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Abstract

Commonly used analytical approaches for estimation of pumping-induced drawdown and stream depletion are based on a series of idealistic assumptions about the stream-aquifer system. A new solution has been developed for estimation of drawdown and stream depletion under conditions that are more representative of those in natural systems (finite width stream of shallow penetration adjoining an aquifer of limited lateral extent). This solution shows that the conventional assumption of a fully penetrating stream will lead to a significant overestimation of stream depletion (> 100%) in many practical applications. The degree of overestimation will depend on the value of the stream leakance parameter and the distance from the pumping well to the stream. Although leakance will increase with stream width, a very wide stream will not necessarily be well represented by a model of a fully penetrating stream. The impact of lateral boundaries depends upon the distance from the pumping well to the stream and the stream leakance parameter. In most cases, aquifer width must be on the order of hundreds of stream widths before the assumption of a laterally infinite aquifer is appropriate for stream-depletion calculations. An important assumption underlying this solution is that stream-channel penetration is negligible relative to aquifer thickness. However, an approximate extension to the case of nonnegligible penetration provides reasonable results for the range of relative penetrations found in most natural systems (up to 85%). Since this solution allows consideration of a much wider range of conditions than existing analytical approaches, it could prove to be a valuable new tool for water management design and water rights adjudication purposes.

Introduction

Stream-aquifer interactions are an important component of the hydrologic budgets of many watersheds and have significant socioeconomic and political ramifications (Bouwer and Maddock 1997). In watersheds undergoing ground water development, pumping-induced water transfers (stream depletion) often comprise a sizable proportion of the stream-aquifer interactions. Many of the approaches currently used to quantify these transfers are based on analytical models of idealized configurations that often bear little resemblance to natural systems. In this article, a transient analytical model that is based on a more realistic representation of the stream-aquifer system is proposed. This model can be used for estimation of the drawdown and stream depletion produced by a pumping well in the vicinity of a finite-width stream of shallow penetration. Although an analytical approach cannot represent stream-aquifer interactions to the same degree of detail as a numerical model, it can provide a useful screening tool for consideration of the influence of various factors and to obtain estimates commensurate with the level of commonly available data.

Over the last 60 years, several analytical models have been developed to assess the influence of nearby streams on ground water development. Theis (1941) was the first to propose a transient model for evaluation of the impact of a stream on pumping activities. This approach, later generalized by Glover and Balmer (1954), is based on a series of idealistic assumptions that include a fully penetrating stream and a perfect hydraulic connection between the stream and adjoining aquifer (Figure 1a). The Jenkins (1968) implementation of this approach has become a standard tool for water management design and water rights adjudication. Hantush (1965) extended this model to consider an imperfect hydraulic connection between the aquifer and the stream (Figure 1b), but this extension has seen relatively little use in practice.

Unfortunately, the idealized configurations of Figure 1 bear little resemblance to many natural systems. For example, streams in the Great Plains region of the United States commonly only partially penetrate the adjoining aquifer. In many cases, the depth of penetration is a few tens of percent or less of the aquifer thickness (Larkin and Sharp 1992). Although not widely known outside the former Soviet Union, a steady-state model of stream-aquifer interactions has been developed that incorporates a simplified representation of a partially penetrating stream (Grigoryev 1957; Bochever 1966). This model, which has been extensively used in the former Soviet Union (Bochever et al. 1969, 1978; Mironenko et al. 1994), is based on the assumptions that the depth of penetration is very small relative to aquifer thickness and that the stream and aquifer are separated by a thin zone of relatively low hydraulic conductivity. Recently, Zlotnik et al. (1999) have developed a tran-

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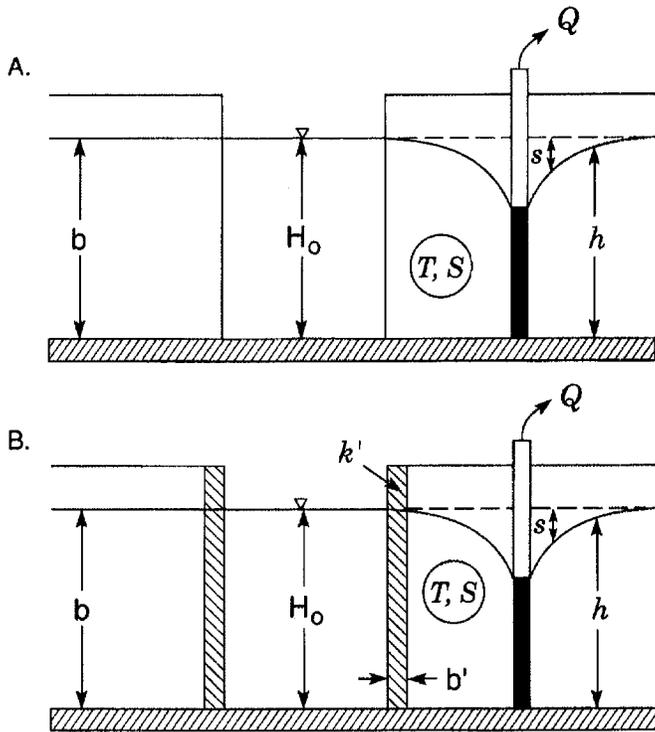[3]Corresponding author: (785) 864-2116; jbutler@kgs.ukans.edu

Figure 1. Cross-sectional views of hypothetical stream-aquifer systems (notation explained in text): (a) fully penetrating stream model of Theis (1941), Glover and Balmer (1954), and Jenkins (1968); (b) fully penetrating stream with low-conductivity streambed model of Hantush (1965).

sient analytical solution for estimation of stream depletion in systems that can be represented by the Grigoryev-Bochever model. Concurrent with that work, Hunt (1999) has proposed a solution for drawdown and stream depletion in an infinitely thin stream variant of this model. In both cases, the authors employ various simplifying assumptions that reduce the general applicability of their solutions.

In this paper, the steady-state Grigoryev-Bochever model is used as the starting point for the development of a general analytical solution for transient drawdown and stream depletion in the stream-aquifer system depicted in Figure 2. The solution will be used to assess the influence of a low-permeability streambed, stream width, lateral boundaries, and stream-channel penetration on drawdown and stream depletion. The degree of error introduced by the fully penetrating stream models of Figure 1 will be emphasized throughout the presentation. The article will conclude with a discussion of the most significant limitations of the solution and the implications of this work for water resources management in interconnected stream-aquifer systems.

