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37

Reflection and Refraction

37-1 Passage of Light through a Medium

In the passage of a beam of light through a medium, some of the radiant energy is absorbed and is transformed into internal energy, while some of it is scattered in all directions. The oscillating electric field associated with the light wave sets some of the electrons of the medium into oscillation, thus giving up some of its energy, and these oscillating electrons subsequently reradiate energy as scattered electromagnetic radiation. Scattering therefore takes place only in the presence of matter. The color of the sky is due to the small amount of scattering of sunlight by the molecules of the air. At high altitudes the number of scattering particles diminishes, and the sky is darker. In the stratosphere the sky appears almost black. The molecules of the air are more effective in scattering short wavelengths, so that the scattered light appears blue. Since the shorter wavelengths are scattered from the direct beam, the beam appears redder as it passes through larger thicknesses of air. It is for this reason that the setting sun looks redder than the noonday sun. The scattering of light by the air is responsible for twilight. In microwave communications the distance between transmitting and receiving antennae is limited by the curvature of the earth, because of the straight-line propagation of electromagnetic waves. By transmitting a stronger initial signal and detecting the scattered microwaves, it has been possible to reduce the number of transmitting and receiving stations in the microwave communications network by increasing the distance between relay stations beyond the line of sight.

If a beam of light of luminous flux F is incident upon a slab of matter of thickness Δx, some of the incident flux is absorbed in the substance as heat, and some is scattered out of the incident beam, as shown in Figure 37-1. The luminous flux reaching the detector is diminished by an amount ΔF in such a way that the fractional loss in luminous flux is proportional to the thickness of the slab. Thus we have

$$\frac{\Delta F}{F} = -\mu \, \Delta x, \tag{37-1}$$

where μ is a constant of proportionality called the *absorption coefficient.* The minus sign appearing in the equation is associated with the fact that a positive increment in thickness is accompanied by a negative increment

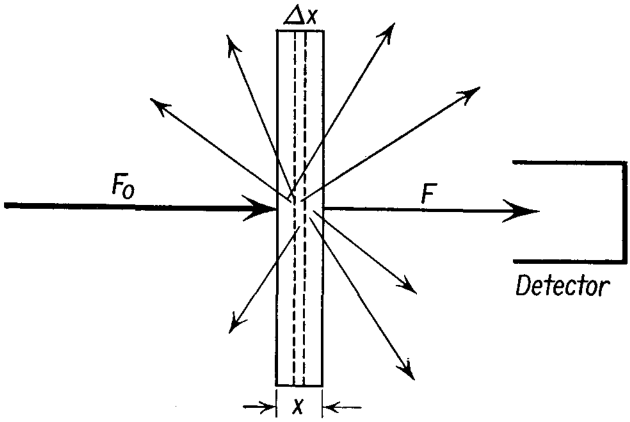

Fig. 37-1

in luminous flux. When a beam of incident flux F_0 is incident upon a slab of thickness x, the emergent beam is of flux F, whose value may be obtained by integrating Equation (37-1). We have

$$\int \frac{dF}{F} = -\int_0^x \mu \, dx,$$

or

$$\ln \frac{F}{F_0} = -\mu x,$$

yielding

$$F = F_0 e^{-\mu x}. \tag{37-2}$$

Thus the luminous flux in a beam of radiation is exponentially absorbed on its passage through matter. The absorption coefficient μ depends upon the wavelength λ and upon the nature of the material through which the radiation is passing.

When a beam of light strikes the surface separating one medium from another—for example, the surface between air and glass—some of the light is reflected back into the first medium at the surface of separation, and the remainder enters the second medium. The light which passes from one medium to another is said to be *refracted.* If the surface of separation between the two media is smooth, the light which is reflected back into the first medium is said to be *regularly reflected* or *specularly reflected.* If the surface is rough, the light is *diffusely reflected.* Unless otherwise stated, we shall assume that the surface between two media is smooth.

In general, smooth polished metal surfaces will reflect about 90 per cent of the incident light, while smooth polished glass surfaces will reflect about 4 to 10 per cent for angles of incidence from 0° to 60°. In the case of a metal, the light which is refracted through the surface is absorbed in a very small thickness of the surface; that is, the absorption coefficient of a metal is very high. Glass, on the other hand, has a very low absorption coefficient for visible light; that is, very little light is absorbed in its passage through reasonable thicknesses of glass.

