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What hadron collider is required to discover or falsify natural supersymmetry?

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What hadron collider is required to discover or falsify natural supersymmetry?

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ABSTRACT

Weak scale supersymmetry (SUSY) remains a compelling extension of the Standard Model because it stabilizes the quantum corrections to the Higgs and W, Z boson masses. In natural SUSY models these corrections are, by definition, never much larger than the corresponding masses. Natural SUSY models all have an *upper limit* on the gluino mass, too high to lead to observable signals even at the high luminosity LHC. However, in models with gaugino mass unification, the wino is sufficiently light that supersymmetry discovery is possible in other channels over the entire natural SUSY parameter space with no worse than 3% fine-tuning. Here, we examine the SUSY reach in more general models with and without gaugino mass unification (specifically, natural generalized mirage mediation), and show that the high energy LHC (HE-LHC), a pp collider with $\sqrt{s} = 33$ TeV, will be able to detect the SUSY signal over the entire allowed mass range. Thus, HE-LHC would either discover or conclusively falsify natural SUSY with better than 3% fine-tuning using a conservative measure that allows for correlations among the model parameters.

The discovery of a new scalar boson $h(125)$ at the CERN Large Hadron Collider [1] (LHC) has cemented the Standard Model (SM) as the appropriate effective field theory describing physics up to the weak scale $m_{\rm weak} \sim 200$ GeV. However, in the SM, the quantum corrections to the Higgs boson mass are quadratically sensitive to the scale of new physics and exceed the observed value of m_h unless the cut-off scale, beyond which the SM ceases to be a valid description, is as low as $\Lambda \sim 1$ TeV. As the cutoff Λ grows beyond the TeV scale, increasingly precise fine-tunings of SM parameters are required in order to maintain m_h at its measured value.

It has long been known that extending the underlying spacetime symmetry from the Poincaré group to the more general super-Poincaré (supersymmetry or SUSY) group tames the quantum corrections to m_h, provided that SUSY is softly broken not very far from the weak scale [2]. Realistic particle physics models incorporating SUSY, such as the Minimal Supersymmetric Standard Model (MSSM), thus require the existence of new superpartners [3], *some of whose masses lie close to the weak scale*, hence the name *weak scale supersymmetry* (WSS); the remaining ones may have multi-TeV masses. Three independent calculations involving virtual quantum effects provide indirect experimental support for WSS. 1) The measured values of the three SM gauge couplings unify at a scale $Q \simeq 2 \times 10^{16}$ GeV in the MSSM but not in the SM, 2) the top quark mass, $m_t \simeq 173$ GeV, falls within the range required by SUSY to radiatively break electroweak gauge symmetry, and 3) the measured value of the Higgs mass, $m_h \simeq 125$ GeV, (which could have taken on any value up to the unitarity limit $\lesssim 1$ TeV in the SM) falls within the narrow range, $m_h < 135$ GeV [4], required by the MSSM.

These considerations led many to expect WSS to be discovered once sufficient data were accumulated at the LHC. However, with nearly 40 fb^{-1} of data at $\sqrt{s} = 13$ TeV, no evidence for superpartner production has been reported. Recent analyses based on ~ 36 fb^{-1} of integrated luminosity have produced mass limits on the gluino $\tilde{g}$ (spin-1/2 superpartner of the gluon) of $m_{\tilde{g}} > 2$ TeV and of the top squark (the lighter of the spin-0 superpartners of the top quark) of $m_{\tilde{t}_1} > 1$ TeV [5] (within the context of various simplified SUSY models), with even stronger limits on first generation squarks. These may be compared with early estimates – based upon the naturalness principle that *contributions to an observable* (such as the Z-boson mass) *should be less than or comparable to its*

