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The Design and Enactment of Modeling Tasks: A Study on the Development of Modeling Abilities in a Secondary Mathematics Course

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THE DESIGN AND ENACTMENT OF MODELING TASKS:
A STUDY ON THE DEVELOPMENT OF MODELING ABILITIES
IN A SECONDARY MATHEMATICS COURSE

by

Danielle Buhrman

A DISSERTATION

Presented to the Faculty of
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Major: Educational Studies

Under the Supervision of Professor Stephen A. Swidler

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This study uses components of action and self-study research to examine the design and enactment of modeling tasks with the goal of developing student modeling abilities. The author, a secondary mathematics teacher, first closely examined the curriculum design and instructional decisions she made as she prepared for a unit on mathematical modeling in her precalculus course. Detailed descriptions of the timeline of events as this unit was enacted is then presented, providing insight into student and teacher interactions that occurred during an intense five weeks of modeling instruction. An analysis of these events identified ways in which students relearned how to learn, the teacher-researcher relearned to teach, and how both students and teacher relearned mathematics within authentic tasks. Finally, the overall development of modeling abilities as a result of this relearning and implications for this research are discussed.
For Brooks and Baby B

May you have an impossible dream that becomes possible.
Acknowledgements

*I can do all things through Christ who strengthens me.*

~Philippians 3:14

Finding the motivation to see this dissertation to the end would not have been possible without the support of countless family, friends, and colleagues, and without the strength of my Lord and Savior Jesus Christ. The timeline of events as they played out during a month of modeling occurred only weeks after my husband and I lost a baby through miscarriage. Feeling broken, this support system never let me give up on myself or this impossible dream.

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# TABLE OF CONTENTS

INTRODUCTION........................................................................................................... 1

My Personal Mathematics Story............................................................................... 2

Undergraduate Education......................................................................................... 3

From Novice to Master Teacher................................................................................ 4

My Reflective Journey in Precalculus...................................................................... 6

Student-Centered Instruction..................................................................................... 7

Authentic Tasks......................................................................................................... 8

Intersecting Ideas: Mathematical Modeling.............................................................. 10

Why Modeling?........................................................................................................... 12

Modeling in Precalculus........................................................................................... 12

My Study.................................................................................................................... 14

PART ONE: EXPANDING MY KNOWLEDGE OF MODELING..................................... 17

CHAPTER 1: PROBLEM AND PURPOSE..................................................................... 18

Modeling Cycle........................................................................................................... 19

Modeling in the Common Core State Standards...................................................... 20

The Modeling Process as Defined by Research Literature...................................... 21

Modeling versus Problem Solving........................................................................... 23

Importance of Modeling........................................................................................... 25

Development of Modeling Abilities.......................................................................... 26

Theories on Student Learning of Modeling............................................................... 26

Developed Student Skills.......................................................................................... 29

Teaching to Develop Modeling Abilities.................................................................... 35
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curriculum</td>
<td>38</td>
</tr>
<tr>
<td>Design Guidelines</td>
<td>39</td>
</tr>
<tr>
<td>Reflecting on Research Problem to Identify Study Purpose</td>
<td>43</td>
</tr>
<tr>
<td>Research Study</td>
<td>44</td>
</tr>
<tr>
<td>Central Research Questions</td>
<td>45</td>
</tr>
<tr>
<td>Sub-Questions</td>
<td>45</td>
</tr>
<tr>
<td>CHAPTER 2: METHODOLOGY</td>
<td>46</td>
</tr>
<tr>
<td>Qualitative Research Methods</td>
<td>46</td>
</tr>
<tr>
<td>Self-Study</td>
<td>47</td>
</tr>
<tr>
<td>Action Research</td>
<td>48</td>
</tr>
<tr>
<td>Design Research</td>
<td>48</td>
</tr>
<tr>
<td>Understanding Methods Leads to Research Plan</td>
<td>49</td>
</tr>
<tr>
<td>Research Plan</td>
<td>45</td>
</tr>
<tr>
<td>Design</td>
<td>50</td>
</tr>
<tr>
<td>Data Collection Instruments</td>
<td>54</td>
</tr>
<tr>
<td>Reliability</td>
<td>62</td>
</tr>
<tr>
<td>Data Analysis</td>
<td>64</td>
</tr>
<tr>
<td>Conclusion</td>
<td>65</td>
</tr>
<tr>
<td>CHAPTER 3: THE PREQUEL TO A MONTH OF MODELING</td>
<td>66</td>
</tr>
<tr>
<td>Development of Design Criteria</td>
<td>68</td>
</tr>
<tr>
<td>Targeting Specific Modeling Abilities</td>
<td>69</td>
</tr>
<tr>
<td>Criteria for Modeling Tasks</td>
<td>76</td>
</tr>
<tr>
<td>Unit Design</td>
<td>80</td>
</tr>
</tbody>
</table>
CHAPTER 6: RELEARNING HOW TO TEACH

Preparing to Teach

What is Mathematics?

Assessing this New Mathematics

Teacher is Not the Only Expert

Reflections on My Preparations to Teach Modeling

Changing Perception of Student Needs

Encouraging Teamwork

Student-to-Student Discourse

Reflections on Support Needed

Activating Students’ Real Life Knowledge

To Develop Mathematical Literacy

To Develop Quantitative Literacy Skills and Reflection

Reflections on the Need for Real Life Knowledge
<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Role of Teacher Questioning</td>
<td>299</td>
</tr>
<tr>
<td>From Funneling to Focusing</td>
<td>300</td>
</tr>
<tr>
<td>Questions to Develop Quantitative Literacy Skills</td>
<td>307</td>
</tr>
<tr>
<td>Reflections on the Role of Questioning</td>
<td>323</td>
</tr>
<tr>
<td>Conclusion</td>
<td>325</td>
</tr>
<tr>
<td>CHAPTER 7: RELEARNING MATHEMATICS WITHIN AUTHENTIC TASK</td>
<td>326</td>
</tr>
<tr>
<td>Authentic Situations</td>
<td>326</td>
</tr>
<tr>
<td>The Need for Identifying and Stating Assumptions</td>
<td>327</td>
</tr>
<tr>
<td>Assumptions in Project 1</td>
<td>328</td>
</tr>
<tr>
<td>Assumptions in Project 2</td>
<td>329</td>
</tr>
<tr>
<td>Assumptions About Data in Projects 3, 4, and 5</td>
<td>345</td>
</tr>
<tr>
<td>Recognizing Limitations to Models Developed</td>
<td>347</td>
</tr>
<tr>
<td>Limitations and Its Relationship to Assumptions</td>
<td>348</td>
</tr>
<tr>
<td>Limitations as a Result of Real Life Implications</td>
<td>349</td>
</tr>
<tr>
<td>Reflections on Limitations Within Authentic Situations</td>
<td>353</td>
</tr>
<tr>
<td>Conclusion</td>
<td>354</td>
</tr>
<tr>
<td>CHAPTER 8: CONCLUSION</td>
<td>356</td>
</tr>
<tr>
<td>Reflections on My Study</td>
<td>357</td>
</tr>
<tr>
<td>Development of Modeling Abilities</td>
<td>358</td>
</tr>
<tr>
<td>Lessons Learned</td>
<td>361</td>
</tr>
<tr>
<td>The It is Important</td>
<td>361</td>
</tr>
<tr>
<td>Modeling is Not Quick and Easy</td>
<td>363</td>
</tr>
<tr>
<td>Authenticity is Essential</td>
<td>365</td>
</tr>
</tbody>
</table>
Mathematical Power and How One Develops It........................................... 367
Communication Needs to be Two-way.................................................... 370
Teaching is Research.............................................................................. 371
How Modeling has Evolved in My Classroom........................................ 372
Expanding Outside My Walls.................................................................. 374
Future Research..................................................................................... 375
In Conclusion.......................................................................................... 376
REFERENCES.......................................................................................... 377
APPENDICES............................................................................................ 402
List of Figures

Figure 0.1 Mathematical modeling seems to result in student outcomes consistent with student-centered instruction and the use of authentic tasks........................................... 11

Figure 1.1 Four-step modeling cycle........................................... 19

Figure 1.2 Modeling cycle involving four basic steps....................... 21

Figure 1.3 Traditional views of problem solving............................. 24

Figure 1.4 Modeling perspectives of problem solving...................... 24

Figure 1.5 Intuition of structure................................................ 27

Figure 1.6 Mathematical modeling schema.................................... 32

Figure 2.1 Triangulation matrix showing data sources, frequency, and duration......................................................... 63

Figure 3.1 Modeling abilities developed from NCTM’s four-step modeling cycle......................................................... 70

Figure 4.1 Samantha’s work showing Team 2’s “guess and check” method using minutes per gigabyte rates....................... 125

Figure 4.2 Evan’s guess and check work....................................... 127

Figure 4.3 Chris’s verification of Evan’s guess and check calculations......................................................... 128

Figure 4.4 Models developed by students using gigabytes per minute rate......................................................... 131

Figure 4.5 Models developed by students using minutes per gigabyte rate......................................................... 132
Figure 4.6  Dylan’s individual work showing the model his team had developed………………………………………………… 137

Figure 4.7  Geometric representation of measurements taken for the Width of Memorial Stadium project……………………… 140

Figure 4.8  Luis’s work to find the width of Memorial Stadium……… 143

Figure 4.9  Emily’s work showing all calculated angles and distances.. 145

Figure 4.10  Megan’s work showing calculations as a result of assuming parallel distances…………………………………… 146

Figure 4.11  Samantha’s individual work showing Team 2’s calculations for finding an average percentage growth in high school enrollment…………………………………… 154

Figure 4.12  Samantha’s work showing their modified model using only the last four years of enrollment figures………………… 156

Figure 4.13  Alicia’s individual work showing Team 3’s incorrect strategy for calculating the average percentage growth…… 157

Figure 4.14  A slide from Team 1’s presentation comparing their linear and exponential model…………………………………… 159

Figure 4.15  Ranking developed by Teams 3 and 5 where property and violent crimes are calculated equally…………………….. 170

Figure 4.16  Ranking developed by Team 2 by weighing violent crimes twice as much as property crimes………………………… 172

Figure 4.17  Ranking developed by Team 1 using a point system………. 173

Figure 4.18  Ranking developed by Team 4………………………………. 174
Figure 4.19  Comparison of rankings developed from teams using unweighted and weighted scores……………………. 175

Figure 4.20  Graph and comparison of sports brands used during Team 1’s presentation…………………………. 190

Figure 4.21  Team 2’s incorrect model finding the number of blades of grass on the football field’s turf……………………. 192

Figure 4.22  Graph of holiday and retail sales linear regression models showing growth of spending……………………. 195

Figure 4.23  Team 3’s incorrect assessment of 19.74% of annual spending coming from holiday sales…………………. 196

Figure 4.24  Team 4’s graph showing three models they created and tested for validity……………………………………. 199

Figure 4.25  Team 6’s graph showing the number of animals arriving, leaving, adopted, and euthanized each year beginning in 2010…………………………………………………………. 203

Figure 4.26  Data set used and graphs of linear models developed by William, Matt, and Elijah to predict when the entire U.S. population will have internet access…………………………. 206

Figure 5.1  Summary of the work done by members of Team 5 to validate their model and predictions……………………. 222

Figure 5.2  The last three cities on the rankings for Team 1 and Team 2 showing how the cities are the same but in a different order………………………………………………………. 234
Figure 5.3  A comparison of Team 4’s ranking of most dangerous cities to a ranking developed by teams with a similar methodology…………………………………….. 236

Figure 5.4  Chris’s individual work showing his calculations for getting an answer of 753.97…………………………………….. 244

Figure 7.1  Chris’s scale drawing of the measurements given for the Width of Memorial Stadium Project………………………… 330

Figure 7.2  Dylan’s picture referencing the triangles Team 5 was using for their calculations………………………………………. 336

Figure 7.3  Picture showing variables used in the models developed by Evan, William, and myself……………………………….. 339

Figure 7.4  Specific case where X and W are known to be parallel…… 343

Figure 7.5  A slide from Team 6 demonstrating the ways in which students identified potential limitations to their model…….. 350
List of Appendices

Appendix A  Modeling Unit Student Reflection………………………… 402
Appendix B  Samantha Interview Protocol………………………………… 403
Appendix C  Chris Interview Protocol……………………………………… 405
Appendix D  Brooke Interview Protocol…………………………………… 407
Appendix E  Matt Interview Protocol……………………………………… 410
Appendix F  Luis Interview Protocol……………………………………… 413
Appendix G  Intro To Modeling Classroom Document…………………… 415
Appendix H  Recording Quality Project 1 Classroom Document……….. 420
Appendix I  Width of Memorial Stadium Project 2 Classroom Document…………………………………… 422
Appendix J  Data for Project 3 Classroom Document…………………… 424
Appendix K  River Junction Enrollment Project 3 Classroom Document……………………………………… 425
Appendix L  Crime Safety Rankings Project 4 Classroom Document…. 426
Appendix M  Student Choice Project 5 Classroom Document…………… 428
Appendix N  Individual Modeling Cycle Rubric…………………………. 430
Appendix O  Team Product Rubric………………………………………. 433
Appendix P  Modeling Cycle Graphic Organizer………………………. 435
Appendix Q  Project 5 Guiding Questions Classroom Document………. 436
Appendix R  Crime Safety Ranking Presentation Graphic Organizer….. 437
Appendix S  Student Choice Presentations Graphic Organizer……….. 438
INTRODUCTION

It has arguably never been so hard to be a mathematics teacher. On a daily basis teachers must juggle demands from government legislation, state assessments, content standards, district mandates, and the needs of an increasingly diverse student population. With additional pressures to be competitive internationally and meet the rising demand for STEM fields, mathematics teachers are seemingly desperate for instructional methods and tasks that help students with varying backgrounds and abilities develop mathematical knowledge going beyond procedural fluency.

Reform efforts brought on by the National Council of Teachers of Mathematics (NCTM) and the Common Core State Standards Initiative (CCSS) focus less on the results of a mathematical computation and more on the student thinking and understanding going into the development and application of the mathematical skill. In NCTM’s (2000) Principles and Standards for School Mathematics process standards of problem solving, communication, reasoning and proof, connections, and representations have just as much emphasis K-12 as specific content skills. The National Governors Association Center for Best Practices and Council of Chief State School Officers’ (2010) eight Common Core Standards for Mathematical Practice mirror these ideas in their use of reasoning, modeling, communication, and application. NCTM’s publication Principles to Actions: Ensuring Mathematical Success for All (Leinwand et al., 2014) suggests in order to address these processes and thinking standards, teachers should implement tasks that promote reasoning and problem solving, facilitate meaningful mathematical discourse, and support productive struggle in learning mathematics.
Applying these instructional techniques can be difficult for some teachers, however, because it may contradict perceptions of how students learn mathematics based on a teacher’s own learning background. For example, I continually struggle to teach mathematics in a way that is different than how I experienced it as a student growing up in the 1990’s and early 2000’s.

My Personal Mathematics Story

For as long as I can remember, I loved math. I grew up in a small, rural farming community and attended a school known for its academic success, particularly in mathematics, which only fueled my love for the subject. I appreciated mathematics rigidity, focus on correct solutions, and procedures I could memorize and apply. I had the same math teacher, Mrs. Anderson, all four years of high school by the time I graduated in 2004, and she instilled in me a strong work ethic combined with high expectations. Each day of class we were expected to take diligent notes, listen intently, answer questions when called upon, and complete a quiz covering the homework assignment given to practice the skills she presented to us. Desks were arranged in rows with enough space between students seeing each other’s work was impossible, and if you talked to your peers during her lecture you were quickly reprimanded. Mrs. Anderson’s class meant business, but students respected her because of the results she got out of them. And if students did not meet expectations, she was willing to be at school until late at night helping you reach the bar she had set. Standardized tests scores at our high school were always highest in mathematics; and after I scored a 35 on the mathematics portion
of the ACT test my senior year of high school, she encouraged me to take advanced mathematics courses at the postsecondary level.

**Undergraduate Education**

During my undergraduate education I continued experiencing mathematics in a similar manner as I pursued a degree in secondary mathematics education. Specifically, I perceived my professor as the generator of knowledge, and it was my duty to process and apply the skills and procedures shown to me. I took notes religiously in class, worked hard, asked questions, and finished my academic coursework with a 4.0 grade point average. I appreciated even more the experiences I had with Mrs. Anderson as they closely mimicked and thus prepared me for my undergraduate mathematics courses. As I began my final student teaching term, I was excited to provide students with the experiences and classroom structures I found so successful with Mrs. Anderson while also providing the support and encouragement needed to meet such rigorous standards.

**Student teaching.** Recognizing, at least, the experiences I had growing up in my small, rural farming community were not likely going to prepare me for the urban populations I was more inclined to teach in, I decided to student teach in one of the largest and most diverse schools in my home state, River Junction Senior High. There I was fortunate enough to learn under my cooperating teacher, Amy Nebesniak, while teaching students in algebra and algebra support classes. Mrs. Nebesniak’s teaching style was about as far from Mrs. Anderson’s as one can be. Students in her class were grouped into teams of four students, cooperative learning was utilized daily, conversations between students were encouraged, relationship building was essential, hands-on
activities and manipulatives were the norm, and conceptual understanding in addition to procedural fluency were expected. Students in Mrs. Nebesniak’s classes were the generators of knowledge. They constructed their own ideas and developed new knowledge by making connections to and building upon previously learned concepts. The learning environment Mrs. Nebesniak had created contradicted all experiences I had as a learner; there were many tears and conflicts as these new teaching experiences challenged my old beliefs, but I realized quickly her instructional techniques were much more effective in helping low income and struggling learners. I had to re-examine my perceptions of the teacher role within the classroom and change beliefs on student learning in order to address the needs of students in her classroom. The teacher in this environment could not be viewed as the ‘keeper of knowledge’ with students passively absorbing this knowledge. These new perceptions and beliefs would continue to develop as I was given my own classroom at River Junction Senior High, allowing me to continue interacting with students who had little in common with my own background and strengths as a learner.

**From Novice to Master Teacher**

Over the course of eight years teaching at River Junction Senior High, I earned a Master’s of Arts in Teaching degree, was selected as the youngest participant in the coveted Noyce Master Teaching Fellowship for my state, and in 2013 achieved National Professional Board Certification in Teaching. At River Junction Senior High, my primary teaching responsibilities included precalculus, geometry, and support and remediation for algebra students. These teaching opportunities coupled with my
experiences as a learner in the classroom continued to develop my beliefs on the teaching and learning of mathematics.

Like Mrs. Nebesniak, in my classroom I keep students in teams of three to four students allowing for conversations and cooperative learning strategies to be the norm. When designing lessons, desired student outcomes must demonstrate conceptual understanding along with procedural fluency. And unlike Mrs. Anderson, I try not to give rigid presentations or lectures; new knowledge is typically gained through explorations, team discussions, or student responses to questions I ask activating prior knowledge. Despite the knowledge I have gained, however, I continue to want more for my students. Balancing more rigorous standards with increasing student needs forces me to constantly reflect, analyze, and expand my beliefs and practices. As I think about my students and their future aspirations, however, I worry I am not providing opportunities for students to experience mathematics in a way that leads to the development of social, problem solving, and critical thinking skills needed for their personal and professional lives. My goal, as Hawkins (2003) suggests, is for children to acquire skills necessary to educate themselves once “the institutional part of education is finished” (p. 56). Changing my practice to account for skills beyond a list of mathematics standards, unfortunately, may require instruction and knowledge outside of my expertise. Therefore in preparation for this research study I found myself asking: (1) how do I expand my own learning in order to help students develop mathematics and thinking skills needed to be productive citizens and (2) on what do I focus this learning?
My Reflective Journey in Precalculus

In an effort to answer these questions, I first embarked on a reflective analysis of my teaching practices in my precalculus courses in order to take informed action (Brookfield, 1995). The primary purpose of precalculus is to prepare students for the skills necessary for successful completion of calculus (Karim, Leisher, & Liu, 2010). Topics in precalculus may mimic those found in college algebra courses focusing on linear, polynomial, rational, exponential, and logarithmic functions and their special features including but not limited to asymptotes, zeros, inverses, and end behavior (Demana et al., 2011). However, precalculus also includes a thorough study of trigonometric functions consisting of methods to solve trigonometric equations, graphs, inverses, applications, and identities (Demana et al., 2011). Students enrolled in precalculus typically do so as the capstone of their mathematics coursework or in preparation for calculus (Karim, Leisher, & Liu, 2010).

I decided to focus on precalculus as I perceived my instruction and curriculum resources in this course as the most traditional. My precalculus resources seemed to focus on specific mathematics skills and lacked opportunities for students to develop more meaningful and overarching skills and connections. In my precalculus courses students typically take a lot of notes, complete lengthy assignments practicing the skills presented in class, work individually, and generally want to be told what to do. In my experience, these students tend to be quick memorizers, like I was in my own mathematics courses. As a result, I found this course the most difficult to move away from the type of instruction I experienced with Mrs. Anderson. However, I needed to
identify specific aspects of my instruction to target. To help me narrow the scope of possibilities, I kept a reflection journal and asked myself ‘what do I struggle with on a day-to-day basis?’ ‘What specifically do I want my students to be able to do?’ ‘How could I focus my attention on improving a singular aspect of my teaching that made the demands of teaching mathematics beyond just the standards feel less daunting?’ I found I particularly desired to improve instruction in precalculus that was student-centered and embedded within real-world (authentic) tasks.

**Student-Centered Instruction**

Despite my best efforts to not be the main focus in my classroom, in my reflective analysis I noticed much of the dialogue and learning in my classroom was teacher-driven. Although I realize that at times teacher-centered approaches can produce worthwhile results, I felt my instruction was too one-sided and I needed to embed more student-centered instruction and tasks. According to Felder and Brent (1996), student-centered instruction substitutes “active learning experiences for lectures, holding students responsible for material that has not been explicitly discussed in class, assigning open-ended problems and problems requiring critical or creative thinking that cannot be solved by following text examples” (p. 1). Similarly, Hannifin and Land (1997) describe student-centered activities as ones which address students’ individual interests while deepening their understanding and thinking about the topic being studied. Several studies have found student-centered instruction has a positive effect on student achievement particularly on minority student populations (Clements, Agodini, & Harris 2013, Brown 2003, Salinas & Garr 2009). Of particular relevance in mathematics is using
problem-based learning to promote student-centered instruction (Savery, 2006, Hmelo-Silver, 2004, Glasgow, 1997) and its positive effects on student achievement (Dochy, Segers, Van den Bossche, & Gijbels 2003). Particularly in my precalculus courses, I struggled turning control over such difficult and rigorous content to students in the way this research suggests; how can I provide opportunities for students to make these connections without deliberately guiding them through the learning process?

**Authentic Tasks**

I believe my role as a math teacher should be to prepare students for the types of mathematical situations they will see in their personal and professional life outside of the math classroom. However, in my reflective analysis I found I did little of this in my teaching. Hestenes (2013) suggests this is because K-12 math teachers are ill-prepared to teach mathematics from this perspective. Much of what I did in the classroom was driven by a state test that was very procedure-driven. At the time, these tests only required students to apply skills like “find arc length and area of sectors of a circle” or “determine the surface area and volume of three-dimensional objects” (Nebraska Department of Education, 2009). When I did embed story problems depicting real life situations, I was baffled the majority of my students could not communicate with me the meaning of their answer or explain connections to learned tasks. As I further analyzed my instructional practices related to this, I realized I was doing little to include opportunities for the development of skills like reflection, reasoning, and quantitative literacy students would need to successfully communicate this type of analysis. As a result, I knew I needed to include more authentic tasks requiring students to:
1. Determine the question needing to be answered;
2. Design a plan and determine appropriate strategies to answer the question;
3. Interpret and connect the results of their analysis back to the situation; and
4. Communicate their strategies and findings.

Forman and Steen (2002) suggest authentic tasks are ones embedded in real life contexts often containing realistic yet sometimes incomplete data and allow for multiple strategies and conclusions. In a comprehensive review of literature, Herrington, Oliver, and Reeves (2003) identified ten characteristics of authentic tasks, including the requirement students define the subtasks needed to complete the overall problem from their own interpretation of the situation culminating in the completion of a valuable product. Authentic tasks in this sense are not included in my curriculum resources or required by district mandates, yet I recognized the potential for relevance and rigor for student learning in addition to the learning outcomes I desired to see.

The importance of giving students opportunities to interact in ‘real’ experiences in school can be traced back to Dewey’s (1933) discussion of the scientific mind and the use of schools to give students training of thought and life experiences. Similarly Forman and Steen (2002) argue secondary school mathematics should “include more authentic tasks that reflect the ways in which mathematics arises in life and work” (p. 6) and be “embedded in real contexts, mixing hard and soft data, amenable to multiple approaches and conclusions” (p. 8). Shephard (2000) also argues that “school learning should be authentic and connected to the world outside of school not only to make learning more interesting and motivating to students but also to develop the ability to use knowledge in
real-world settings” (p. 7) By using authentic tasks, students may also become more “aware of the relevancy and meaningfulness of what they are learning” (Nicaise, Gibney, & Crane, 2000, p. 80).

Another, slightly modified view of authenticity falls into the discipline of mathematics itself. Lajoie (1995) suggests that by studying the underlying implications of NCTM standards, authenticity might simply describe what mathematicians do in their domain and be rooted in the skills of problem solving, communicating, reasoning, and making connections. As I reflected on what I wanted my students to be able to do, I realized I desired to use authentic tasks in a way which connects these two viewpoints. This suggests that an authentic task is embedded in real life, both in the language of the situation itself and the thinking skills students need to apply for successful completion of the task. I was convinced these experiences were important to include in my curriculum, but I did not feel confident in my ability to teach these types of tasks which were outside the experiences I had as a learner of mathematics.

**Intersecting Ideas: Mathematical Modeling**

The topics of increasing student-centered instruction and authentic tasks still seemed too large and daunting for a deliberate and focused effort to improve a component of my instruction while still addressing the daily demands of mathematics teaching and learning. As a result, I analyzed my two goals simultaneously and arrived at their intersection: the use of modeling tasks to generate student learning consistent with student-centered instruction and characteristics found in authentic tasks (see Figure 0.1).
Figure 0.1: Mathematical modeling seems to result in student outcomes consistent with student-centered instruction and the use of authentic tasks. Hestenes (2013) specifically calls modeling instruction “a student-centered inquiry approach” facilitated by the teacher (p. 33). Mathematical modeling can be used to connect problem-solving to the real world (de Corte, Verschaffel, & Greer, 2000) while simultaneously using methods typically employed in student-centered instruction: collaborative group work, practical tasks, problem solving, open tasks, and investigation (Bobis et al., 2011).

Researchers do not seem to agree on a singular definition of what constitutes a ‘model.’ Speiser and Walter (2013) state models are simply tools that help make sense of something. Lesh and Doerr (2003) and Lesh and Fennewald (2013) describe models as conceptual systems written using external notation in order to explain, describe, interpret, or construct behaviors or patterns in other systems so that the other system can be manipulated or predicted in a compelling way. Hestenes (2013) further adds to this definition by clarifying a system as “a set of related objects, which may be real or imaginary, physical or mental, simple or composite” (p. 17). Niss, Blu, and Galbraith (2007) suggest mathematical models map relevant phenomena, assumptions, and
questions from an extra-mathematical domain of interest from the real world to a suitable mathematical domain. These models include both conceptual systems for describing and mathematizing the problem situation and accompanying procedures for achieving the situation’s goals (Lesh & Harel, 2003).

**Why Modeling?**

Identifying my problem of practice on the development of mathematical models in my classroom and the learning outcomes as a result of their use initially was relieving. Modeling is emphasized heavily throughout the Common Core State Standards in Mathematics and specifically in its Standards for Mathematical Practices, which my school district would be adopting versions of in the fall of 2016. Focusing my inquiry on modeling would allow me to experience teaching and learning associated with modeling ahead of my peers, allowing me to provide valuable feedback, professional development, and resources for my district. As a practitioner researcher, I am also in the unique position to analyze the teaching of and learning from modeling tasks as a participant in the process. This allows me to experience and analyze the success, conflicts, and new learning associated with mathematical modeling from an insider’s perspective as opposed to experiencing from the outside looking inwards.

**Modeling in Precalculus**

Because of its focus on exploring features of a variety of functions, modeling in precalculus seems to warrant additional attention, support, and learning. For example, the CCSS-M standard HSF.IF.B.4 states:
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. (Corestandards.org, 2014c)

This standard is stressed throughout an entire precalculus course through the exploration and application of algebraic, numeric, and graphical representations of polynomial, exponential, logarithmic, and trigonometric functions. In precalculus, “Students are exposed to functions as mechanisms for modeling data and are motivated to learn about how various functions can help model real-life problems” (Demana, Waits, Foley, & Kennedy, 2011). However, when exploring modeling tasks embedded within my curriculum resources, I did not feel these tasks provided the authentic experience I yearned for my students. Particularly, these resources seemed to address modeling within a particular mathematical content area, perhaps making an appropriate model more obvious and limit student ideas to focus on only one potential solution strategy. For example, we ask students to design a model but when we ask them to do it in the section on graphing quadratics, students are most likely going to use a quadratic or its graph to model the situation. In the precalculus curriculum resource I currently use in the classroom, a typical modeling problem looks like the following:

The population of River City in the year 1910 was 4200. Assume the population increased at a rate of 2.25% per year.

a) Estimate the population in 1930 and 1945
b) Predict when the population reached 20,000. (Demana et al., 2011, p. 271, #32)

I appreciate the effort made by the sub questions to focus on the usefulness of models to estimate and predict, however in the text this question is listed with the heading “Exponential Growth” which leaves the choice aspect out of it for students. In order to help students transfer math knowledge to authentic situations, I feel they must have to think about what type of model would be appropriate in the given situation. In this example, which occurs in the section titled “Exponential and Logistic Modeling,” the text gives the students too much information by telling them to model this with “Exponential Growth” and even provides sample problems in the text similar to this modeling problem for students to follow. Little research has been done on modeling and precalculus, but studies conducted by Gordon (1994 & 2002) found students completing a reform modeling course in precalculus focusing on conceptual understanding, problem-solving, and realistic applications performed better on common precalculus and subsequent calculus assessments than their peers who had completed a traditional precalculus course.

**My Study**

Unfortunately, this relief at identifying a usable problem of practice was soon replaced by nervousness and hesitation as I reflected back to my original question: how do I expand my own learning in order to help students develop mathematics and thinking skills needed to be productive citizens? Modeling allowed for students to be the focus of the learning process while engaging in and experiencing mathematics from a real world perspective, but I knew little about modeling. How do I design modeling tasks which
provide the best opportunity for student learning and authentic experiences? What does that learning look like? What does modeling instruction need to include?

Before I could begin forming the specifics of my problem of practice and in particular, this research study, I needed to drastically expand my knowledge of modeling both from a review of current literature and as it relates to teaching and curriculum design. Therefore, this study is broken into three parts: expanding my knowledge of modeling, a month of modeling, and an analysis of teaching and learning in a month of modeling.

In Part 1: Expanding My Knowledge of Modeling, I describe the research needed and methodology used for the particulars of exploring the teaching and learning associated with mathematical modeling. Within Part One, Chapter 1 outlines the current research on modeling leading to the specific purpose my study is based upon: to understand how a teacher designs and enacts mathematical modeling tasks in order to develop both teacher and student modeling abilities throughout a unit on modeling. Chapter 2 describes the methodology I used to develop, teach, and study the learning that occurred during a unit on modeling. Chapter 3 illustrates the research and planning I conducted to prepare my modeling instruction.

Part Two: Enactment of a Modeling Unit contains Chapter 4 and describes the timeline of events as they played out in my precalculus classroom during a month long unit of modeling instruction. Finally in Part Three: An Analysis of Teaching and Learning in a Month of Modeling, I dive deeper into the three themes which emerged during the modeling unit: relearning how to learn (Chapter 5), relearning how to teach
(Chapter 6), and relearning mathematics within authentic tasks (Chapter 7). Finally, implications for this research are discussed in Chapter 8.
PART ONE
EXPANDING MY KNOWLEDGE OF MODELING

Part One consists of the research I conducted to further understand my problem of practice associated with the teaching and learning of mathematical modeling and the specific methodology used in this research study. Chapter 1 contains a review of literature including descriptions of the modeling process and skills needed to conducted a modeling analysis. Also included is research on best teaching practices and curriculum focused on the development of modeling skills. Chapter 1 concludes with a description of my research study and the research questions I used to focus my methodology. Chapter 2 contains this methodology and the particulars of my data collection. Chapter 3 provides a thorough overview of the process, design, and research I used to develop the unit on mathematical modeling that was the focus of this study.
CHAPTER 1

PROBLEM AND PURPOSE

According to the Common Core State Standards in Mathematics, modeling is "the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions" (Corestandards.org, 2014a, para. 1). In other words, modeling is the entire cycle of creating, analyzing, and applying “conceptual representations of phenomena” (Weber et al., 2014, p. 26). Modeling begins by analyzing phenomena or a situation and attempting to mathematize it (Kaput, 1998). When using model-eliciting activities, students construct knowledge from their perception and experience (Lesh et al., 2000). As such, model-eliciting is synonymous with thought-revealing as “the descriptions, explanations, and constructions that students generate while working on them directly reveal how they are interpreting the mathematical situations” (Lesh et al., 2000, p. 593). The goal of modeling is for individuals to provide descriptions, explanations, and interpretations of their constructs (Lesh & Doerr, 2003) in order to reveal their thinking (Lesh et al., 2000).

