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Characterizing Mathematics Graduate Student Teaching Assistants' Opportunities to Learn from Teaching

Yvonne Lai
University of Nebraska-Lincoln, yvonnexlai@unl.edu

Wendy Smith
University of Nebraska-Lincoln, wsmith5@unl.edu

Nathan Wakefield
University of Nebraska-Lincoln, nathan.wakefield@unl.edu

Erica R. Miller
University of Nebraska-Lincoln, erica.miller@huskers.unl.edu

Julia St. Goar
University of Nebraska–Lincoln, s-jstgoar1@math.unl.edu

See next page for additional authors

Authors
Yvonne Lai, Wendy Smith, Nathan Wakefield, Erica R. Miller, Julia St. Goar, Corbin M. Groothuis, and Kelsey M. Wells

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Y. Lai (Corresponding author), N. P. Wakefield, E. R. Miller, J. S. Goar, C. M. Groothuis, & K. M. Wells: Department of Mathematics, University of Nebraska–Lincoln, 203 Avery Hall, PO Box 880130, Lincoln, NE 68588-0130, USA; ***e-mail:*** yvonnexlai@unl.edu; nathan.wakefield@unl.edu; erica.miller@huskers.unl.edu; s-jstgoar1@math.unl.edu; corbin.groothuis@huskers.unl.edu; kelsey.wells@huskers.unl.edu

W.M. Smith: Center for Science, Mathematics & Computer Education, University of Nebraska-Lincoln, 251 Avery Hall, PO BOX 880131, Lincoln, NE 68588-0131, USA; e-mail: wsmith@unl.edu

Abstract

Exemplary models to inform novice instruction and the development of graduate teaching assistants (TAs) exist. What is missing from the literature is the process of how graduate students in model professional development programs make sense of and enact the experiences offered. A first step to understanding TAs' learning to teach is to characterize how and whether they link observations of student work to hypotheses about student thinking and then connect those hypotheses to future teaching actions. A reason to be interested in these connections is that their strength and coherence

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determine how well TAs can learn from experiences. We found TAs can connect observations and future teaching, but that the connections vary in quality. Our analysis suggests future revisions to TA development programs, which we discuss in the conclusion.

Keywords: Post-secondary professional development, Mathematics teacher growth, Reflective practitioner, Graduate teaching assistants, TA development

MSC Codes: 97B99, 97B50, 97-xx, 97Axx, 97B40

1 Introduction

One theory for how instructors learn from their own and others' teaching experience is that learning occurs through deliberately connecting future teaching plans and prior experience. Specifically, instructors create opportunities to learn when they articulate future actions in terms of observations based on previous experience (e.g., Hall & Horn, 2012; Horn, Kane, & Wilson, 2015). Under this theory, enhancing the ability to learn from experience requires both improving how instructors conceive of teaching and tightening connections between future plans and current thinking.

Our goal is to improve TAs' ability to learn by reflecting on their experiences. We report on a study of novice mathematics graduate student teaching assistants (TAs), who were teaching college algebra and intermediate algebra and were all enrolled in a seminar as part of a TA development program. The program aims to help TAs to teach from the principles that: (a) student learning occurs through the student's lens, and observation of student learning occurs through the observer's lens; (b) understanding the experiences that shape students' thinking is important to teaching; and (c) learning occurs through building on prior knowledge. Our study explored the question:

> How do these TAs connect observations and beliefs about their students, hypotheses about student thinking, and proposed next teaching actions?

We open this chapter with one TA's reflections about her students' learning, based on a paper written as part of the TA development program. We use this TA's work to illustrate how we model TA thinking so as to study the opportunities they created to learn from experience. After describing our model for TA thinking, we discuss the literature informing our work and the context in which this study took place. We then describe how we collected and analyzed data to study TA thinking. Finally, we describe paths of TA thinking that we found useful in considering how to improve our TA development program. We reflect on our future actions in terms of observations and beliefs about TAs and hypotheses about TA thinking.

