4-2020

Modeling Predator-Prey Interaction in a Two Patch System

Marc Wade

University of Nebraska-Lincoln, mwade@huskers.unl.edu

Modeling Predator-Prey Interaction in a Two Patch System

Glenn Ledder, PhD, Amanda Laubmeier, PhD, Marc Wade, and Clay Cressler, PhD

Background

- What is meant by a two patch system is that prey live in a habitat that consists of type 1 patches with an abundance of food and type 2 patches with no food.
- We will be assuming that predators cannot enter the first type of patch.
- Our assumptions for the model creates a dynamic in which prey are safe from harm while in certain patches, but will eventually suffer from a lack of resources if they continue to stay indefinitely.
- As there is no food in type 2 patches, the amount of time spent in a type 2 patch by prey is solely determined by how long it takes to find a new type 1 patch.
- Our motivating question is: "Under what circumstances can a predator get enough food to survive when the only food available is migrating prey?
- We will be trying to answer this question by creating a model using differential equations and combining three well-established ecological theories: migration theory, optimal foraging theory, and the standard predator-prey Lotka-Volterra equations.
- Migration theory states that organisms will move towards areas that are more suited for survival. In our model, prey moves from exhausted patches towards new patches with unknown resource levels.
- Optimal foraging theory states that in order to survive, an animal will adopt a strategy that provides the most benefit at the least cost.
- In our model, a prey animal will weigh the benefits of increasing its food intake versus moving through areas inhabited by predators.
- Our model's starting point are the Lotka-Volterra equations, which are a set of differential equations that relate predator and prey grow relative to their population and a few other parameters.

Open Space Model

The model as presented here assumes that the number of patches is effectively unlimited. This means that there is no bound to the number of occupied patches, that the number of occupied patches does not affect the rate at which consumers locate new patches, and that patches always have a full vegetation level when initially occupied .

Consumers move from the open space to a patch with mean time t_T and from patches to open space with optimal time t_F . The emigration rate from patches is $\gamma > 1$ relative to the immigration rate to take into account births occurring in the patches. This two-way migration affects both the population in the open space (X) and the number of occupied patches (N), the assumption being that any one patch can only have one consumer. In addition to migration, consumers die of natural causes at relative rate D and are consumed by predators (Y) at a rate proportional to the interaction. The resulting model is

$$\frac{dN}{dT} = -\frac{1}{t_F}N + \frac{1}{t_T}X - DN,$$

$$\frac{dX}{dT} = \frac{\gamma}{t_F}N - \frac{1}{t_T}X - DX - QXY,$$

$$\frac{dY}{dT} = CQXY - mY,$$

where T is the dimensional time. For scaling, let

$$X = \frac{m}{CQ}x, \qquad N = \frac{m}{CQ}n, \qquad Y = \frac{\gamma}{Qt_F}y, \qquad T = t_T$$

and define parameters

$$\psi = \frac{t_T}{t_F}, \qquad \bar{\delta} = Dt_T, \qquad \mu = mt_T.$$

These definitions yield the dimensionless model

$$n' = -(\psi + \bar{\delta})n + x,$$

$$x' = \psi\gamma n - (1 + \bar{\delta})x - \psi\gamma xy,$$

$$y' = \mu y(x - 1).$$

For convenience, we define additional parameters

$$\nu = 1 + \bar{\delta}, \qquad \phi = \psi + \bar{\delta}, \qquad \bar{\gamma} = \psi\gamma,$$

recasting the system as

$$n' = -\phi n + x,$$

$$x' = \bar{\gamma}n - \nu x - \bar{\gamma}xy,$$

$$y' = \mu y(x - 1).$$

In terms of the original parameters, the viability condition is $\gamma > (1 + Dt_T)(1 + Dt_F)$ which has a simple biological interpretation: the population increase ratio in the vegetation patch must be larger than is required to compensate for the natural deaths that occur during one cycle of movement through an open patch and foraging in a vegetative patch. Two parameters in the open space model are linked to the dynamics in the occupied vegetation patches: the foraging time t_F and the population growth factor γ, which we can think of as a function of t_F . The optimal foraging problem is to choose t_F so as to maximize the fitness of the consumer (prey) species. A direct measure of fitness for the consumer species is not obvious; however, an indirect measure of fitness for the consumer is given by the size of predator population that it can support, which is the equilibrium value Y *. Since all parameters other than t_F and γ are presumed to be known, we write the equilibrium population as

$Y^* = F(t_F) * 1/t_T$, where $F(t_F) = \dfrac{\gamma - (1 + Dt_T)(1 + Dt_F)}{Q(1 + Dt_F)}$

Thus, the optimal foraging problem is to find t_F to maximize F, provided the maximum of F is positive. If F is never positive, then the parameters other than t_F and γ are in a region of the parameter space in which the consumer species is not viable.

Occupied Patch Model

The goal of the occupied patch model is to obtain $\gamma = 1 + B$, where B is the expected number of offspring produced by the consumer from resources collected in the patch. We simplify the notation by measuring food and energy in terms of consumer units. We assume an energy loss rate of at that applies both in the patch and the open space. With H as the net resource harvest and b as the number of offspring per unit of resource, we have

$\gamma = 1 - bat_T + bH(t)$

with the dynamics in the patch governed by the consumer resource model

$$\frac{dV}{dt} = rV\left(1 - \frac{V}{K}\right) - SV, \qquad V(0) = K,$$

$$\frac{dH}{dt} = SV - a, \qquad H(0) = 0,$$

where t is the dimensional time since occupation and S > r, else the patches are never depleted. Similarly a < SK, else there is never a positive resource gain. We scale this model with

$$V = Kv, \qquad H = Kh, \qquad \bar{t} = St$$

$$\bar{\alpha} = \frac{a}{SK} < 1, \qquad \rho = \frac{r}{S} < 1.$$

$$\frac{dv}{d\bar{t}} = v(\rho - \rho v - 1), \qquad v(0) = 1,$$

$$\frac{dh}{d\bar{t}} = v - \bar{\alpha}, \qquad h(0) = 0.$$

From here, standard integration techniques were used to find H(t), and therefore a closed form of γ and Y*. From our application of optimal foraging and other hypotheses Y* was found to be concave down. Thus by taking the derivative of Y* and finding its zero, we are able to draw conclusions for the stability of a population that depends on supposedly known parameters.

Conclusions

We had reached a point in our work in which the next step was to write a program in MATLAB to graph our solutions to our system and then draw conclusions from said graphs and potentially revise our model. Unfortunately due to COVID-19 we only managed to get the program written, and not yet reach meaningful conclusions from the model.