But to understand my problem of practice associated with the teaching and learning of mathematical modeling, knowledge of modeling needs to go beyond definitions. In the following review of literature, I outline the modeling cycle learners use as they navigate and mathematize authentic tasks especially as it compares to more general problem solving tasks. I then discuss the importance of modeling within the mathematics curriculum. Following this is a discussion of modeling abilities students develop while engaging in authentic modeling tasks including issues with the
development of mathematical and quantitative literacy skills. I also examined teaching practices, curricular limitations, and design features research suggests may develop modeling abilities. I conclude this chapter by synthesizing this literature to form the purpose of my research study and questions.

**Modeling Cycle**

Mathematical modeling has become an important component of recent reform efforts from NCTM and the Common Core State Standards initiative. NCTM’s publication *Focus in High School Mathematics: Reasoning and Sense Making* (Martin et al., 2009) describes the modeling process as a cycle whereby a person analyzes a real-world situation, builds and tests a model, and interprets the results within the context of the situation (see Figure 1.1).

![Figure 1.1: Four-step modeling cycle. Reprinted from Focus in High School Mathematics: Reasoning and Sense Making (p. 13), by Martin et al., 2009, Reston, VA: National Council of Teachers of Mathematics.](image)

The cyclical nature of this visual representation suggests modeling tasks may require students to repeat various stages of the modeling process as models are tested and refined.
to better represent the real world situation. The writers of *Reasoning and Sense Making* stress mathematical modeling can provide students opportunities to make connections to real-world mathematics along with other mathematical concepts. According to the publication, of particular benefit to students is that modeling tasks provide “incentive and context for, mathematical topics” and allow students to use “knowledge effectively in new situations” (Martin et al., 2009, p.13-14).

**Modeling in the Common Core State Standards**

With the integration of the Common Core State Standards in Mathematics (CCSS-M), and in particular the Standards for Mathematical Practice into mathematics curricula across the country, developing and using modeling tasks has arguably never been more important. CCSS-M defines modeling as “the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions” (Corestandards.org, 2014a, para. 1). CCSS-M clarifies the process through six modeling actions:

1. identifying variables in the situation and selecting those that represent essential features
2. formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables
3. analyzing and performing operations on these relationships to draw conclusions,
4. interpreting the results of the mathematics in terms of the original situation
(5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable,

(6) reporting on the conclusions and the reasoning behind them.

(Corestandards.org, 2014a)

As with other research, CCSS-M suggests this process is not linear and students will move in and out of various components of the cycle as interpretations are changed, perspectives are shared, and models are refined.

**The Modeling Process as Defined by Research Literature**

Lesh and Doerr (2003) describe a four-step modeling cycle involving (a) describing the real world in terms of a mathematical model (formulation), (b) manipulating the model to analyze and answer questions about the original situation, (c) translating (predicting) mathematical results back into real life context, and (d) verifying the usefulness of the models results and actions (p. 17). Figure 1.2 below summarizes and provides a visual for these steps:
Figure 1.2. Modeling cycle involving four basic steps. Reprinted from Foundations of a Models and Modeling Perspective on Mathematics Teaching, Learning, and Problem Solving (p. 17) by Lesh & Doerr, 2003, Mahwah, NJ: Lawrence Erlbaum Associates.

As evident by the visual’s circular effect, the authors believe modelers will run through several of these modeling cycles as “descriptions, explanations, and predictions are gradually refined, revised, or rejected” (Lesh & Doerr, 2003, p. 18).

Another popular cycle description is one offered by Niss, Blum, and Galbraith (2007) which emphasizes the work students do within the real world domain and the mathematical domain:

1. analyze and structure the real world domain
2. decide on a suitable mathematical domain
3. map and translate relevant relations, objects, phenomena, etc. from the real world domain to the mathematical domain
4. work mathematically within the mathematical domain; and
5. interpret and evaluate conclusions from the mathematical analysis with regard to the real world domain. (p. 4)

And as the name modeling cycle implies, Niss, Blum, and Galbraith (2007) also suggests these steps be repeated as needed until one achieves the desired results.

Despite so many modeling cycles, research seems to stress that modeling must include: analyzing the real world situation for key characteristics, assumptions, and objects, mathematizing this relevant real world information into an appropriate model,
refining the model by reflecting on the results of analysis and testing, interpreting the model and its results in terms of the original situation, and repeating through various aspect of the cycle as models are adapted and refined.

**Modeling versus Problem Solving**

Modeling is often viewed as being synonymous to mathematical problem solving. However, research done by Zawojewski (2013) and Lesh and Zawojewski (2007) suggests there are major differences between the two. Problem solving, according to Zawojewski (2013) is person-dependent and focuses on the relationship between the activity and problem solver. Because problem solving depends on the type of thinking that occurs as a person is engaged in the task, a situation may promote problem solving abilities in one person but not in another. In addition, often in problem solving tasks the goal is in generating a solution path which will yield correct procedures and answers (Lesh & Zawojewski, 2007). On the contrary, modeling is task-dependent not person dependent. When a task focuses on developing a productive way to think about a situation that involves interpretation, according to Zawojewski (2013) modeling occurs. The goal is just as focused on ‘wrongness’ in order to test, modify, improve, and refine models.

Lesh and Doerr (2003) suggest an alternative perspective be taken when comparing problem solving and modeling. They state traditionally, applied mathematics is considered a subset or special case of problem solving as shown in Figure 1.3. This perspective, according to the authors, teaches students prerequisite skills and independent
general problem solving processes and then if time allows, asks students to apply these procedures to real life situations that students tend to find difficult:

![Diagram: Problem Solving vs. Applied Problem Solving]

*Figure 1.3. Traditional views of problem solving. Reprinted from Foundations of a Models and Modeling Perspective on Mathematics Teaching, Learning, and Problem Solving (p. 4 ) by Lesh & Doerr, 2003, Mahwah, NJ: Lawrence Erlbaum Associates.*

Analyzing real life problem solving situations has typically involved finding the algorithm hidden and previously determined by the teacher or curriculum (Henn, 2007). In contrast, the authors suggest taking a modeling perspective where problem solving is a special case of model eliciting activities (see Figure 1.4):

![Diagram: Model-Eliciting Activities vs. Problem Solving]

This perspective suggests solving meaningful problems may be easier than in the traditional perspective because students are able to develop systems and constructs that make sense to them. The process of modeling involves several cycles in order to develop a product that includes descriptions, justification, and generalizable constructs, whereas problem solving under the traditional perspective involves only one cycle of identifying, retrieving, and executing procedures aimed at producing ‘an answer’ (Lesh & Doerr, 2003).

**Importance of Modeling**

The need for mathematical modeling may stem from the current shortage of students interested in mathematics and prepared for STEM fields (Leinwand et al., 2014). The economy and workforce seeks employees who can flexibly and creatively apply mathematical problem solving abilities, work collaboratively, adapt to changing technologies by continuously learning new skills, and effectively utilize a variety of technologies in problem analysis and presentations (NCTM, 2000; Soares & Perna, 2014). The mathematics needed by professions in technology, engineering, and business is more “complex, situated, and multidisciplinary” than what is found in conventional problem solving tasks (Lesh & Zawojewski, 2007, p. 781). Lesh and Doerr (2003) stress the world is made up of complex systems including communication, information storage, economic, and financial, and people who can create and understand these systems often have more personal and professional opportunities (p. 15).

Model eliciting activities, in addition to highlighting student experiences, may showcase the nature of mathematics, the usefulness of mathematics, and the contribution
of skills needed in everyday life (Lesh et al., 2000, p. 600). In addition, technology has changed what problem solving looks like in the real world (Lesh & Zawojewski, 2007) and as such may change instruction pertaining to problem solving in the classroom. Similarly, Bonotto (2007) suggests classroom tasks should develop learning processes that more closely reflect out-of-school mathematical situations as “the ability to describe or explain things mathematically is one of the main factors that is needed in order for mathematics to be useful” (Lesh & Fennewald, 2013, p. 8). Yet, according to Niss, Blum, and Galbraith (2007), students do not automatically transfer learned theoretical mathematics to situations in real life that need mathematizing. As a result, teachers need to explicitly help students develop modeling skills and include it in course curriculums (Niss, Blum, & Galbraith, 2007, p. 7).

**Development of Modeling Abilities**

The use of modeling tasks in the classroom is important in order to develop modeling abilities in students and modeling instructional and design skills for teachers. Research has provided some insight on what student thinking, teacher knowledge, and instructional components need to be considered as teachers design and enact modeling tasks within the classroom in order to promote these abilities.

**Theories on Student Learning of Modeling**

Hestenes (2013) suggests mathematical understanding requires students to have both physical and mathematical intuition. Physical intuition requires students to have real world, bodily, and physical understanding in order to match and develop mental
models of these physical systems to mathematical intuition and thus symbolic representations (see Figure 1.5):

![Diagram of physical and mathematical intuition leading to mental models and symbolic representations](image)

*Figure 1.5. Intuition of structure. Reprinted from *Modeling Theory for Math and Science Education* (p. 22) by Hestenes, 2013, Dordrecht, The Netherlands: Springer.*

This relates to Blomhøj and Jensen’s (2003) discussion of the importance of perceived reality in the formulation of appropriate models. Students cannot identify information that can be linked to mathematics without having an understanding of the physical and real life components of the situation. Interestingly, Hestenes (2013) suggests the deficiencies of U.S. mathematics education and the lack of student abilities to develop these mental models from physical intuition stems from the divide of mathematics from physics in schools so that students no longer have the knowledge of experience to draw upon.

Mathematical intuition is used when students identify relevant key components of the situation by looking for familiar mathematical objects that can serve as modeling instruments (Niss, 2013). When a situation is mathematized, students use mental models
developed from physical intuition that translate the real life objects, relations, patterns, actions and questions into mathematical objects, relations, and questions (Lesh & Doerr, 2003; Niss, 2013). According to Niss (2013), when mathematizing situations, modelers must (1) predict the potential for math involvement and (2) determine the usefulness and relevance of this involvement for the model. Students must be able to identify the components of the situation which could be mathematized, determine what representations would be appropriate by connecting knowledge to their physical intuition and mental models, and anticipate how representations or models could be useful to answer questions (Niss, 2013).

Research suggests initial interpretations by modelers of the real life situation are primitive, qualitative in nature, and only include the most obvious physical characteristics (Lesh & Doerr, 2003). Lesh et al. (2000) call these initial interpretations a “hodgepodge of several disorganized and inconsistent ways of thinking” (p. 597) brought about by individual students isolating different information and segregated data. Similarly, Lesh and Harel (2003) found student interpretations of modeling tasks closely resembled general theories on the development of mathematical skills (see Inhelder & Piaget, 1958). Namely, initial interpretations tend to ignore relevant data and use only obvious information.

In order to get past this initial and limited understanding, students need to hear diverse perspectives through the use of collaborative and productive discourse (Lesh & Doerr, 2003). When students interact in these “communities of practice,” peers provide questions and feedback that allow student understanding to be “tested, integrated,
differentiated, extended, revised, or rejected” (Lesh & Zawojewski, 2007, p. 790). As students continue to refine, discuss, and analyze alternative perspectives, they begin to recognize patterns, make more sophisticated mathematical connections, and refine their models (Lesh & Harel, 2003) so that their assumptions, interpretations, and constraints also progress in sophistication (Lesh & Zawojewski, 2007).

**Developed Student Skills**

According to Galbraith, Stillman, and Brown (2013), instruction pertaining to mathematical modeling sets out to accomplish one of two goals. (1) The learning focus is on mathematical content. Modeling is simply used to motivate and further develop the mathematical content within the curriculum. Or (2) modeling is the content. Here the goal is to create better modelers in order to apply math in a real world context. Regardless of the goal, students will progress through a similar modeling cycle and can then be assessed on their modeling competency. According to Blomhøj and Kjeldsen (2013), modeling competencies include the *degree of coverage* to which students are using the modeling process, the *technical level* relating to the types of mathematics used, and *radius of action* to different situations that students can apply their modeling abilities towards.

As students progress through this modeling cycle, research suggests there are specific skills students can develop. When students analyze the real world situation, students need to bridge and connect knowledge from multiple domains because it is almost impossible to just use ideas from a single topic or discipline (Lesh & Harel, 2003). Once students are able to interpret the situation, they need the ability to take
non-mathematized information, unmask the mathematics, and then mathematize the situation (Mousoulides et al., 2013). Through this mathematization process, students can improve capabilities in transferring mathematics to solve and analyze problems outside of the classroom (English, 2003).

To formulate and analyze their model, students need to develop skills in identifying, selecting, and collecting relevant data (Lesh & Doerr, 2003). Throughout this part of the modeling cycle, students gain insight on how to approach mathematics with flexibility and creativity (Lesh & Doerr, 2003; English, 2006) as they learn to work collaboratively in small groups and have productive discourse (Lesh & Doerr, 2003).

After models are formulated and verified, students learn to interpret solutions in context and transfer mathematical knowledge into real-life knowledge (Lesh & Doerr, 2003). This forces them to develop abilities in assessing the usefulness of mathematical ideas, not just thinking about mathematics but how models can be used to solve real problems (Lesh & Doerr, 2003). Finally, as the products of modeling tasks are developed, students’ mathematical vocabulary, language, and fluency may improve (English, 2006) and students learn to express mathematics and results using a variety of representational media (Mousoulides et al., 2013). However, the development of these skills through the modeling process is not guaranteed, especially as they relate to mathematical and quantitative literacy.

**Issues of mathematical literacy.** Burkhardt (2007) states “Modeling everyday life situations is at the heart of functional mathematical literacy” (p. 180). The Program for International Student Assessment (PISA) defines mathematical literacy as “an
individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements, and to use and engage with mathematics in ways that meet the needs of that individual’s life as a constructive, concerned, and reflective citizen” (OECD, 2003, p. 24). According to Romberg (2000), mathematical literacy should include a focus “on mathematical knowledge put into functional use in a multitude of different situations and contexts in varied, reflective, and insight-based ways” (p. 5) so “students learn to learn to use such ideas (mathematical concepts and procedures) to solve non-routine problems” (p. 8). Pugalee (1999) identifies the process of mathematical literacy as representing by translating and constructing mathematical ideas, manipulating through various calculations, reasoning mathematically by drawing logical conclusions and making conjectures, and problem solving by using prior knowledge and skills to formulate a strategy to a problem lacking an obvious solution.

The problem when trying to teach modeling, as indicated by research, NCTM, or the CCSS-M, is students in general struggle transferring math knowledge to real-life situations, or to those having no obvious solution strategy, in order to build appropriate models to solve problems (see Kaiser and Willander, 2005). This may include selecting from a variety of tools or potential strategies. A 2009 PISA assessment found only 27 percent of U.S. students scored at or above proficiency levels for mathematics literacy meaning “they can select and integrate different representations, including symbolic ones, linking them directly to aspects of real-world situations” (Fleischman et al., 2010, p. 19). Similarly, the 2012 PISA assessment found U.S. students were weak in their ability to establish, apply, and interpret mathematical models in context compared to other OECD
32 countries (OECD, 2013a). In addition, in *Mathematics and Democracy*, Ellis (2001) noted students in his algebra classes could solve fewer word problems after his class than prior. He credited this to students thinking they needed to use symbols to solve problems they had previously solved simply by mathematical reasoning. When we force students into particular strategies, this may reinforce to students that mathematics is about a series of steps to be followed and not about building mathematically appropriate models from knowledge about the situation. Thus, teachers may need to be able to know how to design and enact tasks intended to help students transfer mathematical knowledge to everyday problems in order to better develop their mathematical literacy.

**Issues of quantitative literacy.** Another important hindrance to the development of modeling abilities centers on skills leading to the interpretation of symbols, graphs, and solutions in context to the situation and deciding whether a solution is reasonable. If one issue to the development of modeling abilities centers around the creation of models by translating the real life situation into mathematical notation, then it seems natural a second problem arises when students must interpret the mathematics back into the real life situation (see Figure 1.6):

![Figure 1.6](image.png)

*Figure 1.6. Mathematical modeling schema. Reprinted from Effects of Standards-Based Mathematics Education: A Study of the Core-plus Mathematics*
As illustrated in Figure 1.6, modeling requires students to take an applied problem situation, formulate a model through a mathematical activity by applying mathematical literacy, then interpret the results of this mathematics back in terms of the applied situation through the use of quantitative literacy skills. Quantitative literacy is generally considered a *habit of mind* rather than a list of specific skills and can be thought of as the “ability to identify, understand, and use quantitative arguments in everyday contexts” (Hallett, 2003, p. 91). The problem in teaching this is in general, curriculum and teachers do not include it in instruction. Bonotto (2007) states most curriculum word problems leave out considerations of reality and quantitative reasoning entirely. The focus should be on the process and communicating this process and its results and not simply stating ‘an answer’ so problems demonstrate a “more realistic and less stereotyped” view of mathematical connections in the real world (Bonotto, 2007, p. 186).

The ability to interpret mathematics in-context is related to a larger problem of practice in the development of quantitative literacy. Although quantitative literacy encompasses a long list of student skills, knowledge, and beliefs, it also includes students being able to interpret math notation, tools, and strategies in context and recognize potential errors (Steen, 2001). Kaput (1998) claims “Quantitative reasoning ... can be regarded as modeling-building, usually in several cycles of improvement and interpretation, mathematical systems that act to describe and help reasoning about phenomena arising in situations” (p. 16). During the modeling process, students
continuously formulate, test, and revise models by operating on quantities. They test and analyze new quantities at each phase so “the derived quantities that are the products of thought at one state of development (often) become the objects of thought at the next state” (Larson, 2013, p. 116).

Steen (1999) stresses the importance of quantitative literacy since “the age of information is an age of numbers” (p.8). Rivera-Batiz’s research (1992) agreed with this sentiment when he found adults who have skills related to quantitative literacy are more likely to hold a full-time job. However, when students simply give an ‘answer’ to a word problem, they generally do not think about the realistic aspects of the problem that restrict plausible solutions. Silver, Shapiro, & Deutsch (1993) found only 23% of students could provide the correct answer to the question:

An army bus holds 36 soldiers. If 1128 soldiers are being bussed to their training site, how many buses are needed?

The problem is that the division of this problem yields an answer of 31 and 1/3, but only 23% of students correctly realized that 32 buses would be needed. Similar studies done by Palm (2008) and Greer (1993) also found students were unable to reflect on the authenticity and plausibility of solutions. When students view mathematics as a series of steps and procedures to follow, these studies suggest students are then unable to stop and reflect on the limitations of reality. Students may simply follow “mindless calculations” (Palm, 2008, p. 38).
Teaching to Develop Modeling Abilities

Doerr (2007) suggests the limited use of modeling in secondary classrooms is due to a lack of modeling knowledge by teachers. However, current research can provide some suggestions for what knowledge and skills teachers will need in order to help develop modeling abilities and overcome issues relating to mathematical and quantitative literacy. Initially in the modeling cycle as students seek to interpret the real world situation, teachers can help students make sense of simpler, related problems and use the structure of that problem to make progress towards more complicated versions (Speiser & Walter, 2013). Similarly, Lesh and Zawojewski (2007) suggest helping students organize their knowledge and strategies around the “mathematics of the situation” not “problem-solving heuristics” or “conventional mathematical topics” (p. 767). As a result, students will need support in developing skills in recognizing the mathematics hidden within situations so that it can be interpreted, described, and mathematized (Lesh & Zawojewski, 2007).

Generally students struggle managing, organizing, and gathering data (Stillman, Brown, & Galbraith, 2013) as they formulate and analyze their model so instruction should focus on helping students develop skills relating to data collection and analysis. Also, because modeling involves several cycles of testing, adapting, and revising, strategies for teaching should be “aimed at helping students find productive ways to adapt, modify, and refine ideas that they do have” (Lesh & Doerr, 2003, p. 22). To do this, students may need instruction in metacognition alongside the mathematical content (Kramarski, Mevarech, & Arami, 2002; Lesh & Zawojewski, 2007) including skills
related to reflective thinking, breaking tasks apart into subtasks, and self-regulation (Schoenfield, 1992).

Because teachers will need to understand instructionally, conceptually, and from a design perspective how to include modeling tasks in curriculums, teachers may not just be able to “orchestrate” environments and activities that develop modeling skills without training (Niss, Blum, & Galbraith, 2007, p. 7). These types of environments can be uncomfortable and hard for teachers because they are put in a position of “not knowing” (Schoenfeld, 1992, p. 354). Modeling tasks are designed to elicit different student responses based on their perspectives, and as such, does not come with one right answer. As a result, teachers may need experience and confidence in their ability to teach mathematical modeling in order to handle, foster, and develop these individual perceptions (Schoenfeld, 1992). In addition, teachers should observe thinking during the modeling process so they can identify strengths and weakness which can be used to help strengthen student interpretations and arguments (Lesh et al., 2000). Lesh and Kelly (2000) suggest teachers will know when a task is good based on the quality of students responses and the degree to which it helps teachers identify and understand these conceptual strengths and weaknesses (p. 219).

**Knowledge and skills needed.** Specifically, Doerr (2007) provides descriptions of the knowledge and skills teachers will need to enact modeling tasks effectively in the classroom. First, Doerr states teachers will need a “broad and deep understanding of the diversity of approaches that students might take” (p. 76). When students are working on tasks, teachers will need to listen for anticipated misconceptions or problem areas, offer
“useful representations of student ideas”, analyze and share unexpected strategies, and support students in making connections to other perspectives and depictions (p. 77).

Secondly, and perhaps the most difficult aspect for teachers, is creating an environment valuing collaboration while providing students with opportunities to “interpret, explain, justify, and evaluate” the strategies, approaches, usefulness, and accuracy of their models and those of their peers (p. 77). In this type of environment, Doerr suggests students can communicate and provide explanations to each other and evaluate their own ideas and results as opposed to the teacher. Teachers are used to providing this explanation and evaluation; it will require extra care on the part of the teacher to solicit this from students in a productive way.

Because students are at the center of modeling tasks and the modeling process, teachers will need highly developed questioning skills to know “the right questions” to ask at “the right time” (Ng, 2013, p. 433). Professional development done in Singapore and described by Ng (2013) has provided a framework for teacher scaffolding during each phase of the modeling cycle. According to Ng, initially teachers can solicit key ideas, goals, relevant information, and variables from students as they discuss the modeling situation. As students begin planning and formulating models, group collaboration and organization will be important so students can analyze alternative perspectives. During experimentation of the formulated models, teachers can guide students to meaningful symbolic representations based on the devised approaches. As students verify their models to check for accuracy and appropriateness, teachers can ask questions about how students could check for reasonableness or how students might
know revisions should be made. Once models have been verified and students are ready to present and reflect on the situation, teachers can then probe for student interpretations that map the mathematics back to the initial situation or generalize to other situations.

**Curriculum**

A few recent studies about curriculum alignment to CCSS-M have found many textbooks fail to address modeling content to the degree research, NCTM, or CCSS-M intends, or leaves out grade level content entirely (Polikoff 2015, Schmidt & Houang 2014), to the point textbook publishers have “done little more than slap shiny new stickers on the same books they've been selling for years” (Molnar, 2014, para. 7). In a more recent article in *Mathematics Teacher*, Dan Meyer (2015) specifically addresses the lack of complete modeling tasks in relation to CCSS-M’s modeling standards and actions by analyzing two McGraw-Hill textbooks in algebra and geometry. He found the two textbooks addressed the performing operations and interpreting results actions of modeling (actions 3 and 4 in CCSS-M’s modeling process) fairly well. However, only 8% of the modeling tasks within the textbooks had students identify essential variables (CCSS-M’s modeling action 1), 24% asked students to formulate models which were not given or were not provided an example to refer to (action 2), and only 5% had students validate their conclusions or discuss limitations (action 5) (Meyer, 2015). Meyer suggests action 6: *reporting on conclusions* is more a responsibility of the teacher than the textbook as this may be done in a variety of ways.
Design Guidelines

Research has provided some general guidelines and design principles for the creation of tasks that may maximize the development of student modeling abilities. Initially, teachers may need to provide more structure for students with little modeling experience then back-off as students become more familiar with the modeling process and desired products (Ang, 2013). Also, Galbraith, Stillman, and Brown (2013) stress the importance of stating assumptions to help an audience understand choices made and how the situation is related to the model used. Therefore, when teachers scaffold tasks to provide more structure, they should design tasks to keep in mind student reporting of these assumptions.

In addition, the design of tasks should also embed collaboration, discussion, and peer-to-peer questioning. Hearing different student perspectives allows students to “interpret each other’s points of views, engage in discussions that compare and contrast the proposed models, and work to reach a consensus on a group model” (Zawojewski, 2013, p. 240). In other words, because modeling involves multiple cycles of testing and refining, exposing students to multiple perspectives and ideas will contribute to the formulation of more sophisticated and precise models.

Meyer (2015) suggests teachers will need to step in and design questions and tasks that address the modeling actions within CCSS-M’s process that are not emphasized in textbooks. Prompts and questions relating to “What information is necessary, and what information is unnecessary?” may help identify variables through action 1: identifying variables and essential features while providing opportunities for students to
compare their mathematical analysis to real world results may increase their ability to validate their model in action 5: *validating conclusions* (Meyer, 2015, p. 580).

**Six principles for the design of model-eliciting activities.** Specifically, Lesh et al. (2000) identify six principles for designing productive model-eliciting activities teachers and curriculum developers may find beneficial when planning instruction. The authors state the most important criterion for effective modeling is to have students describe the maturation of constructs which are both mathematically significant and practical (p. 608), and if tasks are created using these six principles than the task is more likely to accomplish this goal. Their six principles include:

1. **Model Construction Principle:** Because model-eliciting tasks are designed to be “thought revealing” (p. 608), then the product of these tasks should “include the development of an explicit construction, description, explanation, or justified prediction” (p. 609). These activities should generate a need for students to develop, revise, and refine a model and developers can design such tasks by thinking about situations in real life that require the use of models.

2. **Reality Principle:** Students make sense of the task based on their own knowledge and experiences. Thus, the questions and goals of the task need to make sense in terms of these experiences rather than asking students to turn off this knowledge in order to achieve “school answers” (p. 619). Lesh et al. suggests to accomplish this by giving different versions of a problem to accommodate various perspectives and use newspaper articles to introduce problem situations while posing preparation questions.
(3) **Self-Assessment Principle:** Students should be able to determine when their results are “good enough” (p. 620). The task’s goals and questions are stated in such a way students can assess progress and usefulness of results through various cycles of modeling refinement. The authors recommend using the idea of a client that students are creating a product for. In this manner, students can then assess how well their model and results accomplish the client’s purpose.

(4) **Construct Documentation Principle:** Tasks should require students to externalize their ideas, thinking, interpretations, and goals. Explanations, justifications, and descriptions are an essential part of the task’s final product (p. 623) so “assessing quality of the final result automatically involves assessing the quality of the mathematical reasoning used to produce it” (p. 624).

(5) **Shareability and Reusability Principle:** The task should result in generalizable descriptions, explanations, or prediction procedures (p. 626) so others can use the solution in similar situations. The authors also suggest when students develop “general ways of thinking” (p. 626) it allows for deeper and more thorough mathematical relationships, ideas, and connections; and,

(6) **Effective Prototype Principle:** The task should be memorable for students so students can refer back to them when working on structurally similar situations (p. 626). As a result, tasks are structurally and conceptually complex and meaningful but not necessarily procedurally complex (p. 627).

Finally, Lesh et al. (2000) recommends that for model-eliciting activities to be productive in revealing student thinking, they should not only follow these six design principles but
be highly structured yet “open” in that the right answers are not already figured out (p. 629).

Another perspective on six design principles. Similarly, Galbraith (2006) describes six additional criteria to consider when designing modeling tasks that may be more straightforward than those offered by Lesh et al. (2000):

Principle 1: The task provides a genuine link with the real world of students.

Principle 2: Students have the opportunity to recognize and state mathematical questions from a general problem task.

Principle 3: The solution to the task is feasible and uses mathematics available to students.

Principle 4: The solution of the mathematics is possible.

Principle 5: Students are able to check the accuracy and reasonableness of their mathematical analysis; and,

Didactical Principle: The problem may be organized into scaffolded sub questions that retain the purpose of the real situation and the integrity of the modeling process.

The design principles offered by Lesh et al. (2000) and Galbraith (2006) are generally used as the standard for model task design. There is some overlap in the principles, specifically in relation to mathematizing the situation and self-assessing for reasonableness. However, Galbraith (2006) seems to focus on student confidence and being able to accomplish the goals of the problem and the subsequent mathematics involved. In contrast, Lesh et al. (2000) focuses explicitly on the communication of
student thinking and model development and being able to generalize results to other structurally similar situations.

**Using criteria for desired outcomes.** Criteria for desired outcomes of the modeling process should also be considered when designing productive tasks. Products of modeling tasks should “identify instructional devices for enhancing perspective taking,” since the social process of taking on other points of view facilitates: (1) detecting flaws and deficiencies in current models, (2) comparing and contrasting the strengths and weaknesses of competing models, and (3) generating new ideas for revising models or posing alternative models (Zawojewski, 2013, p. 242). These products must be sharable, reusable, modifiable, and generalizable (Lesh & Doerr, 2003; Lesh & Harel, 2003;) and address the who and why for use of the model given or implied within the situation (Lesh & Harel, 2003).

**Reflecting on Research Problem to Identify Study Purpose**

As indicated by the introduction to this research study, identifying my problem of practice on the use of modeling tasks in my precalculus courses was relieving and anxiety inducing at the same time. Prior to the development of this research study, I had little knowledge of or experience in modeling. Through a review of research literature, however, I was able to expand on the issues associated with my problem of practice in order to form the specific purpose of my research study.

The modeling cycle as outlined by NCTM and CCSS-M implies a cyclical nature of model generating, testing, refining, and interpreting. As students work through this process, they will need to bring in knowledge from multiple domains, participate in
discourse in order to understand different perspectives, and apply problem-solving, mathematical, and quantitative literacy skills. Teaching methods related to questioning tactics and collaborative environments have been suggested by research outlined in this chapter to help in the development of modeling abilities. In addition, design principles offered by Lesh et al. (2000) and Galbraith (2006) describe curricular features of modeling tasks used to promote modeling abilities.

Research seems to be extensive on what modeling is and why it is important to include in curricula materials, yet it still does not seem to be put into practice effectively in most secondary mathematics classroom. The research included in this chapter allowed me to gain a better appreciation for the complexities and difficulties in implementing modeling in my precalculus courses. But how does a teacher design and implement tasks devoted to mathematical modeling as described by current research, NCTM, and CCSS-M while still being conscientious of mandated standards, curriculums, state assessments, and diverse student needs? What does modeling look like in a secondary mathematics classroom and what are the desired outcomes of its use?

**Research Study**

The purpose of this qualitative action research project is to understand how a teacher designs and enacts mathematical modeling tasks within a conventional classroom using ordinary curriculum resources in order to develop both teacher and student modeling abilities. For this study, modeling is defined using the CCSS-M definition as "the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions"
(Corestandards.org, 2014a). A conventional classroom is one that occurs within the regular sequence of a school day and does not include an Advanced Placement or dual credit course. Ordinary curriculum resources are ones easily accessible to teachers including district-mandated textbooks.

**Central Research Questions**

- How does a teacher design and enact authentic modeling tasks in a diverse secondary mathematics classroom?
- What consequences do these modeling tasks have on modeling abilities and concomitant mathematical learnings?

