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An Early Semester Mastery Activity and Intervention in First-Year Calculus

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Abstract

Success in first-year mathematics courses is essential for students to pursue STEM careers, including teaching careers. We investigate a mastery activity given during the first two weeks of a first-year calculus course at the research site. Previous work showed a model using this activity in College Algebra, together with ACT and high school rank, was predictive of student success in precalculus. Here we do a similar analysis for such an activity in calculus, including an intervention for students who do not complete the activity. We also investigate the intervention's effectiveness. These results show that the early mastery activity, especially when combined with other indicators of mathematics readiness, is useful in identifying students at risk of failing calculus. Moreover, descriptive statistics suggest that students who participate in the intervention are more successful than expected, based on their academic backgrounds and other college grades.

Keywords: calculus, mastery grading, diagnostic activity, undergraduate mathematics, academic retention

Introduction

Historically, introductory STEM courses, including first-year calculus, struggle with high D-Fail-Withdraw rates (DFW; the percentage of students earning a grade of D or F, or withdrawing from the course). According to Bressoud and Rasmussen (2015), among students at Ph.D.-granting institutions, the average DFW rate sits at 25 percent. These poor success rates drive away STEM majors, including potential STEM education majors. Improving student success in college calculus courses is necessary to increase the number of teachers, as recognized by the MTE-Partnership's decision to make Active Learning Mathematics (ALM) one of the five initial Research Action Clusters (RACs). Indeed, the ALM-RAC has an explicit goal of improving "student success with undergraduate mathematics, starting with the Precalculus through Calculus 2 sequence (P2C2)."

We study one aspect of a systematic effort to improve student learning in first-year mathematics courses at the research site, a large, public land-grant university in the Midwest: an early semester mastery activity, the Course Readiness Activity (CRA), which is an enforced review of prerequisite material, with multiple attempts possible during the first two weeks of the semester. A different version of the activity has been used successfully in College Algebra and was shown previously to predict student success in College Algebra (Wakefield, Champion, Bolkema, & Dailey, in press). Our first question, building on this previous paper, is:

Question 1: What capacity does a CRA in calculus have for identifying students who are at risk for failing a calculus course, beyond known identifiers, such as high school grades or ACT Math subscore?

The CRA has the additional benefit of engaging students early in the semester. Since there are multiple attempts, the pattern, timing, and performance changes for these attempts provide valuable information about a student's engagement.

Beyond identification, if some students are at-risk, how can that risk be reduced? In the studied course, a simple remediation effort was offered to students who did not complete the CRA. This leads to our second research question:

Question 2: Did this intervention improve outcomes for those who participated?

Since students chose for themselves whether to participate, a major challenge in answering this question is whether such students are systematically different from those who did not participate in the remediation.

Description

Overview of CRA

The first Course Readiness Activity (CRA) at the research site was introduced in College Algebra, based on a similar Gateway exam in the University of Michigan's Precalculus and Calculus I courses. There are now CRAs in four first-year mathematics courses, each of which covers appropriate prerequisite material for that course, at the depth of understanding that is needed for success.

As an activity, rather than an exam, the expectation is that all students will successfully complete the CRA, i.e., it is not an assessment. Nor is the CRA used to revisit placement decisions. Students who fail to complete the CRA are not encouraged to drop the course.

One motivation for the CRA is that students' experience of college calculus convinces them that they were not properly prepared. In the MAA's *Insights and Recommendations from the MAA National Study of College Calculus,* Bressoud (2015) argues that while 80 percent of students enter college believing they are ready for calculus, by the end of the semester only 55 percent believed they were ready. Furthermore, this sample may be biased in that discouraged students would seem less likely to complete the end-of-semester survey and, indeed, significantly fewer students completed the end-of-semester activity. Thanheiser, Philipp, Fasteen, Strand, & Mills (2013) highlight the importance of showing pre-service teachers that their current understanding of mathematics is limited and that they have more to learn. While not a perfect parallel, one goal of the CRA is to encourage students to review and master prerequisite material and to achieve the deep understanding that is essential for effective application of this material in calculus.