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measured value – that the upper bound on $m_{\tilde{g}}$ is ~ 350 GeV and that $m_{\tilde{t}_1} \lesssim 350$ GeV based on no less than 3% fine-tuning [6].[1] Similar calculations seemed to require *three* third generation squarks lighter than 500 GeV [10,11]. Crucially, the analyses leading to these stringent upper bounds assume that contributions to the radiative corrections from various superpartner loops are *independent*. The assumption of independent soft terms is not valid in frameworks where the seemingly independent parameters – introduced to parametrize our ignorance of the underlying SUSY breaking dynamics – are in fact correlated as in a more fundamental theory [12–14]. It has been argued that ignoring these correlations leads to prematurely discarding viable SUSY models; allowing for such correlations leads to the possibility of *radiatively-driven naturalness* [15,16] where large, seemingly unnatural values of GUT scale soft terms (such as $m_{H_u}^2$) can be radiatively driven to natural values at the weak scale due to the large value of the top-quark Yukawa coupling.

Indeed, it has been shown that to allow for the possibility of parameter correlations one should only require that the *weak scale* contributions to m_Z (or m_h) be not much larger than their measured values. From minimization of the MSSM scalar potential, one can relate m_Z to weak scale MSSM Lagrangian parameters

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2. \quad (1)$$

Here Σ_u^u and Σ_d^d denote 1-loop corrections (expressions can be found in the Appendix of Ref. [16]) to the scalar potential, $m_{H_u}^2$ and $m_{H_d}^2$ the Higgs soft masses at the weak scale, and $\tan\beta \equiv \langle H_u\rangle/\langle H_d\rangle$. SUSY models requiring large cancellations between the various terms on the right-hand-side of Eq. (1) to reproduce the measured value of m_Z^2 are regarded as unnatural, or fine-tuned. Thus, natural SUSY models are characterized by low values of the *electroweak* naturalness measure Δ_{EW} defined as [15,16].

$$\Delta_{EW} \equiv \max|\text{each term on RHS of Eq. (1)}|/(m_Z^2/2). \quad (2)$$

Since Δ_{EW}, by definition, does not include large logarithms of the high scale Λ, Δ_{EW} is smaller than the traditional fine-tuning measures Δ_{BG} [6] or Δ_{HS} [10,11]. These logarithms essentially cancel if the underlying model parameters are appropriately correlated, and then the traditionally used fine-tuning measure reduces to Δ_{EW} once these correlations are properly implemented [12–14]. We conservatively advocate using Δ_{EW} for discussions of fine-tuning since this automatically allows for the possibility that underlying SUSY breaking parameters might well be correlated. Disregarding this may lead to prematurely discarding perfectly viable theories because the traditional computation of fine-tuning (ignoring possible parameter correlations) may falsely lead us to conclude that the model is unnatural.

We see from Eq. (1) that the robust criteria for naturalness are the weak scale values:

- $m_{H_u}^2 \sim -(100\text{–}300)^2$ GeV^2, and
- $\mu^2 \sim (100\text{–}300)^2$ GeV^2 [17]

(the lower the better). For moderate-to-large $\tan\beta \gtrsim 5$, the remaining contributions other than Σ_u^u are suppressed. The largest radiative corrections Σ_u^u typically come from the top squark sector. The value of the trilinear coupling $A_0 \sim -1.6m_0$ leads to split TeV-scale top squarks and minimizes $\Sigma_u^u(\tilde{t}_{1,2})$, simultaneously lifting the Higgs mass m_h to ~ 125 GeV [16].

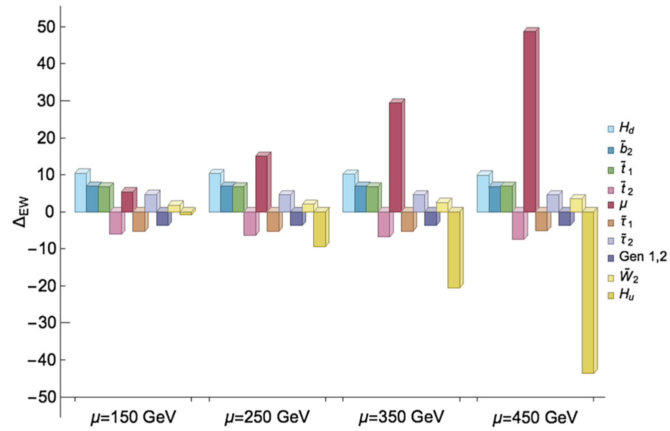

Fig. 1. Top ten contributions to Δ_{EW} from NUHM2 model benchmark points with $\mu = 150,\ 250,\ 350$ and 450 GeV.

