2006

Piezoelectromagnetic waves in a ceramic plate between two ceramic half-spaces

S. N. Jiang
Central South University

Q. Jiang
University of California, Riverside

X. F. Li
Central South University

S. H. Guo
Central South University

H. G. Zhou
University of Nebraska-Lincoln

See next page for additional authors

Authors
S. N. Jiang, Q. Jiang, X. F. Li, S. H. Guo, H. G. Zhou, and J. S. Chang

Available online at www.sciencedirect.com
SCIENCE DIRECT
International Journal of Solids and Structures 43 (2006) 5799–5810
INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES
www.elsevier.com/locate/ijsolstr

Piezoelectromagnetic waves in a ceramic plate between two ceramic half-spaces

S.N. Jiang [a], Q. Jiang [a,b], X.F. Li [a], S.H. Guo [a], H.G. Zhou [c], J.S. Yang [c,*]

[a] *Institute of Mechanics and Sensor Technology, School of Civil Engineering and Architecture, Central South University, Changsha, Hunan 410083, China*
[b] *Department of Mechanical Engineering, University of California, Riverside, CA 92521, USA*
[c] *Department of Engineering Mechanics, University of Nebraska, Lincoln, NE 68588-0526, USA*

Received 12 May 2005
Available online 20 December 2005

Abstract

We analyze the propagation of piezoelectromagnetic waves guided by a plate of polarized ceramics between two ceramic half-spaces. An exact dispersion relation is obtained, which reduces to a few known elastic, electromagnetic, and quasistatic piezoelectric wave solutions in the literature as special cases. Numerical solutions to the equation that determines the dispersion relation show the existence of guided waves. The results are useful for acoustic wave and microwave devices.

Keywords: Plate; Wave; Elastic; Electromagnetic

1. Introduction

The theory of linear piezoelectricity assumes quasistatic electromagnetic fields (Tiersten, 1969). In this theory, although the mechanical equations are dynamic, the electromagnetic equations are static and the electric field and the magnetic field are not dynamically coupled. Therefore it does not describe the wave behavior of electromagnetic fields. Electromagnetic waves generated by mechanical fields need to be studied in the calculation of radiated electromagnetic power from a vibrating piezoelectric device (Mindlin, 1972; Lee, 1989; Lee et al., 1990), and are also relevant in electromechanical devices in which acoustic waves produce electromagnetic waves or vise versa (Oliner, 1978; Scholl et al., 1998). When electromagnetic waves need to be considered, the complete set of Maxwell's equations should be used, coupled to the mechanical equations of motion. Such a fully dynamic theory has been called piezoelectromagnetism by some researchers. For theoretical aspects a variational formulation of piezoelectromagnetism was given by Lee (1991). A few piezoelectromagnetic solutions of plane waves, surface waves, waves in plates, and waves in a plate on a substrate were obtained (Kyame, 1949; Spaight and Koerber, 1971; Sedov and Schmerr, 1986; Schmerr and Sedov, 1986; Li, 1996;

* Corresponding author.
E-mail address: jyang@unlserve.unl.edu (J.S. Yang).

0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2005.10.015

To and Glaser, 2004; Yang, 2000, 2004a,b). In this paper, we analyze the propagation of piezoelectromagnetic waves in a ceramic plate between two ceramic half-spaces. The equations of linear piezoelectromagnetism are summarized in Section 2. Equations for anti-plane motions of polarized ceramics are presented in Section 3. An exact dispersion relation is obtained in Section 4, with discussions and reductions to various special cases in Section 5. A few numerical solutions to the equation that determines the dispersion relation are obtained in Section 6. Finally, some conclusions are drawn in Section 7.

2. Equations of piezoelectromagnetism

For a piezoelectric but non-magnetizable dielectric body the three-dimensional equations of linear piezoelectromagnetism (Mindlin, 1972; Lee, 1991) consist of the equations of motion and Maxwell's equations

$$\begin{aligned} &T_{ji,j} = \rho \ddot{u}_i, \\ &\nabla \times \mathbf{E} = -\dot{\mathbf{B}}, \quad \nabla \times \mathbf{H} = \dot{\mathbf{D}}, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \cdot \mathbf{D} = 0, \end{aligned} \tag{1}$$

as well as the following constitutive relations:

$$T_{ij} = c_{ijkl} S_{kl} - e_{kij} E_k, \quad D_i = e_{ijk} S_{jk} + \varepsilon_{ij} E_j, \quad B_i = \mu_0 H_i, \tag{2}$$

where

$$S_{ij} = (u_{i,j} + u_{j,i})/2. \tag{3}$$

In Eqs. (1)–(3), u_i is the mechanical displacement vector, T_{ij} is the stress tensor, S_{ij} is the strain tensor, E_i is the electric field, D_i is the electric displacement vector, B_i is the magnetic induction, and H_i is the magnetic field. The coefficients c_{ijkl}, e_{kij}, and ε_{ij} are the elastic, piezoelectric, and dielectric constants, μ_0 is the magnetic permeability of free space, and ρ is the mass density. The summation convention for repeated tensor indices and the convention that a comma followed by an index denotes partial differentiation with respect to the coordinate associated with the index are used. The indices i, j, k, l assume 1, 2, 3. A superimposed dot represents differentiation with respect to time t.

