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Physics, Chapter 44: Stable Nuclei

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Stable Nuclei

44-1 Atoms and Nuclei

Atomic and nuclear physics are essentially twentieth-century developments, although these had their origins at the close of the nineteenth century. These developments followed two parallel streams, as shown in Figure 44-1, one dealing with the electronic structure of the atom and the other dealing with its nuclear structure. These two streams served to feed each other and were fed by all other branches of physics, but only a few of the more important contributions concerning the nature of radiation, matter, and energy are shown in the figure.

Before proceeding with a discussion of nuclear properties, it will be worth while to recapitulate some of the phenomena which have been discussed or inferred previously, to emphasize them for the present discussion. There are at present 102 known elements, of which the elements of atomic number above 93 are not found in nature but have been synthetically produced in the laboratory and in the atomic bomb. Each element is characterized by two numbers (a) its *atomic number* and (b) its *atomic weight*. The atomic numbers run consecutively from 1 to 102, at present, and the atomic weights range from 1 to more than 250. We recall that the atomic number was initially simply a serial number which was assigned when the elements were arranged in order of their atomic weights and chemical properties in the periodic table. The work of Moseley on the characteristic x-ray spectra demonstrated that the atomic number Z was a property of fundamental significance. We now know that the atomic number is the number of protons in the nucleus of the atom. At one time in the history of chemistry, the discovery of a new element was extremely important, and the possibility always existed that a large number of elements might be found in one place in the periodic table, as in the case of the rare earths. This possibility no longer exists. There are two scales of atomic weight in general use. The chemical system of atomic weights is based upon the assignment of atomic weight 16 to oxygen, in the isotope