2 Modeling TA Thinking

2.1 One TA's Reflection

At the time of the study, TA12 was a first time instructor who taught College Algebra. She had recently assigned this quiz problem:

Determine whether the following function is a rational function:

$$f(x) = 1 - \frac{6}{x^3} + \frac{-2}{x+4}$$

> *If it is a rational function, write it in the form* $f(x) = p(x)/q(x)$ *, where* $p(x)$ *and* $q(x)$ *are polynomials.*

On the same quiz, she had also asked her students to express the following as a single fraction:

$$1 - \frac{6}{8} + \frac{-4}{6}$$

In a report of her students' performance, TA12 first explained that she had designed the quiz purposefully: the fraction expression is equivalent to evaluating the function f at $x = 2$. She then observed that most students simplified the expression in mathematically valid ways; a typical solution was:

$$1 - \frac{6}{8} + \frac{-4}{6} = 1 - \frac{3}{4} + \frac{-2}{3} = \frac{12}{12} - \frac{9}{12} + \frac{-8}{12} = \frac{12 - 9 - 8}{12} = \frac{-5}{12}$$

However, many students—including some who had performed a valid calculation for fractions—simplified the rational expression as follows:

$$f(x) = 1 - \frac{6}{x^3} + \frac{-2x}{x+4} = \frac{1 - 6 + -2x}{x^3(x+4)}$$

She hypothesized that students' thinking about rational functions did not draw on their experiences with fractions:

> While there are some students who struggle with combining these fractions, most of my students are able to do so successfully. That shows me that they are familiar with and able to use fraction operations, so the root of the misconception in this case is not that they have misconceptions concerning the fraction operations … For some reason, the introduction of variables into the fraction numerator and/or denominator causes a breakdown in their reasoning, which I believe is the root of the misconception. (TA12 Final paper, p. 4)

In TA12's interpretation, students have productive knowledge to build upon, because they can work with closely related numerical expressions in mathematically valid ways. At the same time, students may separate their knowledge of fractions from their knowledge of rational expressions. TA12 then speculated how her own teaching or others' instruction may enforce this separation, calling out the role of emphatically distinguishing operations with numbers from operations with variables (e.g., stressing that $3 + 2 = 5$, but $3a + 2b \neq 5ab$). TA12 thus hypothesized that students may benefit from experiences in which they explicitly connect operations on fractions with operations on rational functions.

As a next step, TA12 proposed to hold a structured conversation with her students. TA12 scripted a hypothetical conversation, a portion of which follows:

> T: Now, even though we used mathematical operations specifically to combine fractions, the truth is that we can use these mathematical operations with any ratio. What exactly are the mathematical operations we used to combine the fractions?

S: First we found a common denominator. Then we changed each individual fraction so that it had the common denominator. Last we added together the changed numerators to get one faction.

T: Exactly! Let's see if we can use those same operations to solve the problem we started with. How could we find a common denominator?

S: I don't really know.

T: How did you find a common denominator of (1, 3, and 4)?

S: I multiplied them together.

T: Exactly! So, you actually did this previously, but we could find the common denominator in our problem by multiplying the denominators together. Just because they have variables in them doesn't change our process. What would our new common denominator be?

S: $x^3 (x + 4)$

T: Correct! So, think back to the second operation you said you used for combining fractions: "change each individual fraction so that it has the common denominator." How do you think we could do this with our problem with variables?

S: We could figure out what we need to multiply each fraction by to get the common denominator!

TA12 envisioned guiding the student in identifying fraction operations used while working with different denominators, and then building concrete connections between fractions and rational expressions. TA12 emphasized that plan was not to stick to the script but rather to ask "purposeful, guiding questions" that allowed as much as possible for "the students to … generate as much knowledge on their own" (TA12 Final paper, p. 8).

2.2 Modeling TA Thinking

To explain how we model TAs' thinking about instruction, we use TA12's reflection as an example. Our model has four components: data, student thinking, hypothesis, and future teaching actions. **Figure 1** displays this model. We define *data* to consist of written and oral expressions made by students that are observed by an instructor. In the case of TA12, the

