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Flow in horizontally anisotropic multilayered aquifer systems with leaky wells and aquitards

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[1] Flow problems in an anisotropic domain can be transformed into ones in an equivalent isotropic domain by coordinate transformations. Once analytical solutions are obtained for the equivalent isotropic domain, they can be back transformed to the original anisotropic domain. The existing solutions presented by Cihan et al. (2011) for isotropic multilayered aquifer systems with alternating aquitards and multiple injection/pumping wells and leaky wells were modified to account for horizontal anisotropy in aquifers. The modified solutions for pressure buildup distribution and leakage rates through leaky wells can be used when the anisotropy direction and ratio (K_x/K_y) are assumed to be identical for all aquifers alternating with aquitards. However, for multilayered aquifers alternating with aquicludes, both the principal direction of the anisotropic horizontal conductivity and the anisotropy ratio can be different in each aquifer. With coordinate transformation, a circular well with finite radius becomes an ellipse, and thus in the transformed domain the head contours in the immediate vicinity of the well have elliptical shapes. Through a radial flow approximation around the finite radius wells, the elliptical well boundaries in the transformed domain are approximated by an effective well radius expression. The analytical solutions with the effective radius approximations were compared with exact solutions as well as a numerical solution for elliptic flow. The effective well radius approximation is sufficiently accurate to predict the head buildup at the well bore of the injection/pumping wells for moderately anisotropic systems ($K_x/K_y \leq 25$). The effective radius approximation gives satisfactory results for predicting head buildup at observation points and leakage through leaky wells away from the injection/pumping wells even for highly anisotropic aquifer systems ($K_x/K_y \leq 1000$).

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1. Introduction

[2] Anisotropy of aquifers is generally characterized by the tensor property of hydraulic conductivity or permeability. In addition to heterogeneity, knowledge of anisotropy, which influences the flow direction of fluids, is important for accurate solution of flow and transport problems in subsurface systems. Principal directions of anisotropy correspond to directions in space where the hydraulic conductivity takes its maximum and minimum values. The ratio of the maximum and the minimum hydraulic conductivities are defined as the magnitude of the anisotropy.

[3] Analytical solutions were developed for flow around a pumping well in the presence of horizontal anisotropy [*Papadopulos*, 1965; *Hantush*, 1966; *Kucuk and Brigham*, 1979; *Mathias and Butler*, 2007]. These solutions allowed for estimating anisotropy and components of hydraulic conductivity tensor by aquifer pumping tests with either three or four wells [*Hantush and Thomas*, 1966; *Neuman et al*., 1984; *Mutch*, 2005]. While these solutions are generally limited to single-layered or two-layered aquifer systems with an assumption of infinitesimal well radii, a few of them involved the effect of wells with finite radii for single-layered aquifer systems [*Kucuk and Brigham*, 1979; *Moench*, 1997; *Mathias and Butler*, 2007; *Fitts*, 2006].

[4] Flow problems in an anisotropic domain can be transformed into ones in an equivalent isotropic domain by using certain relationships. Once analytical solutions are obtained for the equivalent isotropic domain, they can be back transformed to the original anisotropic domain using the inverse of these relationships. Transformation is required for both the governing equations and the boundary conditions. Difficulty arises in this transformation if the well radius cannot be assumed to be infinitesimally small ($r_w \rightarrow 0$). With coordinate transformation, a circular well with finite radius becomes an ellipse in the transformed domain. The confocal ellipses representing head contours in the vicinity of the well in the transformed domain become concentric circles as the distance increases from the well. *Kucuk and Brigham* [1979] and *Mathias and Butler* [2007] developed analytical solutions for elliptical flow

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around a finite radius well in a horizontally anisotropic confined aquifer, and they showed that solutions for a circular boundary (i.e., radial flow solutions) can be used for estimating the head at the well bore of an elliptical shape if the proper effective well radius is used for anisotropic systems.