**Sub-Questions**

- What happens to student modeling abilities as students complete tasks within a unit on modeling?
- What happens to student discourse in the classroom through the use of modeling tasks?
- What pedagogical and mathematical content knowledge does the teacher need?
- What barriers do the teacher and students encounter? How are those barriers addressed?
- How do modeling tasks reshape student and teacher beliefs on what it means to do mathematics?
CHAPTER 2

METHODOLOGY

Qualitative research methodologies were used to analyze the problem of practice focused on designing and enacting modeling tasks because the “key concern is understanding the phenomenon of interest... from the emic or insider’s perspective.” (Merriam, 2009, p. 14). Creswell suggests a qualitative approach be used when “the problem needs to be explored; when a complex, detailed understanding is needed... and when the researcher seeks to understand the context or settings of participants” (Creswell, 2007, p. 51). Of particular importance to teachers is “having an interest in knowing more about one’s practice, and indeed in improving one’s practice” that can often best be “approached through a qualitative research design” (Merriam, 2009, p. 1). Rust (2009) suggests teacher research “draws on traditions of qualitative research in that its focus is process, and because it requires a search for relationships among data sets, it fits within the traditions of quantitative research” (p. 1884). Because my primary research goals are to understand how to design modeling tasks and what happens when those tasks are used in my classroom, a qualitative research methodology provided the best opportunity to provide the type of data that may improve this aspect of my practice.

Qualitative Research Methods

This analysis was rooted in improving my own mathematics curriculum and instruction and contains aspects of self-study, action, and design research. By carefully analyzing the nature of my inquiry, I could employ research types and methods which
may provide the best opportunity for the collection of data leading to insight into my research questions.

**Self-Study**

Self-study methods were used since “the aim of self study research is to provoke, challenge, and illuminate rather than confirm and settle” (Bullough & Pinnegar, 2001 p. 20). Hamilton and Pinnegar (1998) define self-study to be the “study of one’s self, one’s actions, one’s ideas, as well as the ‘not self’” (p. 236) and stress that they can be used to expand our knowledge about teaching and learning. Dinkelman suggests self-studies are a natural complement to the teaching profession: “if teaching is what teacher educators do, and teaching must include reflection, then self-study, as a form of reflection, ought to be an essential part of the activity of teacher educators” (2003, p. 57). Within self-studies, the participant/researcher is interested in how the data collection and analysis connects him or herself to the phenomenon being studied (Pinnegar & Hamilton, 2009).

Throughout the development of my problem of practice, data collection, and analysis for this project, I had to engage in a thorough and thoughtful analysis of my own beliefs and learning associated with the teaching and learning of modeling and mathematics. One central research question asks: how does a teacher design and enact authentic modeling tasks in a diverse secondary mathematics classroom? This required me to expand on and analyze my own mathematical, pedagogical, and content knowledge consistent with self-study research.
Action Research

However, because another central research question focuses on the impact modeling tasks have on modeling abilities and concomitant mathematical learnings, this research could also be characterized as action research. Mills (2007) defines action research in education as an examination of the teaching and learning environment so information may be collected on how teachers teach and students learn in order to gain insight, reflect on practice, and improve learning outcomes. McNiff and Whitehead (2011) describe action research through its component parts. That is, ‘action’ implies “taking action to improve practice” while ‘research’ suggests the participant-researcher will create new knowledge by “finding things out and coming to new understandings” (McNiff & Whitehead, 2011, p. 10). Mertler (2014) suggests action research “allows teachers to study their own classrooms ... in order to better understand them and to be able to improve their quality or effectiveness” (p. 4) while Merriam (2009) adds the participant in the problem being studied is also the researcher. In this study, I embarked on a systematic inquiry into an aspect of my practice in order to improve student learning associated with mathematical modeling. Because my goal was to understand and gain insight into my own practice in order to improve teaching and learning outcomes, this research study follows the purpose and format of action research.

Design Research

The study of the creation of modeling tasks also includes aspects of design research. The goal of design research is to create and evaluate unconventional conditions for learning in order to provide insight into instruction and educational practice
(Schwartz, Chang, & Martin, 2008). It is “directed primarily at understanding learning and teaching processes when the researcher is active as an educator” (Kelly, 2003, p. 3). Its goal is to not show that a design works but to improve instructional practices that support student learning (Cobb & Gravemeijer, 2008).

Design research in teaching experiments focuses not on generalizing students, teachers, or groups but rather on developing ideas so that students and teacher can simultaneously learn (Lesh & Kelly, 2000). When conducting teaching experiments, data analysis goes beyond descriptions of what produces desirable results to identifying and reflecting on what methods stimulate development of knowledge, changes, and improvements (Lesh and Kelly, 2000). The data collection and analysis are on-going throughout the teaching experiment, as the teacher researcher investigates both how students’ constructs develop and change along with how teacher’s constructs change to specifically understand student thinking (Lesh and Kelly, 2000). As suggested by Lesh and Kelly (2000) during teaching experiments, teacher researchers should (1) explicitly reveal current interpretations, (2) test and assess these interpretations, (3) reflect, and (4) refine, extend, or reject the interpretation.

**Understanding Methods Leads to Research Plan**

Once I better understood the nature of my inquiry and its connections to various forms of qualitative research: self-study, action research, and design research, I could then formulate a research plan that was consistent with my role as a teacher-researcher and the research questions I sought to answer. In this chapter, I describe the overall research design, data collection instruments used, how those instruments were used, and
data analysis techniques employed in my study of the teaching and learning of a unit on modeling.

**Research Plan**

This study focused on two aspects of mathematical modeling: designing appropriate tasks and the enactment of these tasks within the classroom. As a result, its goal is on improving teacher instruction and design abilities while focusing on student knowledge development and interactions (see Lesh & Kelly, 2000). I was interested in how I as a teacher can use readily available resources to create modeling tasks which fit into the course of my mathematics curriculum without disrupting its flow and other demands of teaching. I was curious about the process, knowledge, and resources used to create these tasks and therefore studied the decisions and actions I made throughout the design process. Since improving my instruction is done ultimately to improve student knowledge, action research studying how students responded to these tasks during the modeling unit was also a crucial component to the research plan. Consistent with my research questions, I wanted to identify design and instructional practices in the area of mathematical modeling while showing what student learning was possible (Cobb & Gravemeijer, 2008). My purpose in teaching modeling was not to further develop specific mathematical content with students but to develop their overall modeling abilities.

**Design**

In the fall of 2015 I began laying the foundation for this research project to occur in my precalculus classroom. Frustrated with the lack of authentic modeling experiences
I felt my current curriculum resources provided, I wanted to research the design and enactment of modeling tasks through the use of an entire unit on modeling. In order to better address action 2: *formulating a model* within CCSS-M’s modeling process, a modeling unit as opposed to modeling problems within each content area I thought would provide myself and students opportunities to think deeply about the development of models based on mathematical and real life connections. I felt students should have opportunities to generate and analyze various representations of the modeling task based on information within the situation and student knowledge (Christinson et al., 2012) and providing modeling tasks within a unit covering all curriculum content I hoped would better allow for those opportunities. From a practitioner-researcher perspective, I also felt studying a cohesive unit on modeling would provide better opportunities for worthwhile and consistent data collection and results. This allowed me to engaged in “prolonged engagement and persistent observation” (Mertler, 2014, p. 137) in order to gather data on patterns of behavior.

As I began to research and map out my modeling instruction, which is described in Chapter 3, it seemed I would need about a month with my precalculus courses to enact an entire modeling unit in the way I felt was consistent with current research on modeling. This unit ended up consisting of an introduction to modeling and five modeling tasks:

a) *What is modeling?*

b) *Project 1 - The High Definition Trade-Off Dilemma*

c) *Project 2 - Calculating the Width of Memorial Stadium*
d) Project 3 - Enrollment at River Junction Senior High: How big will we get?

e) Project 4 - How safe are we: Developing a crime rank for River Junction

f) Project 5 - Student Choice

Because I was interested in both the design (design and self-study research) and enactment (action research) components of this modeling unit, research was conducted in two phases. In the months of November 2015 to February 2016, I primarily studied myself and the new knowledge I developed as I designed a unit on modeling and the development of these five projects. The enactment phase, or a month of modeling, took place from March 14, 2016 to April 14, 2016 and consisted of 24 class periods. The majority of these class periods were fifty minutes in length unless alternative schedules for special events were used by the school. Follow-up data collection and reflection occurred at the conclusion of the unit through May 2016.

Participants. This study focused on the curriculum, teaching, and students of one honors precalculus course. During the modeling design phase of this study, I primarily studied myself and the actions I took to design tasks and instruction for the modeling unit. During the enactment phase of using these tasks, I studied my actions and thinking in addition to collecting data on students enrolled in my second period precalculus course and their interactions with each other and myself. At the time of this study I taught two sections of honors precalculus. I enacted the modeling unit in both courses, but because of the intensity and amount of data needing to be collected, only my second period students were asked to participate in this research study. Students in this
particular class period had a wider variety of skills, personalities, backgrounds, and interests than in my other precalculus course, and as such, I thought would provide more interesting and worthwhile results for this research study. Twenty-five students were enrolled in this course at the time of the study and were assigned by school counselors and administration. Twenty-three of these 25 students elected to participate in the research component of this study. Eight of the 23 students (35%) were of Hispanic ethnicity, while nine of the 23 students (39%) were female. These students were arranged into groups of three or four (referred to as teams) to develop models for each task in the modeling unit.

From this group of 23 students, I selected five students to participate in a post-unit reflection interview. Samantha\(^1\) was a hardworking, Caucasian female student who typically did well in her mathematics courses but found the most success when she could memorize mathematics procedures. Chris was a high-performing Hispanic student who initially struggled in the modeling unit but provided some insightful learning opportunities. Luis was an average performing Hispanic student representative of most students in the class. Brooke, a lower performing Caucasian female, seemed to find the modeling unit particularly interesting and motivating. And finally Matt, an average performing Hispanic student, who struggled throughout the duration of the unit.

**Location.** This study primarily took place in one mathematics classroom within an urban, Midwestern high school. River Junction Senior High\(^2\) is the only high school in its district and housed around 2400 students with a total district enrollment of about

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\(^1\) All student names are pseudonyms.

\(^2\) All city names are pseudonyms.
9600 students in the time of this study. Approximately 53% of the student population at River Junction Senior High was Hispanic and 64% receive free or reduced-price lunches. The only deviation from the classroom location was during the modeling design phase, which included research and design work done outside the classroom by the teacher-researcher.

**Data Collection Instruments**

Because of the complex nature of understanding both teaching and learning in this research study, multiple data collection instruments were used in an effort to document, analyze, and triangulate results (Creswell, 2007). However, because I was also the teacher at the focus of this study and subject to the responsibilities of that role in addition to my work as a researcher, data collection needed to fit in the daily demands of teaching. As Rust (2009) states:

> Because it is intimately embedded in practice and in the time frames of teachers’ lives in classrooms, teacher research describes a form of qualitative inquiry that draws on techniques that are generally already part of the instructional tool kit of most practitioners. (p. 1883)

As a result, the majority of data collection tools used in this study were ones routinely available to me in my daily role as a mathematics teacher. This section describes the various instruments used and their role in addressing aspects of this research study.

**Personal journal.** A personal journal was kept during all phases of this research study to document new learning, insights, attitudes, and decisions. Mills (2007) recommends journaling in action research to record feelings and observations associated
with the entire research process. He suggests journals can be more than a single data source as “they are an ongoing attempt by teachers to systematically reflect on their practice by constructing a narrative that honors the unique and powerful voice of the teachers’ language” (Mills, 2007, p. 70). My personal research journal served slightly different functions throughout the design, planning, enactment, and reflection of each modeling tasks used in the unit and as such, I responded to set questions prepared in advance to focus my journal entries.

During the design phase of the modeling tasks used in the unit, I primarily kept a journal to thoroughly describe the process I used to develop the learning goals and design principles for the unit, decide on topics for each task, write up each task, and analyze tasks for its alignment to the design principles. For each task used in the unit I focused on the following questions:

1. Where did the idea for the task come from? What’s its ‘story’?
2. What data or outside expertise do/did I need to complete the task?
3. How do I feel the task follows each of my design principles?
4. How do I think this task supports the development of student modeling abilities?
5. What decisions did I make in terms of how much to scaffold the task in advance? What (perhaps) did I consider adding for scaffolding, then decided to hold back and offer that only to students who needed it, rather than to all students?
In this design journal I was sure to document instructional starting points (Cobb & Gravemeijer, 2008) and descriptions on why I felt the task was worthwhile in order to reveal my “own assumptions about the nature of mathematics, problem solving, learning, and teaching” (Lesh & Kelly, 2000, p. 219).

Once I began developing specific lessons and teaching strategies used within the unit, my research journal shifted focus to record decisions teachers typically make while planning instruction. For each task, I responded to the following journal questions:

1. What mathematical knowledge do I think students will need to complete the task?
2. What real-life (contextual) knowledge will students need?
3. How do I think students will approach the task?
4. What do I think will cause students the most trouble?
5. How will I introduce the task?
6. What scaffolding will I provide?
7. What will I do to help students move through the modeling cycle?
8. What questions can I ask to help students move through the modeling cycle?
9. What is my plan for when students get stuck or if a group gets off task?
10. What will it look like if students are successfully working on this task?

Because modeling lessons cannot be truly prepared in advance to account for all perspectives, once I designed the tasks to be used I carefully analyzed and recorded my own ideas on how students might approach the various tasks and potential outcomes so
during enactment I could interpret and determine appropriate responses or interventions (Bonotto, 2007; Doerr, 2007; Schoenfeld, 1992). These particular journal questions focused my attitudes, thoughts, and decisions on how I hoped my written curriculum developed during the design phase would be carried out during the enactment phase.

During the enactment phase of a month of modeling instruction, in order to study student and teacher responses and interactions I kept a daily personal journal responding to the following questions:

1. What is one specific moment in class that sticks out today?
2. What worked today?
3. What didn’t work today?
4. What mathematics and specific modeling abilities did I see students demonstrating?
5. What instructional supports seemed to help students make progress on their task?
6. What revisions do I need to make for tomorrow?

As with most qualitative research, I underwent cycles of design and analysis (Cobb & Gravemeijer, 2008). As a result, my personal journal during the month of modeling was essential in documenting and reflecting on the changes I made stemming from new knowledge of modeling, mathematics, and students. At the conclusion of each week during the five weeks of modeling instruction, I also summarized major learning and events in this personal journal based on the daily responses to these questions.
Finally, after each task within the unit had concluded, I reflected on the design, learning, and barriers which may have occurred by recording in my personal journal responses to the following questions:

1. What knowledge (of mathematical content, modeling, students, vocabulary, etc) did I draw upon?
2. What knowledge would other teachers need to enact this modeling task with students?
3. What mathematics did I learn? How have I looked at math differently?
4. In the future, what sections of precalculus might this modeling task replace or supplement?
5. How would I modify this task for next year (amount of scaffolding, method of introduction, expectation for outcome)?
6. What were the key barriers for this modeling task? What did I decide/do that helped overcome these barriers? What would other teachers need to know or be able to do in order to overcome these barriers?

These questions allowed me to synthesize what my students and myself experienced during the enactment of each task while also thinking about implications for my research in future courses and for other teachers.

**Field notes.** Detailed field notes are necessary in order to make explicit conditions of the learning environment and how it impacts students (Cobb & Gravemeijer, 2008) and describe what parts of this learning environment including the teacher researcher, discourse, peers, and norms were necessary for student learning
results towards the learning goals (Cobb & Gravemeijer, 2008). During each lesson of the modeling unit, I kept a notebook to record observations, moments of student learning or conflict, student and teacher attitudes, and timestamps of conversations I needed to go back and analyze further from the videotape recordings of the lesson. After each lesson I also quickly recorded my overall impressions of what occurred in the classroom which I would expand upon in my daily personal journal. As Mills (2007) suggests, teachers cannot physically keep detailed field notes of all events during an observation, but he suggests to observe and record what one can, look for routines and norms, and record instances outside of the norm that stand out. Keeping detailed field notes while also teaching and responding to student needs proved to be quite difficult but useful in pinpointing and triangulating data gathered from other sources.

**Videotaped lessons.** During the enactment of the modeling unit, four GoPro recording devices were used as a record of daily events and instruction. One GoPro was attached to me to record my interactions with students throughout the class period. Three other GoPros were stationed at three of six teams to record student dialogue and interactions. All dialogue included in this study was transcribed from these videotaped lessons.

It was not feasible for me to acquire and analyze recordings from all six teams, especially since not all students elected to participate in the research component of the modeling unit. However, using purposeful sampling (Creswell, 2007), I situated these three video recording devices at teams with students I felt best represented the variety of students I had in the class. I wanted to be sure to record students with a wide range of
mathematics abilities and backgrounds in addition to those with more specific personality traits such as ones who focused on memorizing mathematics procedures or ones who had high aptitude but did not typically apply themselves on their mathematics coursework. These recordings also supplemented my field notes, allowing me to fulfill my role as a researcher and more meticulously document the observations I made. Between these four recording devices, I captured nearly eighty hours of instruction and dialogue focused on modeling tasks and the development of modeling abilities.

**Artifacts.** Since my ultimate goal is to design instruction in order to develop student modeling abilities, I needed data to document if and how these abilities manifested and evolved through the unit. As a result, I collected student work from each task used in the unit. Collecting student work allowed me to see the modeling process come to fruition including the decisions made throughout the approach and model designed. Although assessment is difficult and a major issue when analyzing and studying modeling tasks (Galbraith, 2007), through the analysis of student work I could more clearly understand student thinking, models developed, reasoning, and rationale. Each task was also instructionally scored against an individual and team product rubric I had designed (see Chapter 3) to evaluate the quality of a student’s progression through the modeling cycle and the development of modeling competencies.

At the conclusion of the unit, I also asked each student to fill out a reflection form recording their overall impressions, new learning, and attitude towards the modeling unit (see Appendix A). Some of these questions included:
1. What is one moment or idea that will stand out to you most from this modeling unit?

2. What do you feel is the most important thing you learned in this modeling unit?

3. What moments caused you frustration throughout the unit? Can you recall them specifically? What helped you get through the frustrations? (Why didn’t you give up?)

This data provided insight into the overall experience and learning students may have encountered throughout an entire month of instruction on modeling. Additional artifacts such lesson plans, student organizers, and group responses were collected if I felt they were related to teacher or student learning in the unit.

**Post unit student interviews.** In late April and early May 2016 after the conclusion of the modeling unit, I selected and interviewed five students who either (1) had unique experiences in the modeling unit or (2) had experiences representative of the majority of students in class. Each student was interviewed one time for approximately 25 to 35 minutes in the Media Center at River Junction Senior High or in my classroom when the Media Center was not available; I then transcribed the interviews. In order to enhance and verify the data obtained from student work, these interviews were invaluable in helping me better understand the development of these students’ modeling abilities or lack thereof, conflicts which arose throughout the unit, and the role the teacher, their peers, and the design of each task played in their new learning. Using a semistructured interview design (Mertler, 2014), I developed common questions asking students to
describe their prior math experiences, reflect on the unit, and articulate their new learning. I also asked additional questions specific to each student’s experience in order to triangulate observations I had made in my field notes, videotaped lessons, or their individual work. The interview protocols for each student can be found in Appendices B through F.

**Reliability**

Throughout the data collection and analysis, I clearly communicated the conjectures, suppositions, and assumptions I used to make interpretations of the modeling design and student learning with the use of multiple data sources, so my conclusions can be subjected to public debate, analysis, and questioning (Cobb & Gravemeijer, 2008). Cobb and Gravemeijer (2008) suggest to ensure results are trustworthy, repeatable, and generalizable in a design research experiment, the researcher needs to be explicit in her criteria for claims and be able to trace claims back through data sources, make known necessary versus contingent aspects of the design, environment, and instruction so others can adapt from it, and explain how it may fit into larger, more global phenomenon.

Triangulation through the use of multiple data sources was also employed to ensure the reliability of data collected (Creswell, 2007; Merriam, 2009; Mertler, 2014; Mills, 2007). This triangulation of data from my research journal, field notes, videotaped class sessions, collection of student work and other artifacts, and student interviews have enhanced and validated the results of this project (Creswell, 2007; Merriam, 2009) by allowing me to document major shifts in student reasoning and learning (Cobb &
Gravemeijer, 2008) and teacher instructional development. Figure 2.1 outlines how I used each of the data sources for this study to gain insight into my research questions:

<table>
<thead>
<tr>
<th>Research Questions</th>
<th>Data Source</th>
</tr>
</thead>
</table>
| How does a teacher design and enact authentic modeling tasks in a diverse secondary mathematics classroom and what are the effects of those tasks on modeling abilities and concomitant mathematical learnings? | 1. Research Journals  
2. Field Notes  
3. Videotaped lessons  
4. Artifacts  
5. Five Student Interviews |

| Frequency                                                                 | Design Phase: when design decision were made  
Enactment Phase: daily |
|--------------------------------------------------------------------------|------------------------------------------------|
| Duration                                                                 | Design Phase: November 2015 to February 2016  
Enactment Phase: March 14, 2016 to April 14, 2016 |
|                                                                          | Enactment Phase: daily  
(24 class periods)                                                   |
|                                                                          | After conclusion of the modeling unit between April 26, 2016 to May 17, 2016 |

*Figure 2.1. Triangulation matrix showing data sources, frequency, and duration.*

Mills (2007) suggests the use of a triangulation matrix to make explicit how each data source was used to answer research questions. Substantial research journals forced me to document my own thoughts and decision both before and during a unit on modeling. Field notes, videotaped lessons, and various artifacts allowed me to corroborate data on what I was seeing versus what students were actually doing. Finally, student interviews gave me the opportunity to confirm with students observations I had made, allowing me to feel confident in the accuracy of themes and patterns which emerged.
Data Analysis

During the month of modeling, data analysis of my research journal, field notes, videotaped lessons, and artifacts occurred simultaneously with data collection (Merriam, 2009), so I could adapt my instruction to new teacher and student learning. This also allowed me to make note of potential patterns and themes that occurred throughout the duration of the unit. However, because of the intensity and demands of being a practitioner-researcher, this data analysis only took the shape of broad reflective notes or memos.

A more in-depth and thorough analysis of all data occurred at the conclusion of the modeling unit in the months of May, June, and July 2016. All journals, field notes, artifacts, videotaped recordings, and interviews were initially coded using words from the original data. Following the qualitative data analysis techniques suggested by Merriam (2009), these codes in addition to the on-going notes I made throughout data collection, allowed me to extract sentences, paragraphs, and occurrences which shared common themes or patterns. I could then group these regularities into categories using words representative of these themes. Examples of these categories include: multiple solutions, authentic mathematics, student-centered, assumptions and limitations, teacher questioning, teamwork, and perseverance. Finally these categories were “reduced and refined and then linked together” (Merriam, 2009, p. 192) into three themes which helped me answer my research questions: students relearning how to learn, relearning to teach, and relearning mathematics within authentic tasks.
Conclusion

The methodology used to understand the design and enactment of modeling tasks allowed me to embrace my new role as a researcher without ignoring the demands of teaching secondary mathematics education on a daily basis. In addition to the precalculus course at the focus of this study and an additional precalculus course not involved in this research, I still needed to teach four classes of nearly one hundred geometry students. I had to design my research plan around data I hoped would answer my research questions, but this data collection also needed to be manageable and not disrupt the learning I was responsible for in my other courses. Fortunately, because of the triangulation through various types of data sources, I was able to gather and analyze more data than was probably necessary to understand teacher and student learning associated with mathematical modeling.

In the following chapters, I discuss and provide analysis on the data collected during a month of modeling. In Chapter 3, I continue expanding my knowledge of modeling in Part One by detailing the research, planning, and preparation that went into the design of the modeling unit. Part Two shifts to describing the ‘what happened’ during five weeks of modeling instruction included in Chapter 4 of this study. Finally in Part Three, Chapters 5 through 7 provide evidence and discussion of the three themes that emerged: students relearning how to learn, relearning to teach, and relearning mathematics within authentic tasks.
CHAPTER 3
THE PREQUEL TO A MONTH OF MODELING

Preparing for an entire unit on modeling in the way research suggested initially seemed impossible. The Common Core State Standards defines the action of modeling as “using mathematics or statistics to describe a real world situation and deduce additional information about the situation by mathematical or statistical computation and analysis” (Common Core Standards Writing Team, 2013, p. 5). It suggests modeling is essential to include in the K-12 curriculum, yet it is difficult since “modeling requires multiple mental activities and significant human skills of abstraction, analysis, and communication” (Common Core Standards Writing Team, 2013, p. 6). Could providing opportunities for students to progress through modeling over several weeks help in the development of modeling abilities? In past years, I was scrambling at the end of the year to cover all learning targets as outlined in my syllabus and district mandated curriculum guides. Now here I was, trying to insert an additional unit into a course already claustrophobic from disconnected and complex topics: polynomial functions, rational functions, exponential and logarithmic functions, trigonometric functions, trigonometric identities, discrete mathematics, statistics and probability, matrix algebra, polar equations, parametric equations, and conic sections. How would I find the time I needed to teach modeling as defined by CCSS-M and provide ample opportunities to overcome potential barriers?

That question was surprisingly easier to answer than I expected. With careful planning and preparation at the beginning of the 2015-2016 school year, nearly seven
months before the unit would commence, I began reorganizing the scope and sequence of my precalculus course. Topics like “Graphing, modeling, and solving polynomial functions of higher degree” I would typically devote three days to would only get two class periods. Sections in our curriculum similar to ones like “Modeling exponential and logistic functions” would drastically get cut down since I felt the modeling unit would have better opportunities to develop these skills. And once I sat down and carefully analyzed my curriculum syllabus, I realized many redundancies within my own course or carried over from other courses including the development of skills related to long and synthetic division, logarithmic properties, matrix operations, and solving systems of inequalities. This is consistent with research which suggests upwards of 60 to 70% of instructional time is spent on topics taught at previous grades (Polikoff, 2012). Other studies have also suggested high levels of redundancy in mathematics courses especially when compared to other countries (see Heibert et al., 2005, Porter et al., 2009, Schmidt et al., 2005). After reducing the scope of most of these redundancies, I was able to free up nearly twenty class periods! I knew I could not account for every unforeseen scheduling conflict such as state accountability testing or a record-setting blizzard dumping twenty inches of snow and canceling school for three days. I also recognized I would need to adapt to my students and their needs throughout the school year perhaps devoting more or less time to certain topics as those needs arose, but twenty class periods was promising! I began to think this unit might actually be possible. I approached my department chair, curriculum coordinator, and associate superintendent with my new and revised plans, and
they gave me their full support to modify my course curriculum and design my own supplemental unit on mathematical modeling.

In this chapter I discuss the modeling abilities I wanted students to develop over the course of a unit on modeling. These modeling abilities lead to descriptions of the design criteria I used to develop my modeling curriculum. I then explain the unit design including the inspiration for and development of each task. I conclude this chapter with a discussion of my plans for assessment and materials used for assessment and student support throughout the unit.

**Development of Design Criteria**

After clearing out enough instructional time for the modeling unit, I spent the months of December 2015 to February 2016 designing and writing the curriculum materials I would use. Having had little modeling experience myself, especially modeling in the way CCSS-M and NCTM intend, I needed to devote significant time and energy to designing tasks which aligned with current research on modeling and my own goals for student learning. Going into this study, I was most nervous about this design phase. I was confident in my ability to teach, but most of what I had taught previously was modified from curricular materials provided to me. Would I be able to design a quality unit on modeling from scratch when my own modeling experiences have been limited? The review of literature I conducted for this study helped me develop the background knowledge I needed to understand what modeling should look like, now I needed to figure out how to create the kind of environment and provide opportunities that might foster these types of modeling skills. Initially I planned to develop tasks aligned
with Galbraith’s (2006) and Lesh et al.’s (2000) design principles as outlined in Chapter 1. However, after feedback and encouragement from my committee members in December 2015, I ended up outlining my own design criteria influenced heavily by Galbraith (2006) and Lesh et al. (2000) based on the specific modeling abilities I identified to target in the modeling unit. From this list of modeling abilities and design criteria I was able to layout a cohesive unit and develop five modeling tasks for students to analyze.

**Targeting Specific Modeling Abilities**

Similar to a list of learning targets outlined in standard curriculum materials, I first wanted to identify the conceptual learning goals for students in this modeling unit (Simon, 2006). The overarching intent for this unit was to help students learn how to model mathematical situations. But in order to accomplish this, my research needed to focus more specifically on how a teacher designs and enacts modeling tasks with the purpose of developing student modeling abilities. However, (1) what are the modeling abilities specifically I wanted students to develop and (2) how are they related to each other? The research literature provides some guidance such as highlighting the importance of data analysis (Lesh & Doerr, 2003), applying and mathematizing everyday knowledge (Lesh & Harel, 2003; Mousoulides et al., 2013), and interpreting mathematical results (Lesh & Doerr, 2003), but I found myself needing a more systematic and related list that could act as targets for both the teaching and learning that would occur in the classroom. Using the research literature as a guide, I first analyzed the modeling cycles illustrated by NCTM and the CCSS-M. Within these cycles, I noted
what modeling abilities students would need to use to successfully navigate each phase and what research supported this development (see Figure 3.1 modifying the four-step modeling cycle as shown in Figure 1.1).

1. Apply everyday knowledge
2. Mathematical literacy
3. Data analysis & collection
4. Flexible and creative thinking
5. Perseverance
6. Quantitative literacy
7. Reflection
8. Reasoning
9. Communication

Figure 3.1. Modeling abilities developed from NCTM’s four-step modeling cycle.


I then adapted and extended each of these identified abilities and developed guiding questions that could support both myself in the design and teaching of a unit on modeling and students as they learn to navigate the modeling cycle.

**Everyday knowledge.** Students need to bring in knowledge of real life and other domains to analyze real life situations for mathematical opportunity. By bringing in everyday knowledge, research finds students are able to enhance their understanding of
mathematics content (Carpenter et al., 1989, Carpenter & Lehrer, 1999, Irwin, 2001). Research has also found students struggle with word problems when it does not allow students to employ their everyday knowledge (Verschaffel, Greer, & De Corte, 2000). As students approach a modeling task, they would use everyday knowledge to initially make sense of the situation and identify opportunities for formal mathematics. They may ask: where do I see math here and where and why would math be useful?

**Mathematical literacy.** The Organisation of Economic Co-operation and Development (OECD) defines mathematical literacy as

An individual’s capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. (OECD, 2013b, p. 25)

In modeling, students need to mathematize situations by translating real life objects and information into mathematical ones. Based on features within the situation or data, students must decide on what mathematics is needed and what variables are appropriate. Stacey and Turner (2015) suggest mathematical literacy is a major component to the modeling cycle since students begin with an authentic situation and then “identifies the relevant mathematics in the problem situation, formulating the situation mathematically by imposing mathematical concepts, identifying relationships and making simplifying assumptions” (p. 25). This description falls within action 2: *formulate a model* as indicated by CCSS-M. As students apply skills related to mathematical literacy one
might wonder: what are the essential features of the situation and how do these features identify and justify the choice of variables?

**Data analysis and collection.** As students analyze an authentic situation in order to develop a viable model, students may need to analyze existing data within the task or collect data in order to gather needed information. In order to do this, students need to understand what data is appropriate, determine how to collect data if it is not given, analyze this data for trends and patterns, and explain real life connections within the data. The National Council of Teachers of Mathematics (2000) states students should be able to collect, organize, and display appropriate data while the Common Core State Standards include an emphasis on analyzing data to make decisions or predictions (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). When analyzing and collecting data students might ask: what information or data will I need and how do I gain access to it? What assumptions am I making about this information?

**Flexible and creative thinking.** Appropriate models are not usually obvious. Students need the ability to be flexible in developing, refining, and testing their models and creative in bringing about various mathematical ideas. Chamberlain and Moon (2005) state creativity in mathematics is “domain-specific thinking processes used by mathematicians when engaged in nonroutine problem solving” (p. 38), and they view creativity as being at the core of modeling tasks. Since students are not following a prescribed procedure in the development of models, students will need to employ their own ideas based on authentic and mathematical features of the situation. This also allows
for the potential for diverse thinking and multiple solution approaches (Lesh & Doerr, 2003). When employing flexible and creative thinking, students would wonder: what mathematical relationships can I use which fit the information or data I have access to?

**Perseverance.** Perseverance seems to be an essential ability to possess in order to analyze a non-routine, authentic problem through the creation and evaluation of a mathematical model. Because models are generally not obvious and may require phases of generating, testing, and refining, students must not easily give up throughout the cycle. Common Core states the importance of perseverance in its first Standard for Mathematical Practice as students should “Make sense of problems and persevere in solving them” (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). Similar in NCTM’s *Principles to Actions*, an effective teacher “provides students with appropriate challenges, encourages perseverance in solving problems, and supports productive struggle in learning mathematics” (Leinwand et al., 2014, p. 11). As perseverance is tested and applied throughout the modeling cycle students may wonder: what about this task motivates me or what do I want to learn?