Problem Statement

The problem of interest here is that of the drawdown (as a function of x, y, and t) and stream depletion (as a function of t) produced by pumping from a fully penetrating well in the configuration of Figure 2. Flow properties are assumed uniform within each zone, and vertical flow within the aquifer is neglected (Dupuit assumptions). The stream and aquifer are separated by a thin zone of relatively low hydraulic conductivity, which is represented mathematically as an incompressible aquitard (Hantush and Jacob 1955). Portions of the aquifer underneath the stream are confined, but can be confined or unconfined elsewhere.
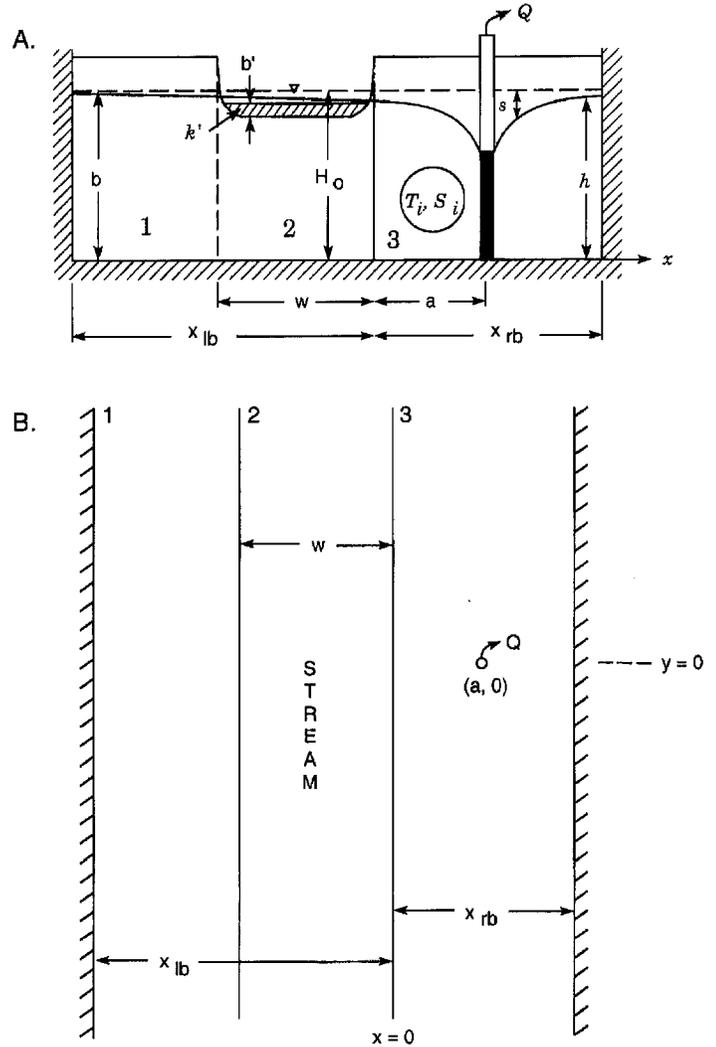

Figure 2. Cross-sectional (a) and areal (b) views of stream-aquifer configuration examined in this paper (notation explained in text; stream depletion in this configuration consists of vertical leakage across the low-permeability streambed).

Governing Equations:

$$\frac{\partial^2 s_1}{\partial x^2} + \frac{\partial^2 s_1}{\partial y^2} = \frac{S_1}{T_1} \frac{\partial s_1}{\partial t}, \; - x_{lb} \leq x \leq - w,$$

$$-\infty < y < \infty, t > 0 \qquad (1)$$

$$\frac{\partial^2 s_2}{\partial x^2} + \frac{\partial^2 s_2}{\partial y^2} - \frac{k'}{b'T_2} s_2 = \frac{S_2}{T_2} \frac{\partial s_2}{\partial t}, \; - w \leq x \leq 0,$$

$$-\infty < y < \infty, t > 0 \quad (2)$$

$$\frac{\partial^2 s_3}{\partial x^2} + \frac{\partial^2 s_3}{\partial y^2} + \frac{Q}{T_3} \delta (x - a) \delta(y) = \frac{S_3}{T_3} \frac{\partial S_3}{\partial t}, 0 \leq x \leq x_{rb},$$

$$-\infty < y < \infty, t > 0 \quad (3)$$

Initial Conditions:

$$s_i (x, y, 0) = 0, - x_{lb} \leq x \leq x_{rb}, -\infty < y < \infty, i = 1,3 \qquad (4)$$

Boundary Conditions:

$$\frac{\partial s_1}{\partial x}(-x_{lb}, y, t) = \frac{\partial s_3}{\partial x}(x_{rb}, y, t) = 0, -\infty < y < \infty, t > 0$$

$$\tag{5}$$

$$s_i(x, \pm\infty, t) = 0, -x_{lb} \le x \le x_{rb}, t > 0$$

$$\tag{6}$$

$$s_1(-w, y, t) = s_2(-w, y, t), -\infty < y < \infty, t > 0$$

$$\tag{7}$$

$$\frac{\partial s_1}{\partial x}(-w, y, t) = \frac{T_2}{T_1}\frac{\partial s_2}{\partial x}(-w, y, t), -\infty < y < \infty, t > 0$$

$$\tag{8}$$

$$s_2(0, y, t) = s_3(0, y, t), -\infty < y < \infty, t > 0$$

$$\tag{9}$$

$$\frac{\partial s_2}{\partial x}(0, y, t) = \frac{T_3}{T_2}\frac{\partial s_3}{\partial x}(0, y, t), -\infty < y < \infty, t > 0$$

$$\tag{10}$$

where i is zone number (i = 1,2,3); $s_i(x,y,t)$ is drawdown in zone i; T_i is transmissivity of zone i; S_i is specific yield or storativity of zone i; k' is hydraulic conductivity of streambed; b' is streambed thickness; Q is pumping rate from well located at (a,0); x_{lb}, x_{rb} are distance from right boundary of stream to left and right lateral boundary of the aquifer, respectively; and w is stream width.

Stream depletion is defined as the total flow across the incompressible streambed

$$\Delta q(t) = \frac{k'}{b'}\int_{-\infty}^{\infty}\int_{-w}^{0} s_2\, dxdy, t > 0$$

$$\tag{11}$$

where Δq is the pumping-induced leakage from the stream (Figure 2 and caption).