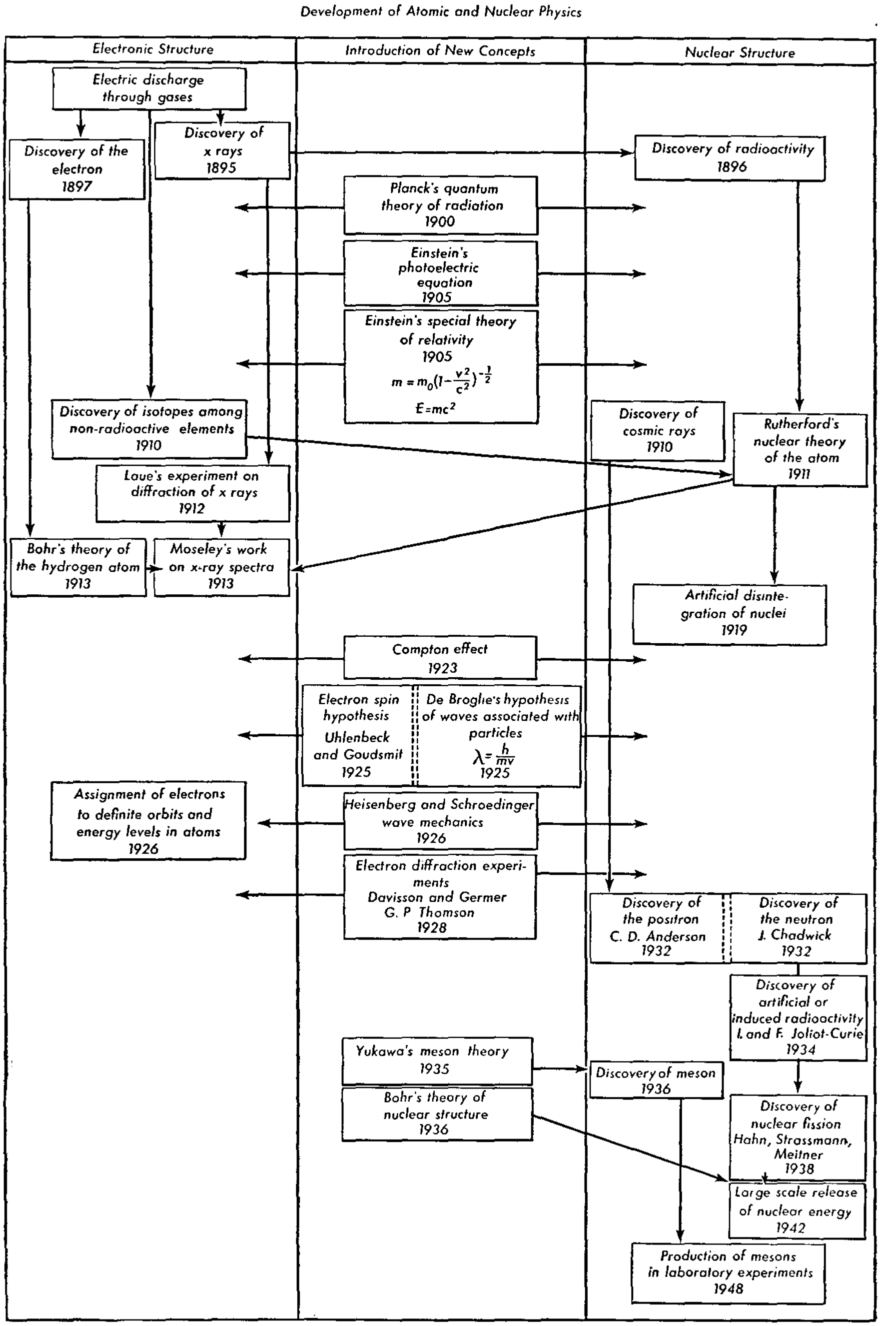

Fig. 44-1 Development of atomic and nuclear physics.

distribution which is found on the earth, while the physical scale of atomic weight is based on the assignment of atomic weight 16 to the most abundant isotope of oxygen. The difference between these scales is quite small; the ratio of the physical to the chemical scale of atomic weights being 1.000275. This difference is significant in physical measurements.

We shall take for granted that each atom consists of a positively charged nucleus of very small diameter, surrounded by an appropriate number of electrons so that the atom is electrically neutral in its normal state. The nucleus is composed of neutrons and protons, where the neutron is an electrically neutral particle of atomic weight 1.008987, and the proton is a positively charged particle of atomic weight 1.007595, whose charge is equal in magnitude to the electronic charge of $e = 4.802 \times 10^{-10}$ stcoul $= 1.602 \times 10^{-19}$ coul.

44-2 Equivalence of Mass and Energy

Until the advent of Einstein's theory of special relativity, one of the fundamental principles of physics and chemistry was the principle of the conservation of mass, which stated that the mass of an isolated system remained constant under all changes of physical states and chemical composition. Einstein recognized that mass was a form of energy, and that the principle of conservation of energy had to be enlarged to include mass. It will be recalled that the principle of conservation of energy was first formulated about 1847, when it was shown definitely that heat is a form of energy. Previous to this period there was a conservation theorem which concerned only the mechanical forms of energy, kinetic and potential energy, for mechanical transformations which occurred in frictionless systems. After the convincing work of Joule on the mechanical equivalent of heat, the concept of energy was extended to include heat. With the formulation of Maxwell's electromagnetic theory of light and the discovery of various forms of electromagnetic radiation, the principle of conservation of energy was readily extended to include electromagnetic radiation among the forms of energy. Now mass is included as one aspect of energy, along with other forms of energy.

Einstein showed that if a mass m is measured in grams or kilograms, the energy $\mathcal{E}$ of this mass can be expressed in ergs or joules by the relationship

$$\boxed{\mathcal{E} = mc^2,} \tag{44-1}$$

where c is the speed of light and is equal to 3×10^{10} cm/sec for conversion from grams to ergs, and is 3×10^8 m/sec for conversion from kilograms to joules.

