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ENERGY TRAPPING OF THICKNESS-SHEAR AND THICKNESS-TWIST MODES IN A PARTIALLY ELECTRODED AT-CUT QUARTZ RESONATOR**

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ABSTRACT The thickness-shear and thickness-twist vibrations of a finite and partially electroded AT-cut quartz resonator are investigated. The equations of anisotropic elasticity are used with the omission of the small elastic constant c_{56} . An analytical solution is obtained using Fourier series from which the free vibration resonant frequencies, mode shapes, and energy trapping are calculated and examined.

KEY WORDS resonator, thickness-shear vibration, thickness-twist vibration, quartz crystal

I. INTRODUCTION

Piezoelectric crystals are widely used to make resonators for time-keeping, frequency generation and operation, telecommunication and sensing. Quartz is the most widely used crystal for resonators. A large portion of quartz resonators operate with the so-called thickness-shear (TSh) modes of a plate^[1,2]. Theoretically, TSh modes can only exist in unbounded plates without edge effects. When a plate is vibrating in TSh modes, motions of the material particles of the plate are parallel to the plate surfaces, and particle velocities vary along the plate thickness only, without in-plane variations. TSh modes are the ideal operating modes of many acoustic wave devices.

In reality, however, due to the finite device sizes, finite crystal plates are used for which pure TSh modes cannot exist because of edge effects. Therefore, in real devices, the actual operating modes in fact have slow in-plane variations. These real modes in finite devices have been referred to as transversely varying TSh modes^[3]. Because of the complications associated with the material anisotropy of quartz, solutions for TSh modes varying in both of the in-plane directions in a finite quartz plate with free edges from the equations of anisotropic elasticity or piezoelectricity cannot be obtained even with various approximate two-dimensional plate equations^[4-6] which are much simpler.

There exist systematic theoretical results from the plate equations for TSh modes with in-plane variation in one direction only, i.e., in the direction parallel to the TSh particle velocity. In this case, the TSh vibration is coupled to flexure and face-shear (FS) modes. Free vibration modes and frequency spectra^[7-9] of finite plates with free edges, energy trapping related to vibration confinement in partially electroded resonators, Bechmann's number regarding electrode dimension and number of trapped modes^[10], monolithic filters with two pairs of electrodes^[11], monolithic arrays of multiple resonators^[12,13], and resonator capacitance from forced vibration analyses have been obtained^[14].

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In the case when the in-plane mode variation is in the direction perpendicular to the TSh particle velocity, the corresponding waves are called thickness-twist (TT) waves^[15,16]. For this case, reported studies are much fewer. Propagating waves in unbounded plates with or without electrodes were studied in Refs.[15–18], with an expression for the Bechmann's number given in Ref.[16]. Since resonators are made smaller and smaller for miniaturization, edge effects have become more pronounced and in-plane variations in both directions are equally important. It is our recent effort to provide new results of TT waves to reach the same level of understanding as that of the knowledge on the in-plane variation in the other direction.

Resonators are usually electroded in the central portion only^[10]. In these partially electroded resonators the electrode inertia is responsible for energy trapping through which the vibration is confined under the electrodes and decays rapidly outside them. Energy trapping is crucial to resonator mounting. In this paper, we study energy trapping of TT waves in a partially electroded quartz resonator. The equations of anisotropic elasticity are used instead of the approximate plate equations. We want to examine how the resonant frequencies, mode shapes, and energy trapping are affected by the mass and dimensions of the electrode. We will show that, when using the equations of anisotropic elasticity, Fourier series can effectively handle the piecewise boundary conditions associated with the partial electrodes and the unelectroded areas at the plate top and bottom surfaces, in a global manner. This is opposite to the situation when using the approximate plate equations where equations and solutions for the electroded and unelectroded regions have to be obtained separately and then joined by continuity conditions.