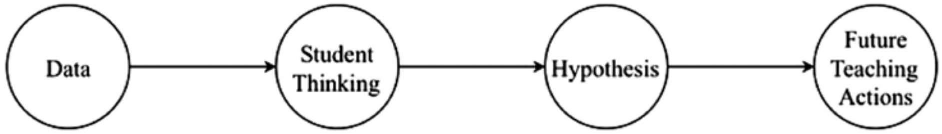

Fig. 1. Model for TA thinking

data are her students' performance on a quiz. *Student thinking* is an interpretation of the data. For example, TA12 interprets the combination of mathematically valid work with fractions and mathematically incorrect work with rational expressions as an indication that her students did not draw on their knowledge of fractions when working with rational expressions. A *hypothesis* is a conjecture about likely experiences that have shaped or could shape the student thinking. TA12's hypothesis is that students may benefit from explicit connections between operations on fractions and on rational functions. *Future teaching actions* describe how the instructor might work with students in the future, given their interpretation of student thinking. TA12 proposed to hold a structured conversation in which she would guide students toward describing properties of rational functions based on properties of fractions, and then give students an opportunity to use these properties.

We use TA12's reflection as an example because it shows how the components of the model fit together, even if there are places where the reflection can be improved or may be unrealistic. The interpretation of the data is reasonable: students are not applying their knowledge of numeric fraction operations to fractions that have variables. The hypothesis addresses the interpretation directly: TA12 interprets that student do not use their knowledge of fractions when working with rational functions, even though this knowledge is useful, and so TA12 proposes that students construct and then use parallels between fractions and rational expressions. The future teaching actions are envisioned to elicit the relevant similarities between fractions and rational expressions. There are places where the dialogue may seem contrived or where the instructor may be appearing to do too much of the students' work. TA12's interpretation that the students separate their knowledge of fractions from rational expressions may be overly simplistic. However, in making these judgments, we should keep in mind that TA12 is a first time instructor, and that on the whole, the components do lead from one to the other. As we discuss later in this chapter, there are examples of TA reflections whose components are not as well connected.

3 Literature Informing the Study

In this section we summarize the literature informing our model for TA thinking and the rationale for drawing on results from K-12 teacher education and professional development. The goal of our model is to describe TAs' claims about future teaching actions as potential opportunities to learn. Thus, our model connects two literature bases: one on argumentation and the other on teachers' opportunities to learn from teaching.

3.1 Literature on Argumentation

Toulmin (1958) is a foundational reference about modeling argumentation. Toulmin originally created his model to analyze legal arguments, and it has since been used for other fields, including mathematics education (e.g., Inglis et al., 2007). The three key components of Toulmin's model are: the grounds, the claim, and the warrant. The grounds are the evidence on which the claim is made, and the warrant is the reason that the grounds support the claim. Toulmin uses the following claim as an example: "I am a British citizen." Possible grounds for this argument include, "I was born in Bermuda;" a warrant could be, "British law states that persons born in Bermuda are British citizens." In our model, we consider both the data and the interpretation to be the grounds of the TA's argument. The future teaching actions are a claim about what instruction may be beneficial. The warrant is the hypothesis about experiences that might shape or have shaped the students' understanding. The way we map our model to Toulmin's components is consistent with the cognitive theory that learning involves the interpretations a person ascribes to their experiences and the inferences made from these interpretations (see Thompson, 2016 for an overview of this theory, which is based on work of the psychologist Piaget).

3.2 Literature on Opportunities to Learn from Experience

Our model is also shaped by studies of K-12 teachers, especially the research of Horn and colleagues (e.g., Hall & Horn, 2012; Horn et al., 2015). Their work focuses on describing the "opportunities to learn" that teachers create in conversation about student data and previous experiences, including how some opportunities may be stronger than others.

3.2.1 Opportunities to Learn

In the theory developed by Horn and her colleagues described in the papers cited above, learning opportunities for improving one's teaching are strongest when: teachers marshal observations and stances about teaching experiences to mobilize themselves for future plans, and these plans represent skillful teaching. "Stances" refer to what the teachers believe is important for them to know about learning and teaching, and how to come by this knowledge. To put this in terms of our model, when the data and interpretation are strongly linked to the hypothesis and future teaching actions, in a way that is consistent with what is known about teaching quality, there is greater opportunity to learn.