Description of CRA

The Calculus CRA consists of 15 questions chosen from such topics as cancellation laws, equations of lines, rational expressions, exponents, inequalities, polynomials, domain and range, piecewise functions, function notation, graphs, logarithms, and trigonometry. There is also a word-problem/modeling question.

An in-class paper CRA is given early in the first week of classes. Students are told of the CRA before the semester starts and course instructors devote a little class time to preparing for the CRA. Instructors grade the CRA and post scores the same evening. Mastery grading is used; that is, students who receive a passing score (80 percent) are given full credit (40 points on roughly an 800-point grading scale). Students who do not achieve 80 percent on the paper CRA can take the CRA online in a university testing center once per day, for the first two weeks of the semester. The online CRA questions are either algorithmically generated or drawn from pools of varied questions covering the same issue. For example, a true-false question on cancellation laws is selected at random from a mix of identities and popular fallacies. The online exam is machine graded with the same requirements as the paper one, except that students receive immediate feedback. Each exam is generated independently, without reference to the student's previous attempts. Students are strongly encouraged to discuss their work with an instructor or a tutor in the tutorial center, particularly if they are unsuccessful in multiple attempts online.

Instructors regularly remind students to complete the CRA. Nevertheless, there is still a contingent of students who do not pass the CRA. Improving success rates means helping struggling students engage with the material more effectively. To this end, the department developed a simple intervention: students who did not pass the CRA were given the opportunity to meet with their instructor and discuss strategies for being more successful in the course. Students who met with their instructor were given back points at their instructor's discretion.

To analyze the predictive power of the CRA, CRA outcomes were compared to overall course grades, scores on individual exams, and institutional registration data, such as ACT Math sub-score and high school percentile rank. In the fall of 2016, complete data was available for 735 students (out of a total enrollment of 837). Among the 735 students in the calculus sample, there were 283 females and 434 males. All but 74 were either 18 or 19 years old and all but 98 were first-time, first-year students. For ethnicity, 79 percent were White/Non-Hispanic, 7 percent Hispanic, 5 percent Asian, 2.3 percent African-American, and 3.1 percent multiple races. The average ACT Math sub-score was 26.4.

Table 1 shows the number of students in each CRA outcome and the course performance for each group. There were five possible outcomes on the CRA: "Passed on Paper" (showed mastery on the paper CRA), "Passed Online" (did not show mastery on the paper CRA exam, but did so on the online exam), "Intervention" (did not achieve mastery but met with instructor to develop a plan to be successful), "Fail" (did not achieve mastery, attempted the CRA online, and did not meet with instructor to develop a plan to be successful), and "Abdicated" (did not achieve mastery and never made an attempt online). Course Success Rate is the proportion of students who achieved a grade of C or better. Exam Success Rate is the proportion of students whose average on course exams, that is, three semester tests and the final exam, was a C (i.e., 70 percent) or better. The course grade was made up primarily of exam scores, but also a variety of items, such as quizzes, participation, and online homework, including, as a small component, the CRA. For statistical analysis, exam performance is used to ensure independence between the CRA and dependent variables.

Table 1

CRA & Course Success (N=735)

CRA OUTCOME	PROPORTION OF STUDENTS	COURSE SUCCESS RATE	EXAM SUCCESS RATE
PASSED ON PAPER	214 (29%)	92%	86%
PASSED ONLINE	397 (54%)	86%	58%
INTERVENTION	30 (4%)	67%	43%
FAILED	70 (10%)	34%	19%
ABDICATED	24 (3%)	20%	17%