II. GOVERNING EQUATIONS

The equations of anisotropic crystals vary considerably according to crystal symmetry. A particular cut of a crystal plate refers to the orientation of the plate when it is taken out of a bulk crystal. As a consequence, crystal plates of different cuts exhibit different anisotropies in coordinates normal and parallel to the plate surfaces. The widely used AT-cut quartz plate is a special case of rotated Y-cut quartz plates which are effectively monoclinic

Fig. 1 A partially electroded plate of rotated Y-cut quartz.

in the plate coordinate system. Consider such a plate as shown in Fig.1. Our analysis differs from most of the results in the literature in that we are interested in the mode variation along x_3 instead of x_1 . In Fig.1, the plate is unbounded in the x_1 direction and does not vary along x_1 . Figure 1 shows a cross section. It carries symmetric electrodes in the central portions of the top and bottom surfaces. The electrodes are assumed to be very thin. Their inertia will be considered but their stiffness will be neglected. Quartz has very weak piezoelectric coupling. For free vibration frequency analysis the small piezoelectric coupling can usually be neglected and an elastic analysis is sufficient. For monoclinic crystals, shear-horizontal (SH) or antiplane motions with only one displacement component are allowed by the linear theory of anisotropic elasticity. The corresponding modes are TT modes in general and taking TSh and FS modes as special cases. SH motions in rotated Y-cut quartz are described by^[15]

$$u_1 = u_1(x_2, x_3, t), \quad u_2 = u_3 = 0 \quad (1)$$

where $\mathbf{u}$ is the displacement vector. The nonzero components of the strain tensor $\mathbf{S}$ and the stress tensors $\mathbf{T}$ are

$$S_5 = 2S_{31} = u_{1,3}, \quad S_6 = 2S_{21} = u_{1,2} \quad (2)$$

$$T_{31} = c_{55}u_{1,3} + c_{56}u_{1,2}, \quad T_{21} = c_{56}u_{1,3} + c_{66}u_{1,2} \quad (3)$$

where $\mathbf{c}$ is the elastic stiffness tensor. The relevant equation of motion is

$$T_{21,2} + T_{31,3} = \rho \ddot{u}_1 \quad (4)$$

The equation satisfied by u_1 is obtained by substituting Eq.(3) into Eq.(4):

$$c_{66}u_{1,22} + c_{55}u_{1,33} + 2c_{56}u_{1,23} = \rho \ddot{u}_1 \quad (5)$$

For the plate in Fig.1, the boundary conditions at the plate top and bottom are^[3]

$$T_{21} = \begin{cases} \mp 2\rho'h'\ddot{u}_1 & (x_2 = \pm h, |x_3| < a) \\ 0 & (x_2 = \pm h, a < |x_3| < b) \end{cases} \quad (6)$$

where ρ' and $2h'$ are the density and thickness of the electrodes. The boundary conditions at the left and right edges are

$$T_{31} = 0 \quad (x_3 = \pm b, |x_2| < h) \quad (7)$$

For AT-cut quartz plates, $c_{55} = 68.81 \times 10^9 \text{ N/m}^2$, $c_{56} = 2.53 \times 10^9 \text{ N/m}^2$, and $c_{66} = 29.01 \times 10^9 \text{ N/m}^2$ ^[11]. c_{56} is very small compared to c_{55} and c_{66} . Therefore, in the rest of this paper, we will adopt the common used approximation of neglecting small c_{56} ^[19].

III. FOURIER SERIES SOLUTION

Here we consider free vibrations, i.e.,

$$u_1(x_2, x_3, t) = u_1(x_2, x_3) \exp(i\omega t) \quad (8)$$

The plate in Fig.1 is symmetric in both x_2 and x_3 . For resonator applications, we are interested in modes that are antisymmetric about the x_3 axis and symmetric about the x_2 axis. Therefore we construct the following solution that is an odd function of x_2 and even of x_3 from the standard procedure of separation of variables based on Eqs.(5) and (7):

$$u_1 = B_0 \sin(\eta_0 x_2) + \sum_{m=1}^{\infty} B_m \sin(\eta_m x_2) \cos \frac{m\pi x_3}{b} \quad (9)$$

