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Effects of Friction on the Dynamic Behavior of Flexible Rocking Bodies

S. Mohammadreza Farooghi Mehr
University of Nebraska-Lincoln, sfarooghimehr2@huskers.unl.edu

Christine E. Wittich
University of Nebraska - Lincoln, cwittich@unl.edu

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S. M. Farooghi-Mehr[1] and C.E. Wittich[2]

ABSTRACT

Flexible rocking bodies are freestanding structures that are free to deform and rock, potentially overturning, when subject to ground excitation. Prior research on the seismic behavior of this type of structure typically assumed sufficient friction at the ground surface to avoid sliding motion. However, experimental observations showed that this assumption was often violated and structures demonstrated non-negligible sliding during their responses. The overarching goal of this study is to evaluate the impact of sliding on the overturning response of flexible rocking bodies. To this end, a two-dimensional analytical model was developed in a Lagrangian formulation, which is presented in this paper. This model was subjected to one-cycle sine pulses of varying amplitude and frequency for several levels of base friction to quantify the impact of sliding on overturning. In general, the results highlight that sliding behavior reduces the overturning demand, however motion-to-motion variability was observed.

Introduction

The dynamic performance of freestanding structures, which are unattached at their base and free to rock, slide, or overturn, has been a point of study since the early 1900s due to observed poor response during earthquake events [1, 2]. In addition, the observations of rocking motion in freestanding structures, particularly rigid blocks, in the 1960 Chile earthquake prompted the foundational study by Housner in which an analytical model for two-dimensional rigid body rocking subjected to ground acceleration was developed [3]. This study included equations of motion, criteria of initiation of rocking motion, and treatment of impact, which was modeled as a coefficient of restitution as lumped energy dissipation. Then, several studies by others have been carried out to further investigate the seismic behavior, such as the effects of variability of ground motion in the failure of rigid bodies [4] and the development of overturning spectra to elucidate the impact of motion amplitude and frequency on overturning rates [2].

Possible modes like sliding, slide-rocking, and free-flight might occur during earthquakes in rigid body motion. Multiple investigations explained the analytical models for each case [5, 6]. In the rigid rocking body studies, researchers typically assumed that there is no sliding displacement during rocking motion by considering sufficient friction at the base of structure. However, Shenton [7] showed the criteria of initiation of modes from rest. The results showed that all considered modes (sliding, rocking, and slide-rocking) could be started from rest. Later, Taniguchi [8] investigated the seismic behavior of rigid slide-rocking bodies subjected to both vertical and horizontal ground motion records. In addition to these and other analytical studies, a handful of experimental tests have been performed [9, 10]. In one particular experimental study, ElGawady et al. [11] carried out tests to recognize the contact surface effect on

[1] PhD Student & Graduate Research Assistant, Dept. of Civil and Environmental Engineering, University of Nebraska-Lincoln, Lincoln, NE, USA (email: sfarooghimehr2@huskers.unl.edu)
[2] Assistant Professor, Dept. of Civil and Environmental Engineering, University of Nebraska-Lincoln, Lincoln, NE, USA (email: cwittich@unl.edu)

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impact mechanism using different materials at the base interface, which is a cause of significant response variability.

Although there are structures for which rigid body analytical models work well, these models do not capture the contributions of variable stiffness and deformation for which many structures require. Hence, the derived model for the flexible rocking body emerged to predict the seismic responses [12]. One of the useful applications of the flexible rocking bodies models is simulating columns of bridge's bent and then carrying out the probabilistic evaluation of bridges under seismic load [13]. One of the earliest contributions in assessing flexible rocking was that by Acikgoz and DeJong [14]. In their work, they presented parametric studies on parameters such as flexibility scale (index to represent the flexibility of structure), damping, and slenderness ratio under trigonometric pulse base excitations. The findings implied that flexible rocking bodies have less overturning vulnerability compared to rigid bodies. Building upon this work, experimental specimens were tested to observe both deformation and rocking responses [15, 16]. While these tests proved the interaction of flexibility and rocking, sliding responses and permanent displacements were still observed. More recently and based upon experimental observations, Vassilou et al. proposed a modified treatment of impact for flexible rocking bodies in which there is a period of full contact (non-rocking behavior) after an impact [17]. As a result of these prior works, this study aims to fill the gap associated with sliding in flexible-rocking models. In this paper, an analytical model is presented including equation of motion, criteria of initiation and modal transition, and impact treatment. Results of this model are presented as overturning spectra for a range of friction to highlight the impact of sliding and characterize the system dynamics.