3. Anti-plane motions of polarized ceramics

For ceramics poled in the x_3 direction the material tensors in Eq. (2) can be represented by the following matrices under the compact matrix notation (Tiersten, 1969):

$$\begin{pmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{31} \\ 0 & 0 & e_{33} \\ 0 & e_{15} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \begin{pmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{pmatrix}, \tag{4}$$

where $c_{66} = (c_{11} - c_{12})/2$. For polarized ceramics, Yang (2000) showed that there exist the following anti-plane modes with only one displacement component, one magnetic field component, and two electric field components (which may be called transverse magnetic modes):

$$\begin{aligned} &u_1 = u_2 = 0, \quad u_3 = u_3(x_1, x_2, t), \\ &E_1 = E_1(x_1, x_2, t), \quad E_2 = E_2(x_1, x_2, t), \quad E_3 = 0, \\ &H_1 = H_2 = 0, \quad H_3 = H_3(x_1, x_2, t). \end{aligned} \tag{5}$$

From Eq. (3) the non-vanishing strain components are

$$S_4 = u_{3,2}, \quad S_5 = u_{3,1} \tag{6}$$

and then from Eq. (2) the non-vanishing components of T_{ij}, D_i, and B_i are

$$\begin{aligned}&T_4 = c_{44}u_{3,2} - e_{15}E_2, \quad T_5 = c_{44}u_{3,1} - e_{15}E_1,\\&D_1 = e_{15}u_{3,1} + \varepsilon_{11}E_1, \quad D_2 = e_{15}u_{3,2} + \varepsilon_{11}E_2,\\&B_3 = \mu_0 H_3.\end{aligned} \tag{7}$$

The non-trivial ones of the equations of motion and Maxwell's equations in Eq. (1) take the following form:

$$\begin{aligned}&c_{44}(u_{3,11} + u_{3,22}) - e_{15}(E_{1,1} + E_{2,2}) = \rho\ddot{u}_3,\\&e_{15}(u_{3,11} + u_{3,22}) + \varepsilon_{11}(E_{1,1} + E_{2,2}) = 0,\\&E_{2,1} - E_{1,2} = -\mu_0\dot{H}_3,\\&H_{3,2} = e_{15}\dot{u}_{3,1} + \varepsilon_{11}\dot{E}_1, \quad -H_{3,1} = e_{15}\dot{u}_{3,2} + \varepsilon_{11}\dot{E}_2.\end{aligned} \tag{8}$$

Eliminating the electric field components from Eq. $(8)_{1,2}$ we obtain

$$\bar{c}_{44}(u_{3,11} + u_{3,22}) = \rho\ddot{u}_3, \tag{9}$$

where $\bar{c}_{44} = c_{44} + e_{15}^2/\varepsilon_{11}$ is a piezoelectrically stiffened elastic constant. Differentiating Eq. $(8)_3$ with respect to time once and substituting from Eq. $(8)_{4,5}$, we have

$$H_{3,11} + H_{3,22} = \varepsilon_{11}\mu_0\ddot{H}_3. \tag{10}$$

Eqs. (9) and (10) govern the mechanical and magnetic fields. Once u_3 and H_3 are determined, E_1 and E_2 can be obtained from Eq. $(8)_{4,5}$.

4. Waves in a plate between two half-spaces

Consider a ceramic plate between two ceramic half-spaces (Fig. 1). This type of structures or some of its special cases are used frequently in acoustic and electromagnetic (microwave) wave devices. Waves propagating in the x_1 direction in such a structure can be classified as guided or radiating modes depending on their behavior in the x_2 direction. Guided modes have exponentially decaying behavior when $|x_2|$ is large.

4.1. Fields in the upper half-space

Consider the following waves propagating in the x_1 direction with:

$$\begin{aligned}&u_3 = U_A \exp[-\eta_A(x_2 - h)]\cos(\xi x_1 - \omega t),\\&H_3 = H_A \exp[-\varsigma_A(x_2 - h)]\cos(\xi x_1 - \omega t),\end{aligned} \tag{11}$$

where U_A, H_A, ξ, η_A, ζ_A, and ω are undetermined constants. The subscript in these constants indicates that they are for ceramic A. Substitution of Eq. (11) into Eqs. (9) and (10) results in

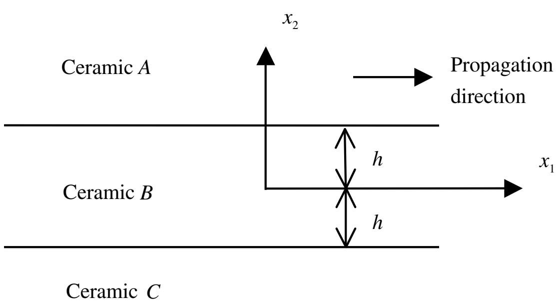

Fig. 1. A ceramic plate between two ceramic half-spaces.

$$\eta_A^2 = \xi^2 - \rho_A\omega^2/\bar{c}_A = \xi^2\left(1 - \frac{v^2}{v_A^2}\right) > 0,$$
$$\varsigma_A^2 = \xi^2 - \varepsilon_A\mu_0\omega^2 = \xi^2\left(1 - \frac{v^2}{c_A^2}\right) > 0, \tag{12}$$

where

$$v^2 = \frac{\omega^2}{\xi^2}, \quad v_A^2 = \frac{\bar{c}_A}{\rho_A}, \quad c_A^2 = \frac{1}{\varepsilon_A\mu_0}. \tag{13}$$

In Eq. (13), v is the guided plate wave speed that is to be determined, v_A is the speed of plane shear waves in the x_1 direction, and c_A is the speed of light in the x_1 direction. The inequalities are for guided waves with decaying behavior from $x_2 = h$. If one or both of the inequalities are violated, the modes become radiating. Clearly, from (12), acoustic radiation occurs before electromagnetic radiation. This observation has important consequences in microwave devices. Some of the materials for microwave devices have electromechanical coupling. A guided electromagnetic wave for a microwave device from a pure electromagnetic analysis may in fact be radiating or leaking energy acoustically due to electromechanical coupling when analyzed from the fully coupled theory of piezoelectromagnetism. From Eqs. $(7)_1$, $(8)_4$, and (11) we obtain

$$T_4 = -\frac{1}{\varepsilon_A\omega}\{\varepsilon_A\bar{c}_A\omega\eta_A U_A \exp[-\eta_A(x_2 - h)] + e_A\xi H_A \exp[-\varsigma_A(x_2 - h)]\}\cos(\xi x_1 - \omega t),$$
$$E_1 = \frac{1}{\varepsilon_A\omega}\{e_A\omega\xi U_A \exp[-\eta_A(x_2 - h)] + \varsigma_A H_A \exp[-\varsigma_A(x_2 - h)]\}\sin(\xi x_1 - \omega t), \tag{14}$$

which will be needed for interface continuity conditions later.