Whenever a substance gains energy, as when it is heated, the substance gains in mass. Whenever a substance loses energy, as in an exothermal chemical reaction, the substance loses mass. In general, the mass change associated with ordinary chemical processes is undetectable; the energy liberated when 1 gm is converted into other forms of energy is

$$\mathcal{E} = 1 \text{ gm} \times (3 \times 10^{10})^2 = 9 \times 10^{20} \text{ ergs} = 2.25 \times 10^{13} \text{ cal},$$

while chemical processes involve energy changes of the order of 10^6 cal/mole. Thus the mass changes in such processes are of the order of 10^{-7} gm in a mole of substance, which is beyond the limit of measurement. It is interesting that the mass changes sought by the advocates of the caloric theory of heat, who tried to determine the weight of caloric, are today required by theory and found in experiment in nuclear reactions.

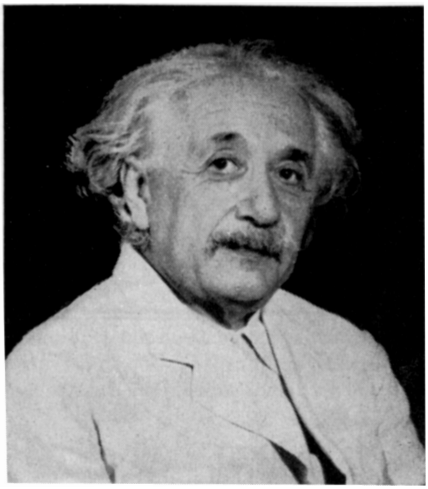

Fig. 44-2 Albert Einstein. He developed the theory of relativity and revolutionized the mode of thinking about fundamental physical problems. One consequence of this theory was the extension of the concept of energy to include mass as a form of energy. Another part of the theory gives us a new insight into gravitational phenomena. He also developed the fundamental equation of the photoelectric effect and the theory of Brownian motion. (Official U.S. Navy Photo from Acme.)

The conversion of mass into other forms of energy is believed to be going on continuously in the sun and other stars. This process is the basis for the construction of nuclear weapons and nuclear reactors. The fundamental mode of conversion of mass into energy is through changes in the nuclear constitution of atoms. An important clue to this process is the precise measurement of the masses of atoms and a comparison of these values with the masses of the constituent particles.

44-3 Positive Ions. Mass Spectrograph

An instrument designed to measure the mass of an ion is called a *mass spectrometer* or a *mass spectrograph*, depending on whether an electrical method or a photographic plate is used to record the ions. Modern mass spectrometers are instruments of very high precision which are capable of determining atomic masses very accurately and also of determining the number and relative abundance of the *isotopes* of which the elements are composed.

The term *isotopes* was introduced by Soddy as a result of the study of the radioactive elements. It was found that several groups of elements having identical chemical properties but different atomic weights were formed in the process of radioactive disintegration. The term "isotopes" was used to designate elements which occupied the same place in the periodic table. It was subsequently found that these elements had the same atomic number but different atomic weights. Because some elements had atomic weights which differed considerably from whole numbers, it was suggested that these elements consisted of two or more different isotopes having different atomic weights. The search for isotopes among the stable elements was begun by J. J. Thomson in 1910, and the first element successfully investigated was neon, whose atomic weight 20.2 differs appreciably from a whole number. By sending the positive ions formed in a gas-discharge tube through electric and magnetic fields, Thomson determined the ratio of the charge to the mass of these ions and found that neon consists of at least two isotopes of atomic masses very close to 20 and 22. Many variations of the original method were made by later investigators to improve the accuracy of this method.

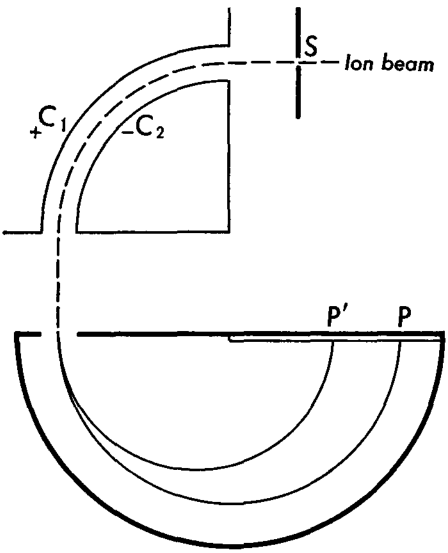

Fig. 44-3 Dempster's mass spectrograph which uses a capacitor with cylindrical plates.