where B_0 and B_m are undetermined constants, and

$$\eta_m^2 = \frac{\rho \omega^2}{c_{66}} - \frac{c_{55}}{c_{66}} \left(\frac{m\pi}{b} \right)^2 = \frac{\pi^2}{4h^2} \left[\frac{\omega^2}{\omega_s^2} - \frac{c_{55}}{c_{66}} \left(m \frac{2h}{b} \right)^2 \right] \quad (m = 0, 1, 2, 3, \dots) \quad (10)$$

$$\omega_s = \frac{\pi}{2h} \sqrt{\frac{c_{66}}{\rho}} \quad (11)$$

Equation (9) satisfies Eq.(5) and (7). ω_s is the resonant frequency of the fundamental TSh mode in an unbounded plate. Quartz resonators are usually with large length/thickness ratios, i.e., $b \gg h$. In this case, η_m^2 is positive when m is not large. We are interested in the first few TSh and TT modes with no more than a couple of nodal points along the x_3 direction for which a large m is not needed. In the case when η_m^2 is nonpositive, the construction of the solution in Eq.(9) will be different. To apply the boundary conditions at the plate top and bottom, we need

$$T_{21} = c_{66} u_{1,2} = c_{66} B_0 \eta_0 \cos(\eta_0 x_2) + c_{66} \sum_{m=1}^{\infty} B_m \eta_m \cos(\eta_m x_2) \cos \frac{m\pi x_3}{b} \quad (12)$$

Substitution of Eqs.(9) and (12) into Eq.(6) gives

$$\begin{aligned} & c_{66} B_0 \eta_0 \cos(\eta_0 h) + c_{66} \sum_{m=1}^{\infty} B_m \eta_m \cos(\eta_m h) \cos \frac{m\pi x_3}{b} \\ &= \begin{cases} 2\rho'h'\omega^2 \left[B_0 \sin(\eta_0 h) + \sum_{m=1}^{\infty} B_m \sin(\eta_m h) \cos \frac{m\pi x_3}{b} \right] & (|x_3| < a) \\ 0 & (a < |x_3| < b) \end{cases} \end{aligned} \quad (13)$$

When symmetry and/or antisymmetry is considered, boundary conditions at $x_2 = \pm h$ lead to the same equation as Eq.(13). We multiply both sides of Eq.(13) by $\cos(n\pi x_3/b)$ ($n = 0, 1, 2, \dots$) and integrate the resulting expression over $(-b, b)$ to obtain

$$c_{66} B_0 \eta_0 \cos(\eta_0 h) 2b = 2\rho'h'\omega^2 \left[B_0 \sin(\eta_0 h) 2a + \sum_{m=1}^{\infty} B_m \sin(\eta_m h) \frac{2b}{m\pi} \sin \frac{m\pi a}{b} \right] \quad (n = 0) \quad (14)$$

$$c_{66}B_n\eta_n \cos(\eta_n h)b = 2\rho'h'\omega^2 \left[B_0 \sin(\eta_0 h) \frac{2b}{n\pi} \sin \frac{n\pi a}{b} + \sum_{m=1}^{\infty} B_m \sin(\eta_m h) C_{nm} \right] \quad (n = 1, 2, 3, \dots) \quad (15)$$

where

$$C_{nm} = \int_{-a}^a \cos \frac{n\pi x_3}{b} \cos \frac{m\pi x_3}{b} dx_3 = C_{mn} \quad (m, n = 1, 2, 3, \dots) \quad (16)$$

Equations (14) and (15) are linear homogeneous equations for B_0 and B_m . For nontrivial solutions the determinant of the coefficient matrix has to vanish, which determines the resonant frequencies. The nontrivial solutions of B_0 and B_m determine the corresponding modes. This is a complicated eigenvalue problem because the eigenvalue or the resonant frequency is present in each η_m .

IV. SPECIAL CASES

In the special case when the plate is unelectroded ($a = 0$), Eqs.(14) and (15) reduce to

$$\cos(\eta_m h) = 0, \quad \eta_m h = n\pi + \frac{\pi}{2} \quad (n = 0, 1, 2, \dots) \quad (17)$$

Equation (17) implies through Eq.(10)

$$\frac{\omega_{mn}^2}{\omega_s^2} = (2n+1)^2 + \frac{c_{55}}{c_{66}} \left(m \frac{2h}{b} \right)^2 \quad (18)$$

The first term on the right-hand side of Eq.(18) determines the frequencies of TT modes odd in x_2 of different orders which are modes of unbounded plates. The last term in Eq.(18) shows the effect of the finite size of the plate which raises the frequencies and disappears when $b \rightarrow \infty$.