4.2. Fields in the lower half-space

For the lower half-space (C) occupying $x_2 < -h$, similar to Eqs. (11)–(14), we have

$$u_3 = U_C \exp[\eta_C(x_2 + h)]\cos(\xi x_1 - \omega t),$$
$$H_3 = H_C \exp[\varsigma_C(x_2 + h)]\cos(\xi x_1 - \omega t), \tag{15}$$

where

$$\eta_C^2 = \xi^2 - \rho_C\omega^2/\bar{c}_C = \xi^2\left(1 - \frac{v^2}{v_C^2}\right) > 0,$$
$$\varsigma_C^2 = \xi^2 - \varepsilon_C\mu_0\omega^2 = \xi^2\left(1 - \frac{v^2}{c_C^2}\right) > 0, \tag{16}$$

$$v_C^2 = \frac{\bar{c}_C}{\rho_C}, \quad c_C^2 = \frac{1}{\varepsilon_C\mu_0}. \tag{17}$$

We can also obtain

$$T_4 = -\frac{1}{\varepsilon_C\omega}\{-\varepsilon_C\bar{c}_C\omega\eta_C U_C \exp[\eta_C(x_2 + h)] + e_C\xi H_C \exp[\varsigma_C(x_2 + h)]\}\cos(\xi x_1 - \omega t),$$
$$E_1 = \frac{1}{\varepsilon_C\omega}\{e_C\omega\xi U_C \exp[\eta_C(x_2 + h)] - \varsigma_C H_C \exp[\varsigma_C(x_2 + h)]\}\sin(\xi x_1 - \omega t). \tag{18}$$

4.3. Fields in the plate

For the ceramic plate (B) the fields can be represented by

$$u_3 = (U_B \cos\eta_B x_2 + V_B \sin\eta_B x_2)\cos(\xi x_1 - \omega t),$$
$$H_3 = (G_B \cosh\varsigma_B x_2 + H_B \sinh\varsigma_B x_2)\cos(\xi x_1 - \omega t), \tag{19}$$

where

$$\eta_B^2 = \frac{\rho_B \omega^2}{\bar{c}_B} - \xi^2 = \xi^2\left(\frac{v^2}{v_B^2} - 1\right),$$
$$\varsigma_B^2 = \xi^2 - \varepsilon_B \mu_0 \omega^2 = \xi^2\left(1 - \frac{v^2}{c_B^2}\right), \tag{20}$$

$$v_B^2 = \frac{\bar{c}_B}{\rho_B}, \quad c_B^2 = \frac{1}{\varepsilon_B \mu_0}. \tag{21}$$

If the η_B^2 and/or ς_B^2 in (20) become negative, the fields in the plate change from sinusoidal to exponential or vise versa but the waves can still remain guided. The field components relevant to boundary conditions are found to be

$$T_4 = \left(-\bar{c}_B \eta_B U_B \sin \eta_B x_2 + \bar{c}_B \eta_B V_B \cos \eta_B x_2 - \frac{e_B}{\varepsilon_B \omega} G_B \xi \cosh \varsigma_B x_2 - \frac{e_B}{\varepsilon_B \omega} H_B \xi \sinh \varsigma_B x_2\right) \cos(\xi x_1 - \omega t),$$
$$E_1 = \frac{1}{\varepsilon_B \omega}(e_B \omega \xi U_B \cos \eta_B x_2 + e_B \omega \xi V_B \sin \eta_B x_2 - G_B \varsigma_B \sinh \varsigma_B x_2 - H_B \varsigma_B \cosh \varsigma_B x_2) \sin(\xi x_1 - \omega t). \tag{22}$$

4.4. Continuity conditions and dispersion relation

At the interfaces $x_2 = \pm h$, according to Jackson (1990) the continuity of u_3, T_4, H_3, and E_1 need to be imposed

