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Reformulating the Dual Graphs of CSPs to Improve the Performance of RNIC

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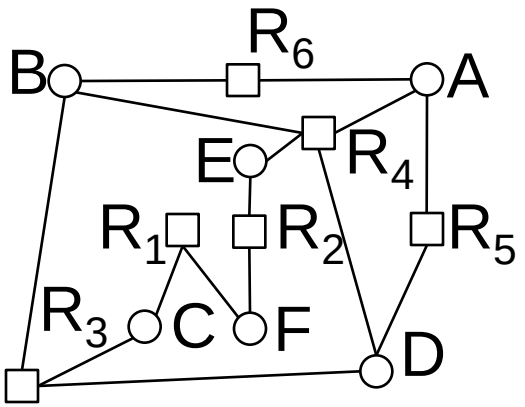
Outline

- Introduction
- Relational Neighborhood Inverse Consistency
  - Property & algorithm
- Reformulating the Dual Graph by
  1. Removing redundant edges, yields property wRNIC
  2. Triangulation, yields property triRNIC
- Selection strategy: four alternative dual graphs
- Experimental Results
- Conclusion

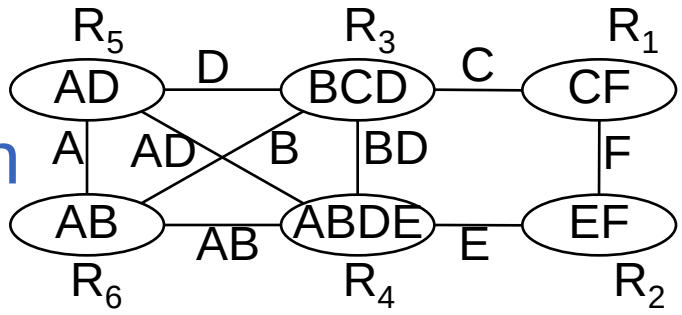
Constraint Satisfaction Problem

- CSP
  - Variables, Domains
  - Constraints: binary / non-binary
- Representation
  - Hypergraph
  - Dual graph
- Solved with
  - Search
  - Enforcing consistency
- Warning
  - Consistency properties vs. algorithms

Hypergraph

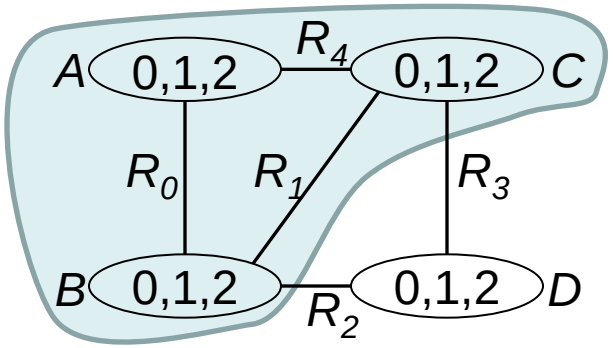


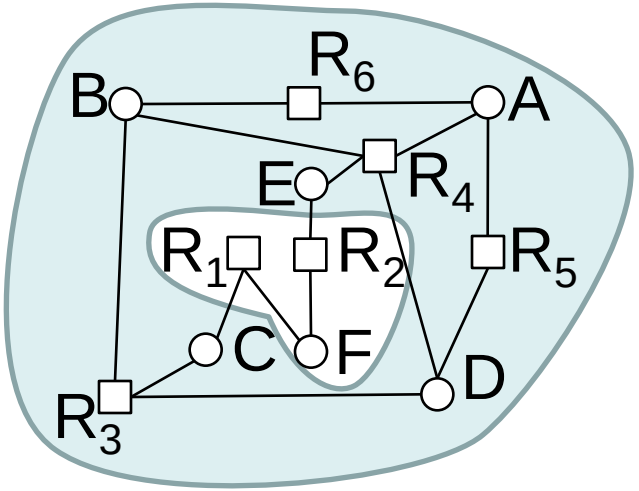
Dual graph



Neighborhood Inverse Consistency [Freuder+ 96]

