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Xunhong Chen
University of Nebraska - Lincoln

Yanfeng Yin
University of Nebraska - Lincoln

ground water

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by Xunhong Chen[1] and Yanfeng Yin[2]

Abstract

In the analysis of streamflow depletion, the Hunt (1999) solution has an important advantage because it considers a partially penetrating stream. By extending the Hunt drawdown solution, this paper presents semianalytical solutions for gaining streams that evaluate the induced stream infiltration and base flow reduction separately. Simulation results show that for a given Δh (the initial hydraulic head difference between stream and aquifer beneath the channel), the base flow reduction is in direct proportion to the product of streambed leakage (λ) and the distance between pumping well and stream (L), and the induced stream infiltration is in inverse proportion to λL. Δh has a significant effect on the ratio of stream infiltration to base flow reduction. The results from the semianalytical solutions agree well with those from MODFLOW simulations. The semianalytical solutions are useful in the verification of numerical simulations and in the analysis of stream-aquifer interactions where water quantity or quality is concerned.

Introduction

Analysis of stream depletion continues to be an important research aspect. Theis (1941) first described a transient process of stream depletion. Later, Glover and Balmer (1954) rederived the Theis solution and presented it in a different form. Hantush (1965) developed a solution for stream-aquifer systems where a semipermeable layer exists. Since the stream is assumed to fully penetrate the aquifer in the Theis and Hantush solutions, application of these solutions to a partially penetrating stream can overestimate the rate of depletion. Hunt (1999) developed a solution for the system where a stream partially penetrates the aquifer and is separated from the underlying aquifer by a low-permeability streambed. Compared to the former two solutions, the Hunt solution made significant progress in that it is more closely representative of a partially penetrating stream. In addition, Hunt (1999) also provided an analytical solution for calculating drawdowns in the aquifer. All of these studies assumed there was no hydraulic gradient between stream and aquifers prior to ground water pumping and, thus, all depletion is from the stream. Other semianalytical solutions include those of Huang (2000) and Butler et al. (2001).

Most streams either gain water from, or lose water to, the aquifer. For example, several streams in the Nebraska Sand Hills receive more than 86% of their water from

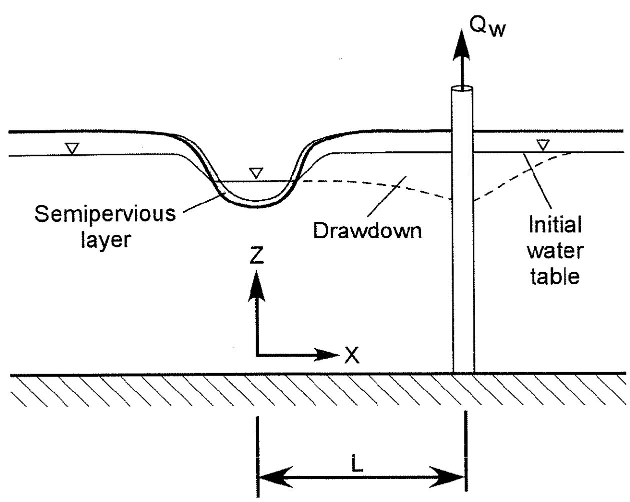

Figure 1. Diagram showing that a partially penetrating gaining stream is affected by pumping of ground water at a nearby well.

[1]School of Natural Resources, University of Nebraska-Lincoln, Lincoln, NE 68588; (402) 472-0722; fax (402) 472-4608; xchen2 @unl.edu

[2]School of Natural Resources, University of Nebraska-Lincoln, Lincoln, NE 68588; (402) 472-5183; fax (402) 472-4608; hgyyin @yahoo.com

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aquifer discharge (Chen et al. 2003). For a partially penetrating gaining stream (Figure 1), pumping of ground water in the nearby aquifer can reduce the base flow, which otherwise would discharge to the stream, and the pumping will then induce stream infiltration to the aquifer when a sufficiently long pumping generates a reversal of hydraulic gradient from the stream to the aquifer. Wilson (1993) demonstrated a separate analysis of the two depletion components is particularly important for areas where water quality issues are concerned. Wilson's (1993) solution, however, is only good for steady-state flow. Chen (2003) developed an analytical solution based on the Theis (1941) solution to calculate stream infiltration and base flow reduction separately, which is only appropriate for a fully penetrating stream. Recently, numerical models are often used to evaluate the stream depletion for more realistic stream-aquifer systems. Examples include Sophocleous et al. (1995), Conrad and Beljin (1996), and Chen and Shu (2002). To the authors' knowledge, however, an analytical solution for the analysis of the stream infiltration and base flow reduction in a partially penetrating stream is still not available.