$$u_3(h^+) = u_3(h^-) : U_A = U_B \cos \eta_B h + V_B \sin \eta_B h,$$
$$u_3(-h^-) = u_3(-h^+) : U_C = U_B \cos \eta_B h - V_B \sin \eta_B h,$$
$$H_3(h^+) = H_3(h^-) : H_A = G_B \cosh \varsigma_B h + H_B \sinh \varsigma_B h,$$
$$H_3(-h^-) = H_3(-h^+) : H_C = G_B \cosh \varsigma_B h - H_B \sinh \varsigma_B h,$$
$$T_4(h^+) = T_4(h^-) : -\frac{1}{\varepsilon_A \omega}\{\varepsilon_A \bar{c}_A \omega \eta_A U_A + e_A \xi H_A\}$$
$$= \left(-\bar{c}_B \eta_B U_B \sin \eta_B h + \bar{c}_B \eta_B V_B \cos \eta_B h - \frac{e_B}{\varepsilon_B \omega} G_B \xi \cosh \varsigma_B h - \frac{e_B}{\varepsilon_B \omega} H_B \xi \sinh \varsigma_B h\right),$$
$$T_4(-h^-) = T_4(-h^+) : -\frac{1}{\varepsilon_C \omega}\{-\varepsilon_C \bar{c}_C \omega \eta_C U_C + e_C \xi H_C\} \tag{23}$$
$$= \left(\bar{c}_B \eta_B U_B \sin \eta_B h + \bar{c}_B \eta_B V_B \cos \eta_B h - \frac{e_B}{\varepsilon_B \omega} G_B \xi \cosh \varsigma_B h + \frac{e_B}{\varepsilon_B \omega} H_B \xi \sinh \varsigma_B h\right),$$
$$E_1(h^+) = E_1(h^-) : \frac{1}{\varepsilon_A \omega}\{e_A \omega \xi U_A + \varsigma_A H_A\}$$
$$= \frac{1}{\varepsilon_B \omega}(e_B \omega \xi U_B \cos \eta_B h + e_B \omega \xi V_B \sin \eta_B h - G_B \varsigma_B \sinh \varsigma_B h - H_B \varsigma_B \cosh \varsigma_B h),$$
$$E_1(-h^-) = E_1(-h^+) : \frac{1}{\varepsilon_C \omega}\{e_C \omega \xi U_C - \varsigma_C H_C\}$$
$$= \frac{1}{\varepsilon_B \omega}(e_B \omega \xi U_B \cos \eta_B h - e_B \omega \xi V_B \sin \eta_B h + G_B \varsigma_B \sinh \varsigma_B h - H_B \varsigma_B \cosh \varsigma_B h),$$

which represent eight equations for U_A, U_B, U_C, H_A, H_B, H_C, V_B, and G_B. For non-trivial solutions of these constants to exist, the determinant of the coefficient matrix has to vanish. This yields the following equation:

$$\begin{vmatrix} 1 & -\cos\eta_B h & 0 & 0 & 0 & 0 & -\sin\eta_B h & 0 \\ 0 & \cos\eta_B h & -1 & 0 & 0 & 0 & -\sin\eta_B h & 0 \\ 0 & 0 & 0 & -1 & \sinh\varsigma_B h & 0 & 0 & \cosh\varsigma_B h \\ 0 & 0 & 0 & 0 & -\sinh\varsigma_B h & -1 & 0 & \cosh\varsigma_B h \\ \bar{c}_A\eta_A & -\bar{c}_B\eta_B\sin\eta_B h & 0 & \frac{e_A\xi}{\varepsilon_A\omega} & \frac{-e_B\xi\sinh\varsigma_B h}{\varepsilon_B\omega} & 0 & \bar{c}_B\eta_B\cos\eta_B h & \frac{-e_B\xi\cosh\varsigma_B h}{\varepsilon_B\omega} \\ 0 & -\bar{c}_B\eta_B\sin\eta_B h & \bar{c}_C\eta_C & 0 & \frac{-e_B\xi\sinh\varsigma_B h}{\varepsilon_B\omega} & \frac{-e_C\xi}{\varepsilon_C\omega} & -\bar{c}_B\eta_B\cos\eta_B h & \frac{e_B\xi\cosh\varsigma_B h}{\varepsilon_B\omega} \\ \frac{-e_A\xi}{\varepsilon_A} & \frac{e_B\xi\cos\eta_B h}{\varepsilon_B} & 0 & \frac{-\varsigma_A}{\varepsilon_A\omega} & \frac{-\varsigma_B\cosh\varsigma_B h}{\varepsilon_B\omega} & 0 & \frac{e_B\xi\sin\eta_B h}{\varepsilon_B} & \frac{-\varsigma_B\sinh\varsigma_B h}{\varepsilon_B\omega} \\ 0 & \frac{e_B\xi\cos\eta_B h}{\varepsilon_B} & \frac{-e_C\xi}{\varepsilon_C} & 0 & \frac{-\varsigma_B\cosh\varsigma_B h}{\varepsilon_B\omega} & \frac{\varsigma_C}{\varepsilon_C\omega} & \frac{-e_B\xi\sin\eta_B h}{\varepsilon_B} & \frac{\varsigma_B\sinh\varsigma_B h}{\varepsilon_B\omega} \end{vmatrix} = 0. \quad (24)$$

The expansion of Eq. (24) yields a rather long expression and is not provided here. When viewed as an equation for ω, a root of Eq. (24) determines a relation between ω and ξ (the dispersion relation). When the relation is linear, the corresponding wave is said to be non-dispersive. Otherwise it is a dispersive wave. Clearly, waves with their dispersion relations determined from Eq. (24) are dispersive in general. The presence of trigonometric functions in Eq. (24) suggests that the dispersion relation may be multi-valued. The existence of roots to Eq. (24) can be seen in various special cases and numerical solutions to be discussed below.

5. Discussions

In this section we examine a few special cases of Eq. (24).

5.1. Symmetric waves

When the two half-spaces are of the same material, i.e.,

$$\rho_A = \rho_C, \quad \bar{c}_A = \bar{c}_C, \quad e_A = e_C, \quad \varepsilon_A = \varepsilon_C, \tag{25}$$

waves determined by Eq. (24) can be separated into symmetric and anti-symmetric ones. For symmetric waves, consider

$$U_A = U_C, \quad H_A = -H_C, \quad V_B = 0, \quad G_B = 0. \tag{26}$$

In this case Eqs. $(23)_{2,4,6,8}$ become the same as Eqs. $(23)_{1,3,5,7}$, and Eq. (24) reduces to

$$\left(\frac{\varsigma_A}{\varepsilon_A}\tanh\varsigma_B h + \frac{\varsigma_B}{\varepsilon_B}\right)(\bar{c}_A\eta_A - \bar{c}_B\eta_B\tan\eta_B h) = \left(\frac{e_A}{\varepsilon_A} - \frac{e_B}{\varepsilon_B}\right)^2 \xi^2 \tanh\varsigma_B h. \tag{27}$$