3.2.2 Features of Skillful Teaching

In our view, which is consistent with the writing of Horn and colleagues, skillful teaching includes: responsiveness to and respect for student thinking; providing opportunities for students to articulate their thinking and respond to others' thinking; maintaining cognitive demand (e.g., if an assigned question is challenging, the teacher helps the student work on the question without stripping away the difficulty); focusing students on core mathematical ideas, especially the meaning behind expressions and procedures; and inclusiveness (all students are attended to). Additionally, when students work on problems based on real-world scenarios, the instructor helps the students understand the real-world context, how mathematics could model this context, and develop common terms to refer to key ideas in the context. Our views are informed by studies of teaching complex tasks (e.g., Jackson et al., 2013; Stein et al., 1996); studies linking qualities of teaching to student outcomes (Learning Mathematics for Teaching Project, 2011); and studies that identify and describe components of tasks of teaching (Boerst et al., 2011; Sleep, 2012).

3.3 Parallels Between K-12 and Post-Secondary Education

Commonalities between pre-calculus courses at the undergraduate level and the high school level make it reasonable to hypothesize that results from K-12 teacher education and professional development are also promising for development of instructors of undergraduates. After all, K-12 teachers and undergraduate course TAs share some challenges, especially when TAs teach a course such as college algebra. Students typically enroll in college algebra through requirement rather than by choice; often they are placed in the course through a combination of assessments and previous coursework. Students are likely to need many opportunities to break unproductive habits. College algebra is also a gateway to many courses needed for scientifically-oriented careers.

We now discuss the particular TA development context in which we collected data on TAs' thinking.

4 Context, Data, and Method

4.1 Context

The TAs in this study were enrolled in a seminar on teaching and learning mathematics at the post-secondary level. The TAs all taught college algebra or intermediate algebra in sections of approximately 40 students, consisting primarily of first-year college students. With few exceptions, every college algebra or intermediate algebra TA participates in the seminar. Each TA is the sole instructor for his or her section. The lessons in all sections feature small group discussions and small group work led by the TAs. Each beginning TA teaches only one section of the course; in subsequent years TAs would teach two courses in the fall and one in the spring. There are a few adjunct instructors (mostly former or current high school mathematics teachers), who teach college algebra, but TAs teach the majority of the sections. The adjunct instructors do not participate in the TA development program.

The seminar met two hours per week in the fall semester, when the study was conducted. To help develop language for reflecting on teaching, TAs read educational literature describing examples and theories for understanding student learning. These include Erlwanger's (1973) classic account of a child's arithmetic understanding, and Tsay and Hauk's (2013) exposition of constructivism. In the seminar, TAs were asked to discuss teaching experiences in terms of the readings.

4.2 Data

We collected final papers written by all 16 TAs enrolled in the seminar. In these papers they were asked to (a) report on student performance on a quiz they assigned, (b) interpret student thinking in the quiz performance, (c) hypothesize about experiences that contributed to the students' thinking, and (d) propose future teaching actions to refine student thinking. In the assignment, the TAs were asked to focus on interpreting student work that was not mathematically valid.

4.3 Rationale

Recall that our aim is to study TAs' thinking, and that we model TA thinking with four components (as shown in Fig. 1): data, interpretation, hypothesis, and future teaching actions. The intention of the assignment was for TAs to hypothesize how or why students may have found their way to mathematically invalid reasoning, and for TAs to describe future teaching actions that are built on productive ways of thinking and provide settings where new ways of thinking might be useful. The assignment is designed to elicit TA thinking for each component and how it related to the previous component: (a) data (b) interpretation (c) hypothesis (d) future teaching actions.

4.4 Analysis

We analyzed the TAs' papers in two parts. First, we examined the components of the model represented, including whether TAs articulated the components and their connections clearly or if components were missing or conflated with other components, and to what degree they represented skillful teaching (as described in Sect. 3.2.2). Second, we examined the internal consistency or inconsistency (i.e., at least two components contradict each other) of the components. As discussed previously (in Sect. 2), TA12's paper is an example of an internally consistent paper. The paper written by TA13 (discussed in more detail in Sect. 5) provides an example of inconsistency as well as conflated components. TA13 interpreted that students see equations as "a string of symbols to memorize." Later TA13 stated several hypotheses, including "When my students see an equation, their first thought is that they have to memorize it" and that students were "uncomfortable" with function notation (TA13 Final paper, pp. 1–2). Thus the hypothesis and interpretation are conflated. Furthermore, she then described future teaching actions that deliberately avoided addressing or using function notation.