A visual display of the top ten contributions to Δ_{EW} is shown in Fig. 1 for NUHM2 benchmark points with $\mu = 150,\ 250,\ 350$ and 450 GeV. For $\mu = 150$ GeV, all contributions to m_Z – some positive and some negative – are comparable to or less than the measured value so the model is very natural. For $\mu = 250$ GeV with $\Delta_{EW} = 15$, we see that some fine-tuning is on the verge of setting in so that the value of $m_{H_u}^2(weak)$ must be adjusted to compensate for such a large value of μ. By the time $\Delta_{EW} \sim 30$, corresponding to $\mu \sim 350$ GeV, cancellation between (presumably) unrelated large contributions is clearly required. This value will therefore serve as a rather conservative upper limit on Δ_{EW} in our study, since – as we are considering "natural SUSY" – we expect the contributions to any observable (in this case m_Z) to be comparable to or less than the value of the observable.[2] To obtain upper bounds on sparticle masses from naturalness, we therefore require $\Delta_{EW} < 30$ (no worse than 3% fine-tuning, even allowing for the fact that model parameters may be correlated).

A large assortment of popular SUSY models with $m_h \simeq 125$ GeV were examined in Ref. [14] where only the two-extra-parameter (compared to the well-known mSUGRA/CMSSM model) non-universal Higgs model (NUHM2) [18] (with the two extra parameters μ and m_A allowed to be free) was found to allow for naturalness. Requiring $\Delta_{EW} < 30$ in the NUHM2 model, then it was found that [16,45]

- $m_{\tilde{g}} \lesssim 5$ TeV (see also Fig. 2),
- $m_{\tilde{t}_1} \lesssim 3$ TeV (with other third generation squarks bounded by ~ 8 TeV) and
- $m_{\widetilde{W}_1,\widetilde{Z}_{1,2}} \lesssim 300$ GeV,

while other sfermions could be in the multi-TeV range. Thus, gluinos and squarks may easily lie beyond the reach of LHC at little cost to naturalness with only the higgsino-like lighter charginos and neutralinos required to lie close to the weak scale.[3] The light-

[1] We recall three cases where naturalness correctly presages the onset of new physics: 1. the classical electromagnetic contributions to the electron energy $E = m_e c^2$ required a relativistic treatment of spacetime and its concomitant *positron* [7], 2. the electromagnetic mass difference of the charged and neutral pions required new physics below ~ 850 MeV (matched by $m_\rho \simeq 770$ MeV) [8] and 3. a computation of the $K_L - K_S$ mass difference required the existence of the charm quark with $m_c \sim 1\text{–}2$ GeV [9].

[2] For concreteness we must choose some upper bound on Δ_{EW}, and there is inherently subjectivity in this choice. Since $\mu \gtrsim 100$ GeV (from LEP2 chargino search limits), then Δ_{EW} is necessarily > 1, while it would be hard to describe $\Delta_{EW} \gtrsim 100$ as "natural". The value $\Delta_{EW} = 30$ corresponds to individual contributions to the right-hand-side of Eq. (1) which exceed a factor of $\gtrsim (3m_Z)^2$.

[3] Our conclusion about the existence of light higgsinos arises from the fact that the higgsino mass is given by the superpotential parameter μ and this same param-