**Quantitative literacy.** Quantitative literacy is considered a habit of mind (Steen, 2001, Hughes-Hallett, 2003) focusing on the “use of fundamental mathematical concepts in sophisticated ways for the purpose of describing, comparing, manipulating, and drawing conclusions from variables developed in the quantification act” (Mayes et al., 2013, p. 6). CCSS-M indicates the importance of “reasoning abstractly and quantitatively” as students create “a coherent representation of the problem at hand;
considering the units involved; attending to the meaning of quantities”
(Corestandards.org, 2014b, para. 3). While modeling, students must be able to transfer
math knowledge back into real-life situations, interpret results of a mathematical analysis
in context of the situation, and decide on the reasonableness of their outcomes based on
limitations to reality. These skills are all ones which promote more general quantitative
literacy abilities. While developing quantitative literacy one may ask: what do the results
of my analysis mean and are the results reasonable based on what I know about the
situation?

Reflection. After assessing reasonableness, students must decide how well their
model accomplishes the goal of the task and if not, they must go back and employ
flexible thinking to reevaluate and modify their analysis. The CCSS Standards for
Mathematical Practice include skills related to reflection stating students should be able
to “monitor and evaluate their progress and change course if necessary”
(Corestandards.org, 2014b, para. 2). When reflecting on their models, students wonder:
what are potential inaccuracies of my analysis and what adjustments can I make?

Reasoning. In NCTM’s publication Focus in High School Mathematics: Reasoning and Sense Making, reasoning is defined as “the process of drawing
conclusions on the basis of evidence or stated assumptions” (Martin et al., 2009, p. 4).
During the validation phase of the modeling process, students need to make conclusions
on if their model works to accomplish the task’s goal, provide rationale on why it works,
and decide if revisions are needed. The importance of reasoning is emphasized heavily
by both NCTM and CCSS-M. Reasoning is stated explicitly in Principles and Standards
for School Mathematics as a foundational skill requiring students to develop, analyze, and test conjectures and arguments (NCTM, 2000). CCSS also stresses the importance of reasoning in its Standards for Mathematical Practices stating students should be able to “distinguish correct logic or reasoning from that which is flawed, and if there is a flaw in an argument, explain what it is” (Corestandards.org, 2014b, para. 4). As students model and apply reasoning skills they make ask: does this work, will it always work, and how do I know?

**Communication.** Because the final phase of the modeling cycle requires one to “report on conclusions,” students must be able to communicate the results of their analysis and modeling process in a way their targeted audience will understand (Corestandards.org., 2014a, para. 6). During this final component, students are also receiving communication from others on the development of models that may be different than their own and therefore must also be able to participate in productive discourse to critique and evaluate the reasoning of others. Like reasoning, communication is a skill heavily emphasized by NCTM and CCSS-M through its inclusion as a process and mathematical practice standard. NCTM states students should be able to communicate their own mathematical thinking coherently using mathematical language while also being able to examine the strategies of others (NCTM, 2000). In CCSS’s Standards for Mathematical Practice students are expected to “justify their conclusions, communicate them to others, and respond to the arguments of others” (Corestandards.org, 2014b, para. 4). As students prepare to communicate their results,
they may ask: how will I communicate and justify the process, solution, and results of my analysis in a way others will understand?

**Next steps.** Although these nine modeling abilities were closely aligned with current research, identifying how they were related to each other and where they might occur in the modeling process helped me as both the curriculum designer and teacher of this unit gain a better appreciation of how a modeling tasks would need to be developed. Once I understood the specific skills students would be working on if they successfully progressed through the modeling cycle, I then could create the design criteria I felt would provide students the best opportunities to develop these abilities.

**Criteria for Modeling Tasks**

After identifying the nine modeling abilities which would serve as the learning targets for a modeling unit, I reflected on what features a task would need in order to give students the best opportunity to apply these skills. I was able to clearly define five design criteria that would act as a checklist for each task I developed for the unit. Although some tasks may highlight certain criterion over another, in order to maximize the potential for each modeling ability to be utilized all tasks would be developed, analyzed, and revised with this set of criteria in mind.

**Criterion 1:** Task generates a need for students to translate and connect knowledge of real life situations and experiences to knowledge of mathematics. Modeling tasks should be authentic. That is, they should be rooted in everyday language and knowledge and require students to translate this common language into a mathematical one.
Based on the review of literature, this seems to be one of the core ideas behind modeling tasks. Both Galbraith (2006) in Principles 1 and 2 and Lesh et al. (2000) with the Model Construction and Reality Principles describe design expectations similar to Criterion 1. Modeling tasks are designed around a real life situation that requires a mathematical analysis to understand it better and make predictions. When tasks contain this design criteria, students should need to apply everyday knowledge and mathematical literacy.

**Criterion 2:** Students have the opportunity to bring in *various* and *available* mathematical experiences, knowledge, representations, and data analysis techniques to develop and refine models.

Essentially, this criteria emphasizes students should have the opportunity to approach a task in a variety of ways. There is not a pre-designed solution or correct procedure to follow. By the mathematics being ‘available’ to students, this emphasizes the mathematics needed should be possible for students using prior real life and math knowledge. This particular criterion seems to not be included in the design principles for Galbraith (2006) and Lesh et al. (2000). However, Galbraith (2006) includes the ‘availability’ of mathematics in Principles 3 and 4 when he states tasks should be feasible and the mathematics possible for students. By including this criterion in task designs, students specifically have the opportunity to develop modeling abilities related to data analysis and collection and flexible and creative thinking.

**Criterion 3:** Students are able to use the task’s information and goal(s) to assess reasonableness of the analysis (model) and if necessary refine the model.
This criterion is consistent with Lesh et al.’s (2000) Self-Assessment Principle and Galbraith’s (2006) Principle 5 which addresses accuracy and reasonableness. The task should be designed in such a way students will be able to determine if their results are plausible and if not, refine their model to make the results more appropriate for the task’s goal(s). Addressing this criterion in the task’s design may help students develop perseverance, quantitative literacy, and skills related to reflection as they go through the cycle of assessment and refinement.

**Criterion 4:** Tasks encourage students to express their own thinking, ideas, and justifications and critique the reasoning of others through the modeling process and in the final product.

As research has stated, discourse throughout the modeling process is essential in helping students develop and refine their models. As a result, tasks should have opportunities for students to explain their thinking and provide feedback to their peers. This criterion is stressed heavily in Lesh et al.’s (2000) Construct Documentation Principle but is left out of Galbraith’s (2006) design principles. Including this criterion should allow students to better develop their reasoning and communication skills. However, in reality this criterion may be more a product of the learning environment that has been established within the classroom as opposed to the task itself. Careful planning and classroom organization needs to also be considered for this criterion to be effective even if it is embedded within the task.

**Criterion 5:** Tasks require students to communicate the results of their mathematical analysis in context of the original real life situation.
In order for students to assess reasonableness effectively, the task’s goals should require students to translate the results of a mathematical analysis back into context of the original situation helping them to develop skills related to quantitative literacy and communication. Specifically, students must be able to interpret and communicate what the mathematics means in a way others not familiar with the mathematics can understand. Communication is a significant part of the modeling process as it is listed specifically by CCSS as the final phase of the cycle and is stressed heavily by standards documents for both NCTM and CCSS-M, yet it is not included in Lesh et al.’s (2000) and Galbraith’s (2006) design principles. The design principles developed by Lesh et al. (2000) and Galbraith (2006) focus on assessing reasonableness and perhaps it is implied that in order to do this students will need to be able to interpret and then communicate their mathematical analysis. However, I think it is important enough to the modeling process and the development of modeling abilities to state it as a specific design criterion.

Although having design criteria does not guarantee the use or development of all nine modeling abilities as students work through each modeling task, the process I went through simply to develop them helped me understand how tasks aligned to CCSS-M and NCTM would need to look. Most of my previous design experience only pertained to problems requiring students to “solve for x.” Accomplishing this type of goal in the past has not required the use of various design elements because the learning target was so straightforward. But because of my own exiguous knowledge, uncomfortableness, and limited experience in modeling, developing and analyzing these five design criteria
proved to be immensely helpful as I learned how to design modeling tasks with the purpose of developing student modeling abilities.

**Unit Design**

Knowing I would be implementing a modeling unit in the middle of second semester, I tried to keep my eyes and ears open for potential ideas several months in advance. I kept a list of topics on my phone so I could conveniently write down thoughts as I had them. After developing the design criteria, I went back to this list and tried to select topics I thought could provide students with substantial opportunities to develop modeling abilities. I also wanted to try to move through the modeling unit from more concrete and scaffolded tasks to more abstract and less structured tasks. As a result, the first few tasks would have more specific results to generate while the remaining tasks truly allowed for multiple solutions. I formed these tasks into one cohesive unit that gradually took control away from me and gave it to students. The unit ended up consisting of an introduction followed by five projects:

a) *What is modeling?*

b) *Project 1 - The High Definition Trade-Off Dilemma*

c) *Project 2 - Calculating the Width of Memorial Stadium*

d) *Project 3 - Enrollment at River Junction Senior High: How big will we get?*

e) *Project 4 - How safe are we: Developing a crime rank for River Junction*

f) *Project 5 - Student Choice*
When I initially set out to design the unit, I hoped to include between four and six projects. This was not based on any research I had found; I only wanted to expose students to a variety of situations and four to six projects seemed reasonable with the amount of time I was able to squeeze out of my curriculum.

For the duration of the unit, students would be working with the same group of three to four students. And because communication is such a critical element to my design criteria, each group would be required to showcase their final model and conclusions. Students would be responsible for their individual work but this work would also lead into a final team presentation to explain, validate, and discuss their mathematical analysis. However, I did not want the presentations for all five projects to look and feel the same. Through the course of my literature review and brainstorming sessions, I had kept a working document with general introduction ideas, a variety of presentation formats, and teaching suggestions that could be used for any of these tasks. From this working document I was able to pull specific design elements to write each task, structure the ending communication of each model, and provide students with a variety of instructional methods.

**Intro to Modeling: What is Modeling?**

Prior to this modeling unit in precalculus, students were familiar with learning targets from our course syllabus that looked like “Use the Fundamental Theorem of Algebra to determine the real and nonreal makeup of the zeros of a polynomial function.” Students would not understand the goal of this target initially unless they were already familiar with the Fundamental Theorem of Algebra, so I would set up opportunities for
exploration and discussion first. Similarly, if I gave students the learning target for this unit “Model authentic situations with mathematics using the Common Core State Standards modeling process so that specific modeling abilities are developed,” students would not understand expectations or how to assess if they achieved the target. In order words, I felt the unit needed an introduction. I did not expect students could jump into a modeling task right away and understand the expectations I wanted for its outcome.

Students first needed to be educated on the process itself and what modeling abilities I hope they developed (Anhalt & Cortez, 2015). I also thought we needed to spend time on establishing relevance to the unit, discuss how this unit may be different than what they are used to in mathematics, understand what modeling looks like, and clarify logistical items such as handling absences and assessment. I planned to spend two days on “What is modeling?” (see Appendix G) which was split up into three parts: why should you care about modeling, what is modeling, and goals and grading.

**Project 1: The High-Definition Trade-Off Dilemma**

During my master’s degree program I had taken a class titled “Using Mathematics to Understand our World.” I also later helped teach an online version of this course. The major premise of the course was to introduce teachers to applied mathematics through the use of modeling tasks. As I was trying to come up with topics to use in this modeling unit with students, I thought I could get some ideas from tasks used in this course. In the first project, teachers enrolled in the course were introduced to a trade-off problem from a letter sent to an instructor titled “Can you Save Tom’s Marriage?” (Hines, 2005). In the letter, Tom asked for help trying to set a recording on his VCR as the higher quality EP
mode took up too much of the tape to record the entire show. Tom needed to know how much he should record in the higher quality EP mode before switching to the lower quality LP mode in order to be able to record the entire show. I thought this type of situation would be a good introduction to modeling as the mathematics required seemed to be more straightforward, requiring primarily the creation of a system of equations, yet it was still embedded within an authentic task.

However, the situation with VCR recordings is outdated for my students so I tried to brainstorm similar trade-off situations my students could relate to. I know a situation I consistently run across is recording video in high definition, so I rewrote the task to showcase recording quality on a GoPro video recorder with the task being introduced through a Facebook post from a made-up friend named Heather (see Appendix H):

“Help me nerdy friends! We are going for a scuba dive in the reef later today (super excited!) and realized my GoPro Hero4 isn’t going to record the entire dive in high-def. Obviously, I want as much of it in 1080p high-def as possible to catch all the awesome colors and wildlife but I want to record the entire 5 hour dive so I don’t miss anything cool...like a reef shark. I have a 64gb memory card in my GoPro, how long should I shoot in 1080p before switching to 720p to maximize the amount of high-def footage I get?” (Classroom Materials, March 16, 2016)

I also included information about the recording times in 1080p and 720p students would need to complete this task. I debated about not including this information to see if
students would ask for it, but decided since this was the first project I would give them enough information to successfully complete the task.

Rather than finding a model that would work with any length recording initially, the situation posed to students in Project 1 was to answer a specific question for Heather about a particular recording length. I did not intentionally set the task up this way; I just wrote the task by mimicking a situation I thought would show up in real life. It seemed more likely someone would want to know about a specific situation as opposed to needing a general model to begin with. But as I expanded on this task to think about other questions someone might want to know about the situation, I realized Heather’s problem created a good springboard for more general questions about recordings on a GoPro device. Additional questions were added to the task requiring students to think about how their model would change for different size memory cards and a different recording length. Therefore, the team’s final product would be to create a letter to GoPro’s corporate office which could provide information to a consumer on how long to record in each mode if the consumer knows the size of the memory card and the total time needed to record. By writing a letter to consumers, students had to think about how to explain their model in everyday language allowing non-mathematicians to understand what to do and why it worked. I also would have teams read each other’s letters to provide additional feedback on this first project.

**Mathematics in Project 1.** To formulate models in Project 1, I first anticipated students would simply use guess and check methods to narrow down a reasonable estimate for recording times in each resolution based on the relationship between units of
space (gigabytes) and time (minutes). In order to do this, students would need to apply CCSS-M standards related to reasoning quantitatively and using units to solve problems. Specifically, they would need to apply standards HSN.Q.A.1 “use units as a way to understand problems and to guide the solution of multi-step problems” and HSN.Q.A.2 “define appropriate quantities for the purpose of descriptive modeling” (National Governors Association Center for Best Practices, 2010).

After recognizing mathematical relationships through guess and check methods, I thought students would set up a system of equations to represent constraints on time and space within the problem. They would then need to solve this system using a method of their preference: substitution, elimination, or intercepts through graphing (see CCSS-M standard HSA.REI.C.6). In CCSS-M standards HSA.CED.A.2 and HSA.CED.A.3, students are required to “create equations in two or more variables to represent relationships between quantities” and “represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable option in a modeling context” (National Governors Association Center for Best Practices, 2010). These standards are typically introduced in algebra then applied and developed further throughout all secondary courses.

**Project 2: Calculating the Width of Memorial Stadium**

I attended a professional development conference at the end of October 2015 and was pleased to see a teaching colleague of mine, Dan Schaben, doing a presentation on a modeling task he had created to find the distance between two towns in his area. He introduced the task with a video of him taking the measurements and then let students...
(his audience in this case) work on the problem. I had been spending a lot of time during this conference thinking about the outline of this modeling unit and the logistics of carrying out this study, taking advantage of the time away from school and home responsibilities. I also had been trying to find an authentic task that would be interesting to students and would require them to apply their knowledge of trigonometry. I thought it fate that this colleague was presenting on modeling and his particular task involved trigonometry! As I thought about his task and presentation I knew I could replicate his idea and was curious if it were possible to develop a general model to find the width of any object using the measurements he provided: two sets of two angle measurements taken along a straight line running north/south or east/west a known distance away from each other.

My husband and I are season ticket holders for the University of Nebraska’s football program at Memorial Stadium. I was sitting in the stadium during a game after this conference and asked myself “I wonder how wide the stadium is.” Then it occurred to me, I could have students calculate the width of Memorial Stadium using a similar strategy to the one my colleague presented and apply this work to determine a more general model. This led to the formation and design of Project 2: Calculating the Width of Memorial Stadium (see Appendix I). Like my teaching colleague, I introduced this project with a video of me taking the measurements. I hoped the video would help students apply mathematical literacy skills correctly as they visualized and mathematized the situation and drew an appropriate representation. Upon developing models to calculate the width of the stadium and attempting to develop more general models to
replicate in similar situations, students would be asked to present their models to the class in small group presentations.

**Issues with Project 2.** I thought this project was going to be the easiest for me to develop. It seemed straight-forward for myself and students to follow. I just needed to take a straight line distance north and south and record two sets of angle measurements from both ends of that line. My husband and I happened to be near Memorial Stadium on a bitterly cold (about -20º F) January day, so using a compass app and GPS distance tracker I had downloaded on my phone my husband quickly shot a video of me explaining the task and taking the measurements at about 40 feet apart in a parking lot outside the stadium. I could not feel my fingers during the course of the video it was so cold. As a result, I was not paying much attention to the measurements I had taken. Later that weekend when we were back at home (90 minutes away) I took the measurements and tried to calculate the width when I realized the angle measurements I had given students would not even produce triangles that were possible. My compass app had apparently not polarized correctly (perhaps due to the cold) and gave me completely inaccurate measurements. As a result we would need to shoot the video again.

The few weeks after this was filled with very mild weather, but of course the next time we went to Lincoln to reshoot the video we had another bitterly cold and blustery day. Standing in the same locations we had the last time, we retook the measurements and video. Being in a hurry to once again get out of the freezing sub-zero temperatures, and apparently not learning my lesson the first time, I waited until later to calculate the
width. When I did, I was getting a width measurement of 100 feet, even though I knew from Google Maps the actual width was 800 feet. The measurements I had taken apparently were still too inaccurate to result in a reasonable width. I played around with the measurements I had taken and realized that a change in the angle measurement by one degree changed the width around 1500 feet. Thinking about why that might be, I hypothesized that the distance I walked to take the angle measurements was simply too short. There would not be a great enough difference in my angle measurements between the two locations if I only walked 40 feet to take those measurements as I was standing nearly a half a mile from the stadium. Any sort of measurement error in my angles would cause the distances to be off by several hundred feet because I was standing so far away from the stadium. I needed to increase the distance between the locations where the angle measurements would be taken.

The third time I shot this video I took measurements which according to my GPS were about 150 feet apart and angle measurements as reported in the project. I also took the measurements first and calculated the width in the parking lot prior to recording the video. The weather this time was not zero degrees but we did have to endure a 30+ mile per hour wind. When I calculated the width of Memorial Stadium this third time I got a width of about 500 feet which is still 300 feet short of the actual measurement. I am almost positive my angle measurements were the problem but the easier value to change was the distance between the two locations. I ended up changing this distance as reported to students to 200 feet which brought the width to about 800 feet.
What I thought would be the easiest and most straightforward project turned out to be the one that caused the most frustrations. After several hours and hundreds of miles of driving just to shoot the videos I needed, I actually thought about scratching this project. But, what I learned is that if I am going to develop authentic tasks I will probably undergo some authentic experiences myself. Real life is not pretty; it often does not come out the way we want it to. When I am trying out new things in my classroom or developing tasks in a manner that is new or uncomfortable to me, I need to be prepared for some frustrations just like my students.

**Mathematics in Project 2.** Students would primarily apply the Law of Sines and Cosines to find the width of the stadium in Project 2. Law of Sines and Cosines is recommended by CCSS-M to be covered in a fourth year secondary mathematics course like precalculus and is included in standard HSG.SRT.D.11. In addition, students also need to be able to draw and represent angle measurements from due north and apply knowledge of geometry including supplementary, vertical, and triangle angle relationships. To create a general model of the task, students also need to apply knowledge of addition and subtraction formulas for sine and cosine in order to simplify expressions of the form:

\[ \sin(180 \pm x) \text{ or } \cos(180 \pm x). \]

This is also recommended by CCSS-M in a fourth year course and included in standard HSF.TF.C.9.
Project 3: Enrollment at River Junction Senior High

At the beginning of the 2015-2016 school year, our school improvement committee showed teachers the current beginning of the year enrollment for kindergarten through 12th grade in our school district. The data immediately caught my attention. Our kindergarten class to start the year had 809 students while our senior class had only 575. Current middle school enrollment had over 700 students for each grade. The group of teachers I was with quickly became alarmed as we thought: where are we going to put all these kids? Our one high school in the school district already had nearly 2400 students enrolled in grades 9-12, and has not had any building additions since the late 1990s when class enrollment was around 350 students per grade. We did not have the space and resources to house students once these kindergarteners entered high school.

Yet, at the time there had been little public discussion about a new high school. The district just passed a $69.9 million bond for construction projects over the next five years (including replacing or adding onto five elementary schools), so another large bond for an additional high school would need to be well researched and thought out. With neighborhoods of economically disadvantaged students throughout town, some residents were worried building another school would also make one school ‘better’ than the other. I thought to myself as I looked at these enrollment figures: What are we as a school district going to look like in ten years? What will it take for the school board to begin making high school space a priority and what does the the public need to know to support such a project?
When I started reflecting on ideas for this modeling unit, this enrollment data continued to stick in my mind as I thought students would find it interesting. Students in our building are constantly complaining about the cramped hallways; I was curious what they would notice in enrollment figures and what opinions they might have about space issues at the high school. I was able to acquire enrollment data for each grade level kindergarten through twelfth grade from the Nebraska Department of Education beginning in the 2000-2001 school year (see data in Appendix J) and from this data designed Project 3 (see Appendix K) with a goal for students to predict the high school’s population in 9 years and 15 years and provide recommendations based on what they learned.

I also arranged for a group of our school’s administrators to listen to student presentations and ask questions about the analysis conducted. Like the format used in Project 1 to produce letters to GoPro consumers, having an outside audience invested in the models produced by students would hopefully help students think about what their model communicates and why it works in everyday, non-technical language furthering the development of skills related to quantitative literacy, reasoning, and communication.

**Issues with Project 3.** I initially worked through the numbers myself to come up with my own predictions so I could design scaffolding questions to help guide student thinking. Students, to my knowledge, had never worked with such a large data set and I was worried about them being overwhelmed with a 14 by 16 array of values to sort through. However, after designing the scaffolding questions and going back through the task, I realized my questions were too leading and focused students too narrowly on one
specific strategy. These questions forced students into the method I used rather than making them develop their own. As a result, I left the guiding questions out of the project I handed to students and kept them only for students I felt needed additional support to successfully accomplish the task’s goals.

**Mathematics in Project 3.** In order to model enrollment data in Project 3, students need to apply knowledge of various previously learned functions including but not limited to linear, exponential, and logistic functions. When students analyze this data they need to look for trends, allowing them to recognize and justify which type of function would be most appropriate. This aligns to CCSS-M standard HSF.LE.A.1 which asks students to “distinguish between situations that can be modeled with linear functions and with exponential functions” (National Governors Association Center for Best Practices, 2010). In addition, they need to create a function that fits the given data, analyze this function to make predictions, interpret this function in context, and validate if their model is appropriate. This forces them to apply CCSS-M standards HSS.ID.B.6 as they “fit a function to the data and use functions fitted to data to solve problems in the context of the data” and “fit a linear function for a scatter plot that suggests a linear association” (National Governors Association Center for Best Practices, 2010).

As they begin to assess reasonableness and interpret features of the function in context, students also apply skills related to standards HSF.IF.B.5 “relate the domain of a function to the quantitative relationship it describes” and HSS.ID.B.7 “interpret the slope and the intercept of a linear model in the context of the data” (National Governors Association Center for Best Practices, 2010). Finally, in order to analyze so much data, I
thought students would need knowledge of mathematical graphing and analysis tools such as their graphing calculator, Desmos, or Geogebra. Using available technology tools to analyze this data also allows students to apply CCSS Mathematical Practice Standard Five: “Use appropriate tools strategically,” and perhaps synthesize knowledge related to computing and interpreting the correlation coefficient to validate a linear model (Corestandards.org, 2014b, para. 6.)

**Project 4: How Safe are We?**

In December 2015, I was reading posts on Twitter when a teaching colleague of mine had posted about the mathematics modeling competition developed by the Consortium for Mathematics and Its Applications (COMAP). I was curious so I visited their website and was able to find additional information on their High School Mathematical Competition in Modeling (HiMCM). In 2015 they held their 18th competition with 719 teams from 144 schools around the world competing (Consortium for Mathematics and Its Applications, 2015a). I was not able to access the papers from the finalists, but after reading the criteria I was impressed with the technical writing skills these students must possess. I was able to access the problems used in the competition every year since 1999 and thought some of these may be modified for use within typical high school classrooms. Obviously, the tasks are written for students who have a substantial amount of modeling experience which my students do not, but I thought some of these tasks could present ideas I could work from. One of the tasks from the 2015 competition titled “Problem: City Crime and Safety” caught my attention as it pertained to issues of citizenship and social justice:
What can we make of the massive amount of crime statistics collected in major cities? Beyond just reporting numbers, how can we use these data to determine the safeness of a city? Assume that you and your modeling team live in My City, a large international hub of commerce, technology, finance and travel, with a current population of 2.8 million people impacted by a metropolitan area of an additional approximately 6 million people.

The data set provided shows two weeks from police reports in My City and includes crimes listed by case number, date of occurrence, primary and secondary crime descriptions, crime location, whether an arrest was made, whether or not this was domestic crime, and the beat number of the police route.

**Part I:** Using mathematical modeling, analyze the data. Create a safety rating for My City. Use your safety rating to specify a measure of how safe My City is.

**Part II:** In addition to the HiMCM contest format, prepare a 1-2 page non-technical report for the Mayor of My City to describe your findings.

(Consortium for Mathematics and Its Applications, 2015b)

The data set provided for the problem was massive and contained nearly 11,000 rows of crime occurrences from within the city. In order to analyze this data set, students would need to be very comfortable and familiar working with and manipulating spreadsheets. Unfortunately, the majority of my students would not be able to manipulate and summarize the data set to make it useful. However, I thought the idea of crime reporting and students developing their own model to assign a safety rating interesting. It gives a lot of mathematical freedom to students; models developed by teams could look entirely
different based on the information they use and the weight they assign that information within their model.

To make it more relevant, I decided students could design a model to assign a safety score to the city we live in, River Junction, based on crime reporting data I found from the FBI and created Project 4 from this idea (see Appendix L). Recently in the news I had seen several articles suggesting River Junction was one of the most dangerous cities in our state and decided I could use these articles as an introduction to the project.

**Issues with Project 4.** I realized after trying to create my own model there is no context to interpret safeness from this score unless you can compare scores from city to city. As a result, I changed the project to focus on creating a model to assign a safety score for major cities in Nebraska and not just River Junction. I also thought it would be interesting to rank these cities based on their scores and compare them to the articles students had to read at the beginning of the project. The project required students to present on their final rankings and report the following information:

1. The information/data you chose to consider in your analysis and *why* you decided that data was important to consider. You may want to conduct research on crime ratings or ask adults what information they would want you to consider in your safety score.

2. A description of your model (i.e., what you are calculating).

3. A thorough description of your methodology and rationale. That is, how did you take into account your data in your model and *why you made the decisions you did regarding this data.*
4. Use your model to find a safety score for River Junction and other major cities in our state (at minimum you must include River Junction, Metroville, Jefferson, Central, Loganview, Saint Peters, Cedar Valley, Platte City, Armstrong, Phillips, and Spring Hill). How do these scores compare to River Junction? Rank these cities from safest to most dangerous. Do you agree with the rankings done in the news?

5. A description of how someone might interpret your data and what the score your model generated for River Junction means. In other words, how safe are we? (Classroom Documents, April 1, 2016).

I was hoping students might notice data can be interpreted in a variety of ways, so it is important to understand the methodology behind the ranking system before making conclusions about the safeness of a city.

**Mathematics in Project 4.** Project 4 applies least to specific mathematics content skills typically found in precalculus or other secondary mathematics courses. However, it addresses larger needs associated with the “mathematics needed for intelligent citizenship” (NCTM, 2000, p. 4). Through the development, communication, and analysis of various types of models assigning a safety ranking for cities in our state, students are gaining skills related to “quantitative sophistication” which are needed to “understand and be able to use mathematics in everyday life” (NCTM, 2000, p.4). As students critique the methodologies presented by others in this task, students are also applying CCSS’s Mathematical Practice Standard Four: “Construct viable arguments and critique the reasoning of others.” Specifically they will need to:
reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and if there is a flaw in an argument, explain what it is. (Corestandards.org, 2014b, para. 4)

My hope for this modeling project was students would learn the importance of knowing about the data in an analysis, where it comes from, and what it does and does not represent. I also want them to question sources in real life and understand the importance of knowing about methodologies. The specific mathematical strategies here are less interesting to me than the quantitative reasoning and issues of social justice I hoped would be discussed.

**Project 5: Student Choice**

In the fall of 2015 as I was digging into research related to designing modeling tasks, I came across Galbraith, Stillman, and Brown’s (2013) chapter “Turning Ideas into Modeling Problems” from *Modeling Students’ Mathematical Modeling Competencies*. In the chapter, they highlight research that was done to determine what types of topics and questions students were interested in modeling and answering. They had given teams complete control over what situations they wanted to model and then studied the results. They noted that “it is not necessary to always focus on developing a complete solution to the problem especially if the pedagogical intention is to be continually developing independent modeling competencies” (Galbraith, Stillman, & Brown, 2013, p. 140). I found this idea of ‘student choice’ an interesting component to consider and thought it
would make a good culminating project at the end of the unit once students understood the modeling cycle and had seen previous examples of situations benefitting from a mathematical model. As a result, I designed Project 5 (see Appendix M) so students would select and research a situation they would like to learn more about through the use of a mathematical model. This task truly embraces my design criterion as the problem students create would not be given to them in a ‘mathematical way’ or using ‘mathematical language.’ It instead requires students to connect, research, and translate an authentic situation then apply mathematics to understand the situation and any questions they might have about it.

From a teaching perspective, I imagined this task would be challenging as I would have several different projects going on at the same time, and I may lack the background knowledge necessary to help students acquire or interpret the information they need about the situation just to create a model. There was little I could do to prepare for this in the design of the task, but it provided me with an opportunity to demonstrate one not needing to be an expert in a particular field to apply mathematics to that field. I was, however, worried about students developing a situation that only required mathematical calculations and not necessarily a mathematical model. I was also concerned students might spend too much time developing an idea from which to model that they would not have enough time to conduct a proper mathematical analysis. To help with this, I developed a list of potential topics and would make this list available to student teams as it was needed.
Assessing Modeling Tasks

One source of conflict and frustration in the design of this unit came from determining the best way to summatively assess students. Formative assessment would be ongoing through dialogue and questioning. But without some form of summative assessment to guide expectations for students, I would have no way of determining if students had made progress towards our learning goals, students would not understand if they are moving in the right direction, and I would not have the means to provide focused and productive feedback (NCTM, 2000, Leinwand et al., 2014). An issue, however, is in order to find evidence of all nine modeling abilities and all phases of the modeling cycle, I did not think I could simply assess the final product each team would produce for each project. I wanted to see how students worked through ideas and mistakes. I wanted to see their thinking. But, the final product submitted by teams will not necessarily illustrate what the students had to go through to accomplish the task’s goals. It will only show their final model and not the previous versions of that model as students develop, test, and refine. The final team product is the culmination of all their work which will hopefully show evidence students understand how to model and communicate that model, but I also wanted to give feedback and assess student’s ability to successfully navigate through the entire modeling process.

As a result, I decided the best way for me to provide feedback on both understanding the modeling cycle and the development of modeling abilities was to provide rubrics for both stages of the modeling task: the process and the final product. NCTM (2000) recommends rubrics especially for feedback as it helps “teachers analyze
and describe students’ responses to complex tasks and determine students’ levels of proficiency” (p. 22). Because of the descriptions within rubrics, students are also able to use them for goal-setting as they understand assessment and task expectations (NCTM, 2000). Therefore during the unit, providing feedback on the modeling process summatively would come from an individual rubric assessment of each student’s individual work. The final product would be a team assessed task primarily providing feedback on how effectively they communicated the results of their analysis. Using rubrics would hopefully help students understand what they must do in order to improve and succeed and track progress towards our desired learning outcomes over the course of individual tasks and the unit as a whole (Arter, 2002). My hope was that using the same two general rubrics for each task could serve as a guide to help students navigate the modeling cycle and formalize their ideas in their team’s final product by providing consistent and clear expectations for each task (Perlman, 2002).

An issue I ran across, however, was that I did not know how to develop a quality rubric. Assessments I have designed in the past typically only focused on awarding points towards completion of a correct answer. In modeling, however, there is no focused procedure to apply or right answer to obtain. As a result, I had to learn more about the purpose and design of rubrics in the research literature.