These future teaching actions are inconsistent with the hypothesis and interpretation, because they seek to address students' use of function notation without opportunities for students to use function notation.

We compared the consistency and connectedness of papers relative to each other, rather than to an external standard. The reasons for this approach were twofold. First, to our knowledge, there is no widely-accepted rubric for judging the coherence of pedagogical argument, though there are theories about the components of such an argument (which we used as a foundation for this study, as discussed in Sect. 3.2). Second, it is a well-established cognitive science result that people are more reliable comparing impressions against one another than judging an impression of quality in isolation (Laming, 1984; Thurstone,1927). The relative comparisons can then be used to sort objects into categories of relative quality and identify attributes contributing to the impression of quality (e.g., McMahon & Jones, 2015). We classified papers into "high", "medium", and "low" connectedness by consensus, in which at least four of the chapter authors weighed in on each paper, with more authors discussing controversial papers. Highly connected papers articulated all four components with internal consistency. Papers with medium connectedness conflated components (for instance, TA13) and were not entirely internally consistent. Low connectedness papers did not specify the reasoning between each component, for instance leaping from data to future teaching actions (as is the case with TA02, to be discussed further in Sect. 5).

During this analysis, we discovered a highly connected paper that did not feature a coherent argument. Although we agreed that the TA attempted to connect all components, and we also agreed on the specific weaknesses of the argument, we disagreed on the plausibility of the connections, and we never resolved our disagreement. We called this type "highly connected with low coherence." We classified two other papers in this way.

5 Results

We asked: How do TAs connect observations and beliefs about their students, hypotheses about student thinking, and proposed next teaching actions? **Table 1** summarizes the TAs' papers by category. **Figure 2**

Table 1. TA papers by category

Connectedness	*Which TAs' final papers exhibited this connectedness*
High	TA04, TA05, TA08, TA11, TA12, TA15
High, with low coherence	TA02, TA10, TA14
Medium	TA01, TA06, TA13, TA16
Low	TA03, TA07, TA09

