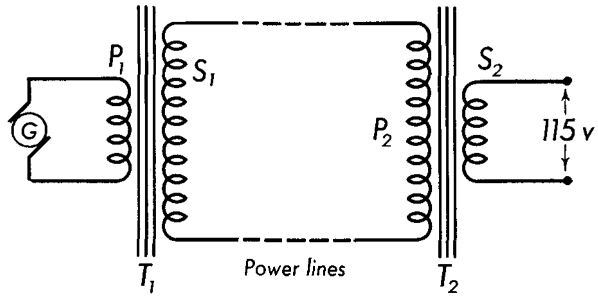

Fig. 34-12 Simple transmission line.

down transformer T_2 at the consumer's end of the line. Because of the voltage drop in the transmission line, the difference of potential at the primary of the step-down transformer may be only 11,500 volts, say. If the ratio of turns between P_2 and S_2 is 100 : 1, the emf at the terminals of S_2 will be about 115 volts, suitable for use with a great many electric appliances.

Problems

34-1. The terminals of a series circuit consisting of a 32-millihenry inductor and a 48-ohm resistor are connected to a source of emf supplying 110-volt, 60-cycle power. Find (a) the reactance of the inductor, (b) the impedance of the circuit, (c) the effective current (d) the maximum current, (e) the phase angle between the current and voltage, (f) the effective voltage across the resistor, (g) the effective voltage across the inductor, and (h) the power dissipated in the circuit.

34-2. A coil with an inductance of 0.020 henry and an internal resistance of 8 ohms is connected in series with a 75-ohm resistor. The series combination is connected to a 110-volt, 60-cycle generator. Find (a) the inductive reactance of the coil, (b) the impedance of the coil, (c) the impedance of the series circuit, (d) the current in the circuit, and (e) the power dissipated in the circuit.

34-3. By setting

$$i = I_m \sin \omega t$$

and

$$v = V_m \sin (\omega t \pm \phi)$$
$$= \pm V_m \cos \omega t$$

and integrating over one cycle, show that the power consumed in an inductor or in a capacitor is equal to zero.

34-4. A 2 μfd capacitor is connected in series with a 36-ohm resistor, and the series combination is connected to a 120-volt, 60-cycle power line. Find (a) the capacitive reactance, (b) the impedance, (c) the current, and (d) the power consumed in the circuit.

34-5. A 40-ohm resistor is connected in series with a 1.8-henry inductor and a 10-μfd capacitor. (a) Determine the inductive reactance, the capacitive reactance, and the impedance of the circuit at 60 cycles. (b) Determine the current in the circuit when it is connected to a 60-cycle, 110-volt source. (c) Determine the effective voltage across the resistor, across the capacitor, and across the inductor. Is the sum of these different from 110 volts? Why?

34-6. A 45-ohm resistor, an 8-μfd capacitor, and an 0.06-henry inductor are connected in series. (a) What is the resonant frequency of this circuit? (b) When connected to a 100-volt power source at this frequency, what is the current in the circuit? (c) Draw a vector diagram of the circuit at resonance. (d) What is the voltage across the inductor at resonance? (e) What is the voltage across the resistor at resonance? (f) What is the power factor at resonance?

34-7. Plot a graph of the impedance of the circuit of Problem 34-6 as a function of frequency, giving particular emphasis to the shape of the curve in the vicinity of resonance.

34-8. An iron-core transformer has 100 turns in the primary winding and 800 turns in the secondary winding and is operated from a 120-volt, 60-cycle generator. Determine (a) the emf induced in the secondary, (b) the current in the secondary circuit when it is taking 2.4 kw of power with a power factor of 0.75, and (c) the current in the primary.

34-9. A transformer is used to step down the voltage of a transmission line from 13,200 volts to 240 volts. (a) What is the ratio of the turns on the two windings? (b) If the secondary supplies 15 amp, determine the current in the primary.

34-10. A capacitor is connected in series with a resistor, and the combination is then connected to a battery whose emf is V. Show that the potential difference across the terminals of the capacitor is given by

$$v = V(1 - e^{-(t/RC)}).$$

[HINT: Follow the development of Section 32-7.]

34-11. An alternator furnishes 80 amp at 240 volts at a frequency of 60 cycles/sec to a shop in which the power consumed is 17.6 kw. What series capacitance must be introduced into the electrical lines to change the power factor to unity?

34-12. A square-wave generator generates current of the wave form shown in Figure 34-13. Find the effective value of the current.

34-13. A series circuit consisting of a 20-ohm resistor and an inductor whose internal resistance is 10 ohms is observed to have an impedance of 50 ohms at a frequency of $100/\pi$ cycles/sec. (a) What is the current in the circuit when the applied emf is 100 volts? (b) What is the inductance in the circuit? (c) What is the impedance of the circuit at a frequency of $300/\pi$ cycles/sec?

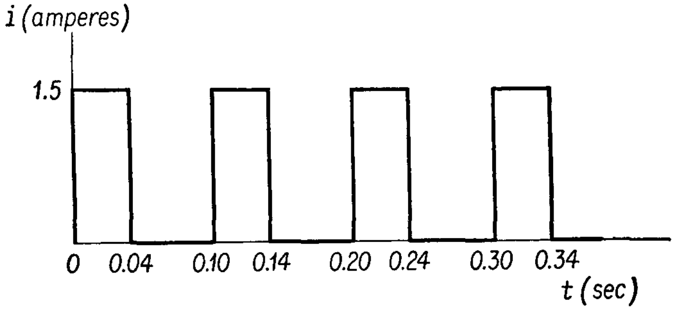

Fig. 34-13

34-14. A series circuit has a power factor of 0.8 with the current lagging the voltage at a frequency of $200/\pi$ cycles/sec. (a) If this circuit has a resistance of 100 ohms, what is the impedance? (b) If this circuit has a capacitance of 10^{-4} farad, what is the inductance? (c) At what frequency will its power factor be zero?

34-15. In a given circuit consisting of a resistor, an inductor, and a capacitor in series, the voltage across the resistor is 100 volts, the voltage across the capacitor is 200 volts, and the voltage across the inductor is 150 volts. The power consumed by the circuit is 150 watts. (a) What is the current in the circuit? (b) What is the power factor? (c) What is the inductive reactance?