Table 1
Definition of Dimensionless Parameters

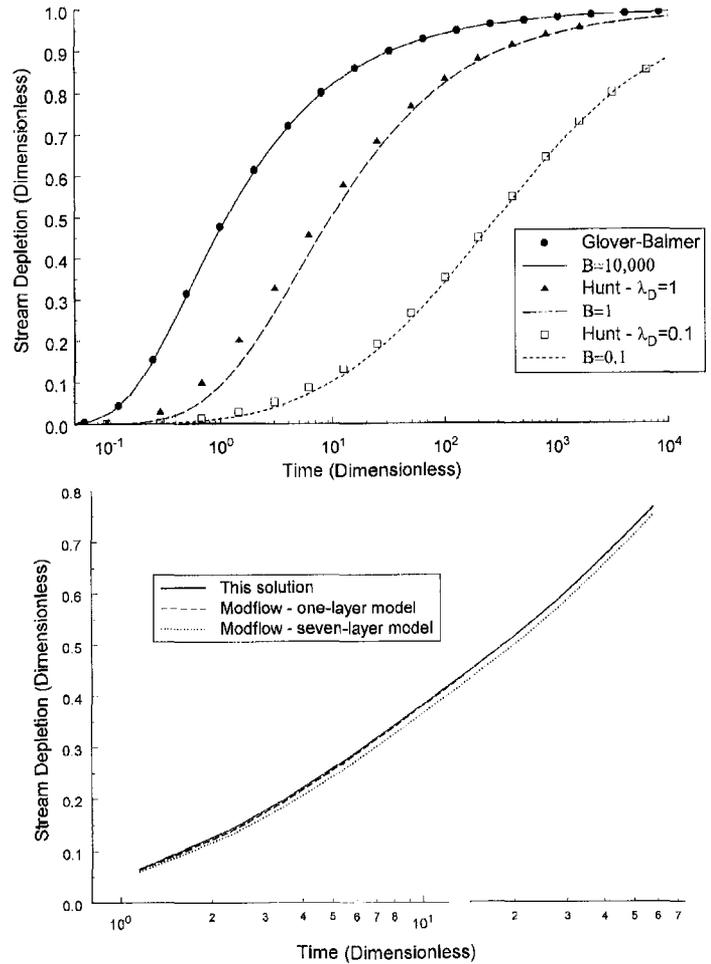| Parameter	Notation	Expression
Drawdown	Φ_i	$s_i T_3/Q$
Stream depletion	ΔQ	$\Delta q/Q$
Time	τ	$(T_3 t)/(w^2 S_3)$
Stream leakance	B	$(k' w^2)/(b' T_2)$
Direction perpendicular to stream	ξ	x/w
Direction parallel to stream	η	y/w
Distance between pumping well and stream	α	a/w
Distance to the left aquifer boundary	X_{LB}	x_{lb}/w
Distance to the right aquifer boundary	X_{RB}	x_{rb}/w

Figure 3. (a) Dimensionless stream depletion ($\Delta q/Q$) versus time ($T_3 t/w^2 S_3$) plot of the analytical models of Glover and Balmer (1954) and Hunt (1999, λ_D = [k' wa]/[b' T]), and the new solution (B = [k' w²]/[b' T₂]) for an aquifer of infinite lateral extent (α = a/w = 1); (b) Dimensionless stream depletion versus time plot of the new solution and single- and seven-layer MODFLOW models for an aquifer of limited lateral extent (α = 1.1, B = 0.58, $X_{LB} = x_{lb}/w$ = 8.9, $X_{RB} = x_{rb}/w$ = 7.9; see Table 2 for grid details).

The mathematical model defined by Equations 1 through 10 was solved using an approach analogous to that of Butler and Liu (1991). Since the solution is a straightforward application of conventional methods, the details are not provided here (Butler and Tsou 2000). Stream depletion was calculated using the approach of Hunt (1999) with the transform-space analog of Equation 11. Transform-space expressions for both drawdown and stream depletion are given in the Appendix, and the numerical inversion of these expressions is implemented in Butler and Tsou (1999).

The drawdown and stream depletion computed using this solution have been compared to results from existing analytical and numerical models. The dimensionless parameters of Table 1 are used to illustrate the results of these comparisons (see Butler and Tsou [2000] for derivation of dimensionless parameters). The new solution will reduce to existing analytical approaches for special cases as shown by the example comparison given in Figure 3a, where normalized stream depletion is plotted versus dimensionless time for a laterally infinite aquifer (solutions of Glover and Balmer [1954] and Hunt [1999] shown for reference). In these conditions, the Glover and Balmer solution is reproduced using a large value for stream leakance (B). Since the Hunt solution is based on the assumption of an infinitely

Table 2
Grid Details and Parameter Values for MODFLOW Model

Size of model. 69 columns (x), 85 rows (y), and either one or seven layers (z).

Spacing in x direction. From x = 0 m to right boundary, spacing is as follows (all distances in meters, number of columns at a particular spacing given in parentheses): $\Delta x = 1.0$ m (23), $\Delta x = 1.5$ m (1), $\Delta x = 2.3$ m (1), $\Delta x = 3.4$ m (1), $\Delta x = 5.1$ m (1), $\Delta x = 7.6$ m (1), $\Delta x = 11.4$ m (1), and $\Delta x = 17.1$ m (1). Note that distance from x = 0 m to left boundary includes stream so nine additional cells ($\Delta x = 1.0$ m) are required.

Spacing in y direction. From center of grid (cell that includes y = 0 m) to top boundary, spacing is as follows (number of rows at a particular spacing given in parentheses): $\Delta y = 1.0$ m (6), $\Delta y = 1.5$ m (4), $\Delta y = 3.0$ m (3), $\Delta y = 5.0$ m (5), $\Delta y = 10.0$ m (5), $\Delta y = 20.0$ m (5), $\Delta y = 40.0$ m (3), $\Delta y = 80.0$ m (3), $\Delta y = 160$ m (3), $\Delta y = 320$ m (3), and $\Delta y = 640$ m (3).

Equal spacing in z direction ($\Delta z = 7.0$ m for one-layer model; $\Delta z = 1.0$ m for seven-layer model).

Model parameters used in simulations are as follows:
  Hydraulic conductivity (uniform) = 20 m/day;
  Specific storage (uniform) = 4.3×10^{-4} m^{-1};
  Stream width = 9 m; streambed thickness = 0.5 m;
  Hydraulic conductivity of streambed = 0.5 m/d (Figure 3B) and 0.1 m/d (Figure 10);
  Pumping rate = 210 m^3/day.

Boundary conditions. No-flow boundaries are defined on all sides of grid (x,y,z). Boundaries in y direction are sufficiently far from the pumping well that they have no impact on the simulation results.

Figure 4. Dimensionless drawdown ($s_s T_3/Q$) versus time plot of the analytical models of Theis (1935, 1941) and the new solution for an aquifer of infinite lateral extent ($\alpha = 1$, $\xi = x/w = 0.5$, $\eta = y/w = 0.0$).

thin stream, the stream leakage is concentrated at x = 0. Figure 3a shows that this assumption can lead to an overestimation of stream depletion when B is large and the pumping well is relatively close to the stream (B > 0.1 for $\alpha = 1$). However, additional work has shown that the Hunt solution is in excellent agreement with the new model for the case of a laterally infinite aquifer when the pumping well is a normalized distance of five or more from the stream ($\alpha \geq 5$). Note that although the Hunt leakance parameter, λ, was not originally defined in terms of physical quantities, Butler and Tsou (2000) show that for a uniform aquifer ($T_i = T$, i =1,2,3) the dimensionless form of the parameter can be written as

$$\lambda_d = (k'wa)/(b'T) = B\alpha \qquad (12)$$

Figure 3b displays the stream depletion computed for a bounded aquifer using the new solution and two MODFLOW (Harbaugh and McDonald 1996) simulations (Table 2). Both the analytical solution and the single-layer MODFLOW simulation neglect the vertical component of flow by using vertically averaged heads to calculate the stream depletion. In the seven-layer simulation, heads vary in the vertical direction and stream depletion is introduced into the top layer. The small difference between the single- and seven-layer results is a consequence of the neglect of vertical flow in the single-layer model. When the hydraulic conductivity of the streambed is small relative to that of the aquifer, the neglect of vertical flow should not be of practical significance except when the aquifer is thick and the pumping well is close to the stream.