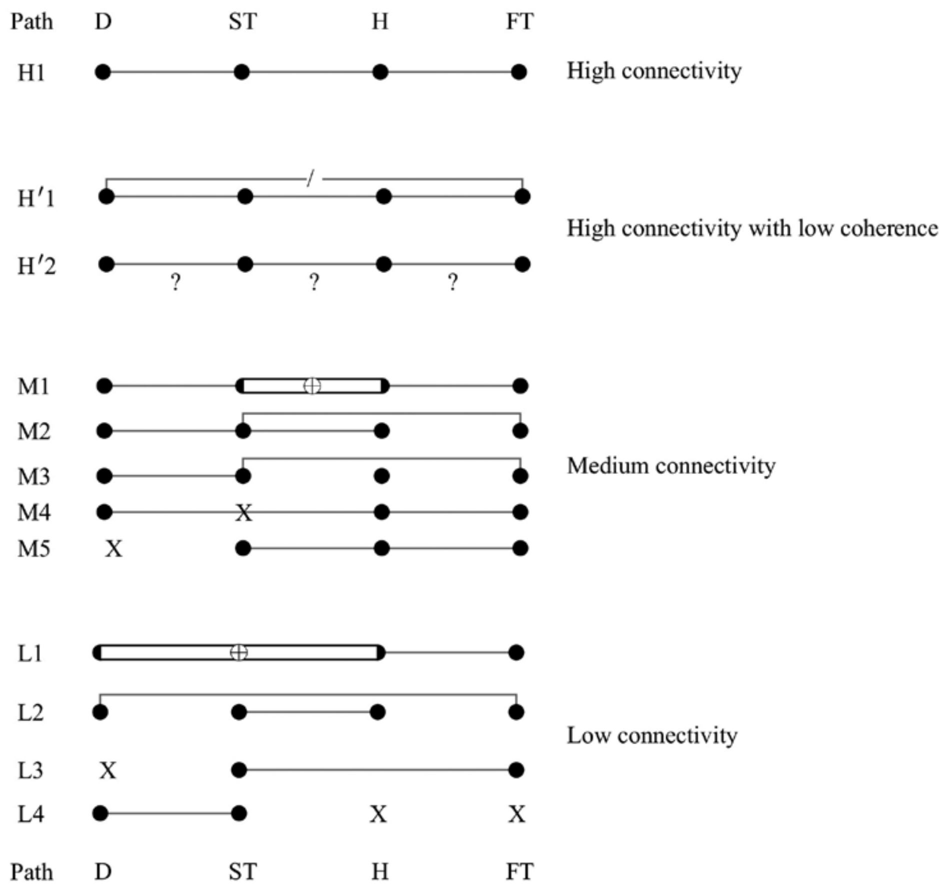

Fig. 2. Connectedness of TA papers. Key*:* *D* = data; *ST* = student thinking; *H* = hypothesis; *FT* = future teaching actions; *X* = component absent; ⊕ = conflated components; •—• = link was attempted and satisfied criteria; •—?—• = no consensus from research team on whether link is plausible; • – / – • = future teaching actions do not plausibly address student thinking identified in data.

shows the connectedness paths exhibited in our data. We now illustrate each path with an example paper. To retain anonymity of the TAs, we use the same pronoun, "she", to refer to all TAs. (Of the 16 TAs in the cohort studied, nine were female.)

5.1 Illustrations of Paths of TAs' Thinking

5.1.1 High Connectedness (H1 in Fig. 2; TA12)

TA12's paper (described in Sect. 2) is highly connected. She described all four components, and the components and her reasoning from one to the next were internally consistent.

5.1.2 High Connectedness with Low Coherence (H'1; TA02)

TA02 posed the quiz problem, "How would you find the y-intercept of a function f? Explain why your method gives the y-intercept." Several students responded similarly to: "You would plug in a zero into the x. By putting zero into the x, the y-intercept would be the only thing left." TA02 concluded, "It seems that they view 'plug in 0 for x' as a way to get rid of the x, rather than a consequence of the fact that if a point is on the y-axis, its x-value must be 0" (TA02 Final paper, p. 2). As a result, TA02 hypothesized, "They see the graph and the equation as two distinct objects—related, because you can sketch the graph given the equation, but not exactly representing the same mathematical relationship. The confusion about how to find the y-intercept is probably a special case of this disconnect" (p. 2). TA02's interpretation and hypothesis are plausibly linked to the data.

TA02 proposed that in the future, she would design a worksheet that asked students to sketch a graph of a given function, complete an input/output table, and sketch vertical lines on the graph. The intention would be for students to experience finding outputs of a function both using its defining equation and using intersecting the graph of the function with vertical lines. While the worksheet does link to the hypothesis, it did not plausibly address the TA's interpretation of student thinking. The first worksheet question asked students to graph a function defined by an equation. However, assuming TA02's interpretation of student thinking, the student most likely would struggle with graphing. It is possible that the remainder of the worksheet would have helped the student recognize the connection between graphs and equations, but this assumes that the student would be able to begin the work. We classified this paper as an example of an H'1 pathway where despite plausible links between components, the argument still does not hold together.

5.1.3 Medium Connectedness (M1; TA13)

Using data from a unit exam question involving revenue, profit, and cost (denoted $R(n)$, $P(n)$, $C(n)$ respectively), TA13 observed:

> … many of them seemed to think that an equation was a string of symbols to memorize, as opposed to something that they were capable of understanding or even constructing on their own. … During the exam I had students raise their hands and tell me, ‘I forgot the formula from class!’ They wrote down things like $P(n) = R(n) + C(n)$, $P(n) = C(n) - R(n)$, … This was very surprising because I am confident that every one of them has an intuitive understanding of the concept of ‘net gain’. This reminded me very much of the situation described in the paper *Mathematics in the Streets and in Schools* [Carraher et al., 1985], in which kids were perfectly capable of doing arithmetic in the marketplace, but when handed pencil and paper and asked to work the same problems out symbolically, were frequently flummoxed. (TA13 Final paper, p. 1)

To address the students’ conception of equation, TA13 proposed to use a story about a renter saving up money for an upcoming vacation to derive a formula involving rent R, living expenses L, monthly income I, savings S, and the cost T of round trip plane tickets. As the students arrived at expressions to solve the problem, TA13 would ask students to justify their findings. TA13 explained that this task

> requires students to create something, as opposed to manipulating a formula that is given … It would get students used to the idea that equations are not divinely inspired, can be written by ordinary people, and used as shorthand to describe events that are entirely understandable (p. 4).