MANOVA Analysis

To answer the first research question, a multivariate analysis of variance (MANOVA) was performed using ACT, high-school percentile, and CRA performance as explanatory variables and exam performance as dependent variables. Visual inspection of the matrix of scatter plots for each exam revealed elliptical scatter plots between dependent variables, correlation between each dependent variable, and a hook shape along each of the diagonals (q-q plot). The elliptical scatter plots suggest correlation between the dependent variables. The hook shape on the diagonal reveals some left skew in the distributions, as is common with exam data. However, the left-skew issues can be overcome by the large sample size. Hence, MANOVA is a reasonable statistical test; see Figure 1.
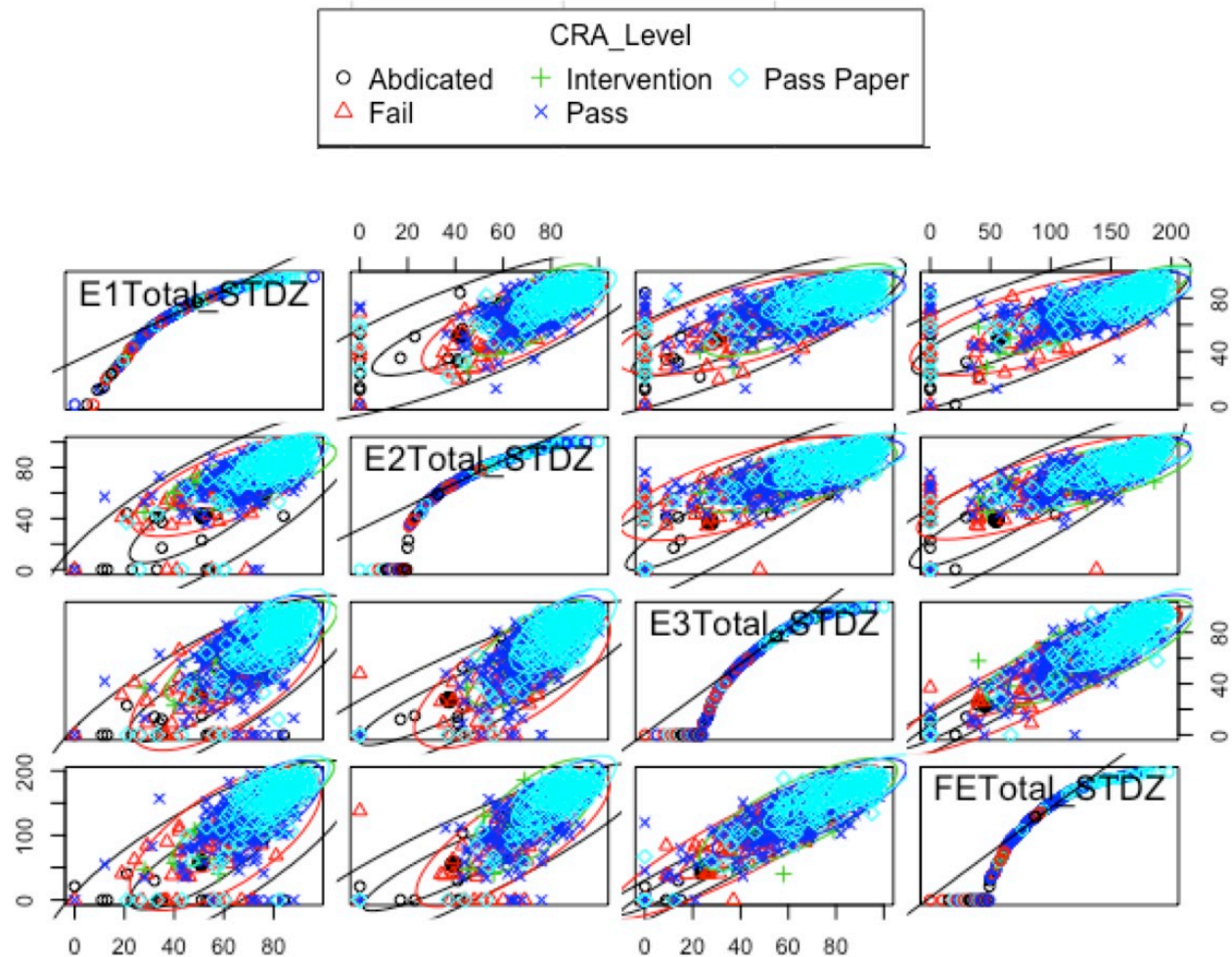
Figure 1: Matrix of scatter plots: Exam 1, Exam 2, Exam 3, Final Exam.