Stream-Aquifer System and the Total Depletion

Figure 1 is a schematic diagram showing a partially penetrating gaining stream. This stream-aquifer system is very similar to the one described by Hunt (1999), except there is no hydraulic gradient between the stream and aquifer in the Hunt model. Other assumptions for the stream and aquifer can be found from Hunt (1999). Since all of the ground water governing equations are linear, the Hunt solution holds for streams that have any combination of initial head distribution that are a gaining, losing, or equilibrium stream-aquifer system. Thus, the total depletion for the given gaining system is (Hunt 1999)

$$\frac{\Delta Q}{Q} = erfc\left(\sqrt{\frac{SL^2}{4Tt}}\right) - exp\left(\frac{\lambda^2 t}{4ST} + \frac{\lambda L}{2T}\right) erfc\left(\sqrt{\frac{\lambda^2 t}{4ST}} + \sqrt{\frac{SL^2}{4Tt}}\right) \quad (1)$$

where ΔQ is the stream depletion rate; Q is the pumping rate; S and T are the aquifer specific yield (or storage coefficient for a confined aquifer) and transmisivity, respectively; L is the distance between well and stream; *t* is the pumping time; and λ is the streambed leakage per unit length of stream reach (Hunt et al. 2001) and is defined by

$$\lambda = \lim_{\substack{w \to 0 \\ k'/b' \to \infty}} \frac{W}{b'} K' \quad (2)$$

where W is the width of the stream, and K′ and b′ are the hydraulic conductivity and thickness of the streambed, respectively.

The drawdown ϕ at the location of x = 0 for the above gaining system can be computed from (Hunt 1999)

$$\phi(0,y,t) = \frac{Q}{4\pi T} \left\{ W\left(\frac{l^2 + y^2}{4Tt/S}\right) - \int_0^\infty e^{-\theta} W\left[\frac{(l+2T\theta/\lambda)^2 + y^2}{4Tt/S}\right] d\theta \right\} \quad (3)$$

where $W(u) = \int_u^\infty \frac{e^{-u}}{u} du$ is the well function.

Calculation of Stream Infiltration and Base Flow Reduction

For a gaining stream (Figure 1), the depletion is made of two components—induced stream water infiltration and base flow reduction. Base flow reduction often occurs before induced stream infiltration and has a long-term effect on streamflow (Chen and Yin 2001).

For a given Δh (the initial hydraulic head difference between the stream and the underlying aquifer), pumping leads to a drawdown of ϕ in the aquifer beneath the stream (at the locations of x = 0 in Figure 2). For a stream segment where $\phi > \Delta h$, infiltration is induced (Figure 2). This infiltration segment incepts near the location x = 0, y = 0, and it expands up- and downstream equally with the increase of pumping time. Outside this segment, the drawdown is < Δh, and the stream continues to receive ground water, but at a lower rate compared to the rate before pumping began. The two ends of the infiltration reach (y′ and –y′) are called dividing points, where there is no vertical ground water movement. The locations of y′ and –y′ vary with pumping time, but can be calculated for a given time, t, using