These comparisons demonstrate that the new solution provides a reasonable representation of stream-aquifer interactions under a wide range of conditions. Therefore, for the remainder of the paper, this solution is used to assess the impact of a variety of fac-
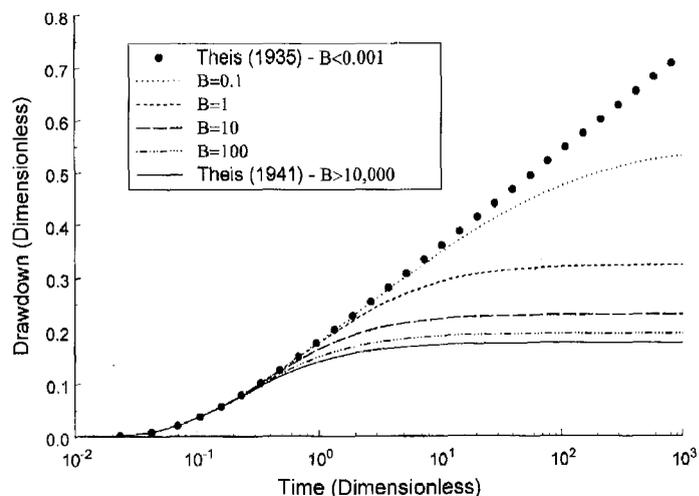
tors on drawdown and stream depletion in stream-aquifer systems. As with the comparisons of this section, dimensionless parameters will be used to increase the generality of the presentation.

Dependence on Streambed Properties

A low-permeability zone in the portion of the aquifer immediately adjacent to the stream appears to be a common feature of many systems. Larkin and Sharp (1992) and Conrad and Beljin (1996) provide estimates of streambed properties that show a dramatic contrast between the permeability of the streambed and that of the aquifer in many cases. The impact of a low-permeability streambed can be readily assessed with this solution by changing the stream leakance parameter (B). If all other dimensionless parameters are kept constant, changes in B equate to changes in the k'/b'ratio. Figure 4 is a plot of drawdown at an observation well midway between the stream and the pumping well. In this case, a seven order of magnitude variation in B spans the range of conditions expected in natural systems. For B < 0.001, the drawdown is equal to that of the infinite-aquifer model of Theis (1935), i.e., the contribution of the stream is negligible. On the other extreme, B > 10,000, the drawdown converges on the fully penetrating stream model of Theis (1941). Figure 5 is a pair of contour plots that graphically display the influence of streambed properties on the areal distribution of drawdown. The propagation of drawdown beneath a partially penetrating stream, such as illustrated in Figure 5a, is commonly observed in the field (Sophocleous et al. 1988). These results, in keeping with findings of earlier investigations, clearly show that the stream leakance parameter (B) is an important control on behavior in stream-aquifer systems. Field investigations should therefore be focused on acquiring data about the quantities comprising this parameter.

The conclusions from the drawdown analysis are further substantiated by consideration of stream depletion. Figure 6a is a stream depletion plot for the conditions of Figure 4. Although increases in B lead to convergence on the Glover and Balmer model, this convergence occurs at extremely large B values for pumping wells close to the stream. As the distance between the stream and the pumping well increases, the influence of streambed properties decreases and the Glover and Balmer model becomes a more reasonable representation of system behavior (Figures 6b and 6c).
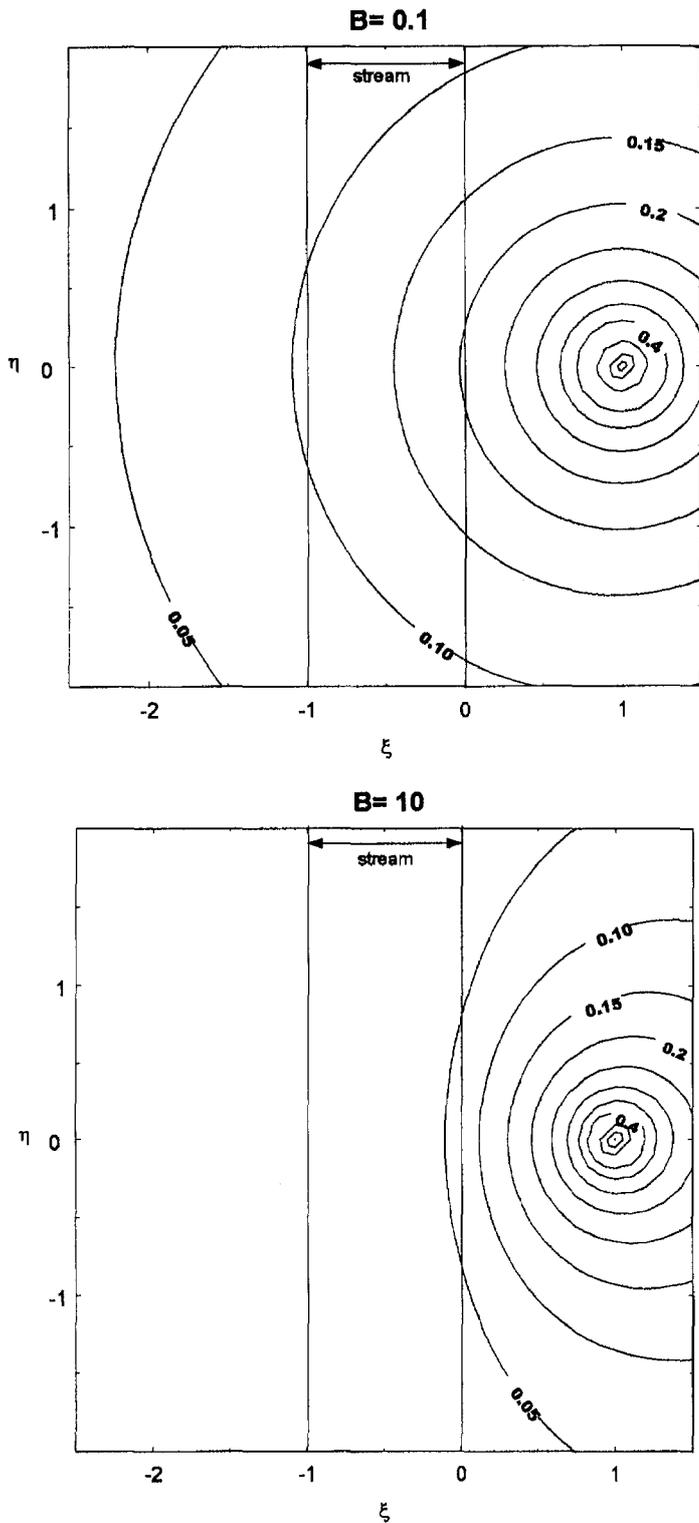
B= 0.1

B= 10

Figure 5. Contour plots of dimensionless drawdown (s_iT_3/Q) for a laterally infinite aquifer $(\tau = T_3t/w^2S_3 = 5, \alpha = 1)$: (a) $B = 0.1$; (b) $B = 10.0$. Contour interval of 0.05 used for drawdown below 0.4 while contour interval of 0.1 used for drawdown above 0.4.