Analytical Model

Equations of Motion

Previously, researchers developed the equations of motion regarding the flexible rocking body [12, 14]. These include equations that correspond to the angular rotation and deformation. The equations were derived in polar coordinates and then converted to intuitive coordinates to facilitate physical interpretation. In this study, those equations were extended to consider the added degree of freedom (sliding motion). Fig. 1 shows the possible motions that are covered herein. The idealized flexible body has stiffness (K), damping coefficient (c), lumped mass (m), width of foundation (B), height (H), and slenderness ratio (α). The motion is shown characterized by two sets of coordinates, including (R, β, d) and (u, θ, d) where R is the distance of the lumped mass from the base pivot, β is the Lagrangian rotation parameter, u is the elastic deformation, θ is the angular rotation or rocking response, and d is sliding displacement.

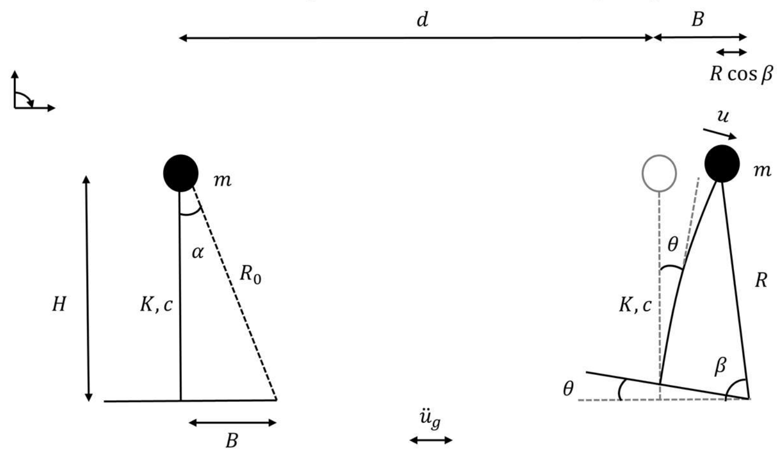

Figure 1. Deformation, rocking, and sliding responses of flexible body.

The derivation of the equations of motion are based on a Lagrangian formulation and are presented in another paper [18] and are reproduced below for brevity. The equations of motion in (u, θ, d) are as follows:

$$\ddot{u} + 2\left(\frac{R_0}{B}\zeta\right)\left(\frac{R_0}{B}\omega_n\right)\dot{u} + \left(\frac{R_0}{B}\omega_n\right)^2 u \pm \frac{{R_0}^2}{B}\left(\dot{\theta} + \frac{H\dot{u}}{{R_0}^2}\right)^2 \mp \ddot{d}\sin(\alpha \mp \theta) = \frac{R_0}{B}(-\ddot{u}_g \sin(\alpha \mp \theta) \pm g\cos(\alpha \mp \theta)) \quad (1)$$

$$\ddot{\theta}+\frac{H}{{R_0}^2}\ddot{u}\mp\frac{2B}{{R_0}^2}\left(\dot{\theta}+\frac{H\dot{u}}{{R_0}^2}\right)\dot{u}+\frac{\ddot{d}\cos(\alpha\mp\theta)}{R_0}=\mp\frac{g}{R_0}\sin(\alpha\mp\theta)-\frac{\ddot{u}_g}{R_0}\cos(\alpha\mp\theta) \tag{2}$$

$$\ddot{d}+\frac{B}{R_0}\ddot{u}\sin(\alpha\mp\theta)-2\frac{B}{R_0}\dot{u}\left(\dot{\theta}+\frac{H\dot{u}}{{R_0}^2}\right)\cos(\alpha\mp\theta)+R_0\left(\ddot{\theta}+\frac{H\ddot{u}}{{R_0}^2}\right)\cos(\alpha\mp\theta)\pm R_0\left(\dot{\theta}+\frac{H\dot{u}}{{R_0}^2}\right)^2\sin(\alpha\mp\theta)=-\ddot{u}_g-\mu_k sign(\dot{d})[g\pm\frac{B}{R_0}\ddot{u}\cos(\alpha\mp\theta)+2\frac{B}{R_0}\dot{u}\left(\dot{\theta}+\frac{H\dot{u}}{{R_0}^2}\right)\sin(\alpha\mp\theta)\mp R_0\left(\ddot{\theta}+\frac{H\ddot{u}}{{R_0}^2}\right)\sin(\alpha\mp\theta)+R_0\left(\dot{\theta}+\frac{H\dot{u}}{{R_0}^2}\right)^2\cos(\alpha\mp\theta)] \tag{3}$$

where ω_n is the natural frequency of the flexible body, ζ is the damping ratio, μ_k is the kinetic coefficient of friction, $\ddot{u}_g$ is the ground acceleration, and g is the acceleration due to gravity.