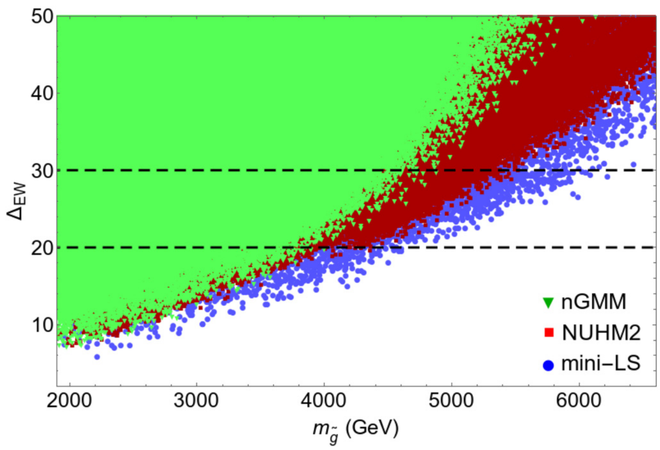

Fig. 2. Plot of $m_{\tilde{g}}$ vs. Δ_{EW} from scan over NUHM2 model (red squares), nGMM model (green triangles) and the mini-LS picture (blue circles). Points with $\Delta_{EW} < 30$ are conservatively regarded as natural. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

est higgsino $\widetilde{Z}_1$ comprises a portion of the dark matter and would escape detection at LHC. The remaining dark matter abundance might be comprised of, *e.g.*, axions [23]. Owing to the compressed spectrum with mass gaps $m_{\widetilde{W}_1} - m_{\widetilde{Z}_1} \sim m_{\widetilde{Z}_2} - m_{\widetilde{Z}_1} \sim 10$–20 GeV, the heavier higgsinos are difficult to see at LHC because the visible energy released from their decays $\widetilde{W}_1 \to f\bar{f}'\widetilde{Z}_1$ and $\widetilde{Z}_2 \to f\bar{f}\widetilde{Z}_1$ (where the f denotes SM fermions) is very small. The NUHM2 model can be embedded in a general $SO(10)$ SUSY GUT.

Keeping in mind that the stabilization of the Higgs sector remains a key motivation for WSS, these upper bounds are vital for testing the validity of the naturalness hypothesis.[4] While the naturalness upper bound is $m_{\tilde{g}} \lesssim 5$ TeV, experiments at the LHC have probed $m_{\tilde{g}} < 1.9$ TeV via the $\tilde{g}\tilde{g}$ production channel. The reach of the high luminosity LHC (HL-LHC) for gluino pair production has recently been evaluated in Ref. [24] (see also [25] and [26]). Using hard $\not\!\!E_T$ cuts, it was found that the LHC14 reach extends to $m_{\tilde{g}} \sim 2.4$ (2.8) TeV for 300 (3000) fb^{-1} – not sufficient to probe the entire natural SUSY range of gluino masses.[5] Moreover, the HL-LHC is expected to probe maximally to $m_{\tilde{t}_1} \sim 1.4$ TeV [25,26], again far short of the complete range of natural models.

This is not the complete story for the NUHM2 framework, because the underlying assumption of gaugino mass unification constrains the wino mass to be $\sim m_{\tilde{g}}/3$. As LHC integrated luminosity increases, wino pair production provides a deeper reach into parameter space, via the clean same-sign diboson (SSdB) channel [28] (from $pp \to \widetilde{W}_2^{\pm}\widetilde{Z}_4$ with $\widetilde{W}_2^{\pm} \to W^{\pm}\widetilde{Z}_{1,2}$ and $\widetilde{Z}_4 \to W^{\pm}\widetilde{W}_1^{\mp}$). This channel offers a HL-LHC 3000 fb^{-1} reach to $m_{1/2} \sim 1.2$ TeV, covering nearly all of the $\Delta_{EW} < 30$ region. Although electroweak production of higgsinos is swamped by SM backgrounds due to the small visible energy release in higgsino decays, higgsino pair production in association with a hard QCD jet – for instance $pp \to \widetilde{Z}_1\widetilde{Z}_2 + jet$ with $\widetilde{Z}_2 \to \widetilde{Z}_1\ell^+\ell^-$ – offers a HL-LHC reach to $\mu \sim 250$ GeV [29]. The presence of the soft dilepton pair with $m_{\ell\ell} < m_{\widetilde{Z}_2} - m_{\widetilde{Z}_1}$ is crucial for limiting the SM background. In general models (see below), where the wino is heavier than its unification value, the SSdB signal would be kinematically suppressed, and at the same time, the mass gap between the higgsinos would be reduced, leading to a diminished efficiency for detection of the soft leptons in the $\ell^+\ell^-$+monojet events just discussed. Thus although these combined channels cover nearly all of $\Delta_{EW} < 30$ parameter space in the NUHM2 model or in the other low $|\mu|$ models with gaugino mass unification [30], they cannot be relied on to guarantee LHC detection in a natural SUSY framework without gaugino mass unification.