$$\phi(0, y, t) - \Delta h = 0 \quad (4)$$

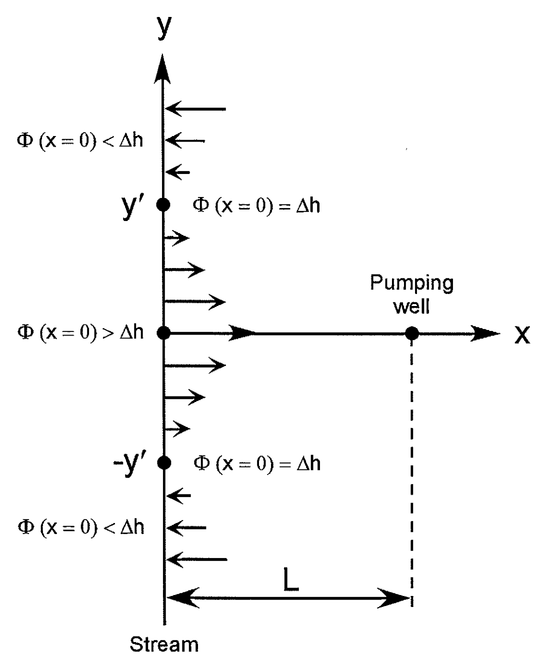

Figure 2. Schematic showing the dividing points (y′ and –y′). Stream infiltration has been induced by the pumping well for the reach between y′ and –y′.

Equation 3 shows the mathematical expression of $\phi(0, y, t)$ beneath the stream.

After the dividing points are determined, the rate of the stream infiltration (Q_s) is calculated by integration between y' and $-y'$ such that

$$Q_s = \lambda \int_{-y'}^{y'} (\phi(0,y,t) - \Delta h)dy \tag{5}$$

The induced stream infiltration, D_s, is equal to Q_s/Q.

Since the rate of total depletion, ΔQ, is the sum of the rates of stream infiltration, Q_s, and base flow reduction, Q_b, and is given by

$$\Delta Q = \lambda \int_{-\infty}^{+\infty} \phi(0,y,t)dy \tag{6}$$

the rate of base flow reduction is calculated from

$$Q_b = \Delta Q - Q_s \tag{7}$$

Similarly, the base flow reduction, D_b, is Q_b/Q.

The total volume of the water infiltrated into the aquifer, V_s, is sum of the product of Q_s and pumping time:

$$V_s = \sum Q_s \Delta t \tag{8}$$

and the total volume of base flow reduction, V_b, is

$$V_b = \sum Q_b \Delta t \tag{9}$$

For water quality issues, y' and V_s provide critical information about the possible contamination zone and the volume of stream water leaked into the aquifer if the river contains contaminants. For a given pumping period, t, the pumped ground water volume, Q_t, is equal to $V_s + V_b + V_{st}$, where V_{st} is the depletion of aquifer storage.

The superpositition technique can be employed in the evaluation of ground water drawdown, as well as for residual effects of the depletion components for a postpumping period. A FORTRAN program was developed to calculate the dividing points and the two depletion components. The golden section search method (Gill et al. 1981) was used for solving Equation 4 to determine the dividing point. We found the convergence of this method during the iteration occurs faster than that of Newton's method.

Results

Relationship Between Stream Infiltration and Base Flow Reduction

Equations 5 and 7 indicate implicitly that the stream infiltration and base flow reduction in a gaining stream are not only affected by aquifer properties and streambed leakage, but also by Δh. A set of dimensionless depletion curves was calculated using the two equations for a given Δh (= 0.056 m). The curves for base flow reduction, stream infiltration, and the total depletion are plotted in Figures 3a to 3c, respectively. The figure shows that for a stream-aquifer system with a large $\lambda L/T$ value, the base flow reduction accounts for a large percentage of the total stream depletion. On the other hand, for a system with a small $\lambda L/T$ value, the stream infiltration accounts for a large part of the total stream depletion. Simulation results indicate that when $\lambda L/T > 9$, all the depletion is from the base flow and the pumping does not induce stream infiltration. This is important information for wellhead protection if prevention of an induced stream infiltration due to pumping is warranted. The total depletion calculated using the Theis (1941) model is plotted in Figure 3c for a comparison; it indicates the Theis model can provide an overestimated depletion when it is used for a partially penetrating stream.