The following observations can be made from Eq. (27):

(i) The waves are dispersive in general.
(ii) The right-hand-side of Eq. (27), which is responsible for the coupling between acoustic and electromagnetic waves, depends on the difference of the ratio of the piezoelectric and dielectric constants. If the plate and the half-spaces are ceramics poled in the same direction, the coupling is not as strong as when they are poled in opposite directions when their piezoelectric constants have opposite signs.
(iii) When the two half-spaces are free space, i.e., $c_A = 0$, $e_A = 0$, and $\varepsilon_A = \varepsilon_0$, Eq. (27) becomes

$$\left(\frac{\varepsilon_B}{\varepsilon_0}\varsigma_A\tanh\varsigma_B h + \varsigma_B\right)\eta_B\tan\eta_B h = -\frac{e_B^2}{\varepsilon_B\bar{c}_B}\xi^2\tanh\varsigma_B h, \tag{28}$$

which is the same as Eq. (27) (Yang, 2004b) for piezoelectromagnetic waves in an unelectroded ceramic plate.

(iv) When the materials are non-piezoelectric, i.e., $e_A = e_B = 0$, Eq. (27) factors into

$$\tan \eta_B h = \frac{\bar{c}_A \eta_A}{\bar{c}_B \eta_B}, \quad \text{or} \quad \tanh \varsigma_B h = -\frac{\varepsilon_A \varsigma_B}{\varepsilon_B \varsigma_A}. \tag{29}$$

We recognize Eq. $(29)_2$ as the frequency equation that determines the dispersion relations for guided electromagnetic waves in an unelectroded dielectric plate (Marcuse, 1982). In terms of the wave speed v, Eq. $(29)_2$ can be written as

$$\tanh \xi h \sqrt{1 - \frac{v^2}{c_B^2}} = -\frac{\varepsilon_A}{\varepsilon_B} \frac{\sqrt{1 - \frac{v^2}{c_B^2}}}{\sqrt{1 - \frac{v^2}{c_A^2}}}, \tag{30}$$

which shows the dependence of the wave speed v on the wave number ξ (dispersion). The dispersion relations determined by Eq. (30) are with an infinite number of branches. It can be expected that the behavior of Eq. (27) will be more complicated.

In terms of the velocity v, Eq. $(29)_1$ can be written as

$$\tan \xi h \sqrt{\frac{v^2}{v_B^2} - 1} = \frac{\bar{c}_A}{\bar{c}_B} \frac{\sqrt{\frac{v^2}{v_A^2} - 1}}{\sqrt{\frac{v^2}{v_B^2} - 1}}, \tag{31}$$

which is the frequency equations that determines the dispersion relations for SH waves in an elastic plate between two elastic half-spaces.

For the special case of an elastic plate alone without the half-spaces ($\bar{c}_A = 0$), Eq. (31) further reduces to

$$\tan \xi h \sqrt{\frac{v^2}{v_B^2} - 1} = 0, \tag{32}$$

which agrees with Graff (1991).

(v) When the speed of light goes to infinity, i.e., $c_A \to \infty$ and $c_B \to \infty$, from Eqs. (12) and (20) we have $\varsigma_A \to \xi$ and $\varsigma_B \to \xi$. Then Eq. (27) reduces to

$$\left(\frac{1}{\varepsilon_A} \tanh \xi h + \frac{1}{\varepsilon_B}\right)\left(\bar{c}_A \frac{\eta_A}{\xi} - \bar{c}_B \frac{\eta_B}{\xi} \tan \eta_B h\right) = \left(\frac{e_A}{\varepsilon_A} - \frac{e_B}{\varepsilon_B}\right)^2 \tanh \xi h, \tag{33}$$

which determines the dispersion relations for symmetric, guided quasistatic piezoelectric waves in a ceramic plate between two half-spaces. This special result appears to be new. Eq. (33) determines acoustic waves only and does not have the electromagnetic branches of the dispersion relation.

5.2. Anti-symmetric waves

For anti-symmetric waves consider

$$U_A = -U_C, \quad H_A = H_C, \quad U_B = 0, \quad H_B = 0. \tag{34}$$

In this case Eq. (24) reduces to

$$\left(\frac{\varsigma_A}{\varepsilon_A} \coth \varsigma_B h + \frac{\varsigma_B}{\varepsilon_B}\right)(\bar{c}_A \eta_A + \bar{c}_B \eta_B \cot \eta_B h) = \left(\frac{e_A}{\varepsilon_A} - \frac{e_B}{\varepsilon_B}\right)^2 \xi^2 \coth \varsigma_B h. \tag{35}$$

Observations similar to Eqs. (28)–(33) can also be made from Eq. (35).

5.3. Love waves

When the upper half-space is a vacuum, Eq. $(23)_1$ should be dropped and Eq. $(23)_5$ should be replaced by $0 = T_4(h^-)$. We also have

$$\bar{c}_A = 0, \quad e_A = 0, \quad \varepsilon_A = \varepsilon_0. \tag{36}$$

The dispersion relation is given by the determinant of the 7×7 elements in the lower-right corner of Eq. (24) which, when expanded, has the following form:

$$\begin{aligned}&\frac{\varsigma_B\cosh 2\varsigma_B h}{\varepsilon_B}\left\{-\bar{c}_B\eta_B\cos 2\eta_B h\left[\bar{c}_C\eta_C\left(\frac{\varsigma_A}{\varepsilon_0}+\frac{\varsigma_C}{\varepsilon_C}\right)-\xi^2\left(\frac{2e_B^2}{\varepsilon_B^2}-\frac{2e_Be_C}{\varepsilon_B\varepsilon_C}+\frac{e_C^2}{\varepsilon_C^2}\right)\right]\right.\\&\quad\left.+\sin 2\eta_B h\left[\bar{c}_B^2\eta_B^2\left(\frac{\varsigma_A}{\varepsilon_0}+\frac{\varsigma_C}{\varepsilon_C}\right)+\frac{\bar{c}_C\eta_C e_B^2\xi^2}{\varepsilon_B^2}\right]\right\}+\sinh 2\varsigma_B h\left\langle-\bar{c}_B\eta_B\cos 2\eta_B h\left\{\bar{c}_C\eta_C\left(\frac{\varsigma_B^2}{\varepsilon_B^2}+\frac{\varsigma_A\varsigma_C}{\varepsilon_0\varepsilon_C}\right)\right.\right.\\&\quad\left.-\xi^2\left[-\frac{2e_Be_C\varsigma_A}{\varepsilon_0\varepsilon_B\varepsilon_C}+\frac{e_C^2\varsigma_A}{\varepsilon_0\varepsilon_C^2}+\frac{e_B^2}{\varepsilon_B^2}\left(\frac{\varsigma_A}{\varepsilon_0}+\frac{\varsigma_C}{\varepsilon_C}\right)\right]\right\}+\sin 2\eta_B h\left\{\bar{c}_B^2\eta_B^2\left(\frac{\varsigma_B^2}{\varepsilon_B^2}+\frac{\varsigma_A\varsigma_C}{\varepsilon_0\varepsilon_C}\right)\right.\\&\quad\left.\left.+\frac{e_B^2\xi^2}{\varepsilon_B^2}\left[\frac{\bar{c}_C\eta_C\varsigma_C}{\varepsilon_C}-\xi^2\left(\frac{e_B}{\varepsilon_B}-\frac{e_C}{\varepsilon_C}\right)^2\right]\right\}\right\rangle\\&\quad=\frac{2\bar{c}_Be_B\varsigma_B\eta_B\xi^2}{\varepsilon_B^2}\left(\frac{e_B}{\varepsilon_B}-\frac{e_C}{\varepsilon_C}\right).\end{aligned} \tag{37}$$

Eq. (37) also appears to be new. It is not comparable but is an addition to the results for the Love waves analyzed in Yang (2004a), where the plate is either a perfect conductor itself or it carries a perfect conductor electrode at $x_2 = h$.

In the special case when all materials are non-piezoelectric, Eq. (37) can be factored into

$$\left[\frac{\varsigma_B}{\varepsilon_B}\left(\frac{\varsigma_A}{\varepsilon_0}+\frac{\varsigma_C}{\varepsilon_C}\right)+\left(\frac{\varsigma_B^2}{\varepsilon_B^2}+\frac{\varsigma_A\varsigma_C}{\varepsilon_0\varepsilon_C}\right)\tanh 2\varsigma_B h\right](\bar{c}_B\eta_B\tan 2\eta_B h-\bar{c}_C\eta_C)=0. \tag{38}$$

The two factors are for uncoupled electromagnetic and elastic waves, respectively. The second factor is the well-known frequency equation that determines the speed of Love waves in an elastic layer over an elastic half-space (Graff, 1991).

5.4. Surface waves

When the ceramic half-space (A) and the layer (B) do not exist, we have surface waves. Setting the relevant material constants of A and B to zero and letting $h = 0$ in Eq. (37), we obtain

$$\frac{\eta_C}{\xi}\left(\frac{\varsigma_C}{\xi}+\frac{\varepsilon_C}{\varepsilon_0}\frac{\varsigma_A}{\xi}\right)=\frac{e_C^2}{\varepsilon_C\bar{c}_C}, \tag{39}$$

which is the result of Yang (2000) for piezoelectromagnetic surface waves.

If the speed of light approaches infinity, Eq. (39) becomes the well-known quasistatic piezoelectric surface wave named after Bleustein (1968) and Gulyaev (1969) with

$$\left(1+\frac{\varepsilon_C}{\varepsilon_0}\right)\frac{\eta_C}{\xi}=\frac{e_C^2}{\varepsilon_C\bar{c}_C}. \tag{40}$$

6. Numerical results

A few branches of the dispersion relation for the symmetric waves determined by Eq. (27) are plotted in Fig. 2 when all ceramics are poled in the same direction. The waves are clearly dispersive. From a plate point of view, the lowest branch of the dispersion relation of symmetric waves is called a face-shear wave with a finite phase speed when $\xi h = 0$. Other higher branches are all called thickness-twist waves with unbounded phase speed when $\xi h = 0$. If the poling direction of the plate is reversed, the dispersion curves are given in Fig. 3. Compared to the curves in Fig. 2 the curves in Fig. 3 are more separated. The relevant material

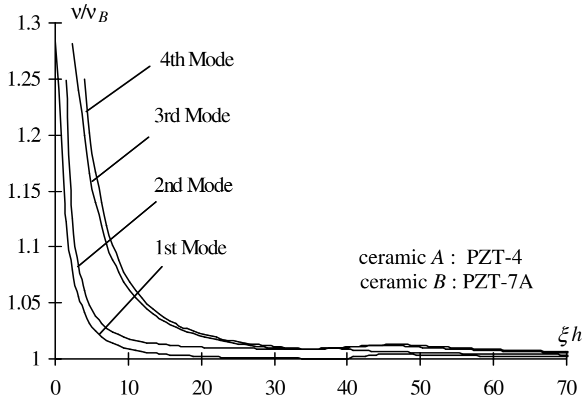

Fig. 2. Dispersion curves for symmetric waves when all ceramics are poled in the same direction.