[5] Sedimentary basins can be conceptualized as multilayered systems with multiple aquifers and alternating aquitards. Regional groundwater production, oil/gas production, and high-volume injection of fluids for liquid waste disposal or geologic sequestration of CO_2 can cause wide-spread pressure perturbations in the multilayered systems [*Zhou et al.*, 2008, 2009]. Many sedimentary basins with multiple aquifers and alternating aquitards have been affected by extensive drilling for groundwater supply and oil/gas exploration and production [*Young*, 1992; *Nicot*, 2009]. Abandoned wells are considered potential conduits for fluid leakage and groundwater contamination [*Gass et al.*, 1977; *Javandel et al.*, 1988; *Lesage et al.*, 1991]. Both diffuse leakage (through aquitards) and focused leakage (through leaky wells) can be important in the case of wide-spread pressure perturbations in the multilayered aquifers [*Birkholzer and Zhou*, 2009; *Cihan et al.*, 2013]. Analytical solutions obtained for simplified geometries can be very useful to gain insights for coupled complex leakage processes in real systems. As alternative to numerical solutions, analytical solutions are computationally very efficient as typically they do not require spatial discretization, and therefore they are very useful in optimization and sensitivity studies or uncertainty quantifications [e.g., *Birkholzer et al.*, 2012; *Wainwright et al.*, 2013; *Azzolina et al.*, 2013]. The analytical solutions are particularly suitable when dealing with a large number of injection and leaky wells in multilayered systems [e.g., *Celia et al.*, 2011], for which numerical simulations would become computationally too expensive as local mesh refinement around each well is required to obtain accurate results.

[6] *Cihan et al.* [2011] developed a set of analytical solutions for transient flow through isotropic multilayered aquifers with alternating aquitards, and injection/pumping and leaky wells. Similar solutions for multilayered aquifer systems were developed using analytical element approaches, which also included heterogeneity by allowing presence of different subdomains with different hydraulic properties [*Bakker*, 2006]. In this note, we extend the solutions given in *Cihan et al.* [2011] to account for horizontal anisotropy in multilayered aquifers. By transformation of the anisotropic governing equations and the boundary conditions, we obtain an equivalent isotropic problem similar to the one solved by *Cihan et al.* [2011]. Above mentioned finite elliptical well boundaries in the transformed domain are approximated by using an effective well radius expression. Applicability of the extended solutions for prediction of pressure changes and leakage in multilayered aquifer systems with various anisotropy ratios is discussed by comparison with either the existing analytical solutions or the numerical solutions.

2. Analytical Solutions

[7] In this note, we consider the similar confined multilayered aquifer system as given in *Cihan et al.* [2011]. The multilayered system consists of N aquifers with alternating aquitards, and any number of pumping/injection and leaky wells. Each of the aquifers (numbered from the bottom aquifer to the top aquifer) is homogeneous and anisotropic with horizontal hydraulic conductivity $\boldsymbol{K}_i$ (L/T), storativity $S_{s,\mathrm{i}}$ (1/L), and constant thickness B_i. Each of the aquitards is also homogeneous with constant thickness, vertical hydraulic conductivity, and storativity. Flow through the aquifers is assumed to be in the horizontal direction only, and flow through the aquitards is assumed to be in the vertical direction only. The assumption of the horizontal flow can be justified as long as the ratio of hydraulic conductivity between the aquifers and the aquitards is larger than 100, as demonstrated by previous studies [e.g., *Neuman and Witherspoon*, 1969].

[8] In the following developments, we use the x, y axes of the horizontal solution domain to coincide with the principal directions of anisotropic horizontal hydraulic conductivity. The principal hydraulic conductivities can be related to the components of the hydraulic conductivity tensor in a given coordinate system [*Bear*, 1972; *Sekhar et al.*, 1994]. Our focus in this note is on transforming the governing equations and the boundary conditions for a horizontally anisotropic multilayered aquifer system into "equivalent" isotropic flow equations and boundary conditions. The equivalent problem can be solved using the analytical solutions in *Cihan et al.* [2011]. For brevity of the analytical derivation and notations, only one well is considered below. However, superposition can be used to solve for pressure buildup and diffuse leakage rates in a system with *multiple* injection and leaky wells as in *Cihan et al.* [2011].

2.1. Governing Equations for Anisotropic Multilayered Aquifer Systems

[9] For an anisotropic multilayered aquifer system, the governing equation for groundwater flow in each aquifer is given by

$$K_{xi}\frac{\partial^2 s_i}{\partial x^2}+K_{yi}\frac{\partial^2 s_i}{\partial y^2}=S_{si}\frac{\partial s_i}{\partial t}+w_i^-+w_i^+;\ i=1,\ldots,N \qquad (1)$$

where $s_i=s_i(x,y,t)$ is the hydraulic head buildup in aquifer i, and w_i^α denotes the rate of diffuse leakage (i.e., specific discharge) through the aquifer-aquitard interface from aquifer i into the overlying $(\alpha=+)$ or underlying $(\alpha=-)$ aquitard, and can be calculated using

$$w_i^\alpha=-\frac{K_i^\alpha}{B_i^\alpha B_i}\frac{\partial s_i^\alpha}{\partial z_{Di}^\alpha}\bigg|_{z_{Di}^\alpha=0} \qquad (2)$$