Prior to describing these future teaching actions, TA13 put forth several hypotheses, including students’ discomfort with the terms “revenue” and “profit,” with function notation, and with the notion of inputs and outputs of a function. However, these hypotheses do not link plausibly to the future teaching actions; TA13 notes that she purposefully designed the worksheet to avoid function notation, even if there are variables used. There is also no mention of revenue and profit.

TA13 proposed one more hypothesis that she emphasized as the most probable cause: “When my students see an equation, their first thought is that they have to memorize it” (p. 1). TA13 described other situations where schooling mandated memorization because the information was in some sense arbitrary, such as naming the 50 states and their capitals. Although this last hypothesis does connect to the future teaching actions proposed, it only restates the interpretation of student thinking.

TA13 interpreted student thinking in a way that was consistent with the data. The future teaching actions are connected to the interpretation of student thinking and the data. Although she attempted to describe experiences that shaped student thinking, the only applicable hypothesis restated the description of student thinking. In other words, TA13’s paper conflated interpretation of student thinking and hypothesis. For these reasons, we categorized TA13’s paper as an example of medium connectedness.

5.1.4 Low Connectedness (L2; TA03)

On a quiz given by TA03, students were unable to articulate the difference between the word “constant” and the phrase “constant rate of change.” Concerned that the students may not understand that these terms denote fundamentally different ideas, TA03 allowed students time to discuss the difference between the terms in groups. However, confusion persisted. TA03 attributed this misunderstanding to lack of precision in language and grammar, leading students to gloss over verbal differences between the two terms. TA03 went on to suggest that lack of precision causes students to group together similar looking functions, even to the extreme of “glossing over the distinction between lines with slope zero and lines with nonzero slope” (TA03 Final paper, p. 1). Generally, TA03 was concerned that imprecise language leads to confused mathematical thinking.

TA03 then proposed that in the future, she would ask students to graph the monthly profits of two businesses in a story problem, one whose monthly profits are a constant function of time ($1000 each month), and the other whose monthly profits have a constant (nonzero) rate of change over time ($1000, $1200, $1400, etc.). TA03 reasoned, "Get students to admit that the second business is much different than the first; in fact, it's much better! Then, and only then, broaden out the discussion to include the actual words 'constant' and 'constant rate of change'. … By building a common starting point through discussing which business is doing better than the other, the teacher can buy themselves enough goodwill to introduce the more abstract terminology" (p. 3). This activity targets the confusion encountered in class with regard to the two terms. However, the activity neither addresses precision of language or precision in a students' view of functions in any significant way. That is, this activity does not build on the hypothesis. Hence, the future teaching actions are connected to the data, but not to any component between data and future teaching actions, despite an effort on the part of TA03 to do so. We speculate that one source of this issue for TA03 is that she genuinely believes that addressing precision of language in general would solve many problems. Perhaps this view pervaded her thinking so strongly that TA03 struggled to identify a hypothesis that provided more guidance for future teaching actions.

5.2 Summary of Findings

We modeled TAs' final papers as a practical argument with four components. We found that components could be present, absent, or conflated, and we found that connections could be present, absent, internally consistent, or internally inconsistent. In some cases, we found TAs connected components that were non-adjacent in our model without connecting adjacent components. We also found one final paper in which the research team arrived at consensus on weaknesses of the TA's argument but could not arrive at consensus on whether links were plausible. In total, our data of the 16 TAs' final papers exhibited 12 paths in four categories. We illustrated one example path for each category.

6 Reflections on TA Education

As we specifically analyzed TAs' written reflections to an assignment from the TA pedagogy course, we detected different levels of TA reflection, as well as different patterns of components and connections. We were able to categorize papers as *high connectedness, high connectedness with low coherence, medium connectedness,* and *low connectedness.*

We believe these results are valuable for TA development. First, in terms of research, they extended theory from the K-12 teacher education literature on opportunity to learn to post-secondary instruction. As far as we know, theorizing on opportunity to learn in the context of TA development is novel. Second, more practically, our results support research into TA learning by describing ways in which TAs may, or may not, connect their experiences to future teaching.