- Property
  - Defined for binary CSPs
  - Every value can be extended to a solution in its variable's neighborhood
- Algorithm
  - \+ No space overhead
  - \+ Adapts to the connectivity
  - − Not effective on sparse problems
  - − To costly on dense problems
- Non-binary CSPs?
  - − Neighborhoods likely too large

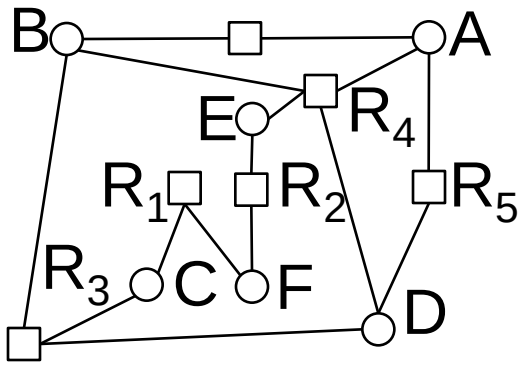




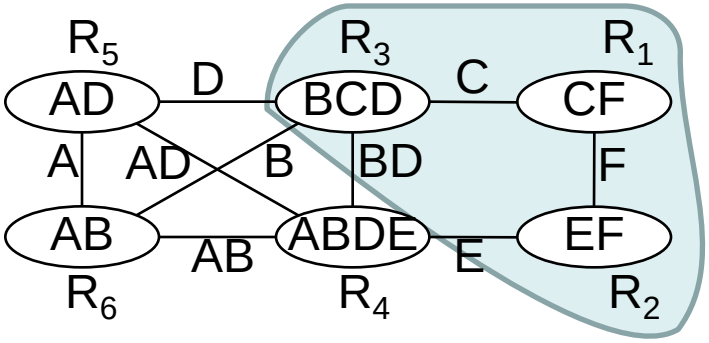
Relational NIC

[Woodward+ AAAI11]

- Property
  - Defined for dual graph
  - Every tuple can be extended to a solution in its relation's neighborhood
- Algorithm
  - Operates on dual graph
  - … filter relations (not domains!)
- Domain filtering
  - Property: RNIC+DF
  - Algorithm: Projection



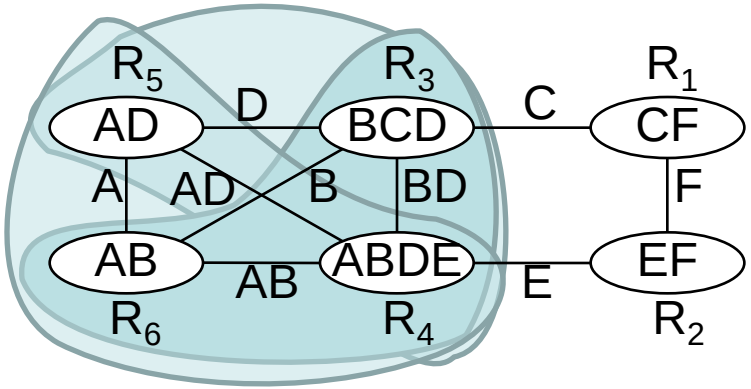
Hypergraph



Dual graph

Reformulation: Removing Redundant Edges

- High density
  - Large neighborhoods
  - Higher cost of RNIC
- Minimal dual graph
  - Equivalent CSP
  - Computed efficiently [Janssen+ 89]
- Run algorithm on a minimal dual graph
  - \+ Smaller neighborhoods, solution set not affected
  - − wRNIC: a strictly weaker property