where $s_i^\alpha=s_i^\alpha(x,y,z_{Di}^\alpha,t)$ is the hydraulic head buildup in aquitard (i, α), z_{Di}^α $(=z_i^\alpha/B_i^\alpha, 0\le z_{Di}^\alpha\le 1$, where B_i^a is the thickness of the aquitard) is the dimensionless local vertical coordinate, and z_i^α is the local vertical coordinate, with $z_i^\alpha=0$ at the interface between aquifer i and aquitard (i, α) and $z_i^\alpha=B_i^\alpha$ at the interface between aquifer $i+\alpha$ and aquitard (i, α) [see *Cihan et al.*, 2011, Figure 1]. Note that $i+\alpha=i+1$ for $\alpha=+$, while $i+\alpha=i-1$ for $\alpha=-$. Aquifers are assumed to have an infinite extent, and

$$s_i(\mp\infty,y,t)=0,\ s_i(x,\mp\infty,t)=0 \qquad (3)$$

[10] The equation for one-dimensional vertical flow through aquitard $(i,\ \alpha)$ is written as

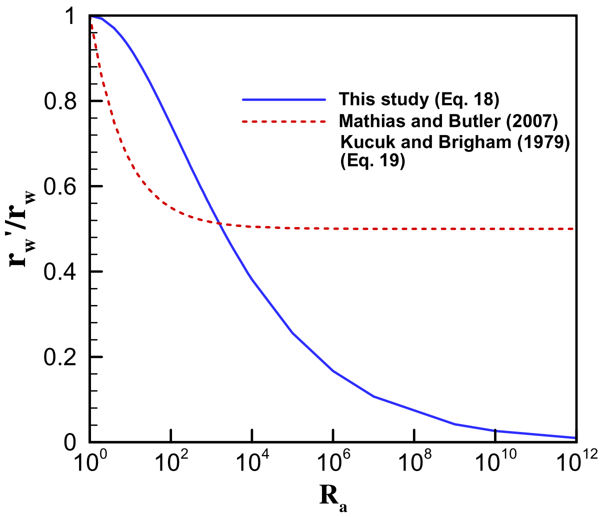

Figure 1. Dimensionless effective well radius as a function of anisotropy ratio based on two different approximate expressions.

$$\frac{\partial^2 s_i^\alpha}{\partial z_{Di}^{\alpha\,2}} = \frac{\left(B_i^\alpha\right)^2}{D_i^\alpha}\frac{\partial s_i^\alpha}{\partial t} \quad 0 \le z_{Di}^\alpha \le 1 \tag{4}$$

with the boundary conditions at aquifer-aquitard interfaces:

$$\begin{aligned} s_i^\alpha(x,y,0,t) &= s_i(x,y,t) \\ s_i^\alpha(x,y,1,t) &= s_{i+\alpha}(x,y,t) \end{aligned} \tag{5}$$

where D_i^α ($=K_i^\alpha/S_{s,i}^\alpha$, where $S_{s,i}^\alpha$ is the storativity of the aquitard) is the hydraulic diffusivity of aquitard (i, α), and there exists a relationship $s_i^+(x,y,z_{Di}^+,t)=s_{i+1}^-(x,y,z_{Di+1}^-,t)$ for $z_{Di+1}^-=1-z_{Di}^+$.

2.2. Boundary Condition at a Well

[11] The boundary condition at an active (injection/pumping) well and a passive (leaky) well is treated in a similar way. For an active well screened at well-aquifer segments the flow rate, $Q(t)$, is known, either constant or time-dependent, while the flow rate, $u(t)$, into or from a passive well, driven by potential head gradients through well-aquifer segments, is unknown and computed as part of the solution [*Cihan et al.*, 2011]. The boundary condition at a cylindrical well bore of an active or a passive well with a radius r_w is expressed as

$$-4B_i r_w \int_0^{\pi/2} K_{ri}\left.\frac{\partial s_i}{\partial r}\right|_{r_w} d\theta = \begin{cases} Q_i(t) & or \\ u_i(t) \end{cases} \quad \text{for } r_w^2 = x^2+y^2 \tag{6}$$

where θ is the angle between r direction and x axis, and K_{ri} is the conductivity in the direction of r in the x-y plane, expressed as [*Hantush*, 1966]

$$K_{ri} = \frac{K_{xi}\sec^2\theta}{1+R_{ai}\tan^2\theta} \tag{7}$$

where $R_{ai}=K_{xi}/K_{yi}$ is defined as the anisotropy ratio. *Cihan et al.* [2011, 2013] also showed modification of the boundary conditions for leaky wells that may be screened or cased at well-aquifer segments or plugged at well-aquitard segments.