Another special case is that the plate is fully electroded at $x_2 = \pm h$ ($a = b$). In this case $C_{mn} = b\delta_{mn}$ where δ_{mn} is the Kronecker delta. Equations (14) and (15) reduce to

$$\cot(\eta_m h) = \frac{\pi^2}{4} \frac{R}{\eta_m h} \frac{\omega^2}{\omega_s^2} \quad (m = 0, 1, 2, \dots) \quad (19)$$

where

$$R = \frac{2\rho'h'}{\rho h} \quad (20)$$

is the mass ratio between the electrodes and the crystal plate. We look for an approximate solution to Eq.(19) when the electrodes are thin and R is small. The lowest-order approximation when $R = 0$ is given by Eqs.(17) and (18). For the next order of approximation we write

$$\eta_m h = n\pi + \frac{\pi}{2} - \Delta_n \quad (n = 0, 1, 2, \dots) \quad (21)$$

where Δ_n is small. Substitution of Eq.(21) into Eq.(19), for small Δ_n and small R , we obtain

$$\Delta_n = \frac{\pi^2}{4} \frac{R}{n\pi + \pi/2} \left[(2n+1)^2 + \frac{c_{55}}{c_{66}} \left(m \frac{2h}{b} \right)^2 \right] \quad (22)$$

Substituting Eq.(22) into Eq.(21) and using Eq.(10), we have, approximately,

$$\frac{\omega_{mn}^2}{\omega_s^2} = (1 - 2R) \left[(2n+1)^2 + \frac{c_{55}}{c_{66}} \left(m \frac{2h}{b} \right)^2 \right] \quad (23)$$

Equation (23) shows that the electrode inertia lowers the frequencies. When $R = 0$ Eq.(22) reduces to Eq.(18).

Fig. 2. Frequency versus plate aspect ratio for a fully electroded plate: comparison between exact and approximate solutions ($n = 0$, $R = 0.1$).

Fig. 3. Frequency versus plate aspect ratio for a fully electroded plate: approximate solutions for different mass ratios ($n = 0$).

V. NUMERICAL RESULTS

For quartz $\rho = 2649 \text{ kg/m}^3$ ^[11]. First, for a fully electroded plate, we plot the exact frequency ω_{mn}/ω_s determined by Eq.(19) and its approximation in Eq.(23) versus the plate aspect ratio h/b in Fig.2 with n is fixed to 0. The TSh mode ($m = 0$) and the first two TT modes ($m = 1, 2$) are shown when $R = 0.1$, which is over estimated from the value in real applications. The exact and the approximate solutions are very close and the frequency axis has to be stretched considerably to see the difference.

In Fig.3 we plot ω_{mn}/ω_s versus h/b from the approximate solution (23) for different values of R . The frequencies increase with h/b because shorter plates have higher frequencies. In the special case of $m = 0$ the frequency is independent of h/b . A larger m means shorter waves and hence higher frequencies. R lowers the frequencies as expected.

Next we examine the effects of partial electrodes on energy trapping systematically. We introduce

$$\bar{\omega}_s = \omega_s(1 - R) \quad (24)$$

which is the resonant frequency of the fundamental thickness-shear mode in an unbounded quartz plate fully covered by electrodes. The resonant frequencies of the modes we are interested in are within $\bar{\omega}_s < \omega < \omega_s$. We fix the plate thickness $2h = 1 \text{ mm}$ and the plate length $2b = 40 \text{ mm}$.