Figures 6a to 6c demonstrate that the Glover and Balmer model may introduce considerable error into stream depletion estimates when the pumping well is relatively close to the stream (α < 20 to 30). In order to assess the magnitude of this error for conditions commonly faced in the field, B estimates based on physically plausible parameter values must be considered. Reasonable values of hydraulic conductivity and aquifer thickness for an unconsolidated sand and gravel aquifer are 50 m/day and 15 m,
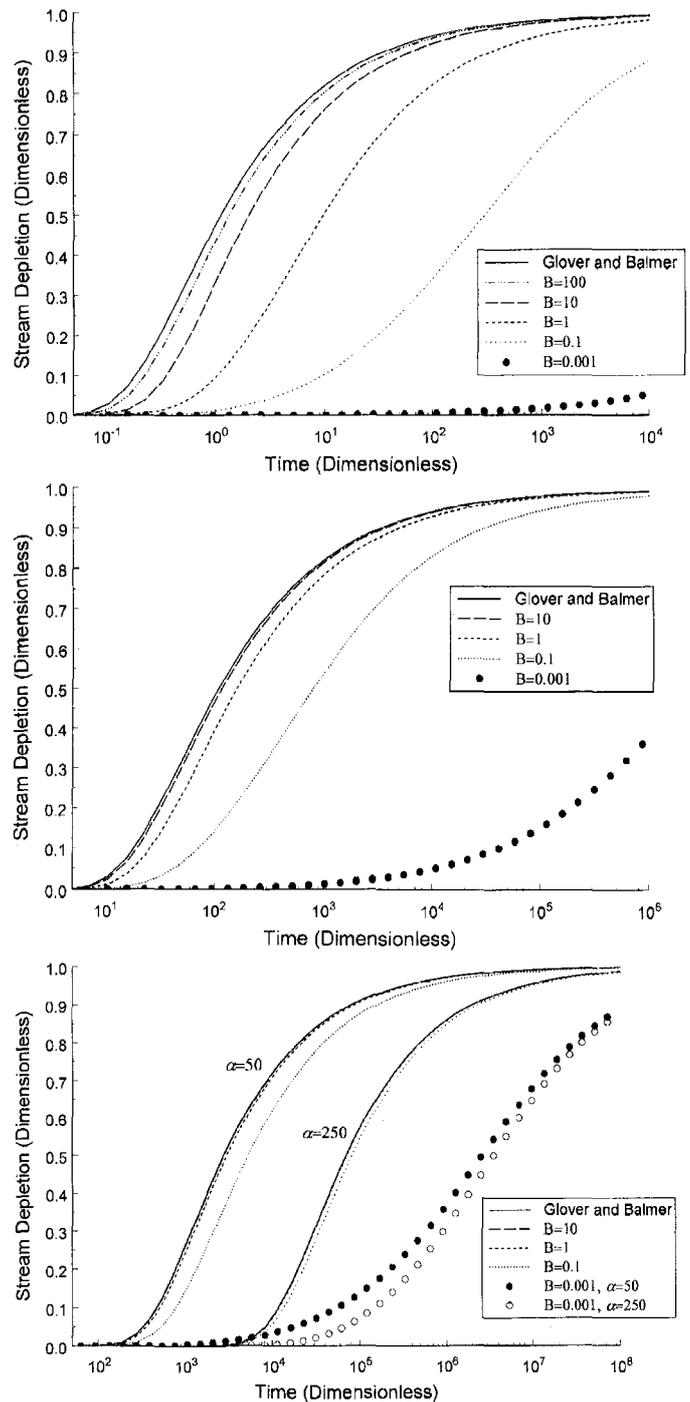

Figure 6. Dimensionless stream depletion versus time plots of the analytical model of Glover and Balmer (1954) and the new solution for an aquifer of infinite lateral extent: (a) $\alpha = 1$; (b) $\alpha = 10$; (c) $\alpha = 50$ and 250.

respectively. Reported values for k' and b' (Conrad and Beljin 1996) indicate that 0.1 to 1 m/day and 0.1 m, respectively, would be plausible values for these quantities. Using these values and a stream width of 10 to 20 m to represent conditions in streams of a moderate size, a range of 0.1 to 5.0 is obtained for B. Figures 6a to 6c show that sizable error (can exceed 1000%) may be introduced when the Glover and Balmer model is used in these conditions. Only when the pumping well is at a relatively large distance from the stream (α > 200 to 250) can the k'/b' ratio be neglected (Figure 6c).

It is important to note that the practical significance of the error introduced by the Glover and Balmer model for a stream of shal-
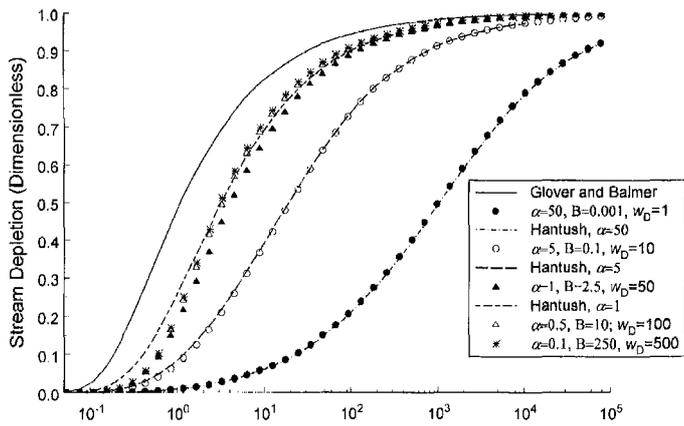
Figure 7. Dimensionless stream depletion versus modified time $(T_3 t/a^2 S_3)$ plot illustrating the dependence of the new solution on stream width for a laterally infinite aquifer (α and B vary only with stream width; $w_D = w/w_{\alpha=50}$; analytical models of Glover and Balmer [1954] and Hantush [1965] shown for reference).

low penetration will depend on the time (duration of pumping) at which the stream-depletion curve for a certain B converges on that for the Glover and Balmer model. Assuming an unconfined aquifer (specific yield = 0.10) and the parameter values of the preceding paragraph, the B = 1 and B = 0.1 curves of Figure 6b (α = 10) will converge (within 20%) on the stream-depletion curve for the Glover and Balmer model after 3.5 and 150 days of pumping, respectively. Clearly, this error can be of considerable practical significance.