The electrical conductivity, the permittivity, and the permeability of a substance all affect the transmission of light through that substance. These properties depend upon the frequency, so that substances which are good conductors at one frequency may be poor conductors at a much higher frequency. As a general statement, electrical conductors transmit light poorly. An optically transparent substance cannot be a good conductor of electricity. Because of the variation of electrical properties with frequency, we find that materials which are transparent to visible light and to radio waves may be opaque to infrared or ultraviolet radiation. While all metals are opaque to visible light, it is possible to transmit a beam of x-rays through them. Sea water, which is a relatively poor conductor, reflects about 80 per cent of the normally incident light. It is possible to coat the surface of glass with a thin layer of silver to make a mirror which will reflect about half the incident light and will transmit the other half. In such cases the layer is very thin, and its electrical resistance is high.

Although reflection and refraction can be studied by considering the detailed interaction between the light and the individual particles of the medium through which it travels, we may obtain useful results by assuming that light travels along straight lines through optically homogeneous media, the direction of motion being indicated by *rays* of light. These rays are drawn at right angles to the *wave front.* The wave front is the envelope of the waves emitted by the particles of the medium; *all points on a wave front are in the same phase.* This method of treating reflection and refraction is a first approximation only. Later, in the discussion of *interference* and *diffraction,* we shall consider some of the modifications that must be made because of the wave character of light. The simpler treatment is called *geometric optics.*

37-2 Laws of Reflection

When a narrow beam of light strikes a smooth surface separating two media, the angles of incidence, reflection, and refraction are all measured from a *normal* to the surface; a normal to a surface at a given point is a line drawn perpendicular to the surface at that point. In Figure 37-2 NP is the normal to the surface at P, AP is the incident ray, PB is the reflected ray, and PC is the refracted ray. The angle of incidence i is the angle between the incident ray and the normal; the angle of reflection r is the angle between the reflected ray and the normal; the angle of refraction r' is the angle between the refracted ray and the normal.

The two *laws of reflection* may be stated as follows: (a) *The incident ray, the normal, and the reflected ray all lie in one plane,* and (b) *the angle of incidence is equal to the angle of reflection.* The laws of reflection are empirical laws and have been known since the tenth century. As we

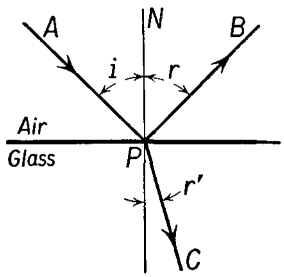

Fig. 37-2 Light reflected and refracted at a surface separating two media.

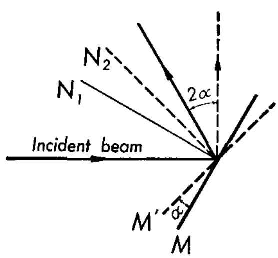

Fig. 37-3 The angle through which the reflected beam rotates is twice the angle through which the mirror rotates.

have seen in Section 20-4, the equality of the angles of incidence and reflection may be derived from Huygens' principle for an incident plane wave. If a wave front is spherical, we can take a sufficiently small portion of the wave and treat it as a plane wave.

Rotating mirrors are often used in physical apparatus to provide an instrument with a long weightless pointer, as in the wall galvanometer. When the mirror turns through a small angle α, the angle of incidence is increased (or decreased) by α so that the angle of reflection is also increased (or decreased) by α. As shown in Figure 37-3, we observe that when a plane mirror, originally in position M, is rotated through an angle α, the reflected beam will rotate through an angle 2α.