Research on Rubrics

Generally for tasks when students are not aiming for a correct answer, holistic rubrics are used (Mertler, 2002). This type of rubric provides a single score and focuses on overall quality of the student product. However, this type of rubric does not take into
account individual parts, only the overall quality. Even though modeling tasks could result in several different outcomes, I decided the rubrics I would include would be analytic rubrics as they would contain separate criterion for me to score. I wanted to be able to provide feedback to students on specific steps of the modeling cycle they were missing or modeling abilities they needed to focus on. A holistic rubric would not allow me to provide that type of feedback effectively, but an analytic rubric would allow me to identify strengths and weaknesses that may help students track progress towards the development of modeling abilities over the course of the unit (Perlman, 2002). When setting forth to actually design the rubrics, I looked at several examples of mathematics rubrics used for performance tasks. I found five rubrics developed by Anhalt and Cortez (2015), Lane et al. (1996), New York Performance Standards Consortium (2010), Examplars (2015), and the State of New Jersey Department of Education (2015) especially helpful and ended up inspiring the general structure used for both the individual and team product rubrics.

**Individual Modeling Cycle Rubric**

Because the overall target for this unit was to learn how to model, I thought the individual rubric should assess and provide feedback on how well students understood and applied the modeling cycle as defined by CCSS-M (see Appendix N). However, I decided to weigh actions within CCSS-M’s modeling cycle differently to emphasize areas of the where students should focus more time and energy (Northwest Regional Educational Laboratory, 2002). For example, in action one students identify variables; this should develop abilities related to mathematical literacy and applying everyday
knowledge but it should not necessarily take a lot of time and energy. By comparison, action two: *formulating a model*, requires students to use abilities related to mathematical literacy, flexible and creative thinking, data analysis and collection, and perseverance, all of which require higher order thinking skills and most likely more work and time in general. It did not seem fair to the student to weigh these the same. As a result, actions requiring more time, work, and higher-order thinking would be weighted accordingly. I thought students would spend most of their time and apply the most modeling abilities to action two: *formulating a model* with action four: *interpreting results* and action five: *validating conclusions* also getting significant attention as students interpret, validate, assess reasonableness, and modify models if necessary. Action one: *identify variables* and action three: *analyze and perform operations* require straightforward mathematical skills and would have the lowest weight towards the overall grade. For each task, action six: *reporting on the conclusions* would be done as a team and as such will form the basis of the Team Product Rubric and not be included here.

Students in my class were already familiar with using a scale from zero to four to identify where they are at in their understanding and application of a particular skill due to our school improvement and professional development being focused on research done by Robert Marzano (see Marzano, 2012). To maintain this consistency, I decided to keep the performance levels for each category of my rubric the same scores. Using a scale developed by teachers as a student growth objective example for the State of New Jersey Department of Education (2015) as a place to start, I modified each dimension and its description to match the work, thinking, and strategies I expected to see. I also brought

To develop the specifics of the rubric I used the “Steps in the Design of Scoring Rubrics” offered by Mertler (2002) as a guide. Mertler (2002) suggested I first reassess my objectives for the modeling tasks from an individual level. I then specified what I expected to see from students when they “Identify Variables,” “Formulate Model,” etc. Once I had determined these specific expectations, I then wrote descriptions of these beginning with excellent work at a score ‘four.’ After this I could describe poor work at a score of ‘one’ (zero work was simply that it was not included). From there it was not difficult to fill in the levels between a one and four based on common errors I expected to see.

**Team Product Rubric**

As I was reading research on rubrics, looking at example rubrics, and re-examining my tasks, I began to realize the final products (presentations, posters, letters, etc.) teams would be making might not necessarily show the struggles individuals and teams had to go through to develop, analyze, and refine their models. In other words, these products would show the final outcome and analysis, but would not necessarily show the modeling cycle as students went through the process. What I expected to see in these final products focused more on interpretation and communication than actual mathematical processes and procedures. The purpose of the final product is different; I wanted to see evidence teams accomplished the task’s goals and communicated the results of their analysis. In the individual modeling rubric I left off the last modeling
action of CCSS-M’s cycle which is to “report on the conclusions and the reasoning behind them” since individuals themselves would not be doing this in their work. All individual work would culminate into the final product teams would create to communicate their results.

As I was doing research on rubrics and continued to look at examples, I particularly liked the rubric used by the QUASAR (Quantitative Understanding: Amplifying Student Achievement and Reasoning) Project, whose goal in the early 1990’s was to improve mathematics instruction in middle schools in disadvantaged schools by using cognitively complex tasks (Silver & Lane, 1993). Because the project also served as a research project, there was a need for “assessment tools to measure growth in high-level thinking, reasoning, and communication as they related to mathematics” (Silver & Lane, 1993, p. 60). They developed a holistic rubric based on conceptual and procedural knowledge, strategic knowledge, and communication and their descriptions seemed to include the use of general modeling abilities. As a result, I decided to use this rubric as the basis for my Team Product Rubric (see Appendix O), but I omitted or modified some the descriptions of each criteria at each level to more closely align to my expectations and changed it to an analytic rubric so that teams would have more feedback from each criteria. Once I reassessed these expectations, I determined descriptions of excellent work and modified them at each score level based on anticipated mistakes and misconceptions. Finally, I weighted the strategic knowledge and communication areas twice as much as the mathematical knowledge section. Students primarily would get assessed on the mathematical knowledge section in the individual assessment rubric. In
the team’s product the focus should be more on developing an appropriate model that accomplishes all of the task’s’ goals and communicating results in a way the identified audience can understand.

Later, as I was developing the Student Choice task and reviewing research I had used in its design, I reread the chapter “Turning Ideas into Modeling Problems” from *Modeling Students’ Mathematical Modeling Competencies* by Galbraith, Stillman, and Brown (2013). The authors described the criteria they gave students during an assignment in which students got to pick the topic. They stated some students may find their models do not work but because of time restrictions they would not have enough time to modify it. As I began preparing to teach this modeling unit, I too was worried about having enough time to accomplish all goals for each task. But because the target of this unit was to develop modeling abilities, I thought it was important to still consider inappropriate models if students could justify why it did not work and provide ideas on where they should have looked next. Galbraith, Stillman, and Brown (2013) state, “it is not necessary to always focus on developing a complete solution to the problem especially if the pedagogical intention is to be continually developing independent modeling competencies” (p. 140). According to these authors, if students can justify why a model does not work, they have demonstrated the development of modeling abilities. Students might not have time to continually modify their model and still produce a product worth sharing to an audience. However, if they can provide rationale and direction for next steps, students have demonstrated the big ideas of this unit and should not be punished because of time constraints I place on them. As a result, I added the
description “Develops a clear & appropriate model for solving the problem or develops an inappropriate model but includes rationale for why it does not work and descriptions for ‘next steps’ to correct the model” within the criteria for Strategic Knowledge in the rubric (see Appendix O).

Assigning Grades from Rubrics

Rubrics can become difficult to use, however, when needing to assign a percentage grade. Research cautions against simply converting rubric scores to percents as it may not accurately reflect a student’s abilities and progress towards the learning targets (Northwest Regional Educational Laboratory, 2002; Mertler, 2002). For example, achieving a ‘two’ on a four point scale would convert to a 50%, but the quality of the work and the description for a ‘two’ probably does not indicate failing. Mertler (2002) suggests the use of a rubric score to grade conversion so the rubric score given more accurately describes student achievement and progress in percentage form. I liked this idea and decided each rubric would contain a grade conversion based on the overall quality of work correlating to a specific percentage grade.

Another problem arose when I reflected on the use of rubrics as a form of assessment because at the beginning of the unit, students will most likely not score well as they are learning how to model and are targeting modeling abilities they need to further develop. I also felt students needed experience using the rubric as a tool to “reflect upon the quality of their work and learn from their successes and failures” before using scores from rubrics to assign summative grades (Brualdi, 2002, p. 4). Therefore, I decided on the first project, I would score students using both rubrics, but this scoring
would only be for feedback purposes and no grade would be assigned. After the first few projects, I would also show anonymous examples of individual work to provide additional opportunities to clarify expectations and improve achievement towards unit goals (Arter, 2002; Brualdi, 2002).

Finally, I needed to determine how to use the grades from these rubrics within our course gradebook. My school district requires me to put formative assessment scores in as 20% of the overall grade and assign summative assessment scores as 80% of student’s grade. Because students were learning how to model and much of the feedback I was giving them is formative in nature, I did not think it was fair to students to include all graded projects in the 80% part of my gradebook. However, I did want my gradebook to reflect students’ mastery of the modeling unit’s learning targets. I decided the final Student Choice project served as the culminating unit assessment and therefore would be considered a summative grade being scored, translated into percentages, and then used in the 80% part of our gradebook. However, because this project is completely developed by students and I was unsure how this freedom would translate into a quality product, I was nervous about this being their only summative grade. This left three remaining projects that could be included for a summative grade. Northwest Regional Educational Laboratory (2002) suggests that students be given a choice on which task best represents their growth and abilities when assigning a grade. Therefore, I decided two of these three remaining tasks would be given scores and grade percentages but only be recorded in the formative assessment (20%) part of our gradebook. Students would pick which one of those three tasks best represented their ability and be scored, graded, and recorded in the
summative assessment (80%) part of our gradebook. Therefore, students would have two summative grades: one under my control but of their choosing while the other in their control but I assigned. The remaining scores would be recorded as formative grades.

**Student Support**

As I was thinking about my students and the ones I thought would enjoy these modeling tasks versus those I thought would struggle, I realized the students I thought would struggle all had one thing in common: they were memorizers. That is, they excelled in situations where they have a set of steps to follow and they can memorize and duplicate those steps. They are diligent students and want to do well, but I knew the structure within modeling tasks may cause frustrations. The modeling tasks I have designed require them to apply their knowledge of mathematics in totality and retrieve that knowledge on their own. I cannot give them an example to follow. However, I could find ways to build in structure to support these students. I made two posters of CCSS-M’s modeling process to reference around the room. I also decided that each day as students were working on their projects, I would list out goals or questions that students should think about in order to keep on track with deadlines. Finally, looking at the expectations in the ‘4’ column of each rubric, I made a graphic organizer (see Appendix P) of questions students could answer to help guide their thinking.

**Conclusion**

I knew going into the design phase of this project I would need expertise on modeling and the modeling cycle. This expertise allowed me to carefully pick apart the modeling process in order to identify specific modeling abilities to serve as learning
targets for this unit and develop design criteria to guide me in the development of modeling tasks. What I was not prepared for, however, was the amount of new knowledge I would need about mathematics curriculum design and assessment in order to stay true to the learning targets I had for students. I had to break myself out of my own experience with mathematics as having a set procedure to follow in order to appreciate the uncertainty authentic mathematics might have. This messiness would need to be reflected in the design of the task by giving students the freedom to develop and test their own ideas and also in the assessment of these tasks by valuing incorrectness and unorthodox. After spending over three months preparing for a month of modeling, I was anxious to see if all the research, revisions, and time spent would result in the student learning I was hoping for. A detailed account of the events in this month of modeling from March 14 to April 14, 2016 is provided in Part Two: Chapter 4 with subsequent analysis of major learning in Part Three.
PART TWO

ENACTMENT OF A MODELING UNIT

Part Two of this research study contains the timeline of events as I enacted a unit on mathematical modeling in my precalculus course from March 14, 2016 to April 14, 2016. It sets the stage and provides readers with the opportunity to experience the modeling unit as it played out in my classroom at River Junction Senior High. Specifically, Chapter 4 provides the sequence of events including dialogue, student work, and precursors to major themes which are discussed in Part Three. In other words, it provides the ‘what.’ Major opportunities for learning and sources of conflict during the unit will be introduced here and then analyzed further in Part Three so the ‘how’ and ‘why’ can be better understood.
CHAPTER 4
A MONTH OF MODELING

I did not sleep the night of Sunday, March 13, 2016 as a month of modeling was set to begin. Despite my months of planning and preparation, I had no way of truly knowing how my students would respond to this new learning environment and the study of modeling over the course of an entire unit. I had no experience teaching modeling tasks in the manner CCSS-M intends, and my students had limited prior opportunities to learn mathematics in this manner. We would all be embarking on a month-long journey through an unknown, full of anxiety and surely frustration. In this chapter, I describe the timeline of what occurred throughout this month of modeling, including depictions of the modeling and mathematical learning outcomes, conversations, and barriers as my students and I learned to navigate in a new environment together while engaging in unfamiliar tasks. This lays out a synoptic map that in subsequent chapters is analyzed for major empirical and theoretical themes in order to provide greater insight and understanding to the challenges of trying to teach in new and ambitious ways.

Introduction to Modeling

Monday, March 14, 2016 and Tuesday, March 15, 2016

We began the modeling unit after students returned from a weeklong spring break vacation. As with any long break, students were a little restless to get back into the swing of things. For two days, March 14th and 15th, we would cover the Intro to Modeling task (see Appendix G) and logistics on how the unit would be laid out and graded. I was hopeful spending two days examining what modeling is, why it is useful, and how it is
different from their previous mathematics experiences would set the stage for the next four and a half weeks.

**What is Mathematics?**

I decided to begin the unit the same way I would end it, by asking two questions to gauge their understanding of and prior experiences in mathematics. I was curious if students would look at mathematics differently after participating in modeling tasks. On March 14, I asked students to record on notecards their responses to two questions: *What is mathematics?* and *What type of questions do we answer using mathematics?* Students, in general, suggested that mathematics was the study or use of numbers, operations, and equations. Chris wrote that mathematics is “using numbers to find answers” (March 14, 2016). Similarly, Emily offered up how mathematics is about “using equations to solve problems including numbers and symbols” (March 14, 2016). The phrase “problem solve” was also included in some students’ definitions. Nicholas, for example, stated that mathematics is “the use of math calculations to problem solve” (March 14, 2016).

Students also had a fairly narrow view of what types of questions can be answered with mathematics. Most focused on the presence of numbers or the use of equations, such as William offering mathematics involves “questions that require a series of equations to find a solution” (March 14, 2016). Ryan elaborated slightly by including specific operations he has used by stating mathematical questions are “problems with answers and numbers, probability, trig, addition, subtraction, etc.” (March 14, 2016). Students’ views were about what I was expecting as they identified answers, operations, and equations; would a month of authentic mathematical experiences change their perception?
What is Modeling?

After discussing their responses, we moved to the Intro to Modeling task. This task took two, fifty minute class periods and primarily involved student discussions about the purpose of modeling and organizational items for the unit. Students began by reading and discussing the article “Career interview: Mathematical modelling consultant” about a consultant named Nira and his modeling experiences (Freiberger, 2007). The article focuses on how Nira applies modeling to a wide variety of domains including automotives, aerospace, global communications, and defense logistics in order to solve problems for his clients. Nira makes the point that he does not need to be an expert in the domain needing modeled. He can learn this knowledge as he finds a need for it, but recognizing opportunities for and applying appropriate mathematics are essential to solving modeling problems.

After reading, students were asked to discuss the following questions: Based on this article, what do you think modeling is? and Did anything surprise you about what Nira was able to do with modeling skills? Teams then posted their responses on an online form for me to see and read through later. After reading and responding to the article, students in Team 1 wrote they thought modeling was about “taking any real life situation and putting it into mathematical terms” (March 14, 2016). Students discussed how they were surprised Nira did not need to be an expert in a particular field in order to apply mathematical ideas and solve problems within that field. Team 2 wrote that Nira “doesn’t have to know anything about the subject going into it. He just learns the facts from the experts as he goes along and problem solves by himself” (March 14, 2016). Other teams,
like Team 4, commented how they were surprised by how many different areas mathematics, and in particular modeling, could apply to. They wrote: “We were surprised about everything you could apply modeling to, such as x-ray vision, corrosions, cars, planes, and pipelines” (March 14, 2016).

These comments suggest students were beginning to find relevance for learning how to model while also realizing mathematical knowledge and problem solving abilities were crucial in the development of a viable model. In reflecting on student conversations and responses, I wrote:

I think the way I decided to intro modeling was successful. The purpose of including the article/discussion questions and the example modeling task/discussion questions seemed to be noticed. The article was a bit long so I don’t think all students had time to get through the entire article, but they seemed to pick up on that modeling will deal with authentic/real life situations and can be used in a lot of different industries and areas. Several students commented on how they were surprised that Nira was able to apply modeling to so many different areas, even areas that he didn’t have a lot of background knowledge about. He wasn’t an expert on oil, automotive, naval carriers, etc. but he was able to gain enough knowledge on the subject from experts in the field to accomplish the tasks he needed to. (March 14, 2016)

At the time, I thought that based on student conversations and written responses, students understood and appreciated how modeling could apply to authentic situations outside of a mathematics classroom. It seems that including the article about Nira seemed to help
establish some relevance to learning about modeling. I was hopeful students would continue seeing connections to real life throughout the duration of the unit perhaps providing them with some motivation necessary to spend several days on one particular task.

Sample modeling task. Once students had adequate time to discuss the article in Part 1 of the Introduction to Modeling task, they were given a formal definition of modeling as “the process of representing a real life situation mathematically in order to better understand the situation and make decisions or predictions related to it” (Classroom Materials, March 14, 2016) and began analyzing a sample modeling task, the Space Shuttle Problem:

Space Shuttle Problem: No More Space Shuttles

On July 21, 2011, the 135th and final US Space Shuttle landed in Florida after its 13-day mission into orbit, complete with a docking at the International Space Station (ISS). NASA will now have to rely on other nations or commercial endeavors to travel into space until a replacement vehicle is developed and constructed. Develop a comprehensive ten-year plan complete with costs, payloads, and flight schedules to maintain the ISS.

Some interesting facts possibly worthy of your consideration:

- The ISS is at full capacity with 6 astronauts, but can surge during shuttle docks to as high as 13.
- The ISS is scheduled to remain in service until at least the year 2020.
Historically, transport to the ISS using US Shuttles has cost between $5000-10,000 per pound. Shuttle missions have lasted approximately 10-14 days in orbit. Missions on board the ISS typically last around six months.

Recently, progress has been made within private industry to launch unmanned rockets into space.

Russia is willing to launch US astronauts into space at a cost of about $60 million each.

(Consortium for Mathematics and Its Applications, 2011; Classroom Materials, March 14, 2016)

After students tried to predict what modeling was based on Nira’s article, I wanted to give students a more formal definition of modeling. However, I think a formal definition of modeling is not useful in isolation, so I wanted to give students an example of a modeling task. By reading through and discussing the Space Shuttle Problem, I wanted to provide students with clearer expectations for this unit and the types of things they will be asked to do in a modeling task. Specifically, I wanted students to see why this was an example of modeling by connecting it to the modeling definition I provided and the examples they saw in Nira’s article. I also wanted students to analyze the task to determine what skills they might need to solve this problem. I expected students would most likely come up with procedural based skills, but my hope was that they begin to understand these tasks will require more higher-order thinking, communication, and reasoning skills than what they are used to in their precalculus tasks.
**Student perspectives on sample task.** When I asked how this example was different than tasks typically given to them in math class most student teams commented on the absence of the phrase ‘solve for x.’ Team 2, for example, discussed this idea:

*Abigail:* How is this task different than other tasks you’ve been given in your mathematics courses?

*Samantha:* There is not as much information, you have to it figure out.

*George:* We are not solving for x.

*Abigail:* It’s a real life situation.

*Samantha:* Yeah it’s a real life situation and we are not solving for x. But usually we’re given so much information that we all have to all make one….

*Noah:* What do you mean?

*Samantha:* You know like on our tests there is like one answer.

*George:* And there could be multiple answers to this.

*Samantha:* Yeah

*Abigail:* And usually we have to memorize like a ummmm a

*George:* Formula

*Abigail:* Formula and this we have to come up with our own way to solve it.

*Samantha:* Yeah and there is so many things you have to consider in real life. Like this could happen, but can that?

*Abigail:* Yeah like what if this happens, or this, but then Russia does this. (March 14, 2016)
Other teams also noticed these differences and included similar observations in their
team’s responses. Team 4 wrote “we are not solving for x, we are solving real life
problems” (March 14, 2016). Teams were also noticing that there could be many
different ways to approach a modeling task. Team 1, for example, wrote “This task
requires more in depth thinking. We have to be able to come up with our own way of
solving because it isn’t going to be like a book problem where there is a set path and a set
answer” (March 14, 2016). Based on student dialogue and responses, providing the
Space Shuttle Problem as an example modeling task and the subsequent discussion
questions was useful in helping students see the major differences in the types of
mathematical questions they are used to answering and the types of situations they would
be analyzing in this unit. In particular, it was encouraging to me they noted there could
be several ways they might approach the task. This would become a key theme
throughout the unit and an underlying implication for what it means to *do* mathematics.

**Modeling Abilities**

We continued this discussion on the Introduction to Modeling on March 15th by
analyzing the nine modeling abilities I targeted in the design of the unit and the modeling
cycle students would be using to approach each task. When asked about how this list of
modeling abilities differed from typical targets in math class, Team 5 and Team 1 wrote
in the online forum how modeling abilities focused more on applying “everyday
knowledge” (March 15, 2016) or “background knowledge” (March 15, 2016) to solve
and reason through problems. I had a conversation with Team 5 which began connecting
these modeling abilities to finding evidence these abilities were used:
Danielle Buhrman (DB): How does this list look different?

Luis: This is deeper stuff, like perseverance. You’ve never really asked about that before.

DB: So a question that might come to your mind, I know it does to mine is, okay reasoning, it’s deeper. (pointing to Luis) It’s not ‘use this to do this.’ It’s almost vague right? So how would I assess you on it? That’s a skill we are going to be working on right, how do I assess you on reasoning?

Elijah: Our explanation?

Dylan: I don’t know, we’d have small arguments I’d say. The reasoning you give yourself like can I do that? Are those right or that one?

DB: Okay so what will be a challenge for some of you? And by you I mean everybody not anybody in particular here.

Elijah: There is no formula to follow or set answer. You just have to figure it out as you go.

DB: And when you hand in your work what am I going to have to make sure I see?

Dylan: Work

Ryan: Steps

DB: Thinking. That’s what I’m assessing you on. I can’t read your mind, I have to see that thinking coming to life. (March 15, 2016)

I later continued this conversation with the class as we wrapped up discussions on modeling abilities to transition to logistics covering grading and rubrics. I asked students
how I would assess something like reasoning or perseverance, and they correctly thought that I would have to really *listen* and *watch* them (from a more informal standpoint) suggesting I would need to be interacting with each team as they communicated their ideas with each other. Brooke brought up the idea that I would know they correctly used these skills “if the reasoning makes sense” to which I responded:

*DB:* What does that mean? Can anyone elaborate on Brooke’s idea?

*Jacob:* If you back up our ideas.

*DB:* How are you going to back up your ideas?

*Various Students shouting out ideas:* Evidence, knowledge, support, examples

(March 15, 2016)

I told them their goal was not about demonstrating proficiency of a particular math skill similar to what they have encountered before; it’s about applying thinking skills so in their work, I needed to see their *thinking* even if their ideas and strategies ended up being incorrect.

**Grading and Expectations**

Because of the conversations we had over the last two days, students did not seem to be too surprised or confused about grading or expectations from the Individual Rubric and Team Product Rubric (see Appendices N & O). For example, at the end of the period we were processing the “4” statement on the Team Product Rubric stating the student “develops an inappropriate model but includes rationale for why it does not work and
descriptions for next steps to correct the model” (Classroom Materials, March 15, 2016).

I asked the class what this meant for them and their work:

_George_: You can be incorrect.

,DB: You can be incorrect and is that okay?

_Several Students_: Yeah.

_DB_: Yeah, that’s a 4 right? What’s important though if you are wrong to get a 4?

_Several Students_: Reasoning.

_Brooke_: You recognize why you are wrong.

_Emily_: What your next steps are.

_Noah_: How you are going to fix it. (March 15, 2016)

Students began to synthesize the conversations they have had over authentic tasks, modeling abilities, and assessment to understand what modeling looks like and in what ways it might be challenging for them. These conversations made me optimistic sacrificing two fifty minute class periods would translate into more opportunities to develop modeling abilities and fewer frustrations because they understood these expectations and goals. Yet, as I describe in the remainder of this chapter, modeling is much more complex than any of us initially realized.

**The High-Definition Trade-Off Dilemma**

**Wednesday, March 16, 2016 - Monday, March 21, 2016**

Based on the conversations students had the previous two days as they processed through modeling as a general learning target, I felt more confident students understood the process they would undergo to accomplish their task’s goal. However, entering the
first project I had a lot of anxiety about their willingness to devote several days to one problem. Up until this point, students worked on problems which typically required only several minutes to complete. To my knowledge, students had never needed to devote so much time to process, analyze, and work through a solution in their mathematics course. Would they be able to meet expectations and how much support would I need to give them?

**Making Sense of the Task**

Class on Wednesday, March 16th was devoted to simply trying to understand and make sense of Heather’s problem in The High-Definition Trade-Off Dilemma (see Appendix H):

“Help me nerdy friends! We are going for a scuba dive in the reef later today (super excited!) and realized my GoPro Hero4 isn’t going to record the entire dive in high-def. Obviously, I want as much of it in 1080p high-def as possible to catch all the awesome colors and wildlife but I want to record the entire 5 hour dive so I don’t miss anything cool...like a reef shark. I have a 64gb memory card in my GoPro, how long should I shoot in 1080p before switching to 720p to maximize the amount of high-def footage I get?” (Classroom Materials, March 16, 2016)

I had given students the project at the end of class the day before in case some students wanted to get started, but had not expected students to dig into it until now. A few students had come to class with rough ideas to build on, but most students were looking at the project for the first time. As a result, students were trying to interpret what
information the task was giving them, identify relationships within the task, and try out ideas. At the beginning of class several students said they were confused and did not feel like they knew where to start. But by the end of class students felt like they had a more solid grasp of what was going on in the situation and most had narrowed down a reasonable estimate for Heather’s recording times.

As students began dissecting and understanding the task, I saw evidence of students applying everyday knowledge. For instance, students were using background knowledge of technology to reason through how the 1080p would take up more space than the 720p on a memory card. Students in Team 1 even discussed converting space available to megabytes rather than leaving it in gigabytes. Most students showed some realization that it would be beneficial to think of all times as minutes rather than hours and minutes for ease of use.

**Development of Models**

After students felt they had a better understanding of what was occurring in the task, students applied skills related to mathematical literacy by recognizing opportunities for relevant mathematics embedded within the authenticity of the task. Specifically, students were successfully applying constraints of space (64 gigabytes) and time (300 minutes) in order to guess and check ideas.

**Minutes per gigabyte.** About half of the class calculated the number of gigabytes used per minute and then plugged in the number of minutes at each resolution until they got a total gigabyte usage of 64. For example, on March 16, Jacob, who was sitting with Team 2 due to several of his teammates being absent for a leadership
workshop, explained to Samantha they could take the total number of minutes the memory card would hold for each resolution and divide it by the 64 total gigabytes to get “4 minutes for one gigabyte (4.09 min/gb) at 1080 and 5 minutes 30 seconds (5.52 min/gb) at one gigabyte of 720.” His team then used his rates to begin processing through the mathematical relationships that existed within the situation:

Jacob: She needs 300 minutes of total time.

Samantha: Why?

Jacob: Because that’s five hours.

Samantha: Ok

Jacob: So you need to figure out a happy medium, like, say you did 200 minutes at 1080 and 100 minutes at

Samantha cuts him off.

Samantha: We should times 4.09 times how many gigabytes, like 55, which is 224 minutes.

Jacob: So you only have 9 gigabytes left.

George: So it would be 9 times 5.52. (March 16, 2016)

Here we see Jacob suggesting they try to guess and check reasonable values to narrow down a solution. Samantha then noticed she could multiply the rate Jacob had figured out times an estimate for the number of gigabytes used at 1080p (55) and it would provide them with how much recording occurred at the 1080p resolution. Since there are only 64 gigabytes total available on the memory card, Jacob stated 9 gigabytes would be remaining to record at 720p. Later, Samantha and George then added the minutes
recorded at each resolution together to get about 275 minutes which they correctly concluded was not enough time.

These students continued to apply these rates and constraints on time and space to narrow down reasonable recording times. Samantha showed her group’s work this way:

\[
\begin{align*}
4.09 \times 85 &= 224.95 \\
5.52 \times 9 &= 49.68 \\
4.09 \times 50 &= 204.5 \\
5.52 \times 14 &= 77.28 \\
4.09 \times 44 &= 179.86 \\
5.52 \times 20 &= 110.4 \\
4.09 \times 40 &= 163.6 \\
5.52 \times 24 &= 137.48 \\
4.09 \times 38 &= 155.42 \\
5.52 \times 26 &= 143.62 \\
4.09 \times 36 &= 147.24 \\
5.52 \times 28 &= 154.86 \\
4.09 \times 37.33 &= 300.37 \\
5.52 \times 27 &= 149.04 \\
\end{align*}
\]

*Figure 4.1.* Samantha’s work showing Team 2’s “guess and check” method using minutes per gigabyte rates. (March 21, 2016)

By applying the relationships between time and memory card space, Samantha, Jacob, and George could narrow down a reasonable estimate by checking values for the number of gigabytes used. In Figure 4.1, their work indicates they were able to get close to 300 total minutes recorded by suggesting you would record for 37 gigabytes or 151.33
minutes at 1080p and 27 gigabytes or 149.04 minutes at 720p. Their work indicated to me at the time the importance of *playing* with the mathematical relationships represented in the problem. As opposed to rushing into the development of equations, using “guess and check” as an initial strategy allowed students to recognize specific relationships that would lead to the development of models. I could then encourage students who were struggling by suggesting they first *play* with the situation in order to identify how constraints of time and space affect recording times in each resolution.

**Gigabytes per minute.** The other half of teams identified a different relationship between time and space by determining the amount of space taken up on the memory card for each minute recorded. Then using this rate, they guess and checked numbers until they reached a total of 300 minutes recorded. By first guessing and checking ideas using the relationships and constraints on time and memory card space, students were gathering data that would lead to the general relationships they would need for their model. Chris, from Team 6, was noticing right away how the constraints could lead to potential equations to use. He told his teammates:

At 720 (720p resolution) it is 0.18 gigabytes per minute. And we need it to add up to 64 gigabytes, but we also need it to add up to 300 minutes. So we could make two equations where the gigabytes add up to 64 and the minutes add up to 300 and where they cross (showing two lines cross with his hands) is the times (recording times). Something like that. (March 16, 2016)

Chris later correctly assessed that Heather will need to switch off from 1080p before 262 minutes since the 64 gb memory card only will hold 262 minutes and it takes up more
space than the 720p resolution. He had calculated 1080p took up 0.244 gigabytes per minute while 720p took up 0.181 gigabytes per minute. Meanwhile Evan, also from Team 6, had come to class with a page full of guess and check work and agreed with Chris’s observation as he claims he got the time at 1080p to be slightly above 152 minutes. In his work (see Figure 4.2), Evan incorrectly referred to the memory card as a battery, perhaps showing a lack of everyday knowledge about the situation, but correctly uses the relationships between space and time to narrow down a probable solution:

1080
100% = 262 (minutes)
720
100% = 353 (minutes)

1080
90% of battery used by 235.8 min.
720
10% left gets 35.3 min.

1080
80% of battery used by 209.6 min.
720
20% left, 70.6 (minutes)

1080
70% battery use, 183.4 m (minutes)
720
30% left, 105.9 m (minutes)

1080
60% use, 157.2 min.
720
40% left, 141.2 (minutes)

1080
57% use, 149.34 (minutes)
720
43% left, 151.79 (minute)
In Figure 4.2, Evan demonstrates how he narrowed down reasonable minutes to record at each resolution by calculating how many minutes he could record in each resolution if a certain percentage of the memory card (he incorrectly used the word “battery”) were recorded at 1080p. Chris immediately grabbed his calculator and used Evan’s ideas about a potential answer and tested it with his own by multiplying Evan’s times by the rates he had developed for each resolution to verify he got around 64 total gigabytes used:

\[
\begin{align*}
.244 \text{ gb/min} & \times 152 = 37.088 \text{ (gigabytes)} \\
.181 \text{ gb/min} & \times 148 = 26.788 \text{ (gigabytes)}
\end{align*}
\]

\[37.088 \text{ (gigabytes)} + 26.788 \text{ (gigabytes)} = 63.826 \text{ (total gigabytes)}\]

Chris had determined Evan was heading in the right direction and the combination of their ideas might allow them to make progress. Evan was not satisfied, however. He stated, “Yeah, but I want a model for this. I don’t know what the model is.” Chris, seeking clarification, asked “Wait, like an equation?” (March 16, 2016). Evan was beginning to see how guessing and checking was getting them to a probable answer but it was an inefficient method.