The discussion of this section demonstrates that the conclusions drawn from consideration of drawdown and stream depletion are quite similar. Thus, for the sake of succinctness, the analysis and discussion will focus on stream depletion for the remainder of the paper.

Dependence on Stream Width

Stream width (w) is not considered in existing analytical approaches for estimation of stream depletion (Jenkins 1968; Hunt 1999). However, for streams of shallow penetration, width can be a factor of some importance (Bochever 1966). Figure 7 is a stream-depletion plot in which w is varied by a factor of 500 but all other quantities are kept constant. The distance between the pumping well and the stream does not vary, but α and B change with increases in w (α decreases while B increases). The Glover-Balmer and Hantush models are shown for reference. Note that a modified dimensionless time parameter using distance to the pumping well as the normalizing length $(T_3 t/a^2 S_3)$ is employed to illustrate the dependence on stream width.

The lack of convergence on the Glover-Balmer solution with increases in w can be explained by considering the definition of stream depletion (Equation 11) and the differences in the hydrogeologic settings (Figures 1a and 2). The Glover-Balmer model assumes a constant-head boundary at the stream edge, so, unless the k'/b' ratio is large, the stream-depletion curves for the new solution will always be offset from the Glover-Balmer model. This offset is a result of the additional time required for the drawdown cone to propagate beneath a large enough area of the streambed to produce the same leakage. With further increases in stream width, the curves for the new solution will converge because the incremental change in stream depletion gradually decreases due to drawdown diminishing with distance from the pumping well. Eventually, w
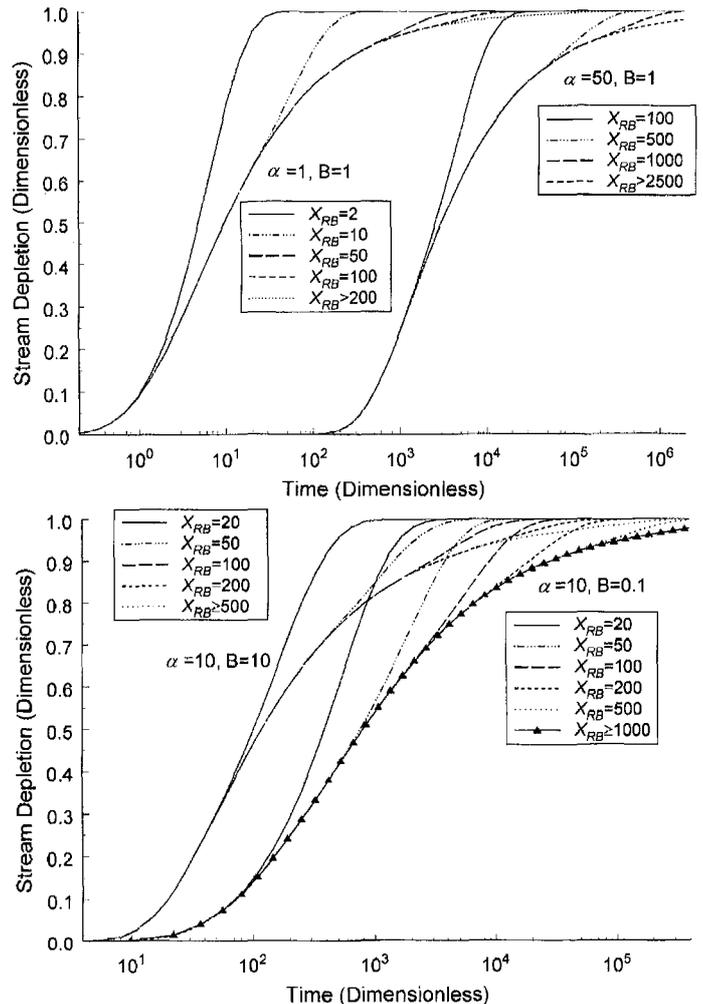

Figure 8. Dimensionless stream depletion versus time plots illustrating the dependence on aquifer width (stream located at valley center): (a) α = 1 and 50, B = 1; (b) α = 10, B = 0.1 and 10.

becomes so large and the drawdown underneath the far side of the stream so small that stream depletion essentially does not change with further increases in width. Thus, an extremely wide stream will not necessarily be well represented by the Glover-Balmer model.

Hunt (1999) describes the correspondence between the Hantush stream leakance parameter and his λ parameter. Butler and Tsou (2000) use this correspondence to derive conditions (aquifer thickness = 0.5w) for which the Hantush solution is an approximation of the solution proposed here. As shown in Figure 7, this approximation is quite reasonable when the pumping well is a normalized distance of five or more from the stream ($\alpha \geq 5$) in a laterally infinite aquifer.

Dependence on Lateral Boundaries

The discussion of the preceding sections focused on aquifers that can be conceptualized as laterally infinite units. In many situations, however, the aquifer is of relatively limited lateral extent (Rosenshein 1988; Larkin and Sharp 1992). Figures 8a and 8b display the dependence of stream depletion on aquifer width for a stream in the middle of an alluvial valley. The width above which the aquifer can be considered as laterally infinite will depend on the normalized distance from the pumping well to the stream (α) and, to a lesser extent, the stream leakance parameter (B). In all cases, as α increases, the width above which the laterally infinite assumption is reasonable increases (Figure 8a). Conversely, as B increases,
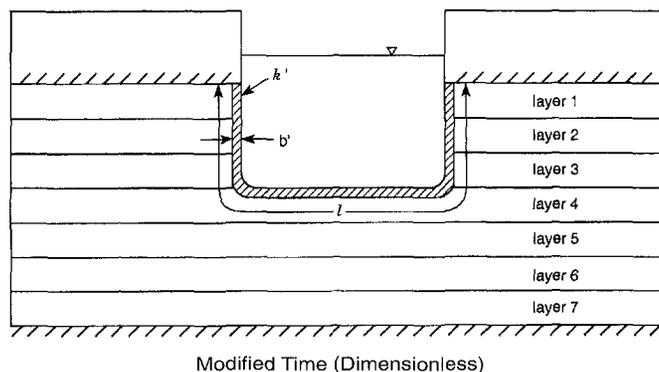
Figure 9. Cross-sectional view of partially penetrating stream configuration investigated with seven-layer MODFLOW model (see Table 2 for model details, notation defined in text).
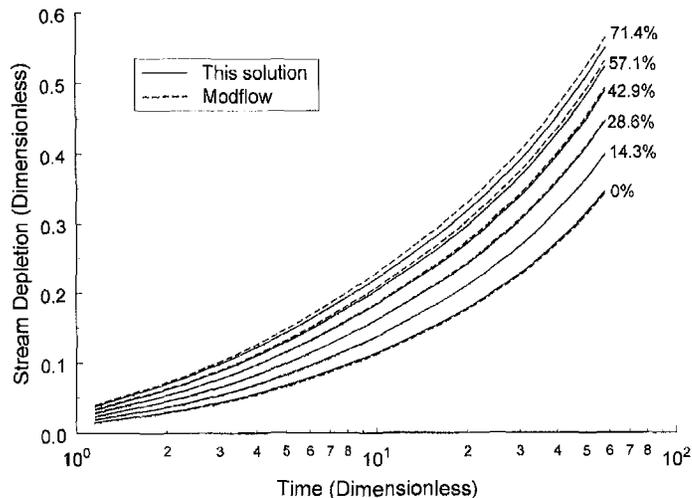