An observer whose eye is directed toward the reflecting surface of a mirror, Figure 37-4, sees rays of light reflected from the mirror. The eye observes the light rays QC and $Q'C'$, and the observer interprets these rays as coming from the point P' behind the mirror. If the observer's head is turned toward the point P, the rays QP and $Q'P$ enter his eye, and he interprets these rays as originating at the point P. If the mirror is perfectly reflecting, the observer sees the real source at P and the virtual source at P' as sources of equal brightness. The point P' is called the image point of P in the mirror. Since the rays QC and $Q'C'$ do not actually come from P' but only appear to do so, the point P' is called a *virtual image* of the point P in the plane mirror. *A plane mirror forms a virtual image of a real object, and the image is located as far behind the mirror as the object is in front of the mirror.*

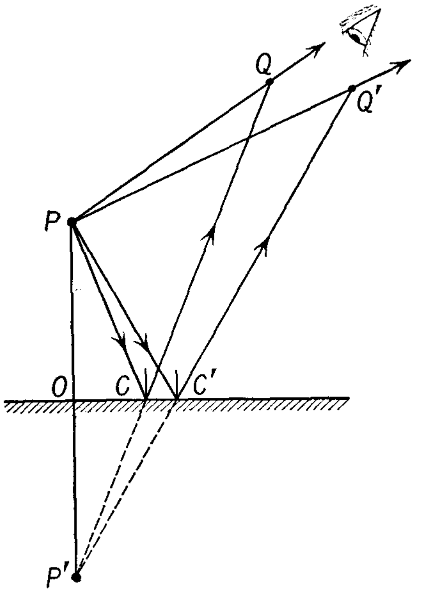

Fig. 37-4 Determining the position of the image formed by a plane mirror.

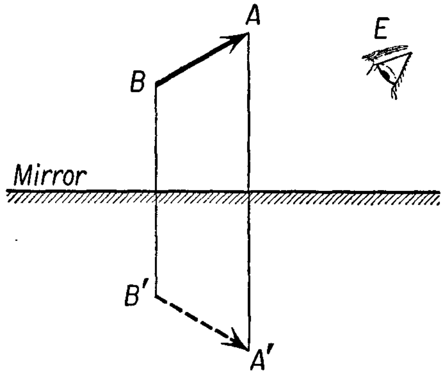

Fig. 37-5 The image formed by a plane mirror appears reversed to an observer at E.

If an object of finite size, such as the arrow BA of Figure 37-5, is placed in front of a plane mirror, each point on the object is imaged behind the mirror. Thus the point B is imaged in the point B', the point A is imaged in the point A', and so on. An observer looking into the mirror sees the virtual image $B'A'$ of the arrow BA, in the position indicated on the figure, no matter from what position he observes the image. An observer at E sees the head of the real arrow AB on his right, while the head of the imaged arrow $A'B'$ appears on his left. The image is said to be *reversed* from right to left. *The size of the image formed in a plane mirror is the same as the size of the object.* The image is formed in a region behind the mirror in which there is no light, so that it is impossible to form the image on a piece of paper or a ground glass. Nevertheless, the eye has interpreted the rays of light reflected from the mirror as though there really were a source of light behind the mirror.

37-3 Refraction of Light

Refraction occurs whenever light passes from one medium to another and is due to the fact that light travels with different speeds in different media. Refraction was studied, and the laws of refraction were derived empirically, long before the cause of refraction was known. Experimental observations on the refraction of light at a plane interface may be summarized in the following *laws of refraction:* (a) *The incident ray, the normal, and the refracted ray are in the same plane, and* (b) *for monochromatic light, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant.* In the form of an equation we have

$$\frac{\sin i}{\sin r'} = n_r, \tag{37-3}$$

in which i is the angle of incidence, r' is the angle of refraction, and n_r is a constant known as the *relative index of refraction* of the second medium with respect to the first medium. Equation (37-3) is often referred to as *Snell's law*, after Willebrord Snell (1591–1626), who is thought to have discovered this empirical relationship in 1621.

We have shown in Section 20-7 that the existence of a relative index of refraction can be derived from Huygens' principle, a derivation first presented by Huygens to the Royal Academy of Sciences at Paris in 1678, and that the relative index of refraction is equal to the ratio of the velocities of light in the two media, as in Equation (20-5)

$$n_r = \frac{v_1}{v_2}, \tag{37-4}$$

in which v_1 is the velocity of light in the first medium, and v_2 is the velocity of light in the second medium.