We begin with the case when the film length $2a = 15 \text{ mm}$ and the mass ratio $R = 3\%$. In this case two resonant frequencies are found within $\bar{\omega}_s < \omega < \omega_s$. When using 9, 10 and 11 terms in the series, the two frequencies are always found to be $\omega_1/\omega_s = 0.9740$ and $\omega_2/\omega_s = 0.9961$. The corresponding modes also converge very well, without noticeable differences. Therefore all calculations below use 11 terms in the series. In this case η_m^2 is positive. More terms will make η_m^2 negative. The modes corresponding to the two frequencies are shown in Fig.4 which is not drawn to scale. The electroded area is represented by the shaded rectangular area in the figure. The first mode has no nodal points along the x_3 direction, which is the mode of interest and the most useful one in applications. It is a transversely varying TSh mode. The second mode has two nodal points along the x_3 direction and is a TT mode. For both of the modes in Fig.4, the vibration is large in the central region and small near the plate edges. In other words the vibration is mainly under the electrodes and decays outside them. This is the so-called energy trapping of TSh and TT modes.

Next we keep $R = 3\%$ and increase the electrode length to $2a = 23 \text{ mm}$. Then within $\bar{\omega}_s < \omega < \omega_s$ there still exist two frequencies $\omega_1/\omega_s = 0.9725$ and $\omega_2/\omega_s = 0.9845$, slightly lower than the previous case because of the longer electrodes and more inertia. The two modes are qualitatively similar to the two in Fig. 4 but have wider distributions along the x_3 direction because of the longer electrodes.

If we still keep $R = 3\%$ and increase the electrode length further to $2a = 26 \text{ mm}$, three resonant frequencies appear within $\bar{\omega}_s < \omega < \omega_s$: $\omega_1/\omega_s = 0.9722$, $\omega_2/\omega_s = 0.9820$ and $\omega_3/\omega_s = 0.9988$. The first two are slightly lower than the previous case as expected. The displacement at the plate top surface of three modes are shown in Fig.5. The first two modes are similar to those in Fig.4. The third mode is new and is a TT mode with four nodal points along the x_3 direction. We note that the second mode is not trapped as well as the first mode, and that the third mode is essentially no longer trapped.

On the other hand, if we keep $R = 3\%$ and reduce the electrode length to $2a = 12 \text{ mm}$, there exists one frequency only within $\bar{\omega}_s < \omega < \omega_s$ with $\omega_1/\omega_s = 0.9753$, slightly higher than the case of $R = 3\%$ and $2a = 15 \text{ mm}$. The corresponding mode has no nodal points along the x_3 direction.

Fig. 4. Two trapped modes when $2a = 15$ mm and $R = 3\%$.

In Fig.6 we summarize the effects of electrode length on the first mode when $R = 3\%$ is fixed and $2a = 15, 23$ and 26 mm, respectively. Clearly, the vibration distribution becomes wider when the electrodes become longer. In the case of $2a = 26$ mm, the length of the unelectroded portions is $b - a = 20 - 13 = 7$ mm, which is seven times of the plate thickness $2h = 1$ mm. In this case the mode still decays to zero at the plate edges and we still have good energy trapping.

Fig. 5. Three trapped modes when $2a = 26$ mm and $R = 3\%$.

Fig. 6. Effects of electrode size on energy trapping of the first mode.

In the following we examine the effect of the mass ratio R . Consider the case when $2a = 23$ mm. We choose different values of $R = 3\%$, 6% , and 10% and examine the corresponding modes. For these three values of R there exist 2, 3, and 3 trapped modes correspondingly. We focus on the first trapped mode for each value of R . The frequencies of these three first modes are $\omega_1/\omega_s = 0.9725$, 0.9452 and 0.9113 . They decrease with the increase of R as expected. The corresponding modes are shown in Fig.7. A larger R leads to better energy trapping, but the effect is not as strong as that in Fig.6 by reducing electrode length.

Fig. 7 Effects of electrode thickness on energy trapping of the first mode.

VI. CONCLUSION

Partial electrodes at the central portion of a plate can produce energy trapping of TSh and TT modes. The trapped modes are sensitive to the electrode inertia and dimension. Typically there exist a few trapped modes near the fundamental TSh frequency. Longer and thicker electrodes tend to trap more modes and lower resonant frequencies. Reducing the electrode length is more effective in improving energy trapping than increasing electrode thickness. Longer electrodes also widen the vibration distribution. Lower-order modes with fewer nodal points along the x_3 axis are trapped better.

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