We also analyzed the impacts of different initial hydraulic head Δh on the ratios of stream depletion to base flow reduction. Simulation results indicate a large Δh leads

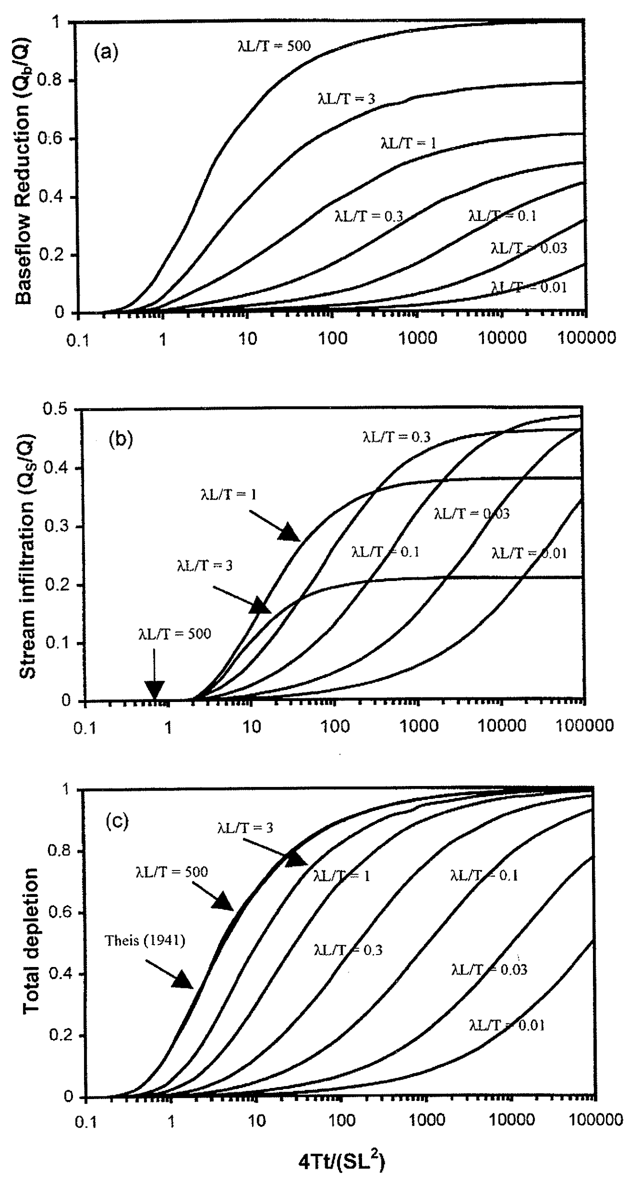

Figure 3. Dimensionless depletion curves: (a) base flow reduction given by Equation 7; (b) infiltration given by Equation 5 (the stream infiltration is zero when $\lambda L/T = 500$); and (c) total depletion given by Equation 6.

to a small stream infiltration and a large base flow reduction. The rate of base flow reduction can be greater than the rate of stream infiltration for some gaining streams.

Comparison to MODFLOW

To verify the new solutions presented in this paper, a stream-aquifer model was built using MODFLOW (McDonald and Harbaugh 1988). The simulation considers 90 days of pumping followed by a 275-day nonpumping period. The model domain is 20,000 m wide in the x-direction (east-west) and 20,000 m long in the y-direction (south-north). The aquifer, 25 m thick, was divided into three layers vertically. The domain's boundaries are designed to be constant head boundaries with a 25 m head along north and south boundaries, and to be impermeable for east and west boundaries. A stream runs from east to west in the middle of the model domain. The values of aquifer and stream parameters for the model are shown in Table 1. In order to generate a constant Δh between the aquifer and the stream, a 730-day nonpumping period was run first for each numerical simulation case to ensure that a ground water flow to the stream has been as steady as possible in the model prior to the pumping, so that the values of Δh used in the analytical solutions and the numerical model are very close to each other.

Figure 4 shows the stream infiltration, base flow reduction, and the total depletion calculated using the equations described in earlier sections and MODFLOW simulations for two gaining streams, Δh = 0.056 and 0.087 m, respectively. Figures 4a and 4b indicate the following. First, the stream infiltration, base flow reduction, and the total stream depletion obtained by the methods presented in this paper are a very good match with those obtained from MODFLOW in both pumping and postpumping periods. Second, a small Δh leads to a small base flow reduction and a large stream infiltration, while a large Δh leads to a large base flow reduction and a small infiltration. Third, the total streamflow depletion is the same for the two gaining streams, even though the initial hydraulic head difference between stream and the aquifer (Δh) differs.