We may define the *absolute index of refraction* n of a medium as the index obtained when light passes from vacuum into the medium. From Equation (37-4) we find that

$$\boxed{n = \frac{c}{v},} \tag{37-5}$$

where c is the velocity of light in vacuum, and v is the velocity of light in the medium. The absolute index of refraction n is customarily found in tables. Since the velocity of light in a medium is dependent upon its frequency, and therefore upon its wavelength, the absolute index of refraction varies with the wavelength, as shown in Table 37-1 for several

TABLE 37-1 ABSOLUTE INDICES OF REFRACTION

Substance	Absolute Indices of Refraction				
Wavelength	7,682 A	6,563 A	5,893 A	4,861 A	4,047 A
Borosilicate crown glass	1.5191	1.5219	1.5243	1.5301	1.5382
Dense flint glass	1.6441	1.6501	1.6555	1.6691	1.6901
Water 20°C	1.3289	1.3311	1.3330	1.3371	
Carbon disulphide 18°C		1.6198	1.6255	1.6541	
Diamond			2.417		

substances. From Equation (37-5) the absolute index of refraction for a medium 1 is given by $n_1 = c/v_1$, while for a second medium, 2, the absolute index of refraction is given by $n_2 = c/v_2$. Dividing the second of these equations by the first, we find

$$\frac{n_2}{n_1} = \frac{c/v_2}{c/v_1} = \frac{v_1}{v_2},$$

so that

$$n_r = \frac{n_2}{n_1}. \tag{37-6}$$

If we relabel the angle of incidence ϕ_1 and the angle of refraction ϕ_2, we may rewrite Snell's law, as given in Equation (37-3), in a more convenient form as

$$\boxed{n_1 \sin \phi_1 = n_2 \sin \phi_2} \tag{37-7}$$

where we may interpret ϕ_1 as the angle the ray makes with the normal in medium 1, and ϕ_2 as the angle the ray makes with the normal in medium 2. From the symmetry of Equation (37-7), we see that the direction of the light beam may be reversed without altering the path taken by the light beam. Thus a beam which makes an angle ϕ_1 with the normal in medium 1 will make an angle ϕ_2 with the normal in medium 2, regardless of whether it passes from the first to the second medium or from the second medium to the first.

A medium whose absolute index of refraction is high is sometimes called an *optically dense medium*, while a medium whose index of refraction is low is called an *optically rare medium*. Optical density bears no relation to actual physical density for different substances. The velocity v of light in a medium is related to its electromagnetic properties through the equation

$$v = (\epsilon\mu)^{-1/2}. \tag{37-8}$$

As we have already seen in Sections 25-6 and 29-8, both the permittivity ϵ and the permeability μ of a medium depend upon the number of molecules per unit volume of a medium. If there are no molecules present, there can be no induced dipole moment. The induced electric and magnetic dipole moments per unit volume vary directly with the number of molecules per unit volume, provided that the molecules are far enough apart so that there is no appreciable interaction between them, as in the case of a gas. Thus we must expect both ϵ and μ to increase with the density of a gas, and consequently we must expect the velocity of light in a gas to decrease with increasing density. A gas whose physical density is relatively high has a correspondingly high index of refraction and is therefore an optically denser medium than the same gas at low density.

37-4 Refraction Effects

When a beam of light from a distant source passes through a transparent medium with parallel surfaces, the beam is displaced parallel to itself, but its direction is unaltered. The beam is *displaced* but is undeviated, as

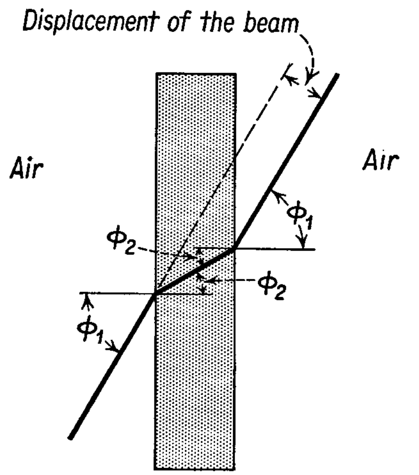

Fig. 37-6 Displacement of a ray of light by a transparent plate with parallel surfaces.