In order to avoid confusion, let us introduce two new terms: (a) the *atomic mass* refers to the mass of an isotope of an element, based upon the physical scale in which the oxygen isotope of atomic mass 16.0000 is taken as the standard; this is the lightest of the three isotopes found in ordinary oxygen; (b) the *mass number* of an isotope of an element refers to the whole number which is nearest to the atomic mass of the isotope.

There are many varieties of mass spectrographs and spectrometers in use in research and industrial laboratories. The essential parts of a mass spectrograph designed by A. J. Dempster are sketched in Figure 44-3. These parts are enclosed in a vacuum chamber. Positive ions from a convenient source pass through the narrow slit S into a radial electric field between two cylindrical plates C_1 and C_2 of a capacitor. The ions are deflected from C_1 toward C_2. Only those ions with appropriate velocity traverse the circular path between the capacitor plates. If E is the radial component of the electric field in this region, and R is the radius of the path, we require that

$$Ee = \frac{mv^2}{R},$$

where e and m are the charge and mass of an ion. The ion beam may originate in an electric discharge in a gas and may contain ions of many different velocities. The capacitor serves as a velocity selector. A short

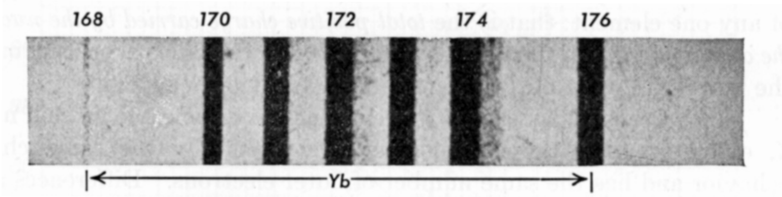

Fig. 44-4 A photograph of the isotopes of ytterbium obtained with Dempster's mass spectrograph. The mass numbers of the isotopes can be obtained from the number scale printed above the lines. (Reprinted from a photograph supplied to the author by Prof. A. J. Dempster.)

distance beyond the electric field the ions enter a uniform magnetic field which is at right angles to the plane of the paper, and, after traversing a semicircular path, strike the photographic plate PP' on which they are recorded. The radius r of the semicircular path is given by

$$\frac{mv^2}{r} = Bev,$$

where B is the magnetic flux density. From these equations we find that

$$v = \frac{ER}{Br},$$

and

$$\frac{e}{m} = \frac{ER}{(Br)^2}, \tag{44-2}$$

where all quantities are expressed in mks units.

A typical spectrogram obtained with this apparatus is illustrated in Figure 44-4, which shows the isotopes of the rare-earth element ytterbium. The mass number of each isotope is shown above the line formed by its ions on the photographic plate. Mass spectrometers are widely used in the petroleum industry as a means of following the refining and cracking processes. Here the molecular weight of large organic molecules is an important factor in the suitability of a fuel for a particular use, and this may be determined most easily with a mass spectrometer.

44-4 Masses of Isotopes and Nuclear Structure

Investigations with the mass spectrograph have established that there are about 300 different stable isotopes among the 102 known elements. The range of mass numbers runs from 1 to more than 250. The atomic masses of these isotopes differ very little from whole numbers. The number of stable isotopes per element varies from 1 for elements fluorine and gold to 10 for element tin. There is one thing which is common to all the isotopes of any one element: that is the *total positive charge carried by the nucleus of the atom*. Hence, in the neutral atom, the number of electrons surrounding the nucleus is the same for each isotope of any one element.