This leads us to examine the natural SUSY parameter space of an alternative framework dubbed natural Generalized Mirage Mediation (nGMM) in which the weak scale gaugino masses have (nearly) comparable values. GMM is a generalization of well-motivated mirage mediation (MM) models [31] that emerge from string theory, with moduli fields stabilized via flux compactification. Gaugino mass unification at the *mirage unification scale* $\mu_{\rm mir}$, is the robust characteristic of this scenario and leads to nearly degenerate gauginos at the weak scale if $\mu_{\rm mir}$ is close to $m_{\rm weak}$. Although MM models that are based on simple compactification schemes appear to be unnatural for the observed value of m_h [14], a more general construction [32] which allows for more diverse scalar soft terms allows $\Delta_{EW} < 30$ with $m_h = 125$ GeV without altering the predicted gaugino mass pattern. Thus nGMM models with low values of $\mu_{\rm mir}$ and $m_{\tilde{g}} = 3$–4.8 TeV may have very heavy winos, suppressing the SSdB signal and leading to very small higgsino mass gaps (2–6 GeV) making the $\ell^+\ell^- j + \not\!\!E_T$ signal challenging to detect. We see that the nGMM model presents a natural, well-motivated framework which may well be beyond the HL-LHC reach.

The string-inspired natural mini-landscape (mini-LS) [33] models, whose phenomenology was recently examined in Ref. [34], is yet another well-motivated example where the spectrum satisfies electroweak naturalness but may not be accessible at the HL-LHC. The mini-LS scenario is closely related to the nGMM model in that gaugino masses maintain the relations of mirage unification – but it differs in that the first/second generation scalar mass soft parameters are significantly larger than those of the third generation and Higgs sector. Models with deflected mirage mediation [35], or models in which the field that breaks supersymmetry transforms as the **75** rep. of $SU(5)$ [36] also lead to a compressed gaugino spectrum which may likewise lie beyond the HL-LHC reach.

To assess the capability of testing SUSY naturalness in a relatively model-independent way, we should not rely on signals which are contingent upon the lightness of the wino relative to the gluino. We have therefore programmed the nGMM model into the Isasugra/Isajet 7.86 spectrum generator [37] (for details on parameter space, see Ref. [32]). This also allows us also to generate the mini-LS spectrum. Next, we have performed detailed scans over the allowed parameter space, requiring $m_{\tilde{g}} > 1.9$ TeV and m_h : 123–127 GeV (allowing for ± 2 GeV theory error in the Isasugra calculation of m_h). We show in Fig. 2 a scatter plot of Δ_{EW} versus $m_{\tilde{g}}$ for both the nGMM model (green triangles), the NUHM2 model (red squares) and the mini-LS picture (blue circles). From the plot, we read off an upper bound $m_{\tilde{g}} \lesssim 4.6$ (5.6)[6.0] TeV if $\Delta_{EW} < 30$ in the nGMM (NUHM2) [mini-LS] model. The bound is only mildly sensitive to the specific assumption about high scale wino and bino masses, but does depend on the hierarchy between first/second generation scalar and the top squark masses. Henceforth we regard the more conservative $m_{\tilde{g}} < 6.0$ TeV as representative of an upper limit on $m_{\tilde{g}}$ in all natural SUSY models and explore prospects for *gluino detection* at a variety of hadron colliders with a view to either detecting or excluding supersymmetry with $\leq 3\%$ electroweak fine-tuning.

eter enters the Higgs boson mass calculation. This situation can be circumvented in extended SUSY models with additional weak scale superfields beyond those of the MSSM that have extra symmetries [19–21] or in models where SUSY breaking higgsino terms are allowed [22]. If these higgsinos couple to SM singlets, such terms would lead to a hard breaking of SUSY.