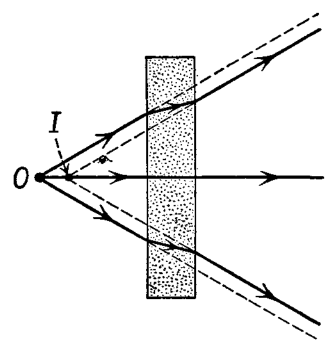

Fig. 37-7 The point of divergence of a beam of light appears displaced after passing through a plate of glass with parallel faces.

shown in Figure 37-6. A beam of light from a source close to the transparent plate must be treated as a divergent beam, made up of spherical waves. Each ray striking the plate is displaced a different amount. If the divergence is not too large, the rays seem to come from a point I, Figure 37-7, displaced from the source of light O by an amount which depends upon the thickness of the glass. By reversing the directions of the rays in this diagram, we note that the effect of a parallel plate of glass on a bundle of rays converging to a point, or to a focus, is to shift the point of convergence away from the plate of glass. This method has been used to change the focal length of camera lenses.

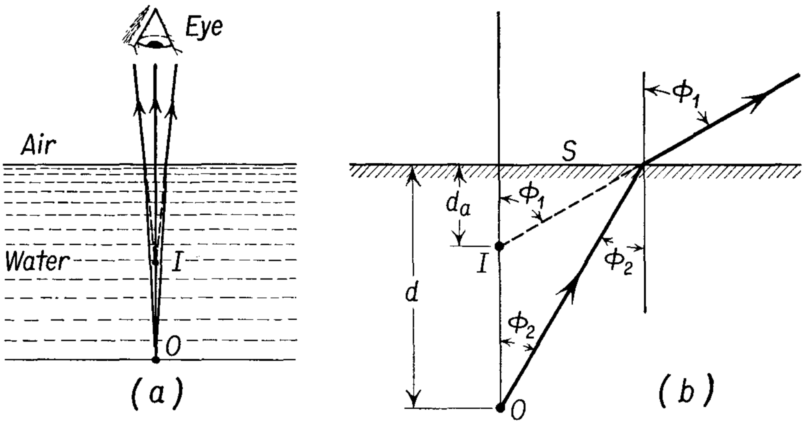

Fig. 37-8 Shallowing effect produced by refraction.

Another interesting effect produced by the refraction of a narrow bundle of rays is illustrated in Figure 37-8(a). Rays from some point O at the bottom of a pool of water are viewed by an observer whose eye is vertically above this point in the air. The narrow bundle of rays entering the eye appears to come from a point I above point O. The pool seems shallower. Redrawing the path of one ray on an exaggerated scale in Figure 37-8(b), we note that

$$n_1 \sin \phi_1 = n_2 \sin \phi_2,$$

and, if the angles ϕ_1 and ϕ_2 are sufficiently small, we have

$$n_1 \tan \phi_1 = n_2 \tan \phi_2.$$

From the figure we note that $\tan \phi_1 = s/d_a$, and $\tan \phi_2 = s/d$,

so that

$$n_1/d_a = n_2/d,$$

and since medium 1 is air with $n_1 = 1$, we may write n for n_2, dropping the subscript 2 to find

$$d_a = d/n. \tag{37-9}$$

Thus the apparent depth d_a of a transparent medium is its actual depth divided by the index of refraction of the medium. This method may be used to measure the refractive index of a glass microscope slide whose thickness is measured with a caliper, and whose apparent thickness is measured by determining the distance the microscope must be lowered when it is first focused on the top surface of the slide and them focused on the bottom surface of the slide.