Since the isotopes of any one element have the same atomic number Z, every atom of the element displays essentially the same chemical behavior and has the same number of outer electrons. Differences among the atomic masses of an element must therefore be due to differences in their nuclei; the fact that the atomic masses of all isotopes are nearly integers suggests that nuclei are made up of particles (called *nucleons*) of approximately unit atomic mass. At present, two particles of nuclear size and mass nearly equal to unity are known. These are the proton and the neutron. The proton is the positively charged nucleus of the hydrogen atom of mass number 1. The hydrogen atom consists of 1 proton as nucleus and 1 electron outside the nucleus. Since the mass of the hydrogen atom is about 1,840 times the mass of the electron, practically the entire mass of the atom is due to the proton. On our scale of atomic units, the atomic mass of hydrogen is 1.00815, the mass of the electron is 0.00055, and the mass of the proton is 1.00760. The total number of neutrons and protons in a nucleus is equal to the mass number A of the isotope of an element. Thus the number of neutrons in the nucleus is $N = A - Z$, and the isotopes of any one element differ only in the number of neutrons in the nuclei of the atoms. Thus helium, the second element in the periodic table, has $Z = 2$. Its most abundant isotope has a mass number $A = 4$; its nucleus has 2 neutrons in addition to 2 protons.

An examination of the known stable isotopes is particularly interesting. Figure 44-5 is a graph of the neutron number N plotted against the proton number Z of the stable isotopes. The region of stability on this neutron-proton diagram is rather narrow. For low mass numbers $N = Z$, while for high mass numbers there are about 1.6 neutrons for each proton. Lines of constant A can be drawn at angles of 135° with the Z axis; such lines pass through *isobars*, that is, nuclei of equal mass. In general, lines of constant A pass through one or two stable isotopes. There are only four cases of such lines passing through three stable isobars, at $A = 96$, 124, 130,

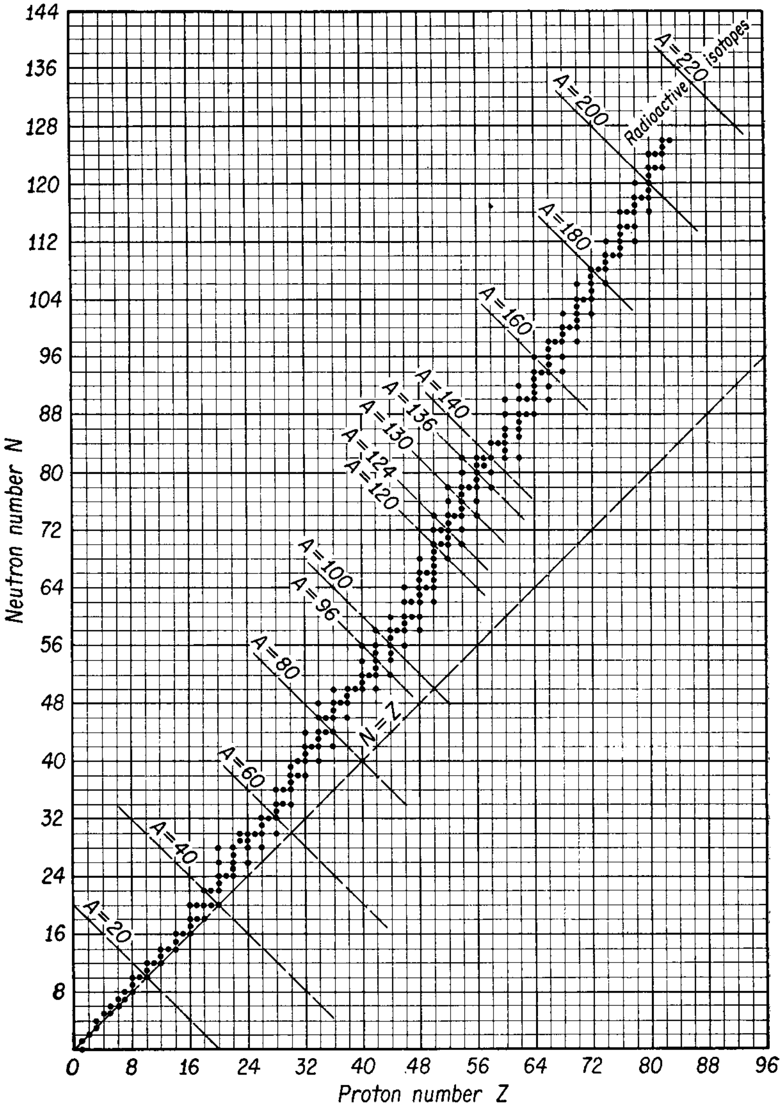

Fig. 44-5 Neutron-proton diagram of stable nuclei.