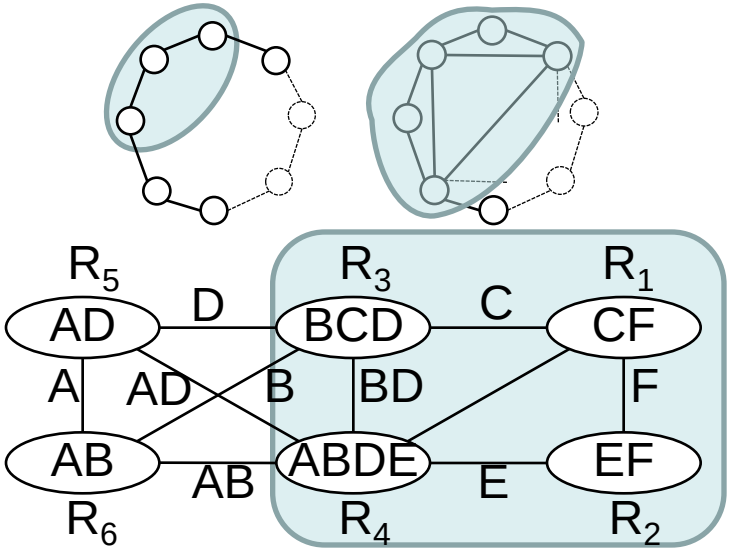
$d^{Go} = 60\%$

$d^{Gw} = 40\%$

wRNIC → RNIC

Reformulation: Triangulation

- Cycles of length ≥ 4
  - Hampers propagation
- Triangulating dual graph
  - Equivalent CSP
  - Min-fill heuristic



- Run algorithm on a triangulated dual graph
  - + Created loops enhance propagation
  - triRNIC: a strictly stronger property

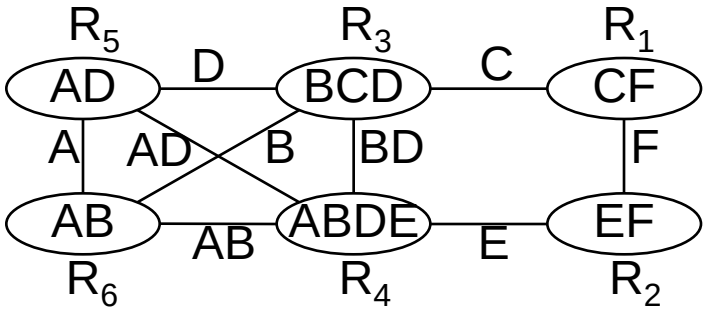
$d^{Go} = 60\%$

$d^{Gtri} = 67\%$

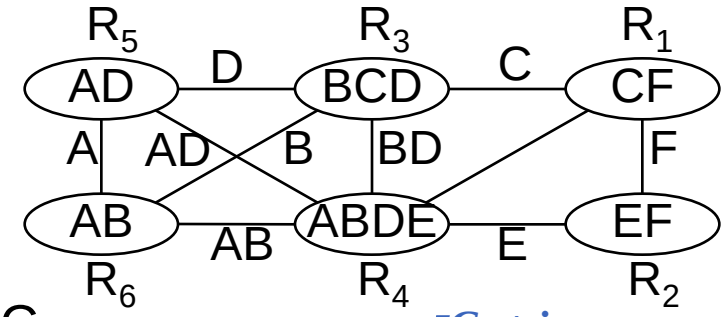
wRNIC → RNIC → triRNIC

Reformulation: RR & Triangulation

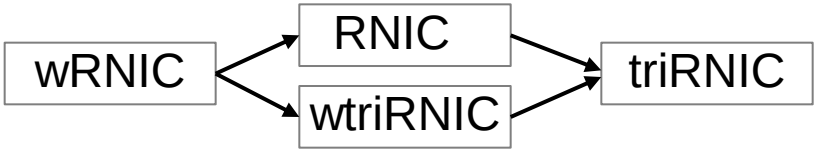
- Fixing the dual graph
  - RR copes with high density
  - Triangulation boosts propagation
- RR+Tri
  - Both operate locally
  - Are complementary, do not ‘clash’
- Run algorithm on a RR+tri dual graph
  - CSP solution set is not affected
  - wtriRNIC is not comparable with RNIC