Learning from experience is the goal of many TA development programs, but as the research in K-12 teacher education and our own results show, the potential for TAs to learn from experience can vary. What we have added to this conversation is particular examples of how TA learning opportunities can vary, even when the TAs are asked to do similar tasks. TAs whose papers were categorized as *high connectedness* were able to clearly articulate the four assignment components, as well as explicit links and connections among the components. TAs in this category have illustrated their capacity to act as reflective practitioners, and use their understanding of student thinking to support student learning. TAs whose papers were categorized as *high connectedness with low coherence* were able to articulate components, but the links were weaker or implicit. These TAs were on their way to becoming more reflective as teachers: they have the components, but need to learn to better articulate connections or links among those components. Given the limitation of analyzing written reflections, we can only conclude the TAs did not write about the connections among the components; it may be the TAs did see those connections, but need further practice in expressing teaching reflections in writing.

Other TAs were still at a stage in which written reflections did not capture the type of components and connections intended, but instead revealed the struggles of novice instructors trying to make sense of student thinking and determining how to respond. These TAs at the *mid* and *low connectedness* prompt us to think about how we might better support TAs in being explicit in their writing, and help TAs to both see and express connections among the components. When we do not see explicit links or components, we do not always have enough information to judge whether the omission was truly a reflection of the TAs' maturity as a practitioner, or instead, a reflection of the TAs' skills in reporting their thoughts and actions in a written reflection. We sometimes find TAs who are pursuing doctorates in mathematics profess to "not be good at writing" and who struggle to express their thoughts in coherent written paragraphs.

In considering what we have learned and others who might learn from our experience, we turn to our model of TA thinking. In this meta analysis, the data are the TAs' final papers. We interpret that while TAs are invested in helping their students learn and are committed to helping students construct knowledge, their proposed teaching actions do not always align with their interpretations of student thinking. We hypothesize that seminar discussions, in which TAs practiced describing their experiences in terms of literature on constructivism, fostered the TAs' dedication to giving students experiences to develop their own mathematical knowledge. Across the cohort of TAs, we saw this evidenced in their proposed teaching. We also hypothesize that these seminar discussions did not support TAs in selecting hypotheses or connecting components because they emphasized the components rather than the connections. Several TAs suggested hypotheses that they did not design instruction to address. In each of these cases, the hypotheses were general statements about students' ways of doing mathematics that would be difficult to mediate in the span of a lesson, rather than hypotheses that specifically applied to student performance on quiz problems.

We propose that in the future, TAs continue to read literature that encourages them to see the value of students discovering mathematics. We also propose that TAs hold seminar discussions in front of their peers, where they make explicit the connections between future teaching actions to hypotheses. In these discussions, the facilitator and peers would help revise one TA's hypotheses and teaching actions to be better defined and more strongly connected. This public revision is reminiscent of discussions between mentor and mentees to design action research, where the goal is to define addressable research questions and design data collection and analysis that address the research questions. In this analogy, research question is to hypothesis as data collection and analysis are to future teaching actions. Holding a public discussion aligns with research on K-12 teachers' learning suggesting that when groups of teachers reflect on experience, they create a "collective zone of proximal development" (Engeström, 1987) where they learn more than would be possible individually.

Finally, we comment that it is reasonable to wonder whether having TAs focus on student error is productive: will that reinforce deficit-views of student thinking? In our experience, many TAs enter graduate school believing that students' mathematical thinking is either right or wrong, and student learning can be accomplished by exposure to "right" ways. For instance, when discussing why students might struggle with composition of functions, TAs at the beginning of the semester have often proposed, "The students just need to learn the rule." We have found that discussing student errors has helped TAs move away from black and white judgments of student thinking. The TAs' final papers, even those of low connectedness, displayed more potential for sensitivity to student thinking than at the start of the semester. We also have the impression that the cohorts have become more sensitive to student thinking over time, which we attribute in part to new graduate students entering a culture where a critical mass of TAs hold a more constructivist orientation. We are optimistic that this trend will grow in the future. To continue encourage this trend to continue, we propose two changes. The first is to focus seminar discussions and assignments on unexpected correct solutions to complex tasks, and the second is focusing TAs' hypotheses more explicitly on what about the student thinking can be built on in future teaching actions.

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