Figure 10. Dimensionless stream depletion versus time plot comparing the extended analytical solution to the seven-layer MODFLOW model ($\alpha = 1.1$; B $= 0.12$ for case of 0% penetration) percent penetration ranging from 0 (stream above first layer of MODFLOW model of Figure 9) to 71.4 (stream above sixth layer of MODFLOW model).

the width above which the laterally infinite assumption is reasonable decreases (Figure 8b). For most conditions, aquifer width must be on the order of hundreds of stream widths before the assumption of a laterally infinite aquifer is appropriate. Given the results shown in Figures 8a and 8b and the fact that the stream channel may often be located near one side of the valley, the impact of lateral boundaries on stream depletion calculations will often be of considerable practical significance.

Dependence on Degree of Penetration

This new analytical model is based on the assumption that the depth of penetration is small relative to aquifer thickness (Figure 2). In many situations, however, the degree of penetration is not negligible. Since a significant degree of penetration will lead to an increase in the perimeter of the streambed relative to the negligible-penetration case, an approximate extension of the model to a significant degree of penetration can be obtained through a redefinition of B:

$$B^* = \frac{Bl}{w} = \frac{k'\,wl}{b'\,T_2} \tag{13}$$

where (l) is the length along the intersection between the aquifer and the streambed (henceforth, the wetted perimeter). A seven-layer MODFLOW model (Figure 9 and Table 2) was used to assess the viability of this approximate extension of the analytical model. The results presented in Figure 10 demonstrate that the approximations used in the extended model do not translate into errors of practical significance for the range of penetrations expected in most natural systems. If errors of practical significance are defined as deviations greater than 20% of the actual stream depletion, this approximate model is valid for relative penetrations as large as 85%. As shown in Figure 10, estimates of stream depletion are dependent on the degree of penetration when the pumping well is close to the stream. This dependence, however, will decrease for pumping wells located at greater distances from the stream.

The results shown in Figure 10 were for B $= 0.12$ and $\alpha = 1.1$. Closer agreement between the numerical and analytical results will be obtained as B decreases and/or α increases. Although the extended analytical model becomes more approximate with increases in B and/or decreases in α, further simulations have demonstrated that this model will be a reasonable approximation at $\alpha = 1.1$ for B values as large as 11.6 and relative penetrations

from 10% to 50%. For B > 10, the Glover and Balmer solution is a reasonable approximation for all ranges of penetrations at larger α values ($\alpha \geq 10$, Figures 6b and 6c).

The results of the numerical assessment indicate that this approximate extension should enable reasonable stream-depletion estimates to be obtained for the range of stream-channel penetrations found in most natural systems. Although these conclusions are based on results for a confined aquifer, they should also be appropriate for unconfined systems except when pumping causes a significant decrease in the wetted perimeter (l).

Model Limitations

The findings of this study must be considered within the light of the major assumptions that are invoked in the analytical model. Assumptions of note include: (1) vertical flow is negligible; (2) the aquifer is isotropic; (3) aquifer heads remain above the stream bottom; (4) the stream level is unaffected by pumping; and (5) the pumping well is fully screened across the aquifer. In the following paragraphs, the ramifications of each of these assumptions will be briefly considered.

A fundamental assumption of this model is that the vertical component of flow can be neglected (Dupuit approximations). Figure 3b shows that neglect of vertical flow does introduce error into stream-depletion estimates. This error can be of practical significance ($> 20\%$) when B is large (> 1) and α is small (< 1). However, neglect of the vertical component of flow should rarely introduce errors of practical significance outside of these conditions. A nonnegligible degree of stream-channel penetration will further diminish the impact of vertical flow.

An anisotropy in aquifer hydraulic conductivity ($K_h \neq K_v$, where K_h and K_v are horizontal and vertical hydraulic conductivity, respectively) can magnify the error introduced by neglect of vertical flow. Several studies (Sophocleous et al. 1995; Chen and Yin 1999) have shown that anisotropy can be an important control on stream-aquifer interactions. Although not emphasized in this previous work, the impact of anisotropy is a function of the distance from the pumping well to the stream. Additional simulations per-

formed with the seven-layer MODFLOW model found that a moderate degree of anisotropy ($K_h/K_v = 10$) can be neglected when $\alpha > 15$. For a high degree of anisotropy ($K_h/K_v = 100$), the impact can be neglected when $\alpha > 75$. If $B < 1$ and/or the degree of stream-channel penetration is nonnegligible, the distances at which anisotropy can be ignored will be much less. Thus, unless anisotropy is large or the aquifer is thick and the pumping well is close to the stream, neglect of anisotropy should not introduce errors of practical significance.

As Rushton (1999), among others, has pointed out, leakage across the streambed will not increase once aquifer heads fall below the bottom of the stream. In that situation, all of the existing analytical approaches will overpredict stream depletion and can produce errors of practical significance. A numerical model should therefore be used when heads have been drawn beneath a significant portion of the streambed.

The viability of the assumption that stream levels are unaffected by pumping depends on the ratio of the pumping rate to stream discharge. When this ratio is large, stream levels may decrease with time leading to decreases in stream depletion. The analytical model can be used to identify the time at which stream depletion becomes a significant fraction of stream discharge, but it should not be used beyond that point.

The validity of the assumption of a fully penetrating pumping well is a function of the distance from the pumping well to both the stream and the observation well, and the degree of anisotropy. If the distance to both the stream and the observation well is greater than $2b(K_h/K_v)^{1/2}$, where b is aquifer thickness, the error introduced by a partially penetrating pumping well will be negligible (Hantush 1964).

Conclusions

A new analytical solution has been developed for estimation of the drawdown and stream depletion produced by pumping in the vicinity of a stream. The solution is based on a model of the stream-aquifer system that is more realistic than that employed in commonly used analytical methods for estimation of pumping-induced water transfers. This new solution was used to assess the influence of a variety of factors on drawdown and stream-depletion calculations, and to demonstrate the magnitude of the error introduced by commonly used analytical methods. The major findings of this study are described in the following paragraphs.

The assumption of a large stream leakance, which underlies the commonly used fully penetrating stream model of Glover and Balmer (1954), can lead to a significant overestimation of stream depletion (100% to 1000%) in many practical applications. The degree of overestimation depends on the value of the stream leakance parameter (B) and the normalized distance from the pumping well to the stream (α). For pumping wells at large distances from the stream ($\alpha > 250$), this overestimation is usually not of practical significance. Although leakage will increase with stream width, a very wide stream will not necessarily be well represented by the Glover and Balmer model.