37-5 Critical Angle. Total Reflection

A ray of light traveling from a region of high refractive index n_2 to a region of low refractive index n_1 may penetrate the surface of separation and enter the medium 1 only if the angle of incidence ϕ_2 is less than a certain angle ϕ_c, called the *critical angle.* If the angle of incidence is greater than the critical angle, the light will not be able to penetrate the surface but will be totally reflected back into medium 2. From Snell's law

$$n_1 \sin \phi_1 = n_2 \sin \phi_2,$$

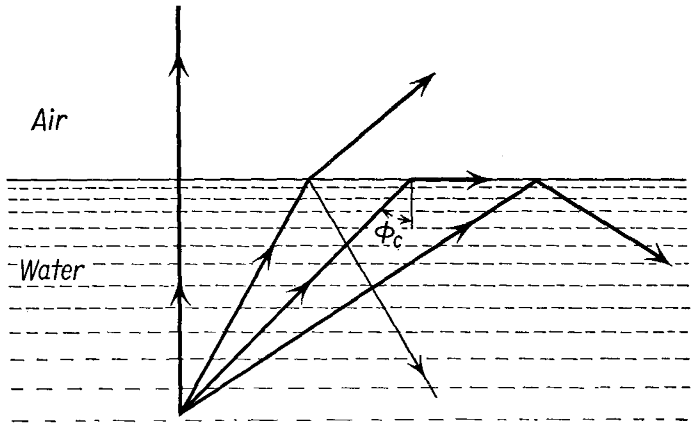

Fig. 37-9 Critical angle of refraction; total reflection.

and since n_2 is greater than n_1, the angle ϕ_1 must be greater than ϕ_2; that is, the ray is refracted away from the normal. The greatest value ϕ_1 may have is 90°. Thus we find that

$$\sin \phi_2 = n_1/n_2$$

for the greatest angle of incidence at which a ray will emerge in medium 1. This angle is the critical angle ϕ_c. In case air is the rarer medium, $n_1 = 1$, and, dropping the subscript 2, we find that

$$\sin \phi_c = 1/n. \tag{37-10}$$

At incident angles greater than the critical angle, Snell's law yields an imaginary solution, for which the sine of the angle of refraction is required to be greater than 1.

As illustrated in Figure 37-9, light incident upon a plane interface between an optically dense and an optically rare medium is partially reflected and partially transmitted as long as the angle of incidence is less than the critical angle. At angles greater than the critical angle, the light is totally reflected.

The property of total internal reflection is applied in optical instruments where glass prisms rather than silvered surfaces, are used to reflect light, for even the best silvered surface absorbs several per cent of the incident light. In prism binoculars a pair of 45° right-angle prisms, called Porro prisms, are used in each monocular to provide two reflections in each prism so that the resulting image is not reversed as shown in

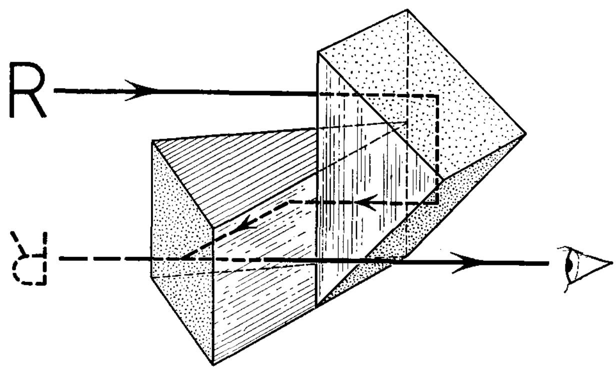

Fig. 37-10 Porro prisms, as used in a prism binocular.

Figure 37-10. The prisms are used to shorten the distance between the objective or front lens of the binocular and the eyepiece. A long piece of plastic rod may be used as a *light pipe*, and the light may be made to bend around corners by the property of total internal reflection. The rod is illuminated at one end and emerges from the other end of the polished rod. This concept is presently being extended in the field of *fiber optics*, where a bundle of fine glass fibers transmits the light incident at one end of the bundle down to the other end. Thus the image focused on one end of the bundle may be made to appear at the other end of a flexible bundle of fibers. This appears to have very promising application in medical research, permitting the visual examination of the interior of the stomach, for example. Display signs of glass or plastic are sometimes illuminated at the edge, and light emerges from the sheet of transparent material where the sheet has been engraved or lettered, so that the lettering is luminous, on a transparent background.