**Formalizing guess and check work.** This guessing and checking method, however, allowed students to more easily define their variables and develop their model as their work directly lead to the equations they had developed. By Thursday March 17th, students were formalizing the work they had done and were beginning to develop equations for Heather’s specific task and modify those models for general cases of time...
and gigabyte usage. They were moving through the middle of the modeling cycle by analyzing, interpreting, and validating their models, and I witnessed modeling abilities that specifically related to theses phases of the cycle. However, some teams had trouble recognizing they actually used two relationships in their guess and check (one for gigabytes used and one for time) and thus would need both relationships represented in their model. For example, Team 4 was working through the work Jacob and Jason had done the day before when Eve and Emily were absent for a leadership convention. Using their repeated guessing and checking work, they developed the equation \(4.09x + 5.52y = 300\) where \(x\) is the minutes of recording in 1080p and \(y\) is the minutes of recording in 720p. They were clearly confused, however, because one equation with two variables is not solvable for numeric solutions. They had not translated the second relationship into a usable equation, so I asked them to think back to the work they had done:

\[\text{DB: So let's back up and I want you to try explaining from the beginning what you were doing here and start processing through what we know. So what are the two pieces of information that we know?}\]

\[\text{Jason: Time}\]

\[\text{DB: We know time, the number of minutes total.}\]

\[\text{Jacob: Device}\]

\[\text{DB: The device can only hold}\]

\[\text{Jason: 64gigs}\]

\[\text{DB: And you know how much 1080p you have at 64 gigs and how much 720p you can have in those 64gigs. So what did you do here? Where is this}\]
work coming from?

*DB points to Jason’s work dealing with the times he had developed.*

Jason: We figured out how many minutes per gigabyte of 1080 and 720. (March 17, 2016)

They then described to me how they used this to develop the equation $4.09x + 5.52y = 300$:

*DB: What does that mean? (referring to the equation)*

*Jacob: 4.09 minutes per gigabyte at 1080 plus 5.52 minutes per gigabyte at 720 equals 300.*

*DB: Does that make sense to everyone? (nods) That is part of your model, but there is a second component here that you are missing though.*

*Jacob: We need to figure out the relationship between the gigabytes.*

*DB: What’s the limit on our gigabytes?*

*Jason: What’s on the card?*

*Emily: 64 gigabytes is the max*

*DB: So you’re defining your, I saw it here.*

*Eve: This (pointing to Jason’s paper)*

*DB: Yes so you’re saying $x$ is the 1080p gigabytes and $y$ is the 720 gigabytes. So what’s the relationship between $x$ and $y$? (pause) If you have a set amount of space, what are you going to do with these?*

*Eve: You have to add them to make them equal to 64.*

*DB: Great, so you have a second relationship here.*
**Eve:** You have two equations.

**DB:** You have two equations. So you’d have this one equal to 300 and you’d have

**Jacob:** $x$ plus $y$ equals $64$

**Emily:** 64

**DB:** Can you solve a system of equations?

**Jacob:** Yes. (March 17, 2016)

Applying skills related to mathematical literacy, or, more accurately, the transferring of this everyday knowledge into mathematical relationships, would continue to cause frustrations as the unit progressed. However, all teams were able to develop usable models to report on in their team’s letter to GoPro. Based on the two different rates (1) gb/min or (2) min/ gb, students came up with the two solution methods shown in Figures 4.4 and 4.5.

**Gigabytes per Minute**

1080p: 0.244 gb/min  
720p: 0.181 gb/min

**Variables:**
x: number of minutes at 1080p  
y: number of minutes at 720p  
G: total gigabytes on memory card  
T: total time needing to be recorded

**Model for Heather’s Situation:**

$0.244x+0.181y=64$  
$x+y=300$

**General Model:**

$0.244x+0.181y=G$  
$x+y=T$

*Figure 4.4.* Models developed by students using gigabytes per minute rate.
Teams 3 and 6 created a system of equations by applying the calculated gigabyte per minute rates each resolution would record. By defining the variables $x$ and $y$ to be the minutes recorded at each resolution, students knew in Heather’s situation $x + y = 300$ would represent the total minutes recorded. Then by multiplying the rates 0.244 and 0.181 by $x$ and $y$ respectively, they could calculate the total space each resolution consumes on the memory card, or 64 gigabytes total. This relationship is represented by the equation $0.244x+0.181y=64$, or more generally, $0.244x+0.181y=G$. I was surprised more teams did not create a similar model, as this was the most straightforward representation of the situation due to the definition of the variables directly resulting in the goal of the task (the minutes at each recording speed).

Using the minutes per gigabyte recording rates, Teams 1, 2, 4, and 5 were able to develop the model shown in Figure 4.5:

**Minutes per Gigabyte**

1080p: 4.09 min/gb  
720p: 5.52 min/gb

**Variables:**

$x$: gigabytes in 1080p  
$y$: gigabytes in 720p  
$G$: total gigabytes on memory card  
$T$: total time needing to be recorded

**Model for Heather’s Situation:**

$4.09x+5.52y=300$  
$x+y=64$

**General Model:**

$4.09x+5.52y=T$  
$x+y=G$

*Figure 4.5.* Models developed by students using minutes per gigabyte rate.
Because students calculated recording rates in each mode based on minutes per gigabyte, unlike Teams 3 and 6, students had to define their variables \( x \) and \( y \) as the number of gigabytes used in each resolution. During their guess and check work as described previously (see Figure 4.1), students noticed they were needing to multiply the rates by the number of gigabytes as opposed to minutes since a rate of minutes per gigabyte multiplied by gigabytes would result in the number of minutes recorded. This is represented by the equation \( 4.09x + 5.52y = 300 \). Students then needed the second equation \( x + y = 64 \) to represent the total gigabytes Heather had on her memory card. However, because of the way students had to define their variables, this model did not yield the desired recording times in each resolution. Because \( x \) and \( y \) represented gigabytes, when students solved their model they knew the number of gigabytes used for each resolution. They would need to include additional instructions in their letter to customers at GoPro on how to convert gigabytes back into minutes by multiplying the gigabytes used in 1080p and 720p by the minute per gigabyte rate at each recording speed. For general models to use in any situations, students at all teams, regardless of method employed, recognized they could replace the 64 with \( G \) for gigabytes on the memory card and 300 with \( T \) for time needing to record in their models. This gave them a model customers at GoPro could use in similar situations when they knew the total size of the memory card (\( G \)) and total time (\( T \)) needing to be recorded.

Because this was the first modeling task and I did not know what mathematical outcomes to expect, I was relieved to see all students develop a usable model. Looking back on it now, I ask myself *usable for whom?* The task asked students to write a letter to
customers at GoPro with rationale for and instructions on how to use their model. Students in precalculus should find solving a linear system of equations to be routine. However, the average customer at GoPro will probably not know how to use these models to narrow down how long to record in 1080p before switching over to the 720p resolution. I should have encouraged students to think about how to make their system of equations more usable for the average person by combining both equations into one and solving directly for the time to record in 1080p.

**Interpreting Models**

As students began thinking about the interpretation and validation components of the modeling cycle, students applied skills related to quantitative literacy. By thinking about numerical relationships within the authentic task and their analysis, students described what their calculations communicated about the situation and could assess the reasonableness of those outcomes. This took more questioning from me and did not occur as naturally as some other skills throughout the modeling cycle, but it was necessary for students to transfer their equations back in context of the situation so they could verify their model worked and communicate why it worked for the letters they would be producing. Specifically, students who had developed a model defining $x$ and $y$ to be gigabytes used had a difficult time realizing their model would not provide an amount of time needing to record in each resolution. In other words, these teams would need to include additional instructions in their letters to customers at GoPro on how to convert the gigabytes used to time in minutes.
Additionally, no team realized limitations to their models included the fact no model would work automatically due to restrictive conditions of the technology. For example, it is not possible to record 300 minutes on a 16 or 32 gigabyte card because it does not have enough storage capacity. If a 64 gigabyte card holds 353 minutes of 720p recording, then a 32 gigabyte card will only hold 176.5 minutes. Yet, their models suggested you could plug in any storage amount and recording time and calculate the appropriate time to record in high definition. Students had not applied quantitative literacy skills related to validation and assessing reasonableness in a way that allowed them to communicate appropriate recording times. The struggles students exhibited and the questions I used to help students more thoroughly develop skills related to quantitative literacy is analyzed further in Chapter 6.

**Making Sense of Models**

The remainder of time spent on this first project was used to write a letter about their models to customers of GoPro and analyze the letters produced by other teams. In reading and giving feedback to other team’s letters, teams were having insightful conversations discussing how models were different and why it worked. Many students were noticing how using different rates caused the models to be different. Megan from Team 6 commented to her teammates “oh, we did gigabytes per minute and they did minutes per gigabyte. That’s why theirs is different” (March 21, 2016).

Several teams were also commenting about the need to thoroughly explain reasoning and why things worked in the letters. Chris from Team 6 commented “I want to know the why!” while reading Team 1’s letter (March 21, 2016). Team 1 combined
their system into one equation $y=4.09((5.51x - z)/1.42)$ where $y$ is the minutes to record in 1080p, $x$ is the number of total gigabytes, and $z$ is the total time to record, yet they did not elaborate on how they developed that equation. In addition, Team 2 was reading the letter from Team 3 and commenting on how Team 3 lacked background knowledge on why their model worked:

*DB*: Did they do the same way you did?

*Several Students*: Yes

*Noah*: I felt like they just threw out their equation though. I feel like you should give a little background on how you got it even though it’s not saying to do that.

*DB*: So in general

*Samantha*: Like how that got it.

*Noah*: So you can trust it.

*DB*: Why do you think that? As a reader?

*Samantha*: If I just pulled up something ...

*Noah*: Why this numbers?

*Abigail*: Like why should I believe it works? (March 21, 2016)

Taking the time to provide opportunities to students to communicate their results and analyze the work of others showed students that when working with a model that is unfamiliar to you, understanding why it works and what the variables represent is essential in helping your audience understand and buy in. Students began to realize that
showing and explaining their reasoning was just as important as the mathematics they provided.

**Issues with individual work.** Unfortunately, the individual work students submitted also did not clearly show the thinking and reasoning that occurred in conversations in class. The work submitted by Dylan, for example, showed the results of his mathematical calculations, but he did not explain how he developed any of his ideas (see Figure 4.6):

![Mathematical calculations and results](image)

*Figure 4.6. Dylan’s individual work showing the model his team had developed.*

(March 21, 2016)

Dylan’s work is representative of what students focused on in their individual write-ups: mathematical calculations and results. During the Introduction to Modeling task on
March 14 and 15, I thought students understood and could communicate to me how modeling was going to require them to extend calculations in order to describe relationships in real life. It was not going to be about the end result, but the process students took to get to that result and what it communicated about the situation. I was disappointed and surprised by Dylan’s work in Project 1 and the majority of students in class as it seemed they actually only thought this meant *show work*. As indicated by the rubrics we had discussed in class, I expected students to extend this work in order to interpret and validate the results of their analysis.

As a result, students on the Individual Rubric scored well on identifying variables, formulating model, and performing operations. Students, however, did not include thinking relating to interpreting results and validating conclusions and overall did not show evidence they had worked through the entire modeling cycle. As we began the next project, I would need to stress conversations students were having with myself and their team be included in their final individual write-up. I felt students demonstrated this thinking verbally but had not thought to include this reasoning in their final report.

**Reflections on Project 1**

Overall, I was excited about how this first project went. My initial fears about students failing to persist in what was for them and me a long and in-depth mathematics task were not realized. However, as students and I began familiarizing ourselves with the modeling process, several moments of relearning needed to occur which will be discussed more thoroughly in subsequent chapters. Specifically, as students responded to such an abstract task with no immediate and obvious solution strategy, the need to verify
their work began to emerge. In Chapter 5, I discuss how students had to learn the validation of models could not come from my approval but their own analysis of the situation.

From a teaching perspective, I learned quickly in order to do modeling well, I needed to provide time for students to explore and apply everyday knowledge before models could be developed. As a result, I had to adjust and add an entire day to my lesson plan during this project. Consistent with research outlined in Chapter 1, students had difficulties applying skills related to quantitative literacy specifically related to recognizing constant recording rates and identifying models which communicated gigabytes used as opposed to the desired minutes at each resolution. Because of this, I came to see that my instruction would need to focus more specifically in each project on helping students successfully translate their mathematics results back in terms of the authentic situation. Finally, as I responded to emerging student needs, my questioning proved to be essential. I noticed towards the beginning of this project I too quickly funneled students into a particular strategy rather than focusing questions on student ideas. These moments of teacher relearning are analyzed further in Chapter 6.

**Calculating the Width of Memorial Stadium**

**Monday, March 21, 2016 - Friday, March 25, 2016**

I introduced the second project at the end of the period on Monday, March 21st by showing a video of me taking angle and distance measurements in front of the University of Nebraska’s Memorial Stadium so students could calculate the width of the stadium from the north to south bleacher sections. I thought by showing a video of me standing in
front of the stadium talking about the measurements, students might have an easier time translating the real life images into geometric pictures. Figure 4.7 shows a geometric representation of the situation described in their handout:

In order to find the width of Memorial Stadium, using a compass I measured the due north angle measurements to North Stadium at 94 degrees and South Stadium at 112 degrees. I then walked a distance of 200 feet straight north in the Haymarket Park parking lot using the GPS tracking on my phone. Finally I measured the angles again from this new location and found that to North Stadium it was now 100 degrees and to South Stadium was 120 degrees.

(Classroom Materials, March 21, 2016)

*Figure 4.7. Geometric representation of measurements taken for the Width of*
Memorial Stadium project.

I wanted to introduce the project before students left on Monday so they could start thinking about it on their own and come back with ideas to get started in class on Tuesday. Tuesday would be the only full class period students would have to work on this project, as state mandated standardized testing in English would be occurring on Wednesday, leaving us with only a 25 minute class period. In addition, Thursday we would lose almost half the period again due to a special school assembly occurring during our usual class time. Finally, Friday when presentations would be occurring, a tornado drill was scheduled, removing about ten minutes of instructional time. As a result of these unforeseen interruptions outside of my control, time would end up being a major factor in this project.

Making Sense of the Task

After reflecting on the High-Definition Trade-Off Dilemma for Project 1, I realized students did not know how to validate their results by assessing its reasonableness in terms of the situation. Within the modeling cycle, validating conclusions is a crucial step, and I wanted students to be able to justify their results both in a mathematical and in a real-life sense. I discuss thoroughly in Chapter 5 the significance of validation as students relearned how to learn in a modeling environment. Because students did not seem to establish this idea in Project 1, I decided it might be useful to actually start with this idea of reasonableness in Project 2. Therefore, at the beginning of class on Tuesday, March 22nd I asked students: *what is a reasonable guess for the width* (of Memorial Stadium) and *why?* Students were to think about these
questions and share their ideas with their team before they started an in-depth mathematical analysis of the situation. By posing this question, I wanted to encourage students to begin using everyday knowledge habitually to analyze the stadium’s width. Students brought in ideas about knowing the width of a football field, adding in end zones, estimating additional space behind the goalposts, estimating how many rows are on each side, guessing how much space is between each row, and adding all that information together to make a reasonable guess. Most students predicted somewhere in the range from 700-900 feet, with one team estimating it most likely was not any more than 1200 feet. This suggested to me that perhaps some students had more knowledge than others about logistics in football, and I should be careful to not assume each student has all necessary background knowledge of the situation to draw upon.

**Development of Models**

As students began digging into the Memorial Stadium task, they had trouble transferring the real life knowledge represented in the video and measurements taken into mathematical relationships. But, other than including the video to help with their visualization skills, I did not structure any specific support in advance. I further analyze and describe the struggles students encountered while applying skills related to mathematical literacy, or the transference of real life knowledge into mathematical relationships, and the teacher learning resulting from it in Chapter 6.

**Developing geometric representations.** Once students were able to translate information within the situation to geometric relationships, they began to apply what I saw as flexible and creative thinking skills. Students applied knowledge of
supplementary angles, adjacent angles, Triangle Angle Sum Theorem, vertical angles, Law of Sines, and Law of Cosines in order to develop several different models to calculate the width of Memorial Stadium.

*Team 5’s diagram and strategy.* Luis and Team 5 ended up applying Law of Sines to find the unknown distances to the north and south sides of the stadium (see Figure 4.8). He then applied Law of Cosines to two different triangles marked in red and blue in Figure 4.8 to validate his method and calculations would work to get a width of 799.3 feet.
Figure 4.8. Luis’s work to find the width of Memorial Stadium. (March 28, 2016)

The first Law of Cosines calculations correlate to the red triangle while the second Law of Cosines calculations correlate to the blue marked triangle.

Team 4’s diagram and strategy. Emily and Team 4 worked through several different strategies in class, but ended up calculating nearly every angle and distance possible within their geometric representation (see Figure 4.9).
Once her team was able to piece together calculated measurements using Law of Sines, they applied Law of Cosines to the triangle marked red in Figure 4.9 to come up with the width of the stadium to be 799.7 feet.

**Strategies from Teams 1 and 6.** Teams 1 and 6, however, developed an alternate strategy based on their assumption that my 200 feet walking distance was parallel to the stadium running due north and south. Megan, from Team 6, demonstrated this method in her individual work (see Figure 4.10).
Figure 4.10. Megan’s work showing calculations as a result of assuming parallel distances. (March 28, 2016)

Using the assumption the distances marked with 200 (the distance I walked) and z (the stadium’s width) were parallel, Megan could easily apply alternate interior angle relationships to find the missing 60 degree angle marked in red. After calculating other unknown distances, she could then apply Law of Sines to the triangle marked in blue to find the width of the stadium to be 753.79 feet. Issues with and the importance of stating assumptions was one of the most significant mathematical learnings for myself and students and will be analyzed further in Chapter 7: Relearning Mathematics within Authentic Tasks. Finally, due to issues with time and unplanned interruptions, not all teams were able to create a general model based on their work and I ended up not making this a required component for this task.

Interpreting and Making Sense of Models

During presentations at the end of the task, students communicated their work and reasoning to other peer groups. Teams who had used models based on parallel
relationships clearly communicated this so those listening to their presentation could follow the mathematical implications of that assumption. I also asked groups who noticed mistakes in my own measurements to communicate those mistakes to their peers and how they noticed potential human errors. I did not feel, however, students did a good job applying skills related to quantitative literacy as they attempted to interpret numerical relationships within their analysis. They were communicating the results as being the width and those that developed general models were sure to state it would only work in a similar situation where you are given the same types of measurements. But no team addressed the triangles they drew were actually imaginary or theoretical triangles and the distances they used in their triangles did also communicate something about the situation: it told you how far away you were standing from the objects being referenced. Students inadvertently found out how far away I was standing from north and south stadium at each location but no one noticed or at least did not communicate that idea during their presentations. I stated this quickly at the end of the period but we were running out of time and I wanted to let students know what the actual width was by showing them a picture from Google Earth with the distance drawn in.

Also, I felt during presentations students did not adequately critique and analyze student work as most of the questions asked came from me and not other students. For instance, I would ask clarifying questions about student’s work if I did not understand what they were referencing in their presentation, but I wanted students to make similar comments to make sure they were following the rationale of each strategy described. If I desired to create a more student-centered learning environment over the course of the
modeling unit, interactions needed to be less teacher-to-student and more student-to-student driven. In my reflection journal (March 26, 2016) I wrote:

In the future I need to do a better job closing out the task and having students really reflect on the work they saw in the presentations. How do I find more time to do this? I feel like this is an important part of the reflection on their own work.

How are these models different? What did I learn from my peers?

These questions were representative of the goals I had for team-to-team discourse, but I was not witnessing in class. I felt this communication piece of the modeling cycle was crucial to experience mathematics authentically with multiple solution strategies. However, based on the lack of student to student interactions and questions I was not sure students were yet benefitting from this communication.

**Perseverance Throughout the Process**

More so in this project than in the last, students exhibited both perseverance and frustration. On Tuesday, March 22nd, Team 4 was trying to work with proportions and determine for every one foot I moved how it would affect my angle measurements and then use those rates to compare and find the width of the stadium. They were getting so frustrated that Jacob was becoming angry and looking like he was about to give up on the task. I came to their team to give them positive feedback on the work they had done and guidance for next steps. I was coming to a genuine realization that authentic mathematics can be messy because students are used to receiving and using directions and procedures. I wanted to ease their frustrations by reassuring them it was okay to be stuck for the moment:
DB: It seems like this idea of proportions is getting us stuck.

Jacob: Yeah

DB: So let’s stop where we are at here and let’s go back and try something else. Maybe you’ll get some ideas there. That’s something we learn when we model. You might be working with something and you’re like, ‘this is not really getting me anywhere.’ And it’s making me more frustrated and more confused than anything. Right?

Jacob: Yeah

DB: So let’s go back to your picture and let’s start thinking of what are some other things we could try. (March 22, 2016)

Emily then suggested perhaps they could go back and find other missing sides within the triangles in her picture. This reassurance that at times we might have to try out multiple ideas in modeling seemed to actually relieve them as they were able to make progress and recognize they could apply Law of Sines and Cosines to find missing distances.

Conflicts Begin to Emerge

A major conflict also became prevalent between Team 1’s Nicholas, William, Brooke, and Matt as they have appeared to essentially separate themselves into two groups with Nicholas and William doing most of the work while Brooke and Matt desiring to participate but not having their voices heard or their input valued. I noted this conflict in my field notes and elaborated in my journal:

I am now questioning why I decided to put this team together. All four of these individuals are unique, and by ‘unique’ I mean they generally see things other
students don’t see...I thought the combination of that would work in their favor and allow them to produce some unique models and perspectives. However, I think they have too strong of personalities to work productively together...Over the course of this project they have fallen apart. (March 24, 2016)

Going into this unit, I carefully considered the placement of each person at a team hypothesizing how essential productive discourse and teamwork would be throughout the unit. It seemed despite my careful planning, I had not anticipated the conflict arising with members of Team 1. William, Matt, and Nicholas worked for two hours in their programming courses the day prior trying to develop a usable model and apparently Matt got so frustrated he left. Similar frustrations were beginning to become prevalent in Team 6 with Monica, Megan, Chris, and Evan as Evan and Chris tend to dominate conversations while leaving Monica and Megan left to try and catch up. I wrote in my journal how I would need to carefully monitor these groups going forward. Teamwork at this point seemed to be an essential component to the quality of mathematics students were producing; a lack of student to student dialogue and feedback I felt would be detrimental to their success in this unit.

Reflections on Project 2

In Project 2, perhaps because of my instructional adjustments, students began to shift the source of model validation. Students were able to apply their own real life knowledge in order to assess the reasonableness of their results and model as opposed to asking for my approval. The importance of applying real life knowledge and providing opportunities for students to apply this knowledge is discussed both in Chapter 5 from the
student perspective and Chapter 6 from the teacher perspective. In addition, students found perseverance despite frustrations to complete the task seemingly because of the authenticity of the task itself. The source of perseverance also extended beyond a grade in class and is discussed more in-depth in Chapter 5. Project 2 also provided the unit’s first major conflict: a lack of positive and productive student-to-student discourse within Teams 1 and 6. These communication issues would prove to escalate in subsequent projects and provide major sources of relearning discussed at great lengths in Chapter 5.

Finally, Project 2 provided students and me our first new insight into the mathematics needed to navigate the modeling process. As students from Teams 1 and 6 developed models based on an assumption my distance was parallel to the stadium’s width, we all became exposed to the significance of stating assumptions and recognizing the implications those assumptions have on the models developed. The mathematics needed to complete a task may look drastically different because of these assumptions. These mathematical ideas are explored further and are the focus of Chapter 7.

Despite their frustrations, generally students fought through and persevered in developing a usable model to find the width of the stadium. Students displayed curiosity to know the result, which seemed to help them find the perseverance necessary to complete the entire modeling cycle. I was asked by several students throughout this week if I had an answer for the width and if I would be sharing. I responded with “Yes, but I’m not sharing until you have your own ideas.” They seem to have become impressed with themselves and the work they were doing. By providing them with opportunities for little successes along the way in addition to asking questions they might
be naturally curious about, students are making progress towards developing modeling abilities.

**Enrollment at River Junction Senior High**

**Monday, March 28, 2016 - Thursday, March 31, 2016**

Of all the projects I had designed for this unit, I was most excited about the possibilities for the development of modeling abilities in the third project analyzing the growth of our high school population. I was hoping by having students analyze a situation they could have real life background knowledge about, they would find success analyzing such a large data set. I was also excited about members of our school’s administrative team observing student presentations and what that might do to focus students on the development of quantitative literacy skills. To introduce the project, I talked about our school district’s recent $69.5 million bond issue in order to build and add on to several elementary and middle schools in our town. Several students, parents, teachers, and community members, however, are concerned about the future of the high school which was largely ignored in this new bond. I wanted students to think about how they would convince the city to spend another $50 million or more to build a new high school or add on for one giant high school. I charged teams with figuring out a model based on trends in the data set (see Appendix J) that could predict enrollment in 9 years and 15 years.

**Making Sense of the Task**

Students were nervous about the size of this data set initially, but once students started applying their everyday knowledge to understand and explain the data, they were
more easily able to identify trends. Mathematically, the majority of teams decided to find the totals for the high school populations each year and only used the rest of the data to justify the validity of their models later. Discussions surrounding the real life knowledge students applied to ease their nervousness about this data set is discussed more thoroughly in Chapter 6: Relearning How to Teach.

**Development of Models**

After students calculated the high school totals, teams began analyzing these totals to find mathematical relationships focusing on either linear or exponential growth. More groups than I expected going into this project were focusing on exponential functions because they have discussed exponential growth for populations before. In other words, the reason they were deciding on exponential was not mathematically based but because they remember doing exponential problems with populations in previous coursework. Megan and Team 6 commented on how they remember doing an enrollment prediction problem with their advanced algebra teacher the year before and that was how they decided to use an exponential model. I asked this team what might be some limitations to using an exponential model and one student incorrectly stated that it would be “half a parabola” (March 29, 2016). It took them awhile to figure out an exponential model will increase faster the further along time goes as the rate of change will eventually be much higher than a linear model, so there is a limitation to that model if you are predicting populations in the future.

**Generating exponential models.** Using exponential functions was causing frustrations for some students as they incorrectly applied fundamental ideas about each
type of function. Unlike Team 6, Teams 2 and 3 were trying to develop an exponential model by hand, but they could not recognize the percentage growth would be calculated by dividing the amount it grew by the previous total high school enrollment. Team 2, for example, found the difference in enrollment for each year, but they incorrectly wanted to find the average change in enrollment numbers which would give them an average linear growth, not a percentage growth. They were unsure how to use the change in enrollment numbers and turn them into percentages. After asking me for help, I had to remind them to calculate a percentage, you must take the value of what you want (the enrollment change from year to year) and divide it by the total (enrollment for the prior year). I did not do a good job accessing their prior knowledge as it was right at the end of the period and we were running out of time, but this team must have realized what I meant by this as they were able to accurately calculate an average percent growth (see Figure 4.11):

<table>
<thead>
<tr>
<th>Year</th>
<th>Enrollment Change</th>
<th>Percent Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>00-01</td>
<td>1694</td>
<td>39/1694 = 0.23</td>
</tr>
<tr>
<td>01-02</td>
<td>1733</td>
<td>93/1733 = 0.064</td>
</tr>
<tr>
<td>02-03</td>
<td>1826</td>
<td>76/1826 = 0.042</td>
</tr>
<tr>
<td>03-04</td>
<td>1902</td>
<td>31/1902 = 0.016</td>
</tr>
<tr>
<td>04-05</td>
<td>1933</td>
<td>-10/1933 = -0.006</td>
</tr>
<tr>
<td>05-06</td>
<td>1923</td>
<td>-8/1923 = -0.004</td>
</tr>
<tr>
<td>06-07</td>
<td>1915</td>
<td>87/1915 = 0.046</td>
</tr>
<tr>
<td>07-08</td>
<td>2002</td>
<td>50/2002 = 0.025</td>
</tr>
<tr>
<td>08-09</td>
<td>2082</td>
<td>(50/2082 = 0.024)</td>
</tr>
<tr>
<td>09-10</td>
<td>2114</td>
<td>-7/2114 = -0.001</td>
</tr>
<tr>
<td>10-11</td>
<td>2185</td>
<td>-49/2136 = -0.023</td>
</tr>
<tr>
<td>11-12</td>
<td>2086</td>
<td>139/2086 = 0.067</td>
</tr>
<tr>
<td>12-13</td>
<td>2226</td>
<td>53/2226 = 0.024</td>
</tr>
<tr>
<td>13-14</td>
<td>2278</td>
<td>52/2278 = 0.023</td>
</tr>
<tr>
<td>14-15</td>
<td>2365</td>
<td>87/2365 = 0.036</td>
</tr>
<tr>
<td>15-16</td>
<td>2393</td>
<td>28/2365 = 0.012</td>
</tr>
</tbody>
</table>
Figure 4.11. Samantha’s individual work showing Team 2’s calculations for finding an average percentage growth in high school enrollment. (April 1, 2016)

Samantha’s work shows the total 9th-12th grade enrollment and the difference in those enrollment figures. In the middle of her work, her team then calculated the percent growth for that year by dividing the difference by the previous year’s enrollment. Finally, they added those percentage growth rates and divided by 15 to get the average growth each year of 2.36%. Team 2 then used the average percentage growth and the initial population in 2000 to develop the following model:

\[ y = 1694(1.0236)^t \]

where \( t \) = years from 2000.

Adjusting exponential model based on real-life knowledge. After analyzing the data again, students in Team 2 noticed dropout rates seemed to decrease within the last four to five years. They asked me why that might be, and I stated a law in our state was passed in 2012 raising the dropout age from 16 to 18. I encouraged them if they had time, to develop a model just looking at the data from these last four years because it would be very interesting to compare to the model they originally came up with. They decided to pursue it by finding the average percentage growth for just the enrollment figures beginning in the 2012-2013 school year:
Figure 4.12. Samantha’s work showing their modified model using only the last four years of enrollment figures. (April 1, 2016)

By changing the definition of their variable \( t \) to be the number of years from the 2012-2013 school year and their initial value (2225) to the actual enrollment in 2012, Team 2 created the following modified model of:

\[
y = 2225(1.0247)^t.
\]

Students in this team then set out to validate this model by plugging in known values and found it to predict exactly the 2015-2016 enrollment figures. Samantha exclaimed, “That’s awesome!” During this team’s presentation, George thoroughly described the reasoning for this change in their model:

The reason we started with 2012-2013 as our base year because when we started with 2000 it had inconsistencies and we realized we were off. So we looked back at what the Nebraska state law was for dropping out, and in 2012-2013 they changed it to where you had to be 18 to drop out. So we changed our model from using 2000 as our base year. (March 31, 2016)
As a result of this decrease in dropouts and larger percentage growth, Team 2’s predictions were higher than other teams. During presentations the next day and when we reflected on those presentations as a class Friday, students appreciated this creativity and analysis of the data. Matt (Team 1) stated, “That was smart of them.”

**Errors in exponential models.** During presentations at the end of this project, students and I noticed the predictions for total enrollment made by Team 3 were much higher than other teams based on a higher percentage rate of increase despite looking at the full 16 years of enrollment figures. In the individual work submitted by members of this team, I noticed the error originated from incorrect calculations towards finding this percentage (see Figure 4.13).
Figure 4.13. Alicia’s individual work showing Team 3’s incorrect strategy for calculating the average percentage growth. (April 1, 2016)

Alicia and Team 3 found the average student growth per year (43.68 students) by dividing the total growth from the 2000 to 2015 school years by the total number of years. To find the percentage growth, they then divided this yearly growth only by the beginning enrollment (1694) as opposed to comparing that growth from year to year. As a result, their growth rate was slightly higher than other teams.

**Generating linear models.** In contrast to exponential models developed by Teams 2, 3, and 6, Teams 4 and 5 used high school enrollment totals to find an average growth per year in order to develop a linear model. Both teams decided to use their graphing calculator to generate linear regression models. Team 4 validated and applied the model \( y = 43x + 1712 \) where \( y \) is the high school population and \( x \) is the years since 2000. Emily (Team 4) stated in her work “We shouldn’t include decimals because you can’t have a decimal of a student” (April 1, 2016). This generated predictions of 2744 students in the 2024 school year and 3002 students in the 2030 school year. Similarly, Team 5 settled on the model \( y = 43.17x + 1712.2 \) using a base year again of 2000. However, Team 5 originally defined \( x \) to be the year itself which resulted in frustrations as they began interpreting the model in terms of the situation. The issues Team 5 exhibited interpreting their models based on how they defined their variables and the supports I used to guide them in correcting their errors is analyzed further in Chapter 6.