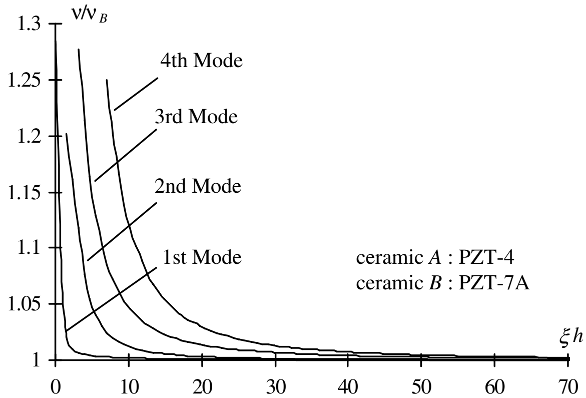

Fig. 3. Dispersion curves for symmetric waves when the half-spaces and the plate are poled in opposite directions (plate poling reversed).

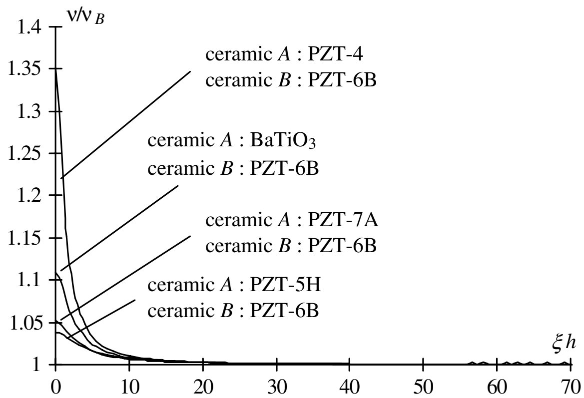

Fig. 4. Dispersion curves for symmetric waves for various material combinations. All ceramics are poled in the same direction.

constants of PZT-4 and PZT-7A are not very different (see the table at the end of this section). The reversal of the poling direction in the plate changes the signs of the piezoelectric constants of the plate. This in some sense makes the material of the plate more different from that of the half-spaces, which has some effect on the dispersion curves. Dispersion relations of the first symmetric mode (face-shear) for different combinations of ceramics are shown in Fig. 4.The figure shows that the wave speeds at zero wave number (the intercepts with the vertical axis in Fig. 4) depend strongly on the material.

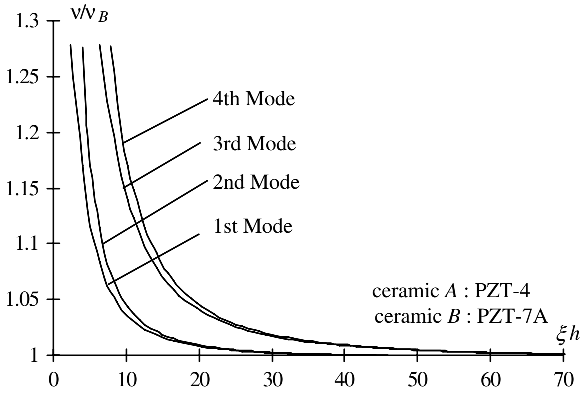

Fig. 5. Dispersion curves for anti-symmetric waves when all ceramics are poled in the same direction.

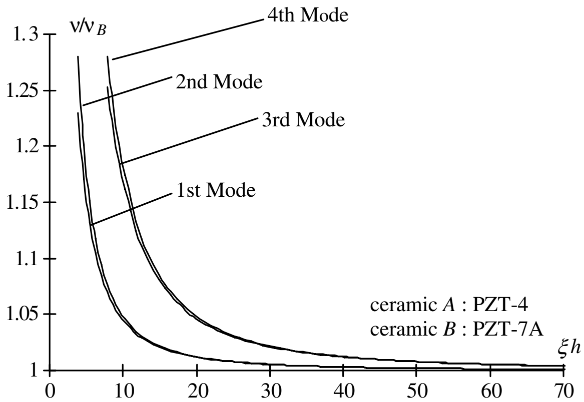

Fig. 6. Dispersion curves for anti-symmetric waves when the half-spaces and the plate are poled in opposite directions (plate poling reversed).

The first four anti-symmetric branches (thickness-twist) of the dispersion relation determined from Eq. (35) are given in Fig. 5 when all ceramics are poled in the same direction, and in Fig. 6 when the poling direction of the plate is reversed. These waves all have unbounded phase velocities at zero wave number. For larger wave numbers (short waves) they seem to approach the same limit speed.

As shown by the numerical examples in Yang (2004a,b), the effect of full electromagnetic coupling on acoustic wave speed is quantitatively small. However, with full electromagnetic coupling and piezoelectric coupling, acoustic and electromagnetic waves are no longer separable. This qualitative change is important for certain phenomena which have consequences in applications. For example, the predicted possible acoustic radiation of electromagnetically guided modes may need to be considered in the design of microwave devices. For device applications usually long waves (wavelength >> plate thickness) are used. We examine Eq. (28) for long waves with $\xi h << 1$. We focus on the lowest acoustic branch (the face-shear wave) of Eq. (28). In this case, in terms of the wave speed v, Eq. (28) assumes the following form:

$$(1+n^2\xi h)\left(\frac{v^2}{v_B^2}-1\right)=-k^2, \tag{41}$$

where

$$n^2=\frac{\varepsilon_B}{\varepsilon_0},\quad k^2=\frac{e_B^2}{\varepsilon_B\bar{c}_B}. \tag{42}$$

Eq. (41) shows that the originally non-dispersive long face-shear wave becomes dispersive due to electromagnetic coupling. We plot Eq. (41) for a few ceramics in Fig. 7. The figure shows that the dispersion is noticeable