2.3. Transformation to an Equivalent Isotropic System

[12] By assuming the principal directions and the anisotropy ratio are the same for all aquifers ($R_{ai}=R_a$ for $i=1$, . . ., N) and introducing the following variables

$$x_i' = x\,Ra^{-1/4},\; y_i' = y\,Ra^{1/4} \tag{8}$$

[13] Equation (1) for flow in the equivalent isotropic aquifers becomes

$$K_i\left(\frac{\partial^2 s_i}{\partial x_i'^2}+\frac{\partial^2 s_i}{\partial y_i'^2}\right) = S_{si}\frac{\partial s_i}{\partial t} + w_i^- + w_i^+ \tag{9}$$

where $K_i=(K_{xi}K_{yi})^{1/2}$ is the equivalent isotropic hydraulic conductivity of aquifer i. The boundary condition at the well bore is transformed by changing the variables (θ, r) in equation (6) to (θ', r'):

$$\begin{aligned} &\theta = \arctan\left(R_a^{-1/2}\tan\theta'\right) \\ &d\theta = \frac{R_a^{-1/2}\sec^2\theta'}{1+R_a^{-1}\tan^2\theta'}d\theta', \quad r' = r\frac{K_i^{1/2}}{K_{ri}^{1/2}} \end{aligned} \tag{10}$$

[14] Then, equation (6) becomes

$$-4B_i r_w K_i^{1/2}K_y^{1/2}\left(\int_0^{\pi/2}\frac{\partial s_i}{\partial r'}\frac{\sec\theta'}{\left(1+R_a^{-1}\tan^2\theta'\right)^{1/2}}d\theta'\right) = \begin{cases} Q_i(t) & \text{or} \\ u_i(t) \end{cases} \tag{11}$$

[15] For small well radii, the shape of the well bore becoming elliptical due to transformation can be approximated with an equivalent circular shape and the flow problem in the equivalent isotropic domain becomes symmetric around the r' axis. In other words, r' becomes independent of θ' and thus $\partial s_i/\partial r'$ can be taken outside of the integral in equation (11) and $\partial s_i/\partial\theta'=0$. Under these conditions, equation (9) can be expressed in radial coordinates by the following transformations

$$x'_i = r'\cos\theta',\; y'_i = r'\sin\theta',\; r'^2 = x'^2_i + y'^2_i,\; \theta' = \tan\left(y'_i/x'_i\right) \tag{12}$$

which leads to

$$\frac{K_i}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial s_i}{\partial r'}\right) = S_{si}\frac{\partial s_i}{\partial t} + w_i^- + w_i^+ \tag{13}$$

[16] The governing equation for flow in aquitards stays the same, and the radial boundary condition at infinity in aquifers is written as

$$s_i(r'\to\infty, t) = 0 \tag{14}$$

and the boundary conditions at aquifer-aquitard interfaces are

$$\begin{aligned} s_i^\alpha(r',0,t) &= s_i(r',t) \\ s_i^\alpha(r',1,t) &= s_{i+\alpha}(r',t) \end{aligned} \tag{15}$$

[17] The integral in equation (11) can be expressed in terms of an elliptic integral as

$$\int_0^{\pi/2} \frac{\sec\theta'}{\left(1+R_a^{-1}\tan^2\theta'\right)^{1/2}} d\theta' = R_a^{1/2}\mathrm{ELK}\,(1-R_a); R_a > 0 \quad (16)$$

where ELK is defined as the complete Elliptic Integral of First Kind [*Abramowitz and Stegun*, 1972]. Then, the boundary condition (equation (11)) at the well bore for the equivalent isotropic problem can be expressed as

$$-2\pi K_i B_i r_w' \frac{\partial s_i(r_w', t)}{\partial r'} = \begin{cases} Q_i(t) \;\; or \\ u_i(t) \end{cases} \quad (17)$$

where we define r_w' as the effective radius of the equivalent circular well bore at which inward or outward flow rate is equal to the flow rate at the elliptical well bore in the transformed domain and express as

$$r_w' = r_w \frac{2}{\pi} R_a^{1/4}\mathrm{ELK}\,(1-R_a) \quad (18)$$

[18] *Kucuk and Brigham* [1979] and *Mathias and Butler* [2007] proposed another effective well radius expression for using radial flow solutions in the transformed isotropic domain by a long time radial flow approximation (i.e., equating steady-state elliptic and radial flow solutions), given as

$$r_w' = r_w \frac{1}{2}\left[1+\left(\frac{K_y}{K_x}\right)^{1/2}\right] \quad (19)$$