**Comparing exponential and linear models.** Finally, Team 1 decided they could develop both a linear and exponential regression on their calculator and compare the two.
Their models were identical to those developed by Teams 5 and 6, but by comparing the two and creating a predicted range for future enrollment they provided a specific example of applying creative and flexible thinking (see Figure 4.14):

![Graphs & Data Tables](image)

*Figure 4.14. A slide from Team 1’s presentation comparing their linear and exponential model. (March 31, 2016)*

Using the two models, they predicted River Junction would have between 2748 and 2879 students in 9 years and between 3007 and 3129 students in 15 years. In addition, three of the students in this team were in physics and decided it would be useful to apply ideas from physics to calculate percent error. Matt stated the exponential model yielded a 2-3% error rate while the linear was only around a 1% error rate, suggesting the linear may be more accurate in the long term.

**Escalating Conflicts**

Despite these insightful mathematical ideas, the conflict between members of Team 1 continued to escalate. Team 1 was not done with their presentation on time, and
they were scrambling to put the rest of it together in between other team’s presentations. During the presentations, as expected, Nicholas and William explained most of the mathematics, not encouraging Brooke or Matt to participate in the discussion. For example, towards the end of this team’s presentation, Matt was going to state his opinion on a question I asked, “Which model [linear or exponential] do you feel will be more accurate in the long run?” (March 31, 2016). Matt was about to say “linear” when he stopped himself as Nicholas quickly chimed in and stated “exponential.” I asked Matt to tell us all what he wanted to say but he just replied with “no.” He did not want to express his ideas as “I don’t think it’s a good idea to disagree with my partner during a presentation” (March 31, 2016). I appreciated the mathematical implications in comparing linear and exponential models and limitations to exponential models, especially over a longer period of time. However, the conflicts arising within Team 1 illustrates the difficulties in transitioning to a student-centered environment especially when students do not feel comfortable expressing or finding validation in their ideas.

**Interpreting and Making Sense of Models**

On the final day of Project 3, school administrators were asked to join groups and sit in on the presentations students gave over their models and enrollment predictions. In preparation for this throughout the week, I encouraged students to think about how they might communicate their analysis and results in a way non-mathematicians would understand. I asked them: *What would an administrator care to know about?* and *What will they understand?* I was pleased to hear during presentations with administrators, all teams communicated they were finding an average growth rate or percent growth each
year. In their totals they understood their values in the end would only be predictions since they were assuming the population to grow the same each year. In Emily’s individual work, for example, she commented: “Our equation won’t be exact because years don’t consistently increase/decrease. The equation is only an estimate, not exact” (April 1, 2016).

During presentations, having an audience that was not a member of our class and familiar with the situation or analysis seemed to force students to think about how they would need to communicate their analysis and results in a way others could understand it. Furthermore, students did a better job in this project engaging in a productive discourse with teams during the question and answer portion of each presentation. I had students document questions on sticky notes during presentations, and some attributed this dialogue to me asking students to deliberately write down questions. Others thought it was because they noticed more differences in the models developed, data used, and validation techniques employed. Therefore, they had more material to ask questions about. And because students had so thoroughly developed, analyzed, and validated their models, they seemed to have few issues answering questions brought up during presentations. For example, Brooke asked some questions to clarify her own understanding of Team 2’s methodology and modified model during their presentation:

Brooke: So is the data you used only the last few years when using that model?

Noah: Yeah, we just used the last four years, just the high school numbers. But we did the whole thing with like 2000 all the way up and we are like 60 off, then 30 off, then 100 off.
George: It got closer the further away from 2000 we went, but we figured out they passed that law where you had to be 18 to drop out.

Samantha: Because if you look at the data, also, between sophomore and junior year there was a major decrease because people were turning 16 and dropping out.

Abigail: Then once they changed that law it was pretty even. (March 31, 2016)

At the time, I was pleased to hear Brooke take the initiative and ask the clarifying questions I desired students use as they engaged in team-to-team discourse. What impressed me even more, however, was how all four members of Team 2 responded to Brooke’s question. One student did not respond, all of them did. The fact that Noah, George, Samantha, and Abigail were able to comment and add to the rationale for their strategy displayed exceptional teamwork and understanding of the situation. After their presentation, I wrote in my reflection journal:

This type of response showed excellent insight and cooperation by all members of this team in understanding the entire process they went to from beginning to end in the development of their model! (March 31, 2016)

Similar dialogue occurred throughout most presentations demonstrating a much more student-centered environment and comfortableness with the material in comparison to presentations for Project 2. Administrators commented on how thoroughly prepared students seemed to be and how appreciative they were in students communicating how their models related to the situation. In my questioning of students throughout the week, I tried to focus on this interpretation piece including the communication of numeric
relationships in the situation and reasonableness of outcomes in order to specifically target skills related to quantitative literacy. Based on the communication outcomes of this project, these questioning skills seemed to have sufficiently prepared students to interpret their models in a way others would understand.

**Reflections on Project 3**

Comparing Project 3 to the two prior modeling tasks, students seemed to more thoroughly apply modeling abilities in order to validate the conclusions and mathematical results of their analysis by using their knowledge of the situation and patterns in the data set. This shift from needing my approval to generating their own opportunities for learning was an important learning component for students in the modeling unit and is a major focus of Chapter 5. In addition, this relearning continued for students through the communication of various models developed to predict enrollment at River Junction Senior High. Students began to change their perspectives on what it means to do mathematics specifically as it relates to the possibility of multiple solution strategies. Because students were allowed to employ their own ideas, they began to find value in different mathematical perspectives and took a lot of pride in their own work, thus further increasing their own perseverance to complete the task.

Discourse, however, proved to be the biggest barrier to the successful development of modeling abilities. Conflicts continued to escalate in Project 3 between students in Teams 1 and 6. The source of these conflicts are examined thoroughly in Chapter 5 in order to better understand productive student-to-student interactions. In reflecting on this and previous projects, I also began to see a need to improve the
discourse occurring between teams during the communication component of the modeling process. In Project 3, I introduced the use of sticky-notes during presentations to encourage students to more deliberately provide feedback to their peers, but conversations seemed to be superficial. In contrast to my planning for this unit, students did not seem to need support with the mathematics, but rather I needed to provide instructional support to encourage both student-to-student and team-to-team discourse. This shift in my own perception of student needs is detailed further in Chapter 6.

**How Safe are We?**

**Friday, April 1, 2016 - Wednesday, April 6, 2016**

Project 4: How Safe are We? (see Appendix L) went through several design versions as discussed in Chapter 3, but I ended up providing a year’s worth of crime data from the FBI and asked students to develop a model that would assign a safety score to various cities in our Midwestern state. I then wanted students to rank these cities from most dangerous to safest based on these scores. I did not think the mathematics students would use would be as complicated as typical topics found in precalculus, but I was eager to find out how students would develop skills related to data analysis, interpreting, and validating conclusions as they dissected each other’s rankings. Will students find value in understanding someone’s methods before making conclusions?

**Changes to Teams**

Going into the fourth project, I decided to make changes to two groups who did not seem to operate as a cohesive team. Up until this point, Matt and Brooke at Team 1 have had major conflicts with their teammates, Nicholas and William, as the latter two
tended to pursue their own ideas without discussing strategies with the rest of the team. Matt and Brooke had both voiced their frustrations to me, at one point even informing me they did not find this unit to be beneficial for their learning. Similarly, Chris and Evan in Team 6 have provided insightful mathematical ideas throughout the unit, but their teammates Megan and Monica never seemed to offer up their own ideas and strategies. That is, they simply went along with whatever Chris and Evan decided. Going into this fourth project, I approached Chris and Evan about switching groups with Matt and Brooke so I had four dominating personalities at one team, while Matt, Brooke, Monica and Megan would form their own team hopefully providing them with greater opportunities to explore and test out their ideas. All students involved agreed to try out the new group arrangement. I was optimistic these changes would provide for a more positive and productive learning environment for the duration of the unit.

**Introducing Safety Rankings**

At the last minute I also decided to introduce Project 4 and the importance of understanding the mathematical methodology behind an analysis differently than I originally intended. Rather than having students read an article about a possible new crime wave developing across the country, I simply projected a set of rankings I had found for the most dangerous cities in our state. In the first ranking, cities were ranked similar to population size with our city River Junction ranked third behind the two most populated cities (Metroville and Jefferson) in our state. I gave teams time to discuss the following questions: *Do you agree with these rankings?* and *Why or why not?* After a class discussion, most students agreed with the rankings as it seemed to match
experiences they had in real life and their perception of ‘dangerous.’ However, some students began asking about how the rankings were developed. Evan commented “crime rate per capita” to which I respond “How do you know? Did I tell you?” (April 1, 2016).

I was not expecting much conflict with the first set of rankings, so after this conversation I projected another set of rankings that had our city in the same rank. However the city of Jefferson, which is four times larger than River Junction, was considered more safe than River Junction and the significantly smaller cities of Central, Loganview, and Saint Peters second, fifth, and seventh most dangerous respectively. As soon as I projected this set of rankings, students had an immediate and loud reaction. Students around the room simultaneously and very powerfully exclaimed “Central?” “Why Loganview?” “No way Saint Peters should be on there!” “I don’t agree with this at all!” (April 1, 2016). Jacob hallowed “Saint Peters has like 45 people!” (April 1, 2016). Brooke offered to the class “Their populations are different so if one crime happens in a little place the crime rate is going to go up” (April 1, 2016). As teams discussed what Brooke meant by this, most identified needing to take into account populations and not just crime occurrences. Noah told his team “How do we know they didn’t already do that?” (April 1, 2016). After these interactions in class, I wrote in my reflection journal:

This intro got students asking the question I wanted them to ask: what are these rankings based on? This project is not very mathematically complicated, but it supports the development of modeling abilities specifically geared towards interpreting and validating conclusions. This is more of a project geared towards
Making students educated citizens not to develop skills found in precalculus. I want them to learn that they can spin data anyway they want to show the result you want so it is very important to understand something about the methodology and data used. By having students discuss these rankings first I think students became curious and wanted to know more. (April 1, 2016)

Introducing this project in this way provided opportunities for students to begin wondering about and questioning the methodologies used to produce these rankings. Especially with the second set of rankings, several of the cities indicated as dangerous clashed with students’ own real life experiences and thus created conflict and discourse. Students wanted to dig into the data I had provided from the 2014 FBI’s Crime in the Country Report in order to discover why these cities were on the list (Federal Bureau of Investigation, 2015). This data consisted of the reported occurrences of various violent and property crimes for the top 45 most populated towns in our state. I then set teams lose to analyze this data and develop their own methodology and set of rankings which would be presented to the class.

**Making Sense of the Task**

As with every other project we have encountered during the unit, the first day focused on students exploring and processing the task at hand. Specific conversations that occurred during this time as students applied their real life knowledge to understand the data is discussed further in Chapter 6. Few potential models are developed until the second day of the project when students start applying mathematical literacy skills and translate their exploration of the authentic task into mathematical relationships.
Unfortunately, I did not escape the modeling unit without unforeseen complications as I developed a terrible case of stomach flu which kept me out of school for the second day of this project. Students still set up two cameras in the room for me to document their progress, however. Interestingly, during this second day, students began applying their methodology to cities outside of our state as they seemed genuinely curious how cities would compare. Some members of Team 1 and 2 were even encouraging each other to look at large cities across the United States. Chris and Evan from Team 1 were discussing data from these cities when Noah from Team 2 approached:

*Chris:* Chicago’s pretty safe man.

*Evan:* No it’s not.

*Chris:* Violent crime rates, they’re safer than Loganview.

*Chis shows Evan his calculator while laughing and Isaiah and Evan smile.*

*Evan:* That’s scary honestly. Look up inner city Chicago.

*Chris:* Inner city Chicago? That’s the whole city. There’s only Chicago, Chicago Heights, and Chicago Ridge.

*Noah came over from Team 2.*

*Noah:* Evan, did you do Detroit? *(Evan shaking his head no).* Do it.

*Evan:* I don’t know if I want to know.

*Noah:* Guess what it is.

*Chris:* Loganview is as deadly as Compton! *(April 4, 2016)*

These conversations illustrate the surprise some students felt by the results they were acquiring. Chris, Noah, and Evan seemed genuinely interested in the data they were
analyzing, even doing more than I originally asked them to do. After watching the videos from the day I was absent I wrote:

I was worried about being gone yesterday but it seems that of all the days I could have gotten sick, it was probably the best one. Students didn’t have a lot of questions and all groups had nearly all or all of their analysis complete...In the videos, students were engaged nearly the entire period in the data they were analyzing by their own choice, perhaps because of their interest in the outcomes and in the data itself. (April 5, 2016)

Although the mathematical skills students needed to apply for this project were much easier than in previous projects, I was still pleased when I returned the next day to see all teams had developed and tested their own model and ranking system. At the beginning of the modeling unit, students seemed to quietly wait for me to reach their team in order to validate the work they were doing. Now, it seemed, students found their own strategies to explore and validated the calculations they had done. Looking back on it now, I am even more pleased this shift occurred as students felt confident in their own work. I was not longer needed.

**Development of Models**

The models developed by teams could be separated into two categories: (1) ones which calculated violent and property crimes equally and (2) ones which weighed violent crimes more heavily than property.

**Violent and property crimes are equal.** Teams 3, 5, and 6 developed models calculating the crime score for a city as \((V + P)/\text{Population} \times \text{Population Weight}\) where \(V\)
is the occurrences of violent crimes, P is the occurrences of property crimes, Population is the population of that city, and Population Weight being some constant used to equalize all crime scores between cities of different sizes. Team 6 used the population weight of 100, Team 5 multiplied by 10,000, and Team 3 used 12,000. In his individual work, Ryan offered an explanation for Team 5’s methodology and the use of 10,000:

Assumption: Will continue crime at same rate.

Work: To start we took the violent crime and divided it by population and then multiplied it by 10,000 because every city was at 10,000 or more for the population. We repeated this for property crime and then (put) both together. Once we did that we got our answers in decimal form making it the crime rate per person, assuming rate was the same. Once we got our number or rate per 10,000 people we made three list: violent list, property list, and then both or overall list.

This shows us our total and final crime ratings. (April 7, 2016)

During presentations later, students noticed these differences in the model but realized it did not affect the final rankings. Luis, for example, commented “They did the model with multiplication by a controlled population, very similar to ours, but a different population. Their listing is the same as ours” (April 7, 2016). Students from Teams 5 and 3 developed the same rankings based on this methodology:

1. Metroville
2. River Junction
3. Spring Hill
4. Loganview
5. Cedar Valley
6. Central
7. Jefferson
8. Armstrong
9. Phillips
10. Platte City

*Figure 4.15.* Ranking developed by Teams 3 and 5 where property and violent crimes are calculated equally. (April 7, 2016).

Team 6 developed a similar model but, at the suggestion of Matt, also created additional information and separate ranks for groups of cities of different sizes. He suggested this as he thought not all residents of our state want to live in bigger cities so comparing the safety of smaller towns might be useful to certain groups of people.

**Violent crimes are more dangerous.** The remaining teams developed a model which weighed the occurrences of violent crimes more heavily than property crimes. As students discovered during presentations, this methodology drastically changed the final rankings and the perception of cities which are ‘most dangerous.’

*Team 2’s model.* Team 2 developed the model \((2V+P)/\text{Population} \times 100 = \)*

Weighted Crime Score where \(V\) is the number of occurrences of violent crimes in the city while \(P\) represented property crimes. In other words, they have a similar model to Teams 3 and 5 but they decided to multiply violent crimes by two. Noah describes the rationale for this in his individual work stating, “These crimes are considered more dangerous to our group. These crimes would have the largest impact on someone interested in moving
to one of these cities” (April 7, 2016). Noah later added in his work assumptions his team made for this model including:

- We did not take into account different areas in larger cities (i.e. - Metroville North, South, etc.).

- Crime will continue at the same rate.

- That people consider violent crimes to be more impactful so we multiplied this crime by two. (April 7, 2016)

Based on this methodology, Team 2 developed the following ranking of dangerous cities:

1. Loganview
2. Metroville
3. River Junction
4. Spring Hill
5. Cedar Valley
6. Central
7. Jefferson
8. Armstrong
9. Phillips
10. Platte City

Figure 4.16. Ranking developed by Team 2 by weighing violent crimes twice as much as property crimes. (April 7, 2016)
In contrast to the ranking developed by Teams 3 and 5 where violent and property crimes are considered equally, Metroville is no longer considered ‘most dangerous.’ Rather, Loganview tops this list yet its population is nearly 1/36th the size of Metroville.

**Team 1’s model.** Using a similar methodology, Team 1 calculated the violent crime, property crime, and total crime rates per person and created three separate rankings based on these rates. The then correlated these ranks to points so a ‘first’ ranking would be awarded one point, but they decided to double the points given from the violent crime rankings. These points were then added together, with the lowest point totals resulting in the most dangerous cities:

1. Metroville
2. Loganview
3. Spring Hill
4. River Junction
5. Jefferson
6. Cedar Valley
6. Central
8. Platte City
8. Phillips
8. Armstrong

*Figure 4.17.* Ranking developed by Team 1 using a point system. (April 7, 2016)

This team did not develop an equation for their model but rather a description:

We decided to make three different point systems. There were three categories for
points: violent crimes, property crimes, and overall crime rate, all of which were
per capita. Since violent crimes are more dangerous, we decided to give it a
weighted score by multiplying it by 2. Although our point system is useless on its
own. The data is only important when all of it is put together. Also, we had ties
for 6th place along with 8th place so it is not perfect since we don’t keep the
starting decimals. (Chris’s Student Work, April 7, 2016)

Because of the methodology developed, in his individual work Chris described why their
rank contained two “6” ranks and three “8” ranks. When the actual per capita crime rates
were converted to ranks and then scores, ties became possible.

**Team 4’s model.** Team 4 created a similar weighted model for violent crimes:

\[(0.25(\text{Property Crimes/Population}) + 0.75(\text{Violent Crimes/Population})) \times 100 = \text{Safety score}\]

with the lower the score, the safer the city (April 7, 2016). Like Teams 1 and 2,
Team 4 emphasized the occurrence of violent crimes more heavily than property crimes,
yet they produced a significantly different set of rankings:

1. Metroville
2. Central
3. River Junction
4. Spring Hill
5. Loganview
6. Jefferson
7. Cedar Valley
8. Phillips
9. Armstrong

10. Platte City

Figure 4.18. Ranking developed by Team 4. (April 7, 2016)

In particular, Loganview was located at the top of rankings produced by Team 1 and 2 yet was located fifth here. In addition, Central was located in the latter half of the rankings submitted by other teams and Team 4 considered it the second most dangerous city in the state. During presentations for this project and after some heated discourse, students realized Team 4 had decided to use data from a different year causing their rankings to be drastically different from the rest of the class. The conflict that emerged from this presentation and the learning which took place as a result of this conflict is discussed more in Chapter 5.

Comparing methodologies. During presentations, students noticed immediately how weighing the violent crimes more heavily created a vastly different set of rankings (see Figure 4.19).

<table>
<thead>
<tr>
<th>Rank from Unweighted Scores</th>
<th>Rank from Weighted Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.  Metroville</td>
<td>1.  Loganview</td>
</tr>
<tr>
<td>2.  River Junction</td>
<td>2.  Metroville</td>
</tr>
<tr>
<td>4.  Loganview</td>
<td>4.  Spring Hill</td>
</tr>
<tr>
<td>5.  Cedar Valley</td>
<td>5.  Cedar Valley</td>
</tr>
</tbody>
</table>

Figure 4.19. Comparison of rankings developed from teams using unweighted and weighted scores.
In particular, Loganview with a population of almost 12,000 was the most dangerous in the weighted ranking system while Metroville with a population of nearly 440,000, and what most students considered as the most dangerous based on their own real life experiences, considered not as dangerous. The issues surrounding Loganview and its surprisingly high violent crime rate was a talking point throughout the project as it conflicted with student’s own experiences and perceptions of residents in these cities.

**Interpreting Models**

Over the course of this project again the biggest source of error originated from issues with quantitative literacy, particularly interpreting what the scores resulting from the model meant about the city’s safeness. Students who had assigned a weight to the violent crimes could not interpret scores as crime per capita. It simply resulted in a scaled score. Scores developed using this methodology could not be interpreted without other scores to compare it to. I had this learning during the design phase of this project as well and as a result altered the project from simply providing a safety score to a safety ranking.

**Issues interpreting scores.** Issues in the interpretation of scores caused Megan and Team 6 frustration. They had not weighed violent crimes more heavily so their scores could actually be interpreted to provide information about each city, but Brooke, Monica, and Matt (Team 6) continually referred to scores as the percent of crime occurring and Megan could not understand why they were using this language. For example, Metroville’s score was being communicated as 4.91%. Megan expressed her frustration in understanding what 4.91% meant for the city of Metroville:
Megan: Instead of saying percents, because percents are confusing me. So like
for Loganview, out of 100 people, 4.5 people will be involved in a crime.

Matt: So how do we change that here (pointing to his spreadsheet).

DB: Well you could just change the column. Crime per

Monica & Megan: 100

Brooke: Okay, I get it. And this would be violent crimes per 100 people. Okay.

(April 5, 2016)

Conversations like the one occurring in Team 6 seemed to have resonated with students
as during presentations all teams accurately described what the score could and could not
communicate about a city’s perceived safeness. In the graphic organizers I asked
students to fill out to reflect on presentations, several also commented on the difficulties
in interpreting scores. Students who had not thought to weigh violent crimes felt that was
a nice idea, but, as Kandy noted, you would “need to have something to compare to
understand” the meaning of the scores (April 7, 2016). Individuals like Abigail who had
weighed violent crimes were concerned about those who did not because “violent crimes
are more dangerous” (April 7, 2016). But, these students recognized the value in having
scores that could be easily interpreted. Chris wrote in his graphic organizer: “It was
effective because it gives you real crime per a certain amount of people” and thus could
be interpreted in isolation (April 7, 2016). By the end of the project, students seemed to exhibited the use of quantitative literacy skills, but it took a lot of questioning on my part. These questioning tactics and supports are analyzed further in depth in Chapter 6.

**Reflections on Project 4**

This project seemed to demonstrate to students the importance of understanding the methodology and reasoning behind a mathematical analysis. In fact, in reflecting on the unit Chris said his most significant learning about mathematics came from this project. He stated he learned most from:

> How safe we are, because a lot of people did different things. And depending on the way you showed your work the information is meaningless...if we just gave them our list, they wouldn’t know how we got to that answer...We have to get the background information before we could judge something. (April 27, 2016)

Despite the success this project seemed to have at stressing the importance of understanding methodologies and the reasoning behind it as Chris describes, in the future I would change the design of this project to provide greater opportunity and freedom for students in the development of their models.

Looking back on this now, I wish I had thought to require students to use multiple years worth of data. I was worried initially the large amount of data required for only an analysis of one year would be a lot for students to handle, but I underestimated what my students were capable of handling. The discourse occurring as a result of the analysis conducted by Team 4 in analyzing a separate year from the rest of the class stressed the importance of analyzing several years worth of data to get a more accurate depiction of
that city’s crime trends. In addition, I would rewrite the task so it is less restrictive.

Specifically, I would not give students specific data to look at. In reflecting on this task I wrote, “I wanted students to understand the importance of asking ‘where did they get their information’ when seeing analysis done in the news, not giving students specific data may actually emphasize this more” (April 9, 2016). Despite my attempts at creating a student-centered environment, I resorted back to teacher driven strategies by inadvertently suggesting criteria, information, and data to consider. If students were to think about crime and factors that can impact crime on their own, they might think to bring in data I would not have thought of and provide greater opportunities for a variety of models and student discourse. I sold my students short and did not provide opportunities for them to share in the ownership of the task by providing too much information to develop their models. I should have allowed students to decide what data to use. The continuous struggles I went through as I transitioned away from being the expert in the room is discussed more thoroughly in Chapter 6.

Finally in Project 4, I felt student discourse provided substantial opportunities for students to develop modeling abilities. Rearranging Team 1 and Team 6 seemed to help Matt, Brooke, Megan, and Monica feel validated in their ideas as evident by all four members contributing to and presenting on their ranking system. Cooperation between the new members of Team 1 (William, Chris, Evan, and Nicholas) also improved as all four boys seemed to feel more comfortable pursuing their strategies without trying to catch each other up on the mathematics. Team-to-team discourse also seemed to be supportive in understanding the different methodologies employed during presentations.
The discourse occurring from within teams and from team-to-team provided essential learning opportunities and is analyzed further in Chapter 5.

**Student Choice**

**Thursday, April 7, 2016 - Thursday, April 14, 2016**

After devoting three and a half weeks exclusively to modeling, I still had a lot of reservations going into the final Student Choice Project (see Appendix M). Students were charged with the following task and criteria:

Over the last several weeks you have developed models for various real-life situations where a mathematical analysis was beneficial in order to better understand that situation. However, these were all tasks Mrs. Buhrman found interesting that she thought you might benefit from knowing more about. She may have failed miserably in her assumptions of what would interest a group of teenagers! Well now it is your turn! Your team will pick and design your own project based on a real-life situation you think could benefit from being *mathematized*. That is, you and your team will pick an event or situation you wish to know more about. You need to be able to identify the goals of your analysis (i.e. what is it that you want to know or understand) and those goals must be accomplished through a mathematical analysis and subsequent model.

(Classroom Documents, April 7, 2016)

I was probably more nervous about this task than any other because of the complete uncertainty of work and outcomes. Even though I had learned throughout this unit that it is nearly impossible to prepare and account for all the different approaches, questions,
and outcomes students may take while working through modeling tasks, I really could not account for any approach, question, or outcome in advance for this final project. I had to trust the work my students had done up to this point prepared them and developed the modeling abilities they would need to be successful in an analysis of their own choosing. I also had to trust in my own learning and experience throughout this unit as I attempted to guide, question, and challenge students to make meaningful real life and mathematical connections.

**Making Sense of the Task**

Going into this final project, I thought students would struggle coming up with a usable situation to model. I know from my own experience designing this unit how difficult it can be to look for examples of meaningful mathematics in the real world. By meaningful, I mean a question or situation which can be better understood through a mathematical analysis that involves more than just basic operations on numbers. Students were initially asking questions that would involve mathematics but not meaningful mathematics. They were asking questions that would not require them to create a model. Team 4, for example, wanted to look at unemployment rates and determine the top 10 states for employment out of college. They were trying to apply what they learned from the Crime Safety Project, but they struggled determining how to create a model in this scenario.

**Encouraging discourse to develop meaningful topics.** In order to stimulate more conversation about ideas and widen the opportunity for productive feedback, on the second day of this project I put groups together consisting of students from different
teams. Each person had to discuss their current idea and their peers had to give feedback on if they thought (1) a model would be necessary to accomplish their goal and (2) what data or information they might want to consider. If the student’s peers did not think the situation was appropriate, this new group brainstormed ways in which the situation could be altered or what new ideas they might consider.

For example, Javier from Team 3 was discussing topics and strategies with Megan (Team 6) and Chris (Team 2). Javier was describing his team’s desire to look at the cause of deaths in various age groups, specifically teenagers, but he was confused about what direction to take with this topic:

Chris: You’ll probably have to look up some statistics of like deaths in (our state) and age groups. I am pretty sure they do that stuff.

Megan: How would you make a model of that though?

Javier: Yeah that’s what I am thinking. (April 8, 2016)

This group of students decide an analysis could be done which was similar to the Crime Safety Project by finding the number of deaths for various causes and calculating the percentage of deaths, but they were not sure this constituted as a model. As a result of this conversation, Team 3 decided to focus their analysis on another topic they had discussed: finding trends in holiday sales. Similar conversations happened with multiple groups and seemed to help tremendously as many peers were pointing out how they did not think the topic students had explored was appropriate for the task at hand. However, all students were able to generate new ideas to take back to their team with the help of feedback from these peers that would better fit the goals of the project.
Refining modeling topics using real life knowledge. Once students returned to their original teams, conversations would continue to focus on how these topics could create opportunities for a meaningful analysis and model. Chris, Evan, and Nicholas in Team 1 were engaged in dialogue about what to analyze and ended up critiquing several ideas over the course of one class period. They started by wanting to analyze the amount of space different apps, photos, and social media used up on phones so you would know how much storage is left and how long you would have access to these apps at current usage rates. Evan did not like the idea, however, as he thought there would be too many variables and factors affecting storage space such as wifi enabled apps versus downloaded apps. After they listened to feedback from their peers, they decided to change directions and began exploring the idea of analyzing the growth of River Junction. They then focused more specifically at analyzing business growth in River Junction including the number of new businesses and types. As they were doing research, they realized they could not easily find the data they needed for their analysis and were having a difficult time figuring out how to use the data they had:

Chris: What if there is more than one business? Are you going to count it twice?

Nicholas: Yeah

Chris: So Bob’s Carwash, you count that thing twice? And Mr. Gee’s Carwash you count that thing twice?

Nicholas: Ummm, I don’t know. I don’t know

Chris: And like Burger King, you want to count that four times?

Nicholas: You guys want to switch to populations?
The three of them appear visibly frustrated.

Evan: Let’s look at population stuff.

Chris: It’s so basic though.

Nicholas: It’d be easier.

Chris: It would be easier, but it’s too basic.

Nicholas: Yeah. How about population and jobs?

Evan: Oh I had this idea (getting excited) and I actually thought it was pretty good. We compare brands and their growth. Say we got like Nike, Under Armour and we look at sales over the years and see if it’s trending upwards or downwards and see which brand will eventually be the top seller in like ten years.

Chris: Yes! (throwing his pencil down). I like it! (He immediately grabs his iPad and starts looking up information). We’re doing that!

Evan: Okay, see if you can find some information on Nike sales for the last 10 years. (April 8, 2016)

The three boys then started looking up sales and revenue reports to gather the data they would need for their analysis. They also decide to include Adidas as another brand to compare. As they were digging into the data they were asking each other good clarifying questions about the data they wanted such as focusing on dollars made or items sold, revenue or profit, and U.S. or global income.
In addition to asking clarifying questions to focus the goal and subsequent model for their analysis, Team 1 began applying their own real life knowledge of the situation to further enhance their analysis. Chris made a comment at one point:

Chris: We could say depending on sponsors their sales can change. Like if Under Armour got Lebron James to sponsor, all the bandwags would go to Under Armour.

Nicholas: There’s also like track athletes sponsored by Nike. (April 8, 2016)

I was excited to see and hear students using everyday knowledge to generate ideas and think about situations that could be better understood through the creation of a mathematical model. Although students in Team 1 worked through various iterations of a potential topic to analyze, without my assistance they settled on a topic that was interesting to them and one in which they could find appropriate data on.

Outline of modeling topics used. Narrowing the scope of student generated topics into manageable questions for this analysis proved to take substantially longer and was much more difficult than I had planned. But, by the end of the second day, all students had a more appropriate question and had a process in place for the data collection and research they would need for their analysis. Each team settled on the following topics:

Team 1 - model and compare growth of net profits for Nike, Adidas, and Under Armour

Team 2 - develop a model to estimate the number of blades of grass on the football field
Team 3 - model and compare holiday spending, annual spending, and credit card debt

Team 4 - model and predict iPhone sales

Team 5 - model and predict immigrant population growth to major cities in our state

Team 6 - model and compare animals coming in versus animals moving out of our local animal shelter

FBLA Team - model growth of internet users to determine when all U.S. citizens will have internet access.

A state Future Business Leaders of America convention pulled William, Matt, and Elijah from class for two days during the brainstorming period, so unfortunately they were forced to form their own group for this project. I was particularly disappointed in this because Matt found validation and inclusion from his new group in the previous project after I moved him away from William; I was worried taking him away from this new environment because of circumstances outside of our control would drastically diminish Matt’s learning and confidence.

I also noticed most teams seemed to be doing an analysis similar to the one they conducted for Project 3: Enrollment at River Junction Senior High. This might be because students seemed to find this project the most interesting, and because this type of data may be the easiest to get ahold of. However, no team needed or asked for the list of potential topics I had created as a backup plan if students were struggling. Once again, students did not need the scaffolding and support I had planned. I did, however, give
students guiding questions (see Appendix Q) to help them organize their goal and data collection plan.

Although teams initially asked questions requiring only mathematical operations and not necessarily meaningful mathematics resulting in a model, they were trying to synthesize their real life knowledge and mathematical knowledge to ask questions about the world around them. The relative ease at which students looked beyond the classroom walls surprised me. In my task reflection journal I wrote, “Students did something in this project that I struggle with: look around you, ask a question, and come up with a mathematical way to answer it” (April 14, 2016).

In my experiences as a student and teacher, I was never forced to think about mathematics the way my students had opportunities to in this project. When I designed and planned this task I was very worried about students being able to come up with an idea and question to pursue that was worth pursuing. Students did not seem to be as frustrated by this as I had planned. Some teams had to play around with ideas for a while and get feedback from other students, but in the end their ideas were their own. And in the case of Team 2 (blades of grass on the field turf) and Team 6 (space issues at the Humane Society), students literally looked out the window and asked a question that was mathematically based. I continued to write in my journal:

More so than in the design of this unit, I learned from students the power of curiosity and how it can relate to mathematics. I learned that I needed to be more curious; to ask more questions and wonder more, and that when I do this most the
time mathematics can be used to find an answer. I just need to ask the question.