4 We stress that WSS always resolves the big gauge hierarchy problem; we are concerned here with stabilizing the weak scale without the need for part per mille fine-tuning.

5 Thus, Ref. [24] and this paper answer the question posed in the Abstract to Ref. [27].

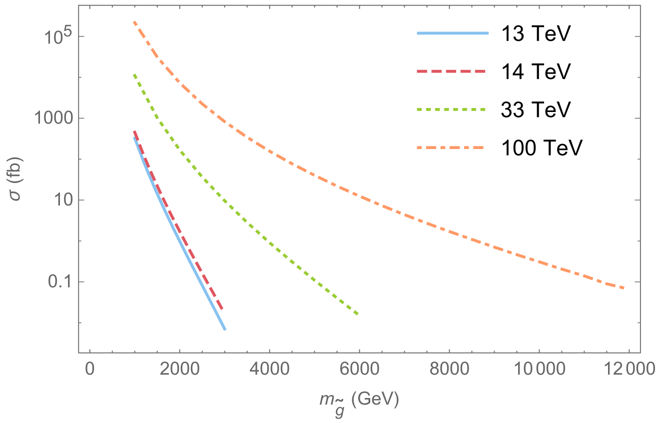

Fig. 3. Total cross section (NLL+NLO) for gluino pair production at various hadron colliders vs. $m_{\tilde{g}}$ for $m_{\tilde{q}} \gg m_{\tilde{g}}$.

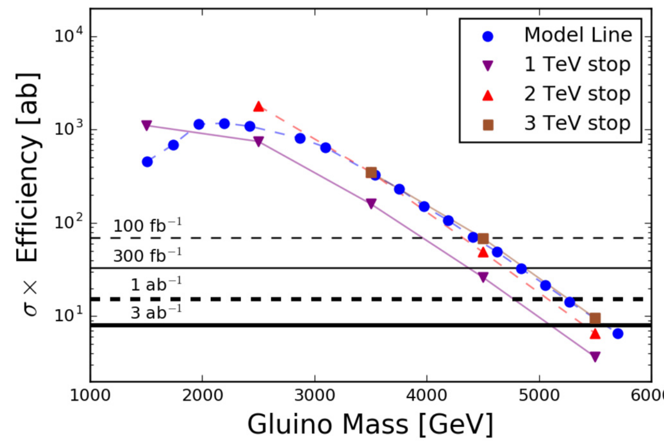

Fig. 4. Plot of cross section after cuts in the 2-tagged b-jet analysis along with 5σ discovery lines for 100, 300, 1000 and 3000 fb^{-1} for the NUHM2 model line introduced above (blue circles), as well as simplified models with $m_{\tilde{t}_1} = 1$ TeV (upside-down purple triangle), 2 TeV (red triangles) and 3 TeV (brown squares). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

In Fig. 3, we show the NLL+NLO evaluation [39] of $\sigma(pp \to \tilde{g}\tilde{g}X)$ versus $m_{\tilde{g}}$ for pp collider energies $\sqrt{s} = 13,\ 14,\ 33$ and 100 TeV. For 3000 fb^{-1} at LHC14, the gluino reach for the NUHM2 model extends out to $m_{\tilde{g}} \sim 2.8$ TeV [24], insufficient to probe the entire natural SUSY parameter space in this channel. Naive scaling suggests that the gluino reach would cover the entire natural SUSY range even at the HE-LHC, a 33 TeV pp collider, for which a peak luminosity of 2×10^{34} cm^{-2} s^{-1}, corresponding to about 100 fb^{-1} per operating year, has been projected [38].

Here, we perform a careful analysis of the natural SUSY reach via gluino pair production at the HE-LHC, assuming the gluinos primarily decay to third generation squarks as expected in natural SUSY models. We have explored the reach in various multijet plus $\not{E}_T$ channels and found that the greatest reach (as measured by statistical significance of the signal over SM backgrounds) is obtained in the $\geq 4j + \not{E}_T$ channel with ≥ 2 tagged b-jets. We use the same b-jet tagging algorithm as in Ref. [24] and find that the reach is nearly optimized with the same set of cuts as in that study, except that we now require jets to have $E_T > 200$ GeV and require $\not{E}_T > 1500$ GeV for the heavier gluinos under consideration.