[19] Figure 1 compares the changes of r_w'/r_w expressed by equations (18) and (19) in *Mathias and Butler* [2007] as a function of the anisotropy ratio. $R_a = 1$ corresponds to an isotropic system. The effective radius approximation of *Mathias and Butler* [2007] approaches 0.5 as the anisotropy ratio goes to infinity, while equation (18) predicts that the effective well radius asymptotically approaches zero with increasing anisotropy ratio. An infinitely large anisotropy ratio physically corresponds to a case where a single vertical fracture elongating in the x direction transmits entire flow in the domain. Mathematically, the 2D flow problem in the x-y plane turns into a 1D flow problem in the x direction ($K_y = 0$), and a cylindrical well can be defined as a point in 1D space. Thus, the prediction of equation (18) for the limiting case of infinite anisotropy ratio is reasonable. In the next section, we will compare the performance of each effective radius approximation.

[20] As stated before, the principal direction of the anisotropic horizontal conductivity and the anisotropy ratio are assumed to be the same in all the aquifers with alternating permeable aquitards. These assumptions guarantee that the radial groundwater flow equations (i.e., equation (13)) for transformed equivalent isotropic aquifers are expressed at the same frame of reference. Under these conditions, the governing equations and the boundary conditions, through equations (13) and (18), become identical to those presented in *Cihan et al*. [2011], and all the solutions in *Cihan et al*. [2011] can be applied by changing the coordinate variables (x, y), the well radii ($r_{wi,m}$ for active wells and $r_{Li,l}$ for passive wells), and the hydraulic conductivity, respectively,

$$x \to xR_a^{-1/4}, y \to yR_a^{1/4} \quad (20)$$

$$r_{wi,m} \to r_{wi,m}\frac{2}{\pi}R_a^{1/4}\mathrm{ELK}\,(1-R_a), \quad r_{Li,l} \to r_{Li,l}\frac{2}{\pi}R_a^{1/4}\mathrm{ELK}\,(1-R_a) \quad (21)$$

$$K_i \to \left(K_{xi}K_{yi}\right)^{1/2} \quad (22)$$

[21] In the presence of strong differences in the principal directions of the anisotropic horizontal conductivity of aquifers with high permeability aquitards, complex groundwater flow patterns (i.e., groundwater whirls) can occur although detection of such systems is challenging in the field [*Hemker et al*., 2004]. In such cases, the horizontal flow assumption in the aquifers may not be applicable.

2.4. Unequal Anisotropy Direction and Ratio in Aquifers Alternating With Aquicludes

[22] The transformation and the solution method described in section 2.3 can be applied to multilayered systems involving aquifers with unequal anisotropy direction and anisotropy ratio when the aquitards are impervious (or aquicludes). When the principal directions of anisotropic horizontal hydraulic conductivity and/or the anisotropy ratios are different at each aquifer, alignments of the coordinate axes in the directions of the principle axes, followed by the coordinate transformations (i.e., equation (8)) create different frame of references (fixed with respect to each other) in the aquifers. When the aquitards are impervious, the governing equations for groundwater flow in the aquifers are decoupled (i.e., without the diffuse leakage coupling terms). This means that before evaluating the specific leakage boundary conditions at the leaky wells, the general solution to the groundwater equation for the head buildup in each aquifer can be expressed at different frame of references, independently from the other aquifers. To demonstrate the solution procedure, we use a simple multilayered system with two aquifers and one aquiclude sandwiched between the aquifers as in *Cihan et al*. [2011]. A constant rate Q of fluid injection is considered into the bottom aquifer (aquifer 1). As a result of the coordinate transformation process, the groundwater flow equation for the head buildup or the drawdown can be expressed in the equivalent isotropic aquifer 1 as

$$\frac{K_1}{r_1}\frac{\partial}{\partial r_1}\left(r_1\frac{\partial s_1}{\partial r_1}\right) = S_{s1}\frac{\partial s_1}{\partial t}; K_1 = \left(K_{x1}K_{y1}\right)^{1/2} \quad (23)$$

and the groundwater flow equation in the equivalent isotropic aquifer 2 is expressed as

$$\frac{K_2}{r_2}\frac{\partial}{\partial r_2}\left(r_2\frac{\partial s_2}{\partial r_2}\right) = S_{s2}\frac{\partial s_2}{\partial t}; K_2 = \left(K_{x2}K_{y2}\right)^{1/2} \quad (24)$$