The assumption of a laterally infinite aquifer is often adopted for stream-depletion calculations (Glover and Balmer 1954; Hunt 1999). Many natural systems, however, are of quite limited lateral extent. Thus, the impact of lateral boundaries on stream-depletion calculations will frequently be of practical importance. The significance of that impact depends on the distance from the pumping well to the stream and the magnitude of the stream leakage. For most cases, aquifer width must be on the order of hundreds of stream widths before the laterally infinite assumption is appropriate.

This new solution is based on the assumption that stream-channel penetration is negligible relative to aquifer thickness. When applied to conditions of nonnegligible penetration, the solution can introduce considerable error into stream-depletion estimates. However, by including the wetted streambed perimeter in a redefinition of the stream-leakance parameter, an extension of the solution can be obtained that is a reasonable approximation for relative penetrations up to and exceeding 85%. Although the approximation must be used with caution when stream leakance is large (B > 1) and the pumping well is close to the stream ($\alpha < 1$), this extension is applicable for a wide range of conditions expected in natural systems.

The findings of this paper demonstrate that this new solution is quite useful for rapid assessment of pumping-induced water transfers in a wide range of stream-aquifer settings. Regulatory agencies throughout the world currently quantify such transfers using analytical models of idealized stream-aquifer configurations that often bear little resemblance to natural systems. Since this solution greatly extends the range of conditions that can be considered with analytical methods, it could prove to be a valuable new tool for water management design and water rights adjudication purposes.

Code and Reference Availability

An executable file for this model and example input and result files can be downloaded from the KGS Stream-Aquifer Interactions Web site (www.kgs.ukans.edu/StreamAq). The Fortran code is available from the KGS Publication and Sales Office for a nominal charge [request Computer Program Series #99-1 (Butler and Tsou 1999)]. Since the Zlotnik et al. (1999) and Butler and Tsou (2000) references are not widely available, downloadable copies of these documents can also be found on the KGS Stream-Aquifer Interactions Web site.

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Appendix

In this section, the transform-space form of the solution to the mathematical model defined by Equations 1 through 11 is presented.

A complete derivation using dimensionless parameters is given in Butler and Tsou (2000).

A solution for drawdown can be obtained from Equations 1 through 10 using integral transforms (Robinson 1968). Except for the addition of lateral boundaries and the stream leakance term, the approach is equivalent to that of Butler and Liu (1991). A Laplace transform in time followed by an exponential Fourier transform in the y direction produces Fourier-Laplace space analogs to Equations 1 through 3. The solution to these equations in Fourier-Laplace space can be written as

$$\overline{\overline{s}}_1(x, \omega, p) = k_1 [e^{2\lambda_1 x_{lb} + \lambda_1 x} + e^{-\lambda_1 x}] \qquad (A1)$$

$$\overline{\overline{s}}_2(x, \omega, p) = k_1 [a_1 e^{\lambda_2 x} + b_1 e^{-\lambda_2 x}] \qquad (A2)$$

$$\overline{\overline{s}}_3(x, \omega, p) = k_1 [c_1 e^{\lambda_3 x} + d_1 e^{-\lambda_3 x}], 0 \le x < a \qquad (A3a)$$

$$\overline{\overline{s}}_3(x, \omega, p) = \left(\frac{k_1 f_1}{g_1}\right)[e^{\lambda_3 x} + e^{2\lambda_3 x_{rb} - \lambda_3 x}], a < x \le x_{rb} \qquad (A3b)$$

where $\overline{\overline{s}}_i$ is the Fourier-Laplace transform of s_i; p is the Laplace transform variable; ω is the Fourier transform variable; $\lambda_1 = (\omega^2 + P_1 p)^{1/2}$; $\lambda_2 = (\omega^2 + L + P_2 p)^{1/2}$; $\lambda_3 = (\omega^2 + P_3 p)^{1/2}$; $P_i = S_i/T_i$; $L = k'/(b'T_2)$; $a_1 = \frac{1}{2}(e^{2\lambda_1 x_{lb} - \lambda_1 w + \lambda_2 w} + e^{(\lambda_1 + \lambda_2)w}) + \frac{\lambda_1}{2\gamma_1 \lambda_2}(e^{2\lambda_1 x_{lb} - \lambda_1 w + \lambda_2 w} - e^{(\lambda_1 + \lambda_2)w})$; $b_1 = \frac{1}{2}(e^{2\lambda_1 x_{lb} - \lambda_1 w - \lambda_2 w} + e^{(\lambda_1 - \lambda_2)w}) - \frac{\lambda_1}{2\gamma_1 \lambda_2}(e^{2\lambda_1 x_{lb} - \lambda_1 w - \lambda_2 w} - e^{(\lambda_1 - \lambda_2)w})$; $c_1 = \frac{1}{2}[a_1 + b_1] + \frac{\lambda_2}{2\gamma_2 \lambda_3}[a_1 - b_1]$; $d_1 = \frac{1}{2}[a_1 + b_1] - \frac{\lambda_2}{2\gamma_2 \lambda_3}[a_1 - b_1]$; $e_1 = -Q/(T_3 \lambda_3 p \sqrt{2\pi})$; $f_1 = c_1 e^{\lambda_3 a} + d_1 e^{\lambda_3 a}$; $g_1 = e^{\lambda_3 a} + e^{2\lambda_3 x_{rb} - \lambda_3 a}$; $h_1 = c_1 e^{\lambda_3 a} - d_1 e^{-\lambda_3 a}$; $j_1 = e^{\lambda_3 a} - e^{2\lambda_3 x_{rb} - \lambda_3 a}$; $k_1 = e_1 g_1 /[f_1 j_1 - h_1 g_1]$; $\gamma_1 = T_2/T_1$; and $\gamma_2 = T_3/T_2$.

A numerical scheme based on that of Butler and Liu (1991) is used to invert the Fourier-Laplace space expressions into real space. This scheme, which is implemented in Butler and Tsou (1999), uses Romberg integration (Carnahan et al. 1969) and the Stehfest (1970) algorithm to evaluate the Fourier and Laplace inversion integrals, respectively.

The solution for stream depletion is found using an approach similar to that of Hunt (1999) to obtain a Laplace-space expression for stream depletion in a laterally bounded aquifer:

$$\overline{\Delta q}(p) = \frac{k_1}{\lambda_2^*}\left[a_1\left(1 - e^{-\lambda_2^* w}\right) - b_1\left(1 - e^{\lambda_2^* w}\right)\right] \qquad (A4)$$

where $\overline{\Delta q}$ is the Laplace transform of Δq and $\lambda_2^* = (L + P_2 p)^{1/2}$ This expression is numerically inverted using the Stehfest algorithm (Butler and Tsou 1999).