After omitting non-significant interaction effects, the MANOVA suggested that there were significant main multivariate effects from each of the explanatory variables on exam scores (p<.001 for ACT Math, HS percentile, and CRA=Pass Paper, Pass, Intervention; p<.01 for CRA=Fail; p<.05 for CRA=Abdicated; multiple R^2=.39, .36, .39, and .40 for the four exams, respectively). Type II multivariate tests for individual explanatory variables indicate that ACT math performance, high school percentile, and CRA performance are all significant at a level of p<0.001 in all four of Pillai, Wilks, Hotelling-Lawley, and Roy's greatest Root.

While it is commonly accepted that ACT mathematics sub-score and high school percentile affect math course outcomes in college, our analysis using MANOVA suggests that the CRA has additional capability of identifying students who are at risk for failing the course early in the semester. That is, the CRA is telling us something more than we know from just the ACT mathematics sub-score and high school percentile. In fact, only 18 percent of students who either failed or abdicated on the CRA successfully completed the course whereas more than 66 percent of students who passed the CRA on paper or online successfully completed the course. Beyond the CRA, high school percentile and ACT math sub-score also provide key indicators of future success.

A natural question is, "What does the CRA measure?" One aspect that the CRA certainly measures is mathematical content. However, it appears the CRA measures more than mathematical content. The CRA correlates with exam performance. However, the CRA does not appear to correlate with ACT Math sub-score. In

fact, running a MANOVA with the CRA outcome and the ACT math sub-score as explanatory variables results in CRA=Pass, and CRA=Pass Paper as the only significant effects ($p<0.05$ and $p<0.01$ respectively). We argue that this is because the CRA, particularly the online CRA, is also measuring engagement in the course.

Analysis of Intervention

We turn now to the second research question: Did this simple intervention improve student success? Recall that the intervention was the opportunity to meet with the instructor and discuss strategies for being more successful in the course, with strong encouragement to use the tutorial center. (Unfortunately, we do not have data about these students' use of the tutorial center.) Students who met with their instructor were given back the CRA points at their instructor's discretion, so participation in the intervention is not fully independent of course grade. Only 30 students participated in the intervention out of the 100 students who attempted the online CRA and did not pass it. As Table 1 shows, students in the intervention were much more likely to have a C or better in the course and more likely to have a C or better average on their exams.

Since exam performance does not include any points for the CRA, we look at box and whisker plots for average exam score and for final exam percentage by CRA outcome, in Figures 2 and 3. Both figures show that students who participated in the intervention did significantly better. In particular, all students in the intervention group did better, by either measure, than the bottom quarter of those who did not participate in the intervention.
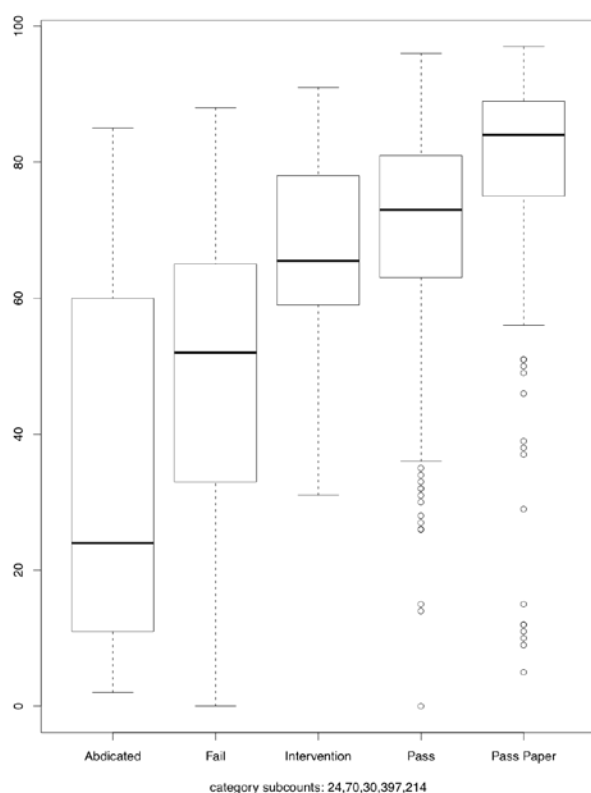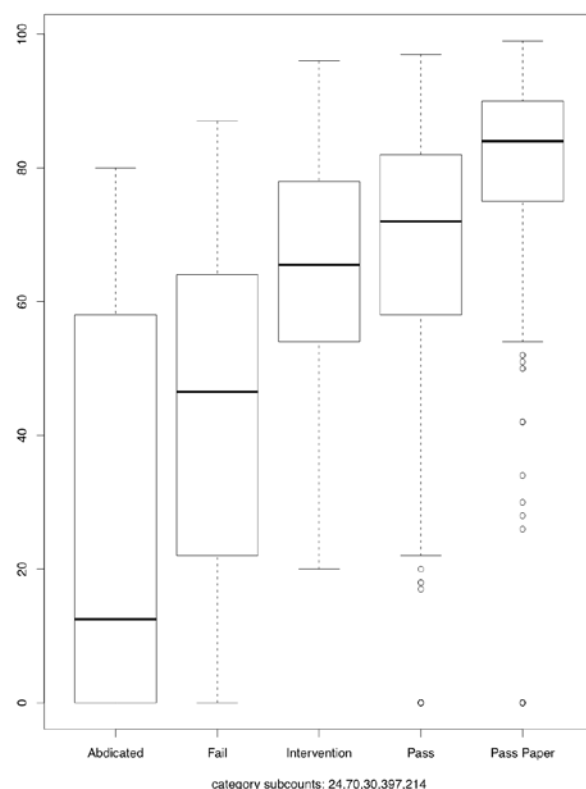