and 136. Another interesting point is that more than half of the stable nuclei have even numbers of protons and neutrons, and are referred to as even-even nuclei. About 20 per cent have even Z odd N, while about the same number have odd Z and even N. There are only four stable nuclei which have odd Z and odd N, namely ${}_1H^2$, ${}_3Li^6$, ${}_5B^{10}$, and ${}_7N^{14}$, where the number appearing as a left-hand subscript represents Z, while the right-hand superscript represents A. In the nucleus, nature has a decided preference for even numbers. We may interpret Figure 44-5 by inferring that points to the left of the stability region have too many neutrons, while points to the right of the stability region have too few neutrons, and that such isotopes will suffer radioactive disintegration.

The particles which constitute a nucleus exert forces on each other. If the only force between nuclear particles were the Coulomb repulsive force between particles of like charge, we would expect to find no nucleus beyond hydrogen. There must be another type of force between nucleons. For want of a better name, we shall call this a *nuclear force*. The exact nature of nuclear forces is not well known and is being extensively investigated by means of high-energy particle accelerators. We may think of the nuclear force as the glue which holds the nucleus together. This glue can be furnished a nucleus without the addition of an electrical repulsive force by adding a neutron rather than a proton.

The nuclear force acts through very short distances, for experiment shows that the nucleus is a compact structure, occupying a volume which is essentially the volume of its A nucleons. The range of the electrical repulsive force between protons is much greater than the range of the nuclear force. Thus we find that the more massive nuclei which have large values of Z must have more neutrons per proton than the lighter nuclei.

It is to be expected that the ideas and concepts that proved so effective in determining the electronic structure of atoms should be carried over into nuclear physics. One of these ideas is that of *shell structure* or *level structure*, with certain shells closed owing to the stability of the system with the given number of particles, much as the stability of the rare gases is determined by the closure of atomic shells. The neutron and proton both have spin quantum numbers of $\frac{1}{2}$, like the electron, so that 2 neutrons or 2 protons can exist in a given orbital energy state. Evidence first advanced by Maria G. Mayer in 1948 showed that nuclei having 20, 50, or 82 protons or 20, 50, 82, or 126 neutrons were particularly stable. The evidence used included the number of stable isotopes in a given category. For example, there are seven stable isotopes with $N = 82$, while there is only one for $N = 81$ and one for $N = 83$. Tin, for which $Z = 50$, has the largest number of stable isotopes, namely 10, of any element. These numbers, sometimes called *magic numbers*, now have some theoretical foundation, based upon the filling of nuclear energy levels in a postulated nuclear force field.

44-5 Binding Energy of Atoms and Nuclei

Let us suppose that a neutral atom of mass number A and atomic number Z is formed by bringing together A nucleons (Z protons plus N neutrons) in the nucleus and Z electrons outside the nucleus. Since this atom is a stable structure, its total energy must be less than that of a system consisting of these same particles separated by such large distances that the effect of the nuclear forces and the electrical forces holding an atom together are negligible. The difference between the total energy of the separated particles and the total energy of the neutral atom composed of these particles is the *binding energy of the atom.* From the principle of equivalence of mass and energy, it can be concluded that the decrease in energy produced in the formation of the atom should be evidenced by a decrease in the mass of the system. The mass of the atom should be less than the sum of the masses of its constituent particles. This was first brought to light as a result of the very precise determinations of the masses of isotopes by means of the mass spectrometer.

As a simple example, let us consider the formation of deuterium $_1H^2$, whose nucleus consists of 1 neutron and 1 proton. The atomic mass of the deuterium atom is 2.01474 atomic mass units (abbreviated amu). The sum of the masses of the constituent particles, in atomic mass units, is

$$\begin{aligned} \text{proton mass} &= 1.00760 \text{ amu}, \\ \text{neutron mass} &= 1.00899 \text{ amu}, \\ \text{electron mass} &= \underline{0.00055} \text{ amu}, \\ \text{total} \qquad & \quad\ 2.01714 \text{ amu}. \end{aligned}$$

Thus the mass of a deuterium atom is less than the sum of the masses of the constituent particles by 0.00240 amu.