37-6 Refraction and Dispersion

As we have seen, the speed of light in a medium depends upon the nature of the medium and upon the wavelength of the light. A medium in which the velocity of propagation of a wave depends upon the wavelength is called a *dispersive medium.* The index of refraction of a dispersive medium also depends upon the wavelength. If a narrow beam of white light is incident upon the face AB of a triangular glass prism, as shown in Figure 37-11, the emergent beam is not white but consists of an array of colors extending from red through orange, yellow, green, blue, and violet. The

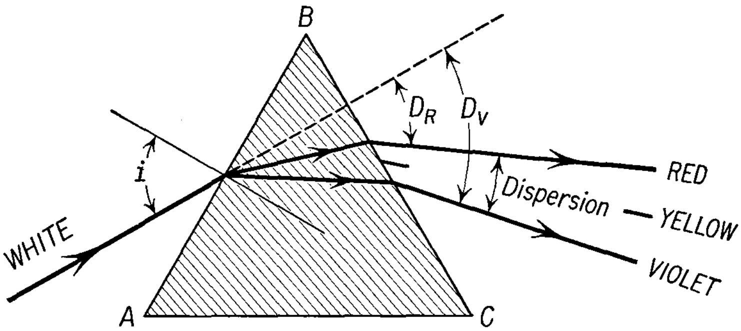

Fig. 37-11 Deviation and dispersion of a beam of white light by a triangular prism. The angle D_R is the deviation of the red light and D_V is the deviation of the violet light.

prism is said to *disperse* the incoming radiation into its spectrum. The white light incident upon the surface AB at some angle i is refracted and dispersed. The different wavelengths, or colors, which constitute the white light are bent toward the normal at the surface AB by different amounts, depending upon the index of refraction at the particular wavelength. The dispersed rays travel through the prism at different speeds and are refracted again at the surface AC, this time being refracted away from the normal to the surface AC, thus deviating the rays still more. The greater the index of refraction, the greater the angle of deviation. Thus red light, for which the index of refraction of glass is smallest, is deviated least by a glass prism, while violet light is deviated the greatest amount, corresponding to its high index of refraction.

If a very narrow beam of *monochromatic* light, that is, light of a single wavelength such as the yellow light from a sodium flame or a sodium arc, is sent through a triangular prism ABC which is made of some transparent substance, the beam will be deviated from its original direction through an angle D, the angle of deviation. If the angle of incidence is changed by rotating the prism with respect to the incident beam, the angle of deviation will also change. It may be shown by a rather tedious computation that the deviation is a minimum when the angle of incidence i and the angle of emergence e are equal, and when the ray within the prism is parallel to the base BC of the prism, as shown in Figure 37-12. There is a simple relationship connecting the *angle of minimum deviation* D_m, the angle A between the two refracting faces of the prism, and the index of refraction n of the material of the prism, for the particular wavelength at which the angle of minimum deviation is observed. In Figure 37-12 we note that D_m is an exterior angle to the isosceles triangle whose base angles are each $i - r'$, and we get

$$D_m = (i - r') + (i - r') = 2(i - r').$$

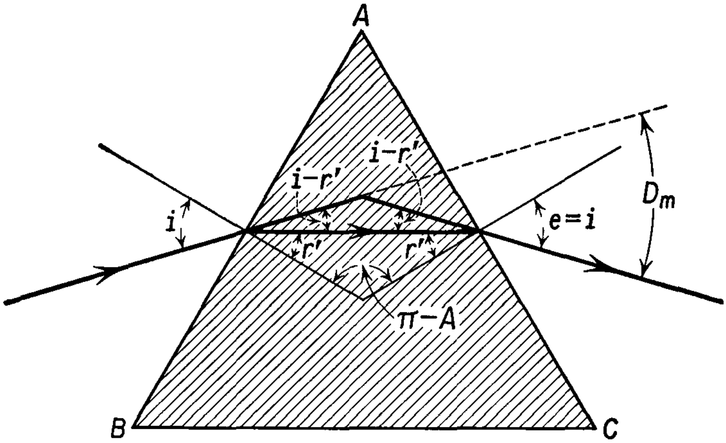

Fig. 37-12 Angle of minimum deviation.