We perform our analysis for several model lines designed to capture features of gluino events in natural SUSY models. We first examine an NUHM2 model line with $m_0 = 5m_{1/2}$, $A_0 = -1.6m_0$, $m_A = m_{1/2}$, $\tan\beta = 10$ and $\mu = 150$ GeV. For this model line, over the mass range of interest (2–6 TeV), the gluino always decays via $\tilde{g} \to \tilde{t}_1 t$, with $\tilde{t}_1 \to b\widetilde{W}_1$ at 50%, $\tilde{t}_1 \to t\widetilde{Z}_1$ at $\sim 25\%$ and $\tilde{t}_1 \to t\widetilde{Z}_2$ at $\sim 25\%$ [40]. The decay products of the daughter higgsinos are essentially invisible. Gluino pair production gives rise to final states with $tttt$, $tttb$ or $ttbb$ plus large $\not{E}_T$. For this model line $m_{\tilde{t}_1}$ increases with gluino mass and is 0.8–1 TeV below $m_{\tilde{g}}$ for $m_{\tilde{g}} = 2$–5 TeV. Since the efficiency for detection after cuts will be sensitive to event kinematics, we have also examined three simplified model lines with $m_{\tilde{t}_1} = 1,\ 2$ and 3 TeV independent of $m_{\tilde{g}}$, where we assume the gluino always decays via $\tilde{g} \to t\tilde{t}_1$ and that the stop decays as in model line 1. We expect that these model lines capture much of the variation expected from natural SUSY models, including the possibility that some fraction of models have a significant (but subdominant) branching fraction for gluino decays to $\tilde{t}_2$ or $\tilde{b}_1$ squarks whose decays also lead to third generation squarks in the final state. We have checked that for most models with $\Delta_{\rm EW} < 30$, $B(\tilde{g} \to \tilde{t}_1 t) \geq 60\%$.

The results of our computation of gluino signal cross section after analysis cuts in the multijet plus $\not{E}_T$ channel with ≥ 2 tagged b-jets is shown in Fig. 4 for the NUHM2 model line introduced above (blue circles), as well as for the simplified models with $m_{\tilde{t}_1} = 1$ TeV (upside-down triangle), 2 TeV (triangle) and 3 TeV (squares). We have checked that the cross section for a simplified model line with $m_{\tilde{t}_1} = 4$ TeV (and large enough gluino masses) is very close to that for the first model line. The horizontal lines denote the cross section levels required for a 5σ signal significance above SM backgrounds from $t\bar{t}$, $t\bar{t}t\bar{t}$, $t\bar{t}b\bar{b}$, $Wt\bar{t}$, $Zb\bar{b}$ and single top production.[6] We see that, with an integrated luminosity of 1 ab^{-1}, the 5σ gluino mass reach at the 33 TeV machine extends to $m_{\tilde{g}} = 4.8$ TeV (and covers the entire $\Delta_{\rm EW} < 20$ part of the allowed mass range) *even with the most pessimistic assumption* for the top squark mass.[7]

It should be kept in mind that this is an extremely conservative estimate of the reach: a 1 TeV stop is just above the current bound, so such scenarios will either be excluded or discovered well before HE-LHC accumulates 1 ab^{-1} of data. We have also checked [34] that in these natural SUSY models, $m_{\tilde{g}} > 4.8$ TeV only if $m_{\tilde{t}_1} < 2$ TeV, and further that the LHC33 reach for top squark comfortably exceeds 2.7 TeV, assuming that the top squark dominantly decays to higgsinos via $\tilde{t}_1 \to t\widetilde{Z}_1$, $\tilde{t}_1 \to \widetilde{Z}_2$ and $\tilde{t}_1 \to b\widetilde{W}_1$ with branching ratios 1:1:2 [44]. It is, therefore, reasonable to conclude that a 33 TeV pp collider will decisively probe almost the entire range of gluino masses available to natural SUSY models with no worse than 3% electroweak fine-tuning, and that if the gluino is too heavy for detection, the signal from the top squark will definitely be accessible.