A minor difference between the results of the semianalytical solutions and the numerical modeling is observed. It may be introduced by the methods of dealing with the streambed leakage. Streambed leakage λ (Equation 2) in the Hunt (1999) model is equivalent to streambed conductance per unit length in MODFLOW. The Hunt (1999) solution assumes zero width for the stream, which is inherited in Equation 5. In contrast, a definite width of stream must be given in MODFLOW for two reasons. First, the calculation of streambed conductance needs the value of stream width. Second, stream width is one of the important references in the determination of grid size near the stream to ensure an accurate numerical result can be achieved. In MODFLOW, the streambed conductance for a unit stream length is

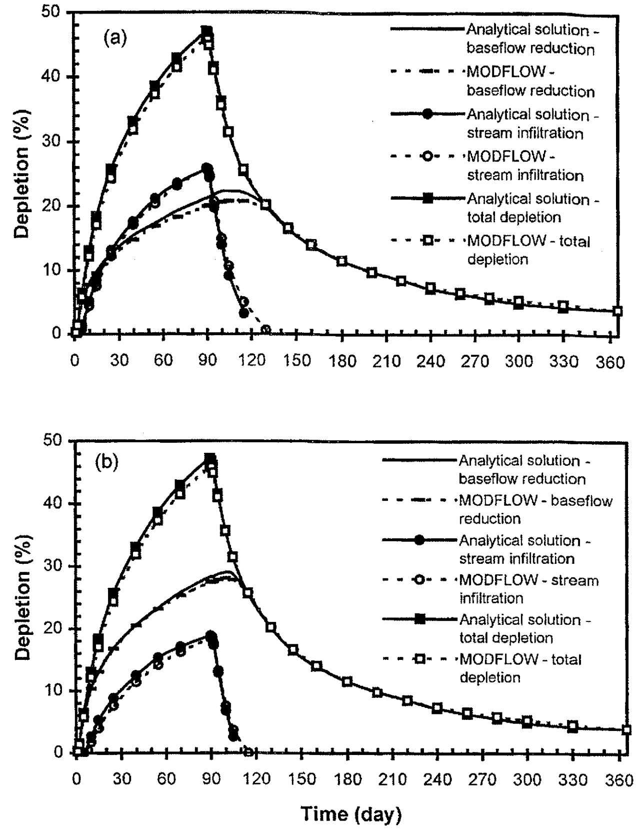

Figure 4. Depletion curves for two gaining streams: (a) Δh = 0.056 m, (b) Δh = 0.087 m. The parameter values for the stream and aquifer are shown in Table 1.

Table 1
Summary of Parameter Values Used in the Stream-Aquifer Model

Parameters	Value
Hydraulic conductivity	100 m/day
Specific yield	0.2
Aquifer thickness	25 m
Pumping rate	4500 m^3/day
Distance between stream and pumping well	300 m
Hydraulic head difference between the stream and aquifer	0.056 m
Stream stage	24.7 m
Stream depth	1.2 m
Unit streambed conductance	5 m/day
Model domain	20,000 × 20,000 m^2

$$C^* = \frac{WK'}{b'} \tag{10}$$

The definitions of W, K′, and b′ in this equation are the same as those in Equation 2.

We also calculated total depletions using Equation 1 and MODFLOW for stream-aquifer systems without base flow. The parameter values are the same as those shown in Table 1, except for $\Delta h = 0$. For the analytical solution (Hunt 1999), λ is 5 m/day. Four simulations, with varied stream width W = 5, 10, 20, and 40 m, were conducted using MODFLOW, but the unit streambed conductance C^* was 5 m/day and remained the same for the four streams. Simulation results indicate the largest difference between the two methods occurs when the stream width is 5 m and the difference value is 1.7%, which is considered an insignificant error. When the stream width becomes large, the difference between the analytical solution and numerical solution decreases. Note that the grid spacing was the same for each of the four simulations.