The angle opposite A in the quadrilateral formed by the sides of the prism and the normals is equal to $\pi - A$. Since the sum of the angles of a triangle is equal to π, we may write

$$r' + r' + (\pi - A) = \pi,$$

so that

$$2r' = A.$$

Hence

$$D_m = 2i - A,$$

or

$$i = \frac{D_m + A}{2}.$$

Using Snell's law,

$$n = \frac{\sin i}{\sin r'} = \frac{\sin \frac{1}{2}(A + D_m)}{\sin \dfrac{A}{2}}. \tag{37-11}$$

If a transparent substance is fashioned in the form of a prism of refracting angle A, its index of refraction for any desired wavelength can be determined by measuring the angle of minimum deviation for this particular wavelength.

Problems

37-1. What is the velocity in water (a) of red light of wavelength 6,563 A and (b) of blue light of wavelength 4,861 A? Use the data of Table 37-1.

37-2. A narrow beam of light, of wavelength 7,600 A, is incident on the surface of water at an angle of 40°. Determine the angle of refraction in the water.

37-3. A narrow beam of white light is incident upon one face of a sheet of crown glass of thickness 5 cm at an angle of 45°. (a) Determine the displacement of the red light of the beam of wavelength 7,600 A. (b) Determine the displacement of the blue light of the beam for which $\lambda = 4{,}000$ A.

37-4. Water is placed in a flat-bottomed jar made of crown glass. The depth of the water is 5 cm, and the thickness of the glass bottom of the jar is 1 cm. Trace the path of a ray of light of wavelength 5,893 A which is incident on the top surface of the water at an angle of 45° and passes through both the water and the glass. (a) What is the angle of refraction in the water? (b) What is the angle of refraction at the glass-water interface? (c) What is the displacement of the beam when it emerges from the glass bottom of the jar? (d) What is the deviation of the beam when it emerges from the glass bottom of the jar?

37-5. The angle between the two refracting surfaces of a crown-glass prism is 60°. Trace the path of a ray of yellow sodium light ($\lambda = 5{,}893$ A) through the prism if the angle of incidence is 55°. (a) What is the angle of refraction at the first surface? (b) What is the angle of emergence at the second surface? (c) What is the angle of deviation of the emergent ray?

37-6. The absorption coefficient of a filter for a particular wavelength of light is 0.2 per centimeter. (a) What thickness of filter is required to reduce the incident flux upon the filter to $1/e$ of its original value? (b) What fraction of the incident flux will pass through a filter 0.5 cm thick? (c) If a thickness t of filter reduces the intensity of the beam by one half, what thickness will reduce the beam by one fourth?

37-7. A sheet of aluminum 0.4 cm thick reduces the flux of an incident beam of x-rays by 30 per cent. What is the absorption coefficient of aluminum for this radiation?

37-8. (a) Determine the angle of minimum deviation for a beam of light of wavelength 5,893 A passing through a 60° glass prism if its index of refraction is 1.65. (b) Determine the angle of incidence of this beam when its deviation is a minimum.

37-9. A low-power microscope is used to measure the index of refraction of a sample of glass which has parallel surfaces and is 1.20 cm thick. The microscope is first focused on a small dot on the lower surface; the microscope tube is raised until a small dot on the upper surface is in focus. The distance the microscope tube has been raised is found to be 0.80 cm. Determine the index of refraction of the glass.

37-10. When the refracting angle A of a prism is small, the angle of minimum deviation is also small. Show that, for a prism of small angle, the angle of minimum deviation is given by $D_m = A(n - 1)$. [HINT: For small angles the sine of θ may be approximated by θ, in radians.]

37-11. Using the equation derived in Problem 37-10, calculate the deviation of a ray of light going through a glass prism having a 5° refracting angle if the index of refraction of the glass is 1.65.

37-12. A drop of water of refractive index 1.33 is placed on the top face of a cube of glass of refractive index 1.66. At what angle should a ray of light be directed into the side face of the cube in order that the ray is reflected from the glass-water interface at the critical angle?

37-13. Show that the path taken by a light ray in reflection from a plane mirror is the path which requires the shortest time. [HINT: Show that the actual path length is a minimum.]

37-14. Show that the path taken by a light ray in its travel between a point A in medium α to a point B in medium β, which is separated from medium α by a plane interface, is the path of shortest time which can be taken by the light ray which passes from A to B.