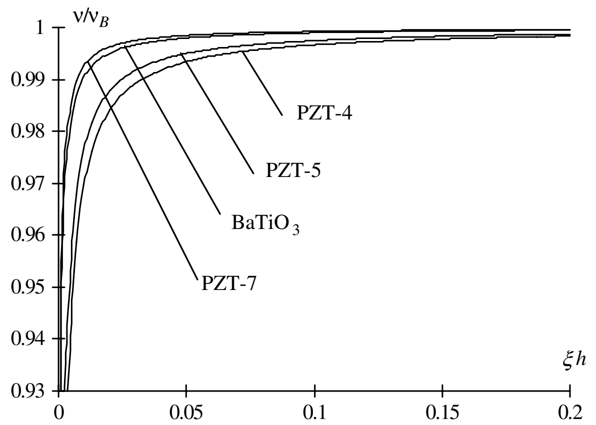

Fig. 7. Dispersion of long face-shear waves in a plate in free space.

only for long waves with $\xi h < 0.1$. This translates into $2\pi h/\lambda < 0.1$, or $\lambda/2h > 10\pi$, i.e., the wavelength is about 30 times the plate thickness.

The material constants used in the calculations are

	ρ (kg/m^3)	c_{44} (10^{10} N/m^3)	e_{15} (C/m^2)	ε_{11} (10^{-8} C/Vm)
PZT-4	7500	2.56	12.7	0.646
PZT-5H	7500	2.30	17.0	1.506
PZT-6B	7550	3.55	4.6	0.360
PZT-7A	7600	2.53	9.2	0.407
$BaTiO_3$	5700	4.39	11.4	0.982
PZT-5	7750	2.11	12.3	0.811
PZT-7	7800	2.50	13.5	1.71

7. Conclusion

Exact solutions are obtained from the three-dimensional equations of linear piezoelectromagnetism for guided waves in a ceramic plate between two ceramic half-spaces. The solutions obtained reduce to a few known elastic, electromagnetic, and quasistatic piezoelectric wave solutions in the literature as special cases. The existence of guided waves is shown through numerical results and special cases with known solutions. Full electromagnetic-elastic coupling may cause acoustic radiation of guided electromagnetic waves in microwave devices. The coupling may also cause additional dispersion in acoustic waves.

Acknowledgements

This work was supported by the US Navy's Metrology R&D Program, the US Naval Surface Warfare Center's Measurement Science Department, AEPTEC Microsystems Inc., and the University of California, MICRO Program. It was also supported partly by the National Natural Science Foundation of China (No. 10272043).

References

Bleustein, J.L., 1968. A new surface wave in piezoelectric materials. Appl. Phys. Lett. 13, 412–413.
Graff, K.F., 1991. Wave Motion in Elastic Solids. Dover, New York.
Gulyaev, Yu.V., 1969. Electroacoustic surface waves in solids. Sov. Phys. JETP. Lett. 9, 37–38.
Jackson, J.D., 1990. Classical Electrodynamics. John Wiley and Sons, New York.

Kyame, J.J., 1949. Wave propagation in piezoelectric crystals. J. Acoust. Soc. Am. 21, 159–167.
Lee, P.C.Y., 1989. Electromagnetic radiation from an AT-cut quartz plate under lateral-field excitation. J. Appl. Phys. 65, 1395–1399.
Lee, P.C.Y., 1991. A variational principle for the equations of piezoelectromagnetism in elastic dielectric crystals. J. Appl. Phys. 69, 7470–7473.
Lee, P.C.Y., Kim, Y-G., Prevost, J.H., 1990. Electromagnetic radiation from doubly rotated piezoelectric crystal plates vibrating at thickness frequencies. J. Appl. Phys. 67, 6633–6642.
Li, S., 1996. The electromagneto-acoustic surface wave in a piezoelectric medium: the Bleustein–Gulyaev mode. J. Appl. Phys. 80, 5264–5269.
Marcuse, D., 1982. Light Transmission Optics. Van Nostrand Reinhold, New York.
Mindlin, R.D., 1972. Electromagnetic radiation from a vibrating quartz plate. Int. J. Solids Struct. 9, 697–702.
Oliner, A.A., 1978. Acoustic Surface Waves. Springer-Verlag, New York.
Schmerr Jr, L.W., Sedov, A., 1986. Some exact solutions for the propagation of transient electroacoustic waves. II: Plane interface between two piezoelectric media. Int. J. Eng. Sci. 24, 921–932.
Scholl, G., Schmidt, F., Ostertag, T., Reindl, L., Scherr, H., Wolff, U., 1998. Wireless passive SAW sensor systems for industrial and domestic applications. In: Proc. IEEE Int. Frequency Control Symp. IEEE, New York, pp. 595–601.
Sedov, A., Schmerr Jr, L.W., 1986. Some exact solutions for the propagation of transient electroacoustic waves I: piezoelectric half-space. Int. J. Eng. Sci. 24, 557–568.
Spaight, R.N., Koerber, G.G., 1971. Piezoelectric surface waves on $LiNbO_3$. IEEE Trans. Sonics Ultrasonics 18, 237–238.
Tiersten, H.F., 1969. Linear Piezoelectric Plate Vibrations. Plenum, New York.
To, A.C., Glaser, S.D., 2004. On the quasi-static assumption in modeling shear horizontal (SH) waves in a transversely isotropic (6 mm) medium. Available from: <http://www.ce.berkeley.edu/~albertto/piezo.pdf.>.
Yang, J.S., 2000. Bleustein–Gulyaev waves in piezoelectromagnetic materials. Int. J. Appl. Electromagnet. Mech. 12, 235–240.
Yang, J.S., 2004a. Love waves in piezoelectromagnetic materials. Acta Mech. 168, 111–117.
Yang, J.S., 2004b. Piezoelectromagnetic waves in a ceramic plate. IEEE Trans. Ultrasonics, Ferroelectrics, Frequency Control 51, 1035–1039.