Criteria of Initiation and Modal Transition

The flexible structure will initially respond in a purely deformation mode and may initiate into either rocking or sliding when subject to the applied ground motion. The criteria of initiation into sliding and rocking, respectively, are:

$$m\left|\ddot{u}_g+\ddot{u}\right|>\mu_s mg \tag{4}$$

$$mH\left|\left(\ddot{u}_g+\ddot{u}\right)\right|>mg(B\mp u) \tag{5}$$

where μ_s is the static coefficient of friction. Slide-rocking motion may begin from either rocking or sliding by satisfying the modal transition criteria. That is, once the flexible body is sliding, it undergoes slide-rocking motion if Eq. (6) is satisfied. The same process happens for slide-rocking from rocking with Eq. (7). These equations are:

$$\left|mH\left(\ddot{u}_g+\ddot{u}+\ddot{d}\right)\right|>mg(B\mp u) \tag{6}$$

$$\frac{\ddot{u}_g-\frac{B}{R_0}\ddot{u}\sin(\alpha\mp\theta)\pm2\frac{B}{R_0}\dot{u}\left(\dot{\theta}+\frac{H\dot{u}}{{R_0}^2}\right)\cos(\alpha\mp\theta)-R_0\left(\ddot{\theta}+\frac{H\ddot{u}}{{R_0}^2}\right)\cos(\alpha\mp\theta)\mp R_0\left(\dot{\theta}+\frac{H\dot{u}}{{R_0}^2}\right)^2\sin(\alpha\mp\theta)}{g\pm\frac{B}{R_0}\ddot{u}\cos(\alpha\mp\theta)+2\frac{B}{R_0}\dot{u}\left(\dot{\theta}+\frac{H\dot{u}}{{R_0}^2}\right)\sin(\alpha\mp\theta)\mp R_0\left(\ddot{\theta}+\frac{H\ddot{u}}{{R_0}^2}\right)\sin(\alpha\mp\theta)+R_0\left(\dot{\theta}+\frac{H\dot{u}}{{R_0}^2}\right)^2\cos(\alpha\mp\theta)}>\mu_s \tag{7}$$

Impact Treatment

During rocking motion, changing the pivot point and impact phenomenon are possible. In the classical formulation [3, 6], the energy dissipation of rocking motion was considered in analytical models through the coefficient of restitution as the impact treatment. This mechanism reduces the post-impact velocities. The equation related to post-impact deformation velocity from [19] was used in this study because it provided a reliable revisited formulation based on experimental observations. During slide-rocking responses in this study, it was assumed that the body maintains full contact during impact ($\dot{\theta}_2=0$) and the sliding velocity does not change ($\dot{d}_2=\dot{d}_1$).

Investigation on Effects of Friction

As previously mentioned, prior flexible rocking studies assumed sufficient friction to prevent sliding. In this study, sliding was introduced to the flexible rocking model to capture its effects on the dynamic performance. To address this aim, the analytical model (Eq. 1-3) was solved numerically in MATLAB [20] by using a 4^{th}-5^{th} order Runge-Kutta numerical integration scheme. The applied load was a one-cycle sine pulse with various frequencies and amplitudes. The results provide overturning spectra and sliding spectra, which can lead to an understanding of the failure modes as a function of friction.

Fig. 2 displays these spectra for a model with $\alpha=0.3$, $\omega_n/p=9$, ζ = 0.05 and various coefficients of friction. In Fig. 2, A is amplitude of the sine-pulse excitation, ω is its frequency, and p is frequency parameter of the structure that is given by $p=\sqrt{g/R_0}$. The results show the model with high friction ($\mu_s=0.8$) has similar responses for both rocking and sliding compared to flexible rocking body ($\mu_s=\infty$). Furthermore, these spectra illustrate that by considering a wide range of frictions, the failure modes may change. For instance, although mode 1 (red area – failure with one impact) was increased in lower frequencies and amplitudes by decreasing the coefficient of friction, this area was reduced in higher frequencies, and also the sliding spectra depicted high sliding displacement in low friction. These findings imply that friction leads the flexible body to have less overturning vulnerability. It should be mentioned that there was no difference in mode 2 (blue area – failure without impact). Fig. 3 shows the time histories of deformation, rotation, and sliding for two low frequencies in model with $\mu_s = 0.4$ to describe the increase in mode 1 at low frequencies for the lowest friction model. While the two low frequencies are close, the difference in response can be attributed to modal transition where the model under $\omega/p = 1.3$ features failure through mode 2 in rocking motion, and the model under $\omega/p = 1.5$ features mode 1 failure through slide-rocking motion.