$d^{Go} = 60\%$

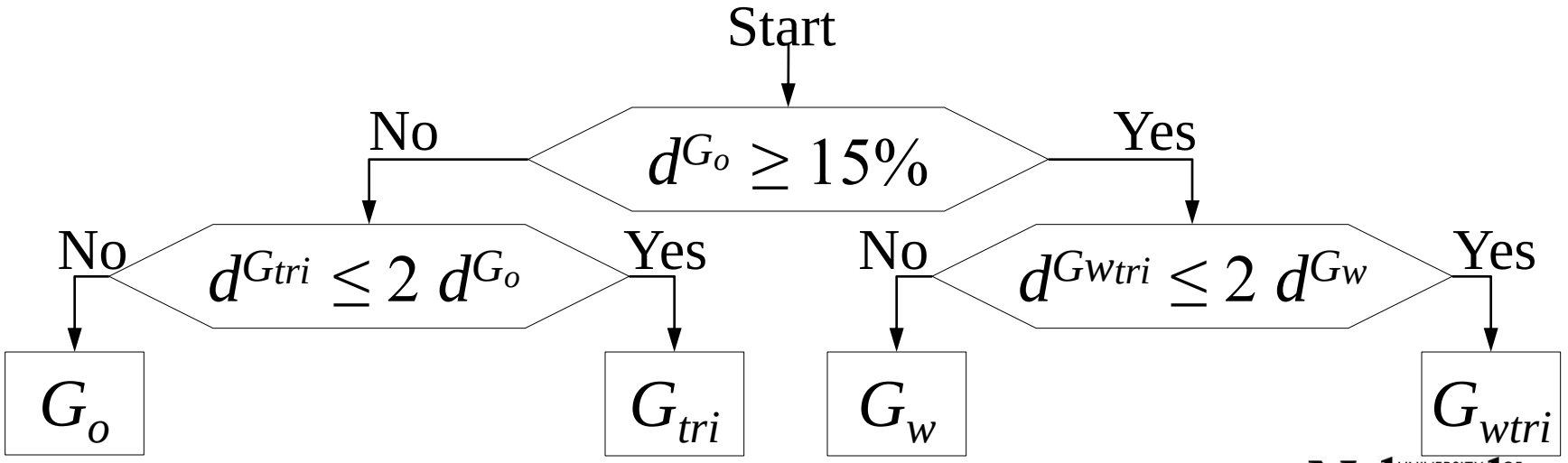


$d^{Gwtri} = 47\%$



Selection Strategy: Which? When?

- Density $\geq 15\%$ is too dense
  - Remove redundant edges
- Triangulation increases density no more than two fold
  - Reformulate by triangulation
- Each reformulation executed at most once



Experimental Results

- Statistical analysis on CP benchmarks
- Time: Censored data calculated mean
- R: Censored data rank based on probability of survival data analysis
- S: Equivalence classes based on CPU
- S_B: Equivalence classes based on completion
- #C: Number of instances completed
- #F: Number of instancesfastest
- #BF: # instances solved backtrack free

Algorithm	Time	#F	R	S	#C	S_B	#BF
169 instances: aim-100,aim-200,lexVg,modifiedRenault,ssa							
wR(*,2)C	944924	52	3	**A**	138	B	79
wR(*,3)C	925004	8	4	B	134	B	92
wR(*,4)C	1161261	2	5	B	132	B	108
GAC	1711511	**83**	7	C	119	C	33
RNIC	6161391	19	8	C	100	C	66
triRNIC	3017169	9	9	C	84	C	80
wRNIC	1184844	8	6	B	131	B	84
wtriRNIC	937904	3	2	B	144	B	129
selRNIC	**751586**	17	**1**	**A**	**159**	**A**	**142**

Conclusions

- Contributions
  - Algorithm
    - Polynomial in degree of dual graph
    - BT-free search: hints to problem tractability
  - Various reformulations of the dual graph
  - Adaptive, unifying, self-regulatory, automatic strategy
  - Empirical evidence, supported by statistics
- Future work
  - Extend to constraints given as conflicts, in intension
  - Extend to singleton type consistencies