where r_1 and r_2 represent radial distances in the transformed domains of the aquifers represented by two different fixed reference frames, (x_1, y_1) in aquifer 1 and (x_2, y_2) in aquifer 2. The time changes are the same in the two aquifer domains. The general solutions in the Laplace domain, as given by *Cihan et al*. [2011], can be expressed by using the Theis solution [1935] and the superposition principle for multiple wells. For example, in the presence of an injection well and a leaky well, the head buildup distribution in aquifer 1 according to the (x_1, y_1) frame of reference is given by

$$\bar{s}_1^T(x_1,y_1,p)=\frac{Q/p}{2\pi K_1B_1}\mathrm{K}_0\left[r_1^I(p/D_1)^{1/2}\right]+\frac{\bar{u}_1}{2\pi K_1B_1}\mathrm{K}_0\left[r_1^L(p/D_1)^{1/2}\right];$$
$$r_1^I=\left[(x_1-x_{1,I})^2R_{a1}^{-1/2}+(y_1-y_{1,I})^2R_{a1}^{1/2}\right]^{1/2}$$
$$r_1^L=\left[(x_1-x_{1,L})^2R_{a1}^{-1/2}+(y_1-y_{1,L})^2R_{a1}^{1/2}\right]^{1/2} \quad (25)$$

where K_0 is the zeroth-order modified Bessel function of second kind. The head buildup in aquifer 2 according to the (x_2, y_2) frame of reference is

$$\bar{s}_2^T(x_2,y_2,p)=\frac{\bar{u}_2}{2\pi K_2B_2}\mathrm{K}_0\left[r_2^L(p/D_2)^{1/2}\right];$$
$$r_2^L=\left[(x_2-x_{2,L})^2R_{a2}^{-1/2}+(y_2-y_{2,L})^2R_{a2}^{1/2}\right]^{1/2} \quad (26)$$

where r_i^I is the distance between any observation point (x_i, y_i) and the injection well ($x_{i,I}$, $y_{i,I}$) in the aquifer i, and r_i^L is the distance between any observation point (x_i, y_i) and the leaky well ($x_{i,L}$, $y_{i,L}$). We assumed here for brevity the radii of both the injection and the leaky wells are negligibly small (i.e., $E_1^I=E_1^L=E_2^L=1$ in equations (27)–(29) of *Cihan et al*. [2011]). $\bar{u}_1$ and $\bar{u}_2$ in equations (25) and (26) are the unknown leakage rates at leaky well-aquifer intersections in Laplace domain as functions of time or Laplace parameter p. The solutions to the leakage rates in the Laplace domain are obtained from the equations below (equation (27)) stating that the flow rate through the leaky well is proportional to the head difference and inversely proportional to the resistance, Ω, along the well (note that storage at the well is neglected, and equation (27) is equivalent to Darcy's law or laminar free flow and defines the boundary conditions at leaky well-aquifer intersections.):

$$\bar{u}_1(p)=-\frac{1}{\Omega}\left(\bar{s}_1^T\big|_{r_{L1}}-\bar{s}_2^T\big|_{r_{L2}}\right);\ \bar{u}_2(p)=\frac{1}{\Omega}\left(\bar{s}_1^T\big|_{r_{L1}}-\bar{s}_2^T\big|_{r_{L2}}\right) \quad (27)$$

[23] Equation (27) involves two equations for two unknowns. The right-hand side of the equations includes the head buildup values evaluated, irrespective of their reference frames, at the leaky well bore—aquifer 1 and—aquifer 2 intersections. For very small well bore sizes, the effective radii, r_{L1} and r_{L2} can be calculated using equation (18) at each aquifer and can be substituted into $\bar{s}_1^T$ and $\bar{s}_2^T$ in equations (25–27). The solution for the leakage rate through the anisotropic aquifers in the Laplace domain can be expressed as

$$\bar{u}_1=-\bar{u}_2=\frac{-Q/(2\pi K_1B_1)\mathrm{K}_0\left\{\left[(x_{1,L}-x_{1,I})^2R_{a1}^{-1/2}+(y_{1,L}-y_{1,I})^2R_{a1}^{1/2}\right]^{1/2}(p/D_1)^{1/2}\right\}}{p\left\{\Omega+K_0\left[r_{L1}(p/D_1)^{1/2}\right]/(2\pi K_1B_1)+\mathrm{K}_0\left[r_{L2}(p/D_2)^{1/2}\right]/(2\pi K_2B_2)\right\}} \quad (28)$$

[24] After substitution of equation (28) into equations (25) and (26), the head buildup or the drawdown can be calculated at any observation point (after back transforming to the time domain) as a function of time for the two aquifers whose anisotropy directions and ratios are different. If desired, the solutions at each aquifer can be represented under the same frame of reference by a transformation (x_2, y_2) → (x_1, y_1). The solution technique presented above, as demonstrated in *Cihan et al*. [2011], can be easily applied to solution of problems with more than two aquifers with multiple injection and leaky wells.