To appreciate the meaning of these numbers, let us convert the atomic mass unit into more commonly used units. Now 1 amu is one sixteenth of the mass of an oxygen atom of mass number 16 and atomic mass 16. Since there are N_0 atoms in 16 gm of oxygen, 1 atom has a mass of 16 gm/N_0, so that 1 amu, which is one sixteenth of this, is simply

$$1 \text{ amu} = \frac{1 \text{ gm}}{N_0} = \frac{1 \text{ gm}}{6.025 \times 10^{23}},$$

from which

$$1 \text{ amu} = 1.66 \times 10^{-24} \text{ gm}.$$

Using the relationship between mass and energy given in Equation (44-1), we find that

$$1 \text{ amu} = 1.49 \times 10^{-3} \text{ erg}.$$

Another convenient unit of energy is the electron volt where

$$1 \text{ ev} = 1.60 \times 10^{-12} \text{ erg}.$$

Thus

$$1 \text{ amu} = 931.2 \times 10^{6} \text{ ev} = 931.2 \text{ Mev},$$

where Mev stands for million electron volts.

The binding energy of the deuterium atom which has been determined to be 0.00240 amu is therefore equal to 2.23 Mev. In order to separate deuterium into its constituent particles, an amount of energy equal to 2.23 Mev must be supplied. This may be done by irradiating deuterium with gamma rays. When these rays have energy greater than 2.23 Mev, it is observed that deuterium is decomposed into its components, that is, into 1 neutron and 1 proton. The binding energy of the electron is very nearly equal to the energy required to ionize a hydrogen atom, or 13.6 ev. This energy is negligible compared to the total binding energy of deuterium. Practically the entire binding energy is that of the two nucleons in the deuterium nucleus. This calculation gives us an indication of the relative magnitudes of the energies involved in atomic and nuclear processes. Normal chemical reactions which deal with the outer electrons of an atom are associated with energies of the order of 10 ev/atom. In nuclear reactions the energies involved are of the order of millions of electron volts per nucleon. While the nucleus does not participate in a chemical reaction, it exercises a controlling influence on any chemical process through its determination of the valency of the atom. Conversely, the state of chemical combination exercises little or no influence on nuclear processes.

The binding energy $\mathcal{E}_B$ of a nucleus is given by

$$\mathcal{E}_B = ZM_H + NM_n - M, \tag{44-3}$$

where M_H is the mass of a hydrogen atom, M_n is the mass of the neutron, and M is the mass of the atom as determined in the mass spectrograph. The mass of a hydrogen atom rather than the mass of a proton is used in order to correct for the number of electrons. If the total binding energy of the nucleus is divided by A, the mass number, we obtain the average binding energy per nucleon, which is plotted in Figure 44-6. Here we see that the average binding energy per nucleon is about 8 Mev for elements of mass number 20 or greater. Of course the binding energy of the hydrogen nucleus is zero. We note that the binding energy per nucleon is about 7.6 Mev for uranium, while it is about 8.7 Mev for manganese. When a uranium nucleus is split into several smaller parts, this difference in binding energy results in the liberation of about 1 Mev/nucleon, or about 200 Mev/nucleus. We see that a given mass of uranium liberates millions of times as much energy through nuclear processes as through chemical processes. Thus nuclear fuels are millions of times more compact than chemical fuels. A greater amount of energy is liberated when protons and neutrons are converted into helium, for here the binding energy is about 7.2 Mev/nucleon. This is the basis of the sun's energy and of thermonuclear processes, such as the hydrogen bomb.