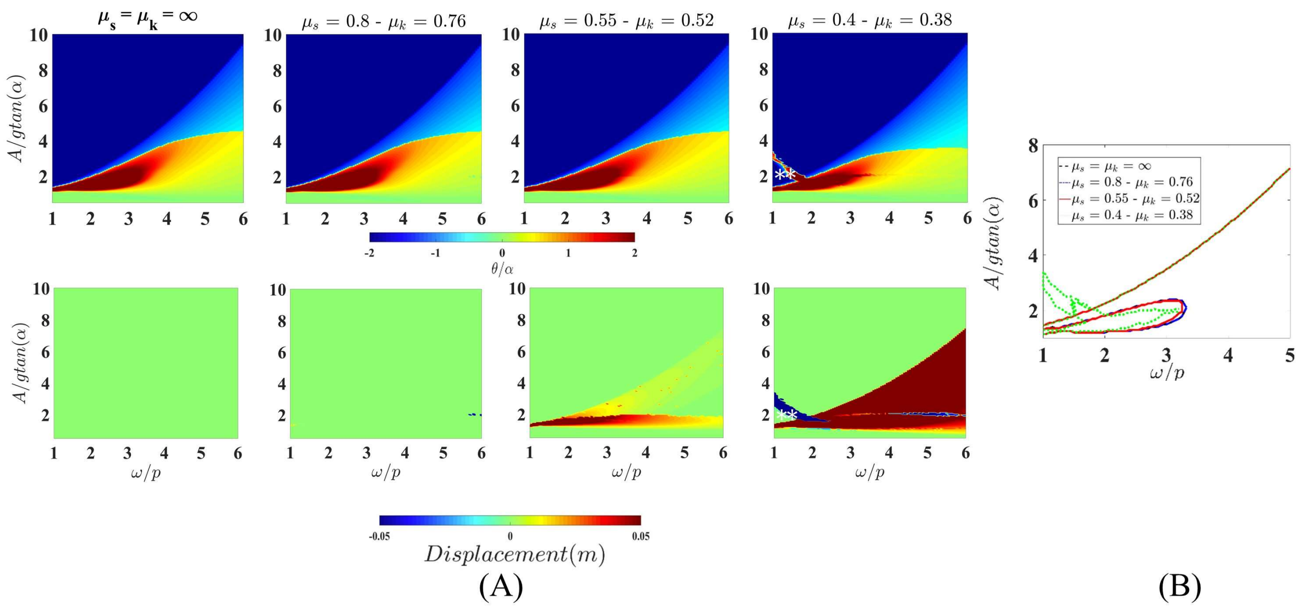

Figure 2. (A) Overturning and sliding spectra of a model characterized by $\alpha = 0.3$, $\omega_n/p = 9$, $\zeta = 0.05$ and various frictions; (B) Overlaid contours of the overturning spectra for each friction case.

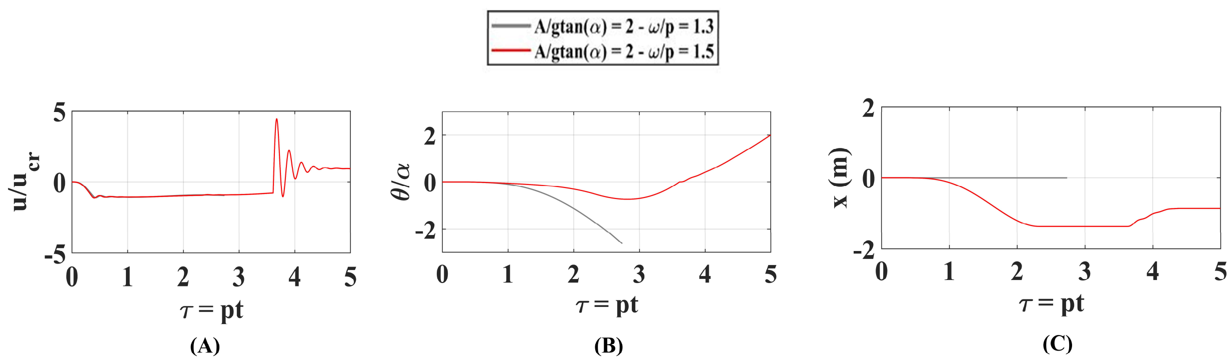

Figure 3. Time histories of (A) normalized deformation, (B) normalized rocking, and (C) displacement for two motions of variable frequency ($\mu_s = 0.4$ - asterisk points in Figure 2)

Conclusions

The flexible rocking body has been examined in the literature by taking sufficient friction to prevent the sliding motion. In this study, an extended analytical model was introduced to find the effects of sliding motion on flexible rocking body. The results expressed that sliding motion affects the failure with one impact or mode 1 failure in both lower and higher frequencies of one-cycle sine pulse ground motion. In general, the overturning vulnerability of flexible body was reduced once the body experienced sliding motion.

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