Additional simulations were conducted for λ = 20 m/day using Equation 1 and C^* = 20 m/day using MODFLOW. The largest difference between the Hunt solution and numerical solution also occurs for the narrowest stream, i.e., W = 5 m, and the difference increases to 3.4%. Therefore, both the unit streambed conductance and the stream width have some effect on the differences between the analytical solution and numerical solution, even though the distance between stream and pumping well is five times the stream width or more, especially when the stream width is small (< 10 m).

Summary and Conclusions

Semianalytical solutions have been developed for a partially penetrating gaining stream to determine two depletion components—induced stream infiltration and base flow reduction. Simulation examples show the induced stream infiltration, the base flow reduction, and the total depletion obtained by these solutions have a very good match with those obtained by MODFLOW for both pumping and post-pumping periods. The analytical solutions are useful in the verification of results from numerical simulations.

Simulation results indicate if the initial hydraulic head difference between the stream and the aquifer beneath the stream (Δh) is given, the base flow reduction is in direct proportion to the product of streambed leakage (λ) and the distance between pumping well and stream (L); the infiltration is in inverse proportion to λL. Δh has a significant effect on the ratio of stream infiltration to base flow reduction. A large Δh leads to a small stream infiltration, but a large base flow reduction. The analyses of the relationship between the two depletion components also provide important information for areas where wellhead protection is concerned.

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References

Butler, J.J, V.A. Zlotnik, and M. Tsou. 2001. Drawdown and stream depletion produced by pumping in the vicinity of a partially penetrating stream. *Ground Water* 39, no. 5: 651–659.

Chen, X.H., and Y. Yin. 2001. Streamflow depletion: Modeling of reduced baseflow and induced stream infiltration from seasonally pumped wells. *Journal of the American Water Resources Association* 37, no. 1: 185–195.

Chen, X.H., and L. Shu. 2002. Stream-aquifer interactions: Evaluation of depletion volume and residual effects from ground water pumping. *Ground Water* 40, no. 3: 284–290.

Chen, X.H. 2003. Analysis of pumping-induced stream-aquifer interactions for gaining streams. *Journal of Hydrology* 275, no. 1–2: 1–11.

Chen, X., X.H. Chen, C. Rowe, Q. Hu, and M. Anderson. 2003. Geological and climatic controls on streamflows in the Nebraska Sand Hills. *Journal of the American Water Resources Association* 39, no. 1: 217–228.

Conrad, L.P., and M.S. Beljin. 1996. Evaluation of an induced infiltration model as applied to glacial aquifer systems. *Water Resources Bulletin* 32, no. 6: 1209–1220.

Gill, P.E. 1981. *Practical Optimization.* New York: Academic Press.

Glover, R.E., and C.G. Balmer. 1954. Stream depletion resulting from pumping a well near a stream. *American Geophysical Union Transactions* 35, no. 3: 168–470.

Hantush, M.S. 1965. Wells near stream with semipervious beds. *Journal of Geophysical Research* 70, no. 12: 2829–2838.

Huang, H. 2000. Evaluation of stream-aquifer interaction considering streambed sediment and stream partial penetration effects. M.S. thesis, Department of Geosciences, University of Nebraska-Lincoln.

Hunt, B. 1999. Unsteady stream depletion from ground water pumping. *Ground Water* 37, no. 1: 98–104.

Hunt, B., J. Weir, and B. Clause. 2001. A stream depletion field experiment. *Ground Water* 39, no. 2: 283–289.

McDonald, M.G., and A.W. Harbaugh. 1988. Modular three-dimensional finite-difference ground-water flow model. Techniques of Water-Resources Investigations of U.S. Geological Survey. Book 6.

Sophocleous, M.A., A. Koussis, J.L. Martin, and S.P. Perkins. 1995. Evaluation of simplified stream-aquifer depletion models for water rights administration. *Ground Water* 33, no. 4: 579–588.

Theis, C.V. 1941. The effect of a well on the flow of a nearby stream. *American Geophysical Union Transactions* 22, no. 3: 734–738.

Wilson, J.L. 1993. Induced infiltration in aquifers with ambient flow. *Water Resources Research* 29, no. 10: 3503–3512.