3. Applicability of the Effective Well Radius Approximation

[25] In order to test the radial flow approximation with the effective well radius formulations for an elliptical flow in the vicinity of wells, we used the results from the single-layered exact solution of *Kucuk and Birgham* [1979] as well as numerical simulation results conducted using the COMSOL Multiphysics package.

[26] *Kucuk and Birgham* [1979] tabulated their exact solution results for dimensionless pressure buildup/drawdown at the well bore of a pumping/injection well operating at a constant rate in a confined anisotropic aquifer. For anisotropy ratios ≤25, the analytical solution results with the effective radius approximation (equation (19)) differ from the exact results given in Table 1 less than 5% on the average. However, at very early times, the difference can be as high as 50%. Equation (18) presented in this study results in slightly better approximations to the exact solution than equation (19). Both approximations appear to perform poorly at the well bore for higher anisotropy ratios. However, at as close as 2 m away from the injection/pumping well, the analytical solution with both the effective radius approximations results in very accurate results even for an extreme case of anisotropy ratio (R_a = 1000) with a slight discrepancy at very early times (Figure 2a), because the effect of finite well radius is important only in the vicinity of the wells. Based on the comparisons with the numerical simulations, we found that when $[x^2R_a{}^{-1/2}+y^2R_a{}^{1/2}]^{1/2}/r_w{}' > 10$, in general the error using the analytical solution with the effective radius approximation is ≤5%.

[27] We also tested the applicability of the analytical solution with equation (18) for the case of a leaky well in a horizontally anisotropic two-aquifer-one aquitard system. The upper aquifer is 30 m thick and the lower aquifer is 20 m thick, and the aquifers are separated by an impervious aquitard of 15 m thickness. The maximum hydraulic conductivity of each aquifer is K_x = 0.017 m/d, and the storativity values of the lower and the upper aquifers are, respectively, 2.5 × 10^{-5} 1/m, and 1.67 × 10^{-5} 1/m. An injection well located at x = 0 and y = 0 injects fluids into the lower aquifer at a unit rate (1 m^3/d). A leaky well with a hydraulic conductivity of 17 m/d located at x = 20 m and y = 0 provides a vertical leakage path between the two aquifers. Both of the wells have a radius of 0.15 m. Figure

Table 1. Dimensionless Head Buildup/Drawdown s_D Versus Dimensionless Time t_D at the Well Bore of an Injection Well for Different Anisotropy Ratios ($s_D = s2\pi KB/Q$ and $t_D = tS_s r_w^2 (R_a - 1)/K_x$)[a]

t_D	Exact [Kucuk and Birgham, 1979]	Equation (18)	Equation (19)
		$K_x/K_y = 2.25$	
0.1	0.28	0.29	0.34
1	0.74	0.76	0.86
10	1.55	1.58	1.73
100	2.60	2.64	2.81
1000	3.75	3.78	3.95
10000	4.90	4.93	5.10
		$K_x/K_y = 4$	
0.1	0.35	0.37	0.48
1	0.88	0.94	1.12
10	1.77	1.89	2.09
100	2.86	3.05	3.20
1000	4.00	4.26	4.35
10000	5.13	5.47	5.50
		$K_x/K_y = 25$	
0.1	0.45	0.67	0.89
1	1.09	1.44	1.78
10	2.08	2.49	2.86
100	3.20	3.62	4.00
1000	4.35	4.76	5.16
10000	5.50	5.92	6.31
		$K_x/K_y = 100$	
0.1	0.47	0.98	1.20
1	1.15	1.91	2.19
10	2.17	3.01	3.31
100	3.30	4.15	4.45
1000	4.45	5.30	5.60
10000	5.60	6.45	6.76

[a]The analytical solution results with the two different effective radius approximations are compared with the exact values tabulated in Table 1 of *Kucuk and Birgham* [1979].

2b presents leakage rate into the upper aquifer through the leaky well as a function of time for the anisotropy ratios $R_a = 25$, 100, and 1000. The approximate analytical solution compares well with the numerical solutions for all the anisotropy ratios except at very early times for $R_a = 1000$. Little discrepancies at early times for extreme anisotropy cases vanish as the distance between the injection and the leaky wells increase.