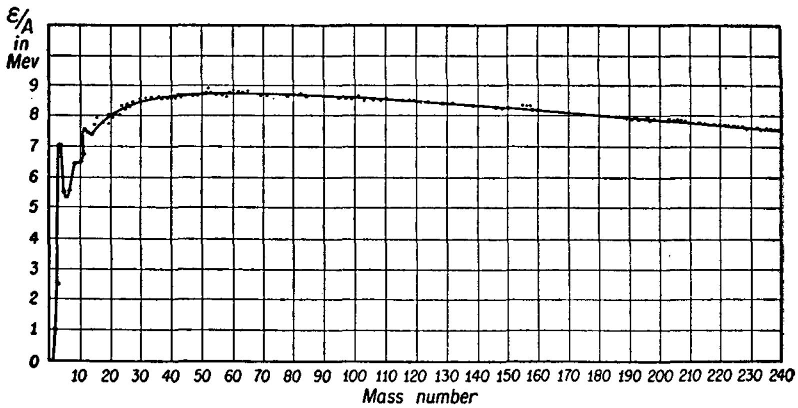

Fig. 44-6 Average binding energy per nucleon as a function of the mass number A.

In Figure 44-6 we note the presence of binding-energy peaks in the light elements at intervals of $\Delta A = 4$. If we imagine that there are energy levels in the nucleus for both neutrons and protons, we would expect these levels to be filled by pairs of neutrons and pairs of protons. According to the patterns established in the atom, a filled level is a more stable structure than an unfilled level. Thus we expect a preference for even numbers of neutrons and protons, and for systems which contain multiples of the helium nucleus. This is indeed the case until we have accumulated sufficient protons in the nucleus for the electrical repulsive force to become a dominating factor. In this connection it is interesting that the isotope $_4Be^8$ is unstable and is not found in nature, for the saturation of nuclear forces is so great in helium that a combination of two helium nuclei is not a stable form. These characteristics of the nucleus and the fact that a relatively constant binding energy per nucleon is observed in the heavier nuclei indicate that each nucleon is bound only to its near neighbors, and that the nuclear force is a short-range force.

The atomic masses of the isotopes are given in Table 3, Appendix A.

Problems

44-1. Calculate the difference between the binding energy of a nucleus of C^{12} and the sum of the binding energies of three alpha particles. (An alpha particle is a He nucleus.)

44-2. Using the data given in Table 3, Appendix A, calculate the binding energy of the last neutron in a $_3Li^7$ nucleus (a) in atomic mass units and (b) in million electron volts. (c) What is the voltage which must be applied across an x-ray tube in order that the x-rays emitted have sufficient energy to decompose this nucleus into $_3Li^6$ + a neutron?

44-3. Singly charged lithium ions of mass numbers 6 and 7, liberated from a heated anode, are accelerated by means of a difference of potential of 400 volts between the anode and the cathode, and then pass through a hole in the cathode into a uniform magnetic field perpendicular to their direction of motion. If the magnetic induction is 800 gausses, determine the radii of the paths of these ions.

44-4. Calculate the mass energy of a proton, in million electron volts.

44-5. Uranium isotopes of mass numbers 235 and 238 are to be separated from a piece of uranium by using a mass spectrometer which will deflect them through 180° into two collectors 4.0 cm apart. If the singly charged ions have energies of 2,000 ev when entering the magnetic field, calculate (a) the magnetic induction necessary to achieve this separation and (b) the radii of the paths of the ions.

44-6. Calculate, in electron volts, the electrostatic potential energy of two protons when their centers are 1.5×10^{-12} cm apart.

44-7. From Figure 44-6 calculate the energy released per atom when a uranium nucleus for which $A = 235$ splits into two fission products whose mass numbers are 72 and 163, respectively.

44-8. Let us suppose that a gram of uranium $A = 238$ and $Z = 92$ splits into fragments, each of mass number 119 and atomic number 46, and that the resulting material is a gas. Assuming that no energy is lost from the system and that the temperature is sufficiently high that each of the 92 electrons of a uranium atom acts as a free particle, calculate the final temperature of the resulting gas.

44-9. (a) How much energy, in calories, is liberated when 1 gm of hydrogen is converted into helium? (b) The heat of combustion of bituminous coal is approximately 10,000 Btu/lb. In this fusion process, how much coal is energetically equivalent to 1 gm of hydrogen?