Figure 2. Exam average by CRA category.

Figure 3. Final exam percentage by CRA category.

An almost immediate concern, given that students self-selected to participate in the intervention, is that there is some significant difference in the background or engagement level of the two groups of students, so that

those who participated in the intervention would have been more successful, with or without the intervention. While it is impossible to answer this definitively in the absence of a randomized trial, we have some evidence that the two groups of students are similar in important ways. Consider the box and whisker plots for ACT mathematics sub-score and high school percentile rank by CRA outcome, in Figures 4 and 5. In each case, the fail and intervention groups appear broadly similar, although the intervention group has a slightly higher average in each case. But the substantial difference in Figures 2 and 3 is not likely explained by these small differences.

Similarly, we can compute these students' semester GPA, excluding the calculus course. We take this 'nonmath GPA' as a proxy for a student's overall academic success in college. Looking at box and whisker plots for nonmath GPA by CRA outcome, students in the Intervention group have a similar average to those in the Fail group, although with a somewhat higher distribution overall. Therefore, the Intervention students are not simply better at college or, at least, not sufficiently better to explain the large differences in Figures 2 and 3.
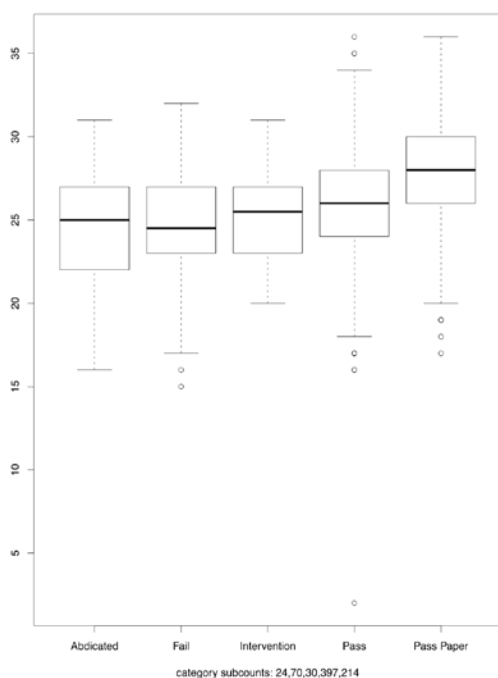