4. Summary and Conclusions

[28] By coordinate transformation, the existing solutions presented by *Cihan et al.* [2011] for isotropic multilayered aquifer systems with alternating aquitards and multiple injection/pumping wells and leaky wells were modified to account for horizontal anisotropy in aquifers. The modified solutions can be used for multilayered aquifers alternating with aquicludes, even when both the principal direction of the anisotropic horizontal conductivity and the anisotropy ratio are different in each aquifer. However, in the presence of permeable aquitards, the modified solutions can be used only when the anisotropy direction and ratio (K_x/K_y) are assumed to be identical for all aquifers. With coordinate transformation, a circular well with finite radius becomes an ellipse, and thus in the transformed domain the head contours in the vicinity of the well have elliptical shapes. However, as the distance increases from the well, head contours in the vicinity of the well in the transformed domain become concentric circles. Through a radial flow approximation around the wells, in the modified solutions of *Cihan et al.* [2011], we approximated the elliptical well boundaries in the transformed domain by a new effective well radius expression. The analytical solutions with the effective radius approximations were compared with the exact

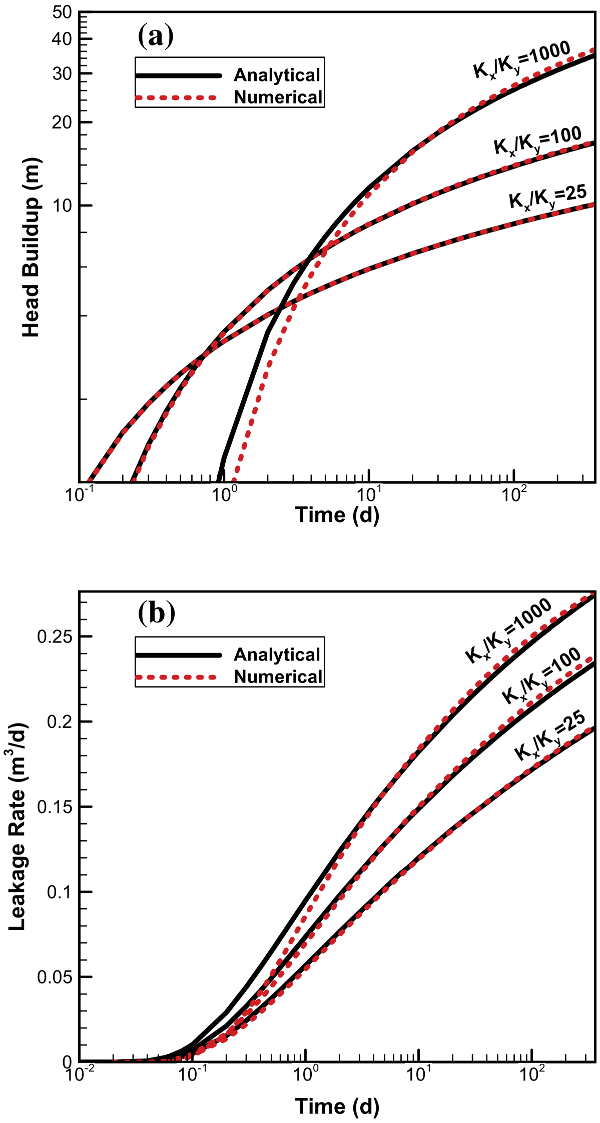

Figure 2. Comparison of head buildup and leakage rate values predicted by the analytical solution in this study with the effective radius approximation and the numerical solution. (a) Head buildup was computed at 2 m distance in the y direction ($x = 0$, $y = 2$ m) from the injection well ($x = 0$, $y = 0$), injecting fluid into a single-layered confined anisotropic aquifer with a unit volume rate (1 m^3/d). (b) Leakage rate through a leaky well (located at $x = 20$ m and $y = 0$) from an injection aquifer into an overlying aquifer as a function of time. An injection well (at $x = 0$ and $y = 0$) injects fluids into a confined aquifer, which is overlaid with an impermeable aquitard and a permeable aquifer, respectively.

solutions for elliptic flow and the numerical solutions. The effective well radius approximation can be used to predict the head buildup at the well bore of the injection wells for moderately anisotropic systems ($R_a < 25$). The effective radius approximation predicted satisfactorily head buildup and leakage through leaky wells away from the injection wells, i.e., when $[x^2 R_a^{-1/2} + y^2 R_a^{1/2}]^{1/2} / r'_w > 10$, even for highly anisotropic aquifer systems.

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