In Fig. 5, the bars show several 5σ gluino discovery and 95%CL exclusion reaches in natural SUSY models for various pp collider options via the channel $pp \to \tilde{g}\tilde{g}$ along with the naturalness upper bound on $m_{\tilde{g}}$. We expect that this upper bound is insensitive to the details of the model as a pMSSM scan with $\Delta_{\rm EW} < 30$ also yields the same bound [45]. The region below the gray band is considered not fine-tuned while the region beyond is fine-tuned. We see that the HE-LHC discovery reach with $\sqrt{s} \sim 33$ TeV and 1000 fb^{-1} will just about cover the entire natural SUSY parame-

[6] If the background is underestimated/overestimated by factor f, these horizontal lines will shift up/down, by about a factor $\lesssim \sqrt{f}$. For $f = 2$ the reach projection is affected by only ≈ 100–150 GeV for ab^{-1} scale integrated luminosities. The effects of event pile-up depend on details of both machine and detector performance and thus are beyond the scope of the present analysis. A discussion of pile-up for CMS at LHC14 is given in Ref. [41].

[7] Our LHC33 reach values are comparable to those previously calculated for hadronic channels in the context of simplified models in Refs. [42,43].

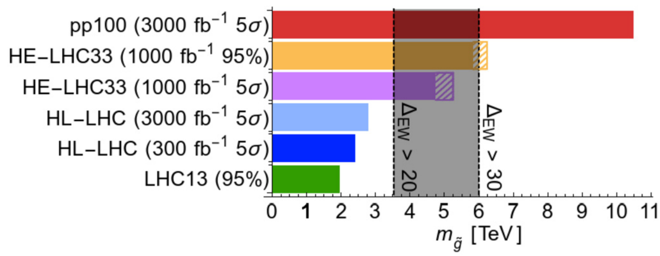

Fig. 5. Reach of various hadron collider options for natural SUSY in the gluino pair production channel compared to upper bounds on $m_{\tilde{g}}$ (gray band) in natural SUSY models. The hatches reflect some model dependence of the HE-LHC reach where the lower edge is *very conservative* since the light stops (for which the lower edge is calculated) offer an independent SUSY discovery channel [43].

ter space as conservatively defined by $\Delta_{EW} < 30$. Moreover, if the gluino is too heavy to be discovered, the top squark signal will be accessible. Thus, HE-LHC should suffice to either discover or falsify natural supersymmetry. We also show the reach of a proposed $\sqrt{s} = 100$ TeV *pp* collider (the FCC-hh or *SppC*) within the context of a simplified model assuming gluino three-body decay to massless quarks [43]. The 100 TeV *pp* collider can probe to values of $m_{\tilde{g}}$ over 10 TeV. (This is likely a conservative value since the projected reach would likely extend to somewhat larger values if instead gluinos are assumed to dominantly decayed to third generation squarks.) However, we note that HE-LHC should already be able to discover or falsify natural SUSY within the context of the MSSM at a small fraction of the cost of a 100 TeV machine.

In summary, supersymmetric models with weak scale naturalness are well-motivated SM extensions with impressive indirect support from measurements of gauge couplings and the top-quark and Higgs boson mass. While the HL-LHC appears sufficient to probe natural SUSY models with gaugino mass unification, we have shown that HE-LHC with $\sqrt{s} = 33$ TeV is required to either discover or falsify natural SUSY (with $\Delta_{EW} < 30$) even in very general – but equally natural – SUSY scenarios such as nGMM with a compressed gaugino spectrum. Alternatively, an e^+e^- collider with $\sqrt{s} \sim 0.5$–0.7 TeV would be sufficient to either discover or falsify natural SUSY via pair production of the required light higgsinos [46]. Discovery of natural SUSY via either of these machines would then provide enormous impetus for the construction of even higher energy machines which could then access many of the remaining superpartners.

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