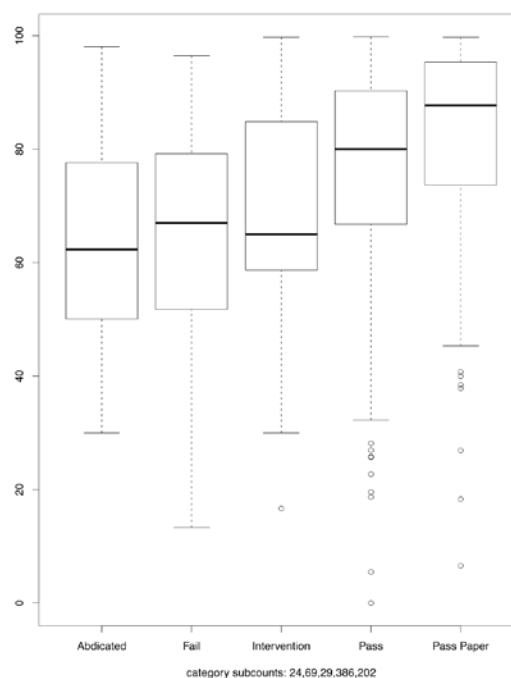*Figure 4.* ACT Math subscore by CRA category.

Figure 5. High school percentile rank by CRA category.

Smith, W. M., Lawler, B. R., Bowers, J., & Augustyn, L. (Eds.). (2017). *Proceedings of the sixth annual Mathematics Teacher Education Partnership conference.* Washington, DC: Association of Public and Land-grant Universities.
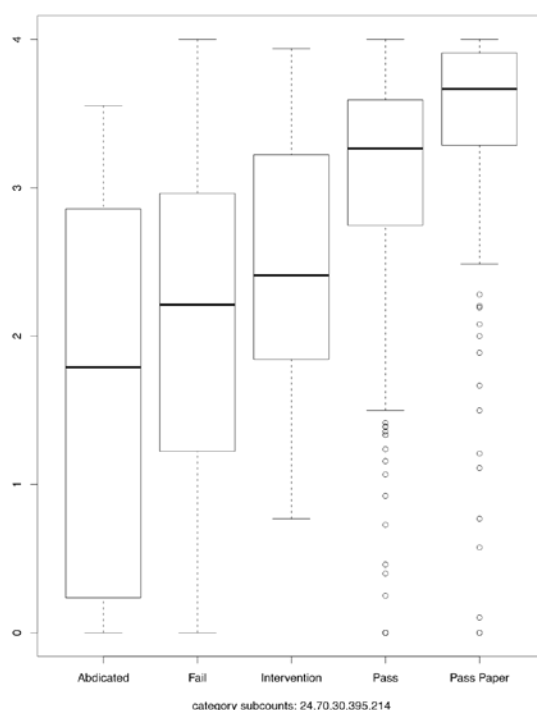
Figure 6. Non-math GPA by CRA category.

Conclusion

Based on the previous study for the College Algebra CRA, we expected that a Calculus CRA would also be able to identify at-risk students. Moreover, we introduced a simple intervention to help those students. If we accept that the CRA is also measuring engagement, then interventions should aim to improve students' engagement with the course.

A statistical analysis of the students' performance demonstrated a strong relationship between the students' CRA performance and their subsequent achievement in the course. The MANOVA analysis shows the CRA provided additional information beyond students' prior academic record. Descriptive statistics suggest the intervention improved grades for participants.

Based on our experience with the CRA, we have a few suggestions for anyone desiring to implement such an activity. To begin, two warnings: first, although students can take the exam once per day for the first two weeks of the semester, many wait until close to the deadline to make their first online attempt at the exam. Second, we framed the CRA as an activity and emphasized that all students should complete it, to avoid anxiety about an "exam," particularly an exam in the first few days of class. Furthermore, by emphasizing the benefits of starting class work early, we try to both reduce anxiety and improve engagement. We often remind students that they have the opportunity to repeat the activity multiple times until they are successful. We want students to use the activity as a way to start the course off with a good grade. For those willing to try, the CRA provides a first step toward success in calculus.

Finally, an important aspect of the CRA is that it provides a basis for engaging at-risk students based on their work in the course, rather than asking them to participate based on prior academic results. Indeed, we don't have access to such information, nor do we wish to "label" students in ways that might compromise instructor's

expectations. We continue to refine both the CRA and the intervention, with the goal of ensuring that students, even those at risk of failure, know what they need to do to succeed and are encouraged to do so.

For More Information

Please contact either author; for details of CRA, please contact Nathan Wakefield.

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