(April 14, 2016)

When I first began embarking on a thorough analysis of my own teaching practices, creating opportunities for authentic mathematics within a student-centered environment became my priority. Mathematical modeling was the avenue I chose for me to learn how to design and teach in this type of environment and for my students to develop skills which seem to only be found in this type of environment. The opening two days of this final project provided me with evidence this teaching and learning dream had come full circle. My classroom was turned into the type of learning environment where students were the primary source of feedback and inspiration resulting in meaningful mathematics learning.

**Development of Models**

Once students had narrowed down their topic and a goal for their analysis, students spent a considerable amount of time researching to expand their background knowledge and construct an appropriate data set. After they had the necessary information about the topic, the majority of students used regression curves with technology to pinpoint appropriate trends and generate models. This process took up the majority of our class time together (nearly an additional 2 class periods), unfortunately not leaving as much time as I would like for the interpretation and validation components of the modeling cycle. My role over the analysis portion of this project was simply assisting students with their data collection process. Some students had trouble accessing the data they needed for their analysis. Because I was not an expert in the field they were
interested in, I became a researcher with my students, sitting with them as we searched for the type of data and information they would find useful. After four days of brainstorming and discussing potential topics, revising those ideas, gathering appropriate data for an analysis, and developing an appropriate model based on that data, students were ready to present their work to the class.

**Team 1: Comparison of sports brands.** Team 1’s goal was to analyze Nike, Adidas, and Under Armour net profits in order to model projected growth. During presentations, they communicated how they only could consistently access data beginning in 2010 so they would use that as their base year. Defining $x$ to be the years since 2010 and $y$ to be the predicted net dollar profit, they developed the following linear regression models:

- **Nike:** $y = 250,628,571.4x + 1,825,761,905$
- **Under Armour:** $y = 33,931,142.86x + 64,600,476.19$
- **Adidas:** $y = 21,449,450x + 652,623,583.3$.

Using Desmos, they plotted their data sets and graphed each regression model creating an interesting discussion topic and visual during their presentations:
During presentations, Chris, Evan, and Nicholas gave the audience information about top endorsement deals affecting each company’s profits. They also correctly stated this as a limitation to their model because of their inability to predict future popular athletes and which company will sign them for endorsements. Students also mentioned how difficult Adidas was to model because their profits jumped up and down during this time period, so that specific model is limited. They concluded based on these models Nike is “killing the game” and pointed out how many students in the room were wearing a Nike branded item. Chris said:

Adidas and Nike have been around longer than Under Armour. Under Armour is a new company only around since 1996 which accounts for why they have such low profits compared to Nike or Adidas. But Under Armour is growing much
faster than Adidas so eventually they are going to overpass Adidas but Nike will not be overpassed by any of them because they are just too good. (April 13, 2016)

During their presentation, Chris, Evan, and Nicholas could analyze the graph shown in Figure 4.20 to indicate growth, but they did not adequately apply quantitative literacy skills related to interpreting their model in specific terms of the situation. They simply concluded the general observation Under Armour was growing faster than Adidas and would eventually become the second largest sports brand company.

**Presentation questions for Team 1.** During presentations this group was asked several questions by students to clarify the data they used for their analysis. For example, Megan asked why they only used data since 2010. Evan explained that because Under Armour was so new and has only recently become a relevant brand they decided to focus only on trends in recent years as they thought that would be a more accurate projection of future growth. Brooke asked “How do the prices differentiate like on the things they are selling between brands?” George added “Like Nike is super expensive while Under Armour is relatively reasonable.” I chimed in about how expensive Jordan’s are compared to Steph Curry’s Under Armour shoes. Evan replied “We didn’t take that into account even though Nike’s shoes are more expensive retail it doesn’t seem to affect their sales” (April 13, 2016). Team 1 students, particularly Evan, were able to bring in their real life knowledge to justify and enhance the data they used for their analysis, thus making it particularly interesting to other students.

**Team 2: Estimating the blades of grass on the football field.** Team 2 asked the question “How many pieces of grass are on the football field?” During their presentation,
Noah described how when they were discussing ideas with other teams he jokingly asked this question and then decided they should try and answer it. I love the randomness of the topic and how they genuinely tried to create a mathematical model to answer a question about something this young man encounters everyday. This showed great curiosity and application of mathematics to real life.

**Mathematical errors with Team 2’s model.** Team 2 described their process as counting the number of pieces of grass in a one inch by one inch square and then scaling it up, but unfortunately their model contained major errors. I was worried about this as I did not have a chance to really sit down with each team and talk to them about their model. I was simply spending the last few days discussing data with teams who needed help. In their presentation, they showed the slide in Figure 4.21 containing their initial computations and efforts to create a general model:

*Figure 4.21. Team 2’s incorrect model finding the number of blades of grass on the football field’s turf. (April 13, 2016)*
Students initially counted 50 blades of grass in a one inch by one inch section of the field, then multiplied it by 12 because they knew 12 inches were in a foot and again by three since there are three feet in one yard. Because they knew the dimensions of a football field in yards (including the endzones) to be 53 ⅓ yards by 120 yards, they took the number of blades of grass in a yard and multiplied it by those dimensions to get the blades of grass in the width and length of the field. During the presentation, Abigail concluded to find the total then they needed to take the length times the width (indicated by the $x \times y = z$ in the final line in Figure 4.20), but in their calculations they show to take the blades of grass in the width (95,994) and multiply by the width again. Both methods are incorrect, yet no students during the presentations noticed or commented on it.

**Lack of real life knowledge.** I hypothesize that since no other students analyzed this situation and had no prior experience or knowledge of the situation, the one to two minutes students spent presenting this slide was not enough time to adequately process and think through the method this team presented. I also think the lack of real life knowledge about this topic made it hard for students to catch this mistake. Students had no idea if 20 billion blades of grass was reasonable or not. I assume they thought it was high, but without the background knowledge needed to adequately assess the validity of their model, this mistake went unnoticed by everyone seemingly in class but me. However, I did not notice the mistake from the blade count. I noticed it within the calculations themselves.

**Mathematical corrections needed for Team 2’s model.** To help develop this model, students should have thought more geometrically. In their presentation, they
referred to the number 600 as the blades of grass in a foot, when really it is the blades of grass in a one inch by one foot strip of the field. When students took 1800*120=216,000, they were figuring out the blades of grass in a one inch section of the length of the football field. Finally, they should have multiplied the 216,000 blades of grass per 1 inch section of the length times the number of inches in the width, or 1920, as opposed to multiplying the number of blades of grass in the length times the blades of grass in the width. The correct calculations would yield a total grass count of 414,720,000 blades. Team 2 got about 20 billion for a grass count, nearly 50 times too much.

**Modeling abilities despite errors.** Despite the incorrect calculations, members of this team did show evidence of all phases of the modeling cycle. George did a nice job stating the assumptions for his model as ignoring the fact the field would have higher traffic areas because “you lose turf to football cleats tearing it up” (April 13, 2016). They also assumed you would use this model with turf that is the same brand, but the process could be replicated to estimate the pieces of carpet on the floor, capacity in a classroom, or how many George’s fit on a football field. In the end, I was impressed they had a specific question they wanted to answer and they worked to come up with a model to answer that question!

**Team 3: Holiday versus regular spending.** Team 3 presented their models on holiday and annual spending. They specifically asked three questions: how do holiday sales compare to annual spending, what percent of the annual sales are holiday sales, and has retail spending contributed to credit card debt? They developed linear models for holiday sales and annual retail sales:
Holiday Sales: $y = 18x + 425.371$

Retail Sales: $y = 72x + 2404.545$

and could clearly describe their variables:

$y$: total spending in billions

$x$: years from 2004.

**Issues with Team 3’s interpretation of models.** Despite the reasonable models students developed, the visual they used as their talking point referred to a scale not realistic for the situation (see Figure 4.22).

*Figure 4.22. Graph of holiday and retail sales linear regression models showing growth of spending. (April 14, 2016)*

The definition of the variables implies the graph shows spending growth out to 50,000 years past 2004, making this visual hard to interpret at this scale. In addition, when showing the visual of the models these students did not elaborate or make any comparisons between the models. There was no mention of the growth rates or how
these lines communicate the growth in spending. They simply pointed out the lines represented the equations they developed. Similar to the analysis done by Team 1, students here spent so much time gathering and analyzing data they did not properly analyze and interpret mathematical implications for the models they developed.

In addition, Team 3’s second goal was to determine what percent of annual spending comes from holiday spending, but they incorrectly calculated this percent for only one year (see Figure 4.23).

**Calculating the Percentage**

<table>
<thead>
<tr>
<th>General model:</th>
<th>Example: Year 2017</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h=18x+425.371$</td>
<td>$h=18(13)+425.371$</td>
</tr>
<tr>
<td>$a=72x+2404.545$</td>
<td>$a = 72(13)+2404.545$</td>
</tr>
<tr>
<td>$h/a=y$</td>
<td>$659.371/3340.545$</td>
</tr>
<tr>
<td>$y=$ percentage of holiday spending.</td>
<td>$y= 19.74%$ of annual spending is holiday sales</td>
</tr>
</tbody>
</table>

*Figure 4.23. Team 3’s incorrect assessment of 19.74% of annual spending coming from holiday sales. (April 14, 2016)*

The problem in doing this is this percent rate does not stay constant, which one can also see from the graph. In 2004 ($x = 0$), the percentage of holiday spending would take into account only $425.371 / 2404.545 = 0.1769$ or 17.69% of total annual spending. Because the growth rates of holiday spending and annual spending are different, this percentage will not stay constant. As time goes on, this percentage will approach the ratio of the growth rates: $18 / 72 = 0.25$ or 25%.
Applying real life knowledge. What this group did do well, however, was bring in real life knowledge to explain and enhance their results. Alicia had the idea to compare this spending to credit card debt hypothesizing that as spending increased so would credit card debt. They were surprised to find out credit card debt had actually gone down. They did not create a model for credit card debt, however, and only looked at the data and current debt figures. In addition, in contrast to work done in the first several projects, this team could correctly point out the following assumptions and limitations to their model and analysis during their presentation:

● Assumed the spending grew at a constant rate
● Assumed holiday spending was just on retail
● Assumed it was more accurate to include the years of the 2008-2009 recession
● Limited to not take in account (future) recessions
● Revised to a linear regression because exponential was only accurate for a short term. (April 14, 2016)

In the end this team also showed how they validated their model using known data to assess its reasonableness.

Presentation questions for Team 3. Again students in the audience asked questions about where the data came from, what classified as retail or holiday sales, or how far back they could find data. I asked the question “What other factors do you have to consider when looking at money trends over time?” Students correctly identified inflation would affect sales trends. I suggested an interesting sidebar to this particular analysis would be to also include a model tracking inflation rates and compare it to the
sales growth. This would allow students to see if sales are rising naturally with the
growth of inflation in which case sales are not actually increasing, or if the sales growth
is higher than what they should be considering inflation rates.

**Team 4: Projected iPhone sales.** Team 4 presented their model on the growth of
iPhone sales. They had researched and gathered data on iPhone sales worldwide
beginning in 2007 when the iPhone was launched. Eve stated:

> Our model will show the future growth rates for iPhones per year. You’ll be able
to plug in any year for x, year 0 being 2007, since that was the first year an iPhone
was released, to be able to get an outcome of the projected iPhone sales. (April
13, 2016)

They also clearly articulated their variables as $x$ being the years since 2007 and $y$ as the
projected units sold in millions. What I appreciated about this group was their desire to
generate and test out several types of regression models. Figure 4.24 contains a visual of
the different models these students created and analyzed.
**Linear, Quadratic or Logistic?**

**Linear:** $y = 28.94x - 24.44$

**Quadratic:** $y = 2.34x^2 + 10.24x - 2.62$

**Logistic:** $y = \frac{265.5}{1 + 34.18e^{-0.636x}}$

X: Year (Year 0 being 2007)  
Y: Projected iPhone sales in millions

*Figure 4.24.* Team 4’s graph showing three models they created and tested for validity. (April 13, 2016)

Team 4 decided to go with the quadratic model as it fit best when they tested its validity versus known data. However, they recognized using a quadratic was a limitation since, as Jason pointed out in real life, “it would level off eventually but not as sharp as the logistic would” (April 13, 2016).

*Applying real life knowledge.* Like other groups, students in Team 4 could identify limitations to their model such as not being able to predict future technology or its desirableness to customers. Team 4 then went on to use their quadratic model to predict iPhone sales in 2025 and made the comment that as the population increases in general iPhone sales will also increase due to a higher consumer base.

*Presentation questions for Team 4.* Questions asked after the presentation again focused on clarifying what their data represented or thoughts on comparing iPhones to
Android operating systems. Like other presentations, students brought in their own real life knowledge to ask and answer questions about the situation. Eve commented how they wanted to compare this model to Android sales but they ran out of time to make an analysis. Jacob added “We did a graph and Android was about the same. They increase at the same rate. But when the new Galaxies came out Android was a lot higher than the iPhones I am pretty sure. It was gaining popularity faster” (April 13, 2016). I appreciated the insight and connection to real life knowledge Jacob was able to bring in to elaborate on Eve’s comment. Unfortunately, a lack of time seemed to cause Team 4 from taking their analysis further.

**Team 5: Modeling immigration growth.** Students in Team 5 looked at annual immigration growth data for five counties in our state. During their presentation, students were able to articulate their data began in 2010 so they used that as their base year (year zero) for the input variable. They created five linear regression models for these five counties and graphed these separately onto five scatterplots of the data. Ryan described how they had a hard time putting all five on the same graph and window because Metroville County’s population was so much higher than the rest. He also correctly interpreted data they found for Central County as there was one year where more immigrants left the county than came in showing a negative growth. In addition, Luis commented they did not know where the immigrants were coming from specifically in the data they had collected. Like other teams, these students addressed limitations and assumptions to their model including the fact there would not actually be a constant growth rate and having only five years worth of data limited their analysis.
Also like students in Team 1 and 3, students in Team 5 did not elaborate on the mathematical implications of their model. Specifically, they wanted to know about the growth of immigrants in these counties but they never actually described how the growth rates compared. They discussed all other aspects to their data collection, analysis, development, assumptions, limitations, and outside research they did to validate the growths, but they did little discussion of the models themselves. Luis elaborated on why they did not go further into their models in his individual work. He stated:

The data we acquired was only from 5 years which not only limited our model, but also the data only represents the change in population from each year instead of the total immigrant population from each year. This implies that our model is only able to predict the change of that year and not the total population. (April 14, 2016)

Luis viewed this as a limitation to their models, but I saw it as strong evidence that students developed skills pertaining to data collection and analysis. Originally this team wanted to be able to predict future populations, and they realized during their analysis the data they found would not allow them to do that. It would only show growth and not total population. Rather than explaining these insightful connections, this team simply seemed to brush over it during their presentation.

Presentation questions for Team 5. As with other presentations, most questions posed to Team 5 after their analysis focused on real life knowledge of the situation and the data collected. Evan asked if they had taken into account illegal immigrants. Ryan responded with “There is no way to calculate that (census data), because it’s illegal”
Dylan added “we could estimate the numbers but that is so hard to figure out. They aren’t going to sign up for the census if they are illegal. We don’t know about them” (April 13, 2016).

Jason asked a question specific to their mathematical analysis and model development. As Team 5 presented their models and graphs, Metroville County did not appear linear. Jason asked “Why did you choose linear for Metroville County?” Dylan comments “It looks like an exponential” and brought back up the data to show the class (April 13, 2016). He points out:

It is a pretty constant growth until you get to 2015 and so like with the 5 years of data, it was more useful to use a linear graph instead of an exponential because we only had 5 years of data. So it was hard to determine which to use. Platte City County and Metroville County were the two that looked exponential ... but it was easier with the five years of data to just use linear. (April 13, 2016)

Jason’s question followed by Dylan’s response pointing out limitations to their analysis demonstrates the growth students have made throughout the unit as they encountered authentic situations and learned how to apply the mathematics they have learned. Prior to this unit, students had little experience in particular collecting and analyzing data. Here, students were engaged in productive discourse analyzing each other’s work and justifying the decisions they made.

Team 6: Animal shelter trends. In their presentation, Brooke from Team 6 stated they acquired data from our local animal shelter in order to understand something
about “the animals they were taking in versus the amount of animals that were going out” (April 13, 2016). Brooke went on to elaborate further:

We wanted to know how big they were going to get because they have this new plot of land they are still deciding what to do with. We were going to see if they needed more space, if they needed more kennels on the new plot of land. (April 13, 2016)

Using data sent to these students by the director of the animal shelter, Brooke, Megan, and Monica created a graph overlaying the various information they received. In Figure 4.25, the top purple plots and curve represents animals coming into the shelter every year since 2010, the blue markings indicate animals leaving the shelter, the green plots represent animals being adopted, and the black plots represent the number of animals getting euthanized:

![Graph showing number of animals arriving, leaving, adopted, and euthanized each year beginning in 2010.](image)

*Figure 4.25. Team 6’s graph showing the number of animals arriving, leaving, adopted, and euthanized each year beginning in 2010. (April 13, 2016)*
**Inconsistencies in data.** Because they wanted to understand how animals coming in and out of the animal shelter compared, students in Team 6 tried to develop appropriate models for data representing these totals (indicated by the purple and blue curves in Figure 4.25). However, because of inconsistencies in the data from 2010 and 2011 they could not explain, this team had difficulties fitting any sort of curve to this data. As a result, Brooke explained how they did not take into account the 2010 data because including it did not allow them to find appropriate trends. They settled on the following quadratic models after removing the 2010 data because as Brooke described, “That was the one that would fit the best” (April 13, 2016):

\[ \text{Animals In} : \ y = 51.21x^2 - 386.59x + 3000.2 \]

\[ \text{Animals Out} : \ y = 42.71x^2 - 359.49x + 2926.2 \]

Brooke also added “If we had more data we would be able to predict if it would keep going up, but we don’t” (April 13, 2016). During the presentation these students forgot to describe what the variables in these models represented. However, their peers asked about these variables during the question and answer portion of the presentation. Megan communicated that \( x \) was the years since 2010 and \( y \) was the number of animals in or the number of animals going out depending on the model used.

**Issues with interpretation of models.** Megan was able to communicate assumptions and limitations to their model such as “the growth of the Humane Society will keep increasing fairly rapidly because we chose to do a parabola” and “we don’t know what’s going on during the spikes and drops so we can’t take that into account either” (April 13, 2016). Similar to other presentations, Team 6 did not seem to use
their models to further analyze or better understand if additional space would be needed for animals and when that space might be needed. The only interpretation of this model came from Megan as she commented the coefficients in each model were similar to each other which “is a good thing, most of the animals that are going in are going out” (April 13, 2016). Team 6 did not come up with specific numbers or predictions from their model about additional animals or space needed. They simply recommended based on the growth they had seen in the data, the Humane Society should use the plot of land to build additional kennel space.

**FBLA group: Modeling internet users.** Finally, William, Matt, and Elijah created their own group due to a Future Business Leaders of America convention pulling them from class for several days. All three students have interests in coding and technology, so they focused their analysis on predicting when the entire U.S. population will be connected to the internet and thus become an internet user. They titled their presentation IoE (Internet of Everything), and they captured their audience’s attention by titling it this way. Matt said “It’s the idea that someday our clothes will be connected to the internet, the lanyards we wear, the shoes we wear, anything we buy will somehow be connected to the internet” (April 14, 2016).

Unlike most other presentation, Elijah, Matt, and William had a clear question they were trying to answer. Elijah explained how they wanted to know when “the total users equal the total population, that will mean that everyone has access to the internet” (April 14, 2016). Their team developed two linear models, one modeling the total U.S. population \(z\) and one calculating the total number of internet users \(y\), \(x\) years after
2000:

U.S. Population: \( z = 2,597,007.674x + 282,895,741 \)

Internet Users: \( y = 9,144,800.735x + 121,869,116 \).

They found the best fit linear regression lines for each group of people beginning in 2000, but interestingly, William explained how they revised and shifted the linear model for the number of users down so the y-intercept matched the base year in 2000 (or the number of users in 2000). In addition, Matt provided a graph showing both models, their corresponding data points, and the intersection of these models (see Figure 4.26).

```
System.out.println("Graph/Data");
```

Figure 4.26. Data set used and graphs of linear models developed by William, Matt, and Elijah to predict when the entire U.S. population will have internet access. (April 14, 2016)

Matt pointed out they intersected around the year 2024 matching research they found predicting the entire U.S. population would be connected to the internet between the
years 2020-2030, further providing validity to their model.

Applying real life knowledge. During their presentation, William stressed a limitation to their model: “One of the limitations to our models is when they do intersect at about 2024 we are basically saying every human being alive in the U.S., even babies, are a user of the internet” (April 14, 2016). Students seemed to appreciate the analysis done by this group of students and the specificity of the question they wanted to answer. Most questions focused on the theoretical principles behinds IoE as students debated the possibilities and ethics of everyday objects being connected to the internet.

Reflections on Project 5

Overall in Project 5, students best exhibited skills related to applying everyday knowledge, mathematical literacy, data analysis and collection, flexible and creative thinking, and perseverance. This makes sense considering the amount of time students spent developing and researching the question they wanted to explore for their analysis. Skills related to communication and productive discourse was most improved in this last project compared to the previous four. Student-to-student questioning and discourse was evident throughout all presentations. Most students focused questions on where the data originated from and real life factors the students did or did not consider. Teams presenting seemed to have no issues answering these questions, showing understanding of the research they had done and a reflection of the amount of time devoted to understanding the background knowledge and data necessary for successful model development of their situation. These improvements in team-to-team discourse are analyzed further to show the progression and significance of communication throughout
this modeling unit in Chapter 5.

Students were not as consistent or developed in their use of quantitative literacy, reflection, and reasoning, however. I have documented throughout this process how difficult skills related to quantitative literacy such as interpreting mathematics in terms of the situation and assessing reasonableness is for students, but I also think students spent so much time researching to expand their real life knowledge of the situation they did not leave enough time to properly analyze and reflect on their results. I also did not have many opportunities to sit and hear about each analysis before presentations as I too was helping students track down the background information they needed to develop a model.

In the end, few groups spent time interpreting the models themselves back in terms of the situation. For example, students only discussed and used the rate of change of linear models developed to explain features of the situation studied when students in the audience specifically asked about it. Students explained most of their interpretation and connections to the situation from the graphs and visuals they had prepared of their model. Few teams actually used their models to answer specific questions or make predictions. It seemed most students just developed a model with the data they collected but did not use that model to further enhance their understanding of the situation. As with all other projects, these issues related to quantitative literacy are explored further in Chapter 6. However, I was impressed with the depth and consistency at which students addressed assumptions and limitations to their model considering students had no experience thinking about these prior to this modeling unit. This shows great growth in their relearning of mathematics as is discussed more in Chapter 7.
Student Reflections on a Month of New Learning

After the conclusion of Project 5, I wanted to revisit the same questions we began the unit: What is mathematics? and What type of questions do we answer using mathematics? I was curious if the authentic tasks students engaged in over the course of this unit changed their perceptions of what it means to do mathematics. Overall, students seemed to focus less on procedural and traditional mathematical notation and operations and more on general mathematical reasoning and problem solving skills. For example, at the beginning of the unit George described mathematics as “numbers and letters being used in calculations” (March 14, 2016). After the unit he viewed mathematics as “a system to predict, solve, or prove something” (April 14, 2016). Similarly, prior to his modeling experience, Nicholas viewed mathematics as “the use of math calculations to help problem solve” (March 14, 2016) and later modified this definition to “the use of mathematical reasoning to solve a question” (April 14, 2016). Abigail added “Mathematics is being able to look at a problem and be able to use your prior knowledge to problem solve” (April 14, 2016).

Prior to this unit, most students focused on operations and numbers. It seems the modeling unit did change some perceptions by expanding mathematics from focusing on specific calculations to more general thinking, reasoning, and problem solving skills. Students also expanded their views on what types of questions could be answered using mathematics. Noah commented how mathematics can be used on “Virtually anything. You can create a question out of about any scenario” (April 14, 2016). Jason agreed stating, “Any question can be answered using math!” (April 14, 2016).
Similar themes emerged in reflection questions I asked students to fill out at the conclusion of the modeling unit (see Appendix A). Alicia wrote, “It was interesting to actually analyze real data and find answers to apply real life situations. It was way more interesting for me rather than solving made up problems and situations” (April 14, 2016). Similarly, Jacob commented, “Life is full of issues and problems and you can solve them using math” (April 14, 2016). After hearing and reading student perceptions of their own learning throughout the modeling unit, I was pleased to see all students commenting on how valuable these experiences were. Every student indicated they had never experienced modeling tasks like those in this unit in prior courses, and they all felt it was important these tasks be utilized more throughout high school. Chris justified this as “You get a real taste of problems. Going into a problem without certain instruction to solve gives a good experience” (April 14, 2016). Even Matt agreed saying “It’s important we learn to utilize our skills outside of a textbook” (April 14, 2016). It was a relief knowing the time, energy, and work I had put into providing this opportunity for students was well-received and found to be valuable by my students.

Conclusion

Over the course of nearly five weeks, my students and I developed a new appreciation for what it means to be teachers and learners of mathematics within a student-centered environment while engaging in authentic tasks. Each student took a separate and unique journey through this modeling unit as they developed modeling abilities, but common themes and experiences emerged that shaped new knowledge of self, each other, and mathematics. However, perhaps I took on the biggest role as a
learner as I navigated a foreign practice and provided experiences contradictory to my own as a mathematics learner. In Part Three of this study, three major themes will be clarified and analyzed more purposefully in order to understand the new learnings and conflicts which emerged: students had to relearn how to learn, I had to relearn how to teach, and we both learned something new about mathematics as it occurs within authentic situations.
PART THREE

ANALYSIS OF TEACHING AND LEARNING DURING

A MONTH OF MODELING

In Part Two, I described the timeline of events including the foreshadowing of three major themes: students relearning how to learn, relearning to teach, and relearning mathematics within authentic tasks. Part Three provides additional data and further analyzes these major themes in order to better understand how and why these themes emerged. I title these themes as “relearning,” however, this relearning includes insight into new learning for both students and teacher. In Chapter 5, I discuss ways in which students had to relearn how to learn during a month of modeling including: recognizing their own ability to create learning opportunities, transitioning from right and wrong to multiple solutions, and finding perseverance from authentic mathematics. Chapter 6 analyzes how I as the teacher had to relearn how to teach both in my preparation to teach and teaching practices. Chapter 7 provides insight into new mathematical knowledge developed as both my students and I relearned mathematics within authentic situations. Finally, Chapter 8 connects these themes to the overall development of modeling abilities and discusses implications of this research for the larger educational and research community.
CHAPTER 5

RELEARNING HOW TO LEARN

I came to realize while researching and developing a unit on modeling just how unique of an experience this would be for my students. As indicated in their unit reflections, students had infrequently participated in tasks that required as much mathematical application, interpretation, reasoning, and communication as all five projects required of them. Their prior experiences tended to be, as Chris described in his interview, a teacher “teaches it, shows it on the board, and if you have questions you can ask” (April 27, 2016). Going into this unit, the mathematical operations required in each task was the least of my worries. I knew my students could ‘do the math.’ I was more concerned with how they would respond to the authenticity of each situation, if they had the perseverance to complete the entire modeling process, and how group dynamics would affect potential learning opportunities. To put it simply, in this unit I asked students to function in an environment which was foreign to them while completing tasks they had never encountered before; I was worried how they might respond.

Consequently, in order to find success completing authentic modeling tasks it seems natural students had to relearn how to learn in their mathematics classroom. In this chapter I discuss three of these new learnings for students: recognizing their own ability to create learning opportunities, transitioning from right and wrong to multiple solutions, and finding perseverance from authentic mathematics. I conclude this chapter with a discussion of a student, Matt, who engaged in conflict with each of these new learnings as he struggled to meet these new expectations.
Students Creating Opportunities to Learn

Students in my classroom typically work in groups of three to four students. In these groups they would ask each other questions about their homework, engage in discussion questions designed to activate prior knowledge or develop a particular skill, and participate in dialogue focused on conceptual understanding. Yet, the group dynamic needed to change for this modeling unit to be successful. Students would need to rely on each other for several days at a time for successful completion of the modeling task. Specifically, they would need to generate, discuss, narrow down, validate, and communicate their own strategies and defend their conclusions to those not familiar with their analysis. Opportunities for learning could not flow through me as the teacher; they needed to be created by students and valued by peers.

Eliminating the Need for Teacher Validation

As the modeling unit began, it became abundantly clear a shift in learning would need to come from my students and their constant need for my validation. Over the course of the school year, I have given students a solution guide for their mathematics textbook and access to a website I created with videos and additional help. As a result, students could always check if they were doing a math problem correctly. Knowing how post-secondary education would most likely look for my students, the goal of these resources was to teach students self-assessment skills. They would know if a mistake had occurred, try to pinpoint the source of that error, or focus questions on more particular aspects of their misconceptions or errors. In the process of trying to help students, I may have inadvertently created an obstacle for the modeling process.
Blum and Ferri (2009) found in their study of student responses to modeling tasks that validating results was especially problematic for students. Most students did not check whether their results were reasonable based on the situation, and the students primarily looked to the teacher to determine whether their work was right or wrong. Similarly, as students began work in the modeling unit for my precalculus course, they consistently needed validation their ideas or strategies would work to accomplish the task’s goals. Students needed to learn how to apply their own real life and mathematical background knowledge to verify their work and results rather than gaining this validation through teacher approval.

**Project 1.** Towards the conclusion of Project 1: The High Definition Trade-Off Dilemma (see pages 121-139), I noticed several groups sitting around quietly and staring aimlessly around the room as though they were waiting for my assistance. I commented in my field notes how they seemed to want validation they were correct so they could move on to other aspects of the task. I wrote in my research journal: “More individuals are sitting around waiting for me to verify models and needing to know they are doing things ‘correctly.’ How can I help them get in the habit of validating their own conclusions?” (March 17, 2016). The next day I decided to begin the period by pointing this out to the class. I told students if validating conclusions is an essential step to the modeling cycle, they need to brainstorm what they could do as a team to verify their models without my validation. Students correctly suggested they could plug in specific values or go back to the original task and use Heather’s 300 minutes of recording time on a 64 gigabyte card to make sure the model worked to communicate recording times in
each resolution. At the time, I was pleased to hear students generate appropriate ideas on ‘what could have been done,’ but I wanted to see these observations translate to unprompted actions as students worked through the modeling cycle.

**Project 2.** After reflecting on student work in Project 1: The High-Definition Tradeoff Dilemma, I thought students did not seem to address how they knew their results were reasonable in terms of the situation. Reasonableness is an implicit goal of math learning in general and modeling in particular. Within the modeling cycle, validating conclusions is a crucial step, and I wanted students to be able to justify their results by applying both mathematical and real life knowledge. Because students did not seem to establish this in Project 1, I decided it might be useful to actually start with the idea of reasonableness in Project 2: Calculating the Width of Memorial Stadium (see pages 139-152). Therefore, at the beginning of class on Tuesday, March 22nd I asked students: *What is a reasonable guess for the width* (of Memorial Stadium) and *why?* Students were to think about it and share their ideas with their team before they even started on an in-depth mathematical analysis of the situation. By posing this question initially, it forced students to begin applying everyday knowledge immediately to analyze the modeling task. Students brought in ideas about knowing the width of a football field, adding in end zones, estimating additional space behind the goalposts, estimating how many rows are on each side, guessing how much space is between each row, and adding all that information together to make a reasonable conjecture.

**Example of applying real life knowledge.** By applying real life knowledge to Project 2, most students were anticipating somewhere in the range from 700-900 feet for
the width of Memorial Stadium with one team estimating it most likely was not more than 1200 feet. The following conversation occurred at Team 2 and summarizes the ideas most students were generating. In this moment, George grabs a markerboard and marker and begins to draw:

*George:* So if you think about it, the center is 100 yards.

*Abigail looks at him with a confused expression.*

*George:* A football field is 100 yards.

*Abigail:* Well it will be a little bigger than that. *(referring to the end zones)*

*George:* A football field is 120 yards.

*Samantha:* That’s the whole entire thing?

*George:* Plus you have to add the bleachers.

*Abigail:* So it’s more than 120.

*George:* So it’s going to be *(trails off)*

*Abigail:* Let’s try to draw it a little bit.

*George* starts drawing on the markerboard and Samantha draws in her notebook. *(March 22, 2016)*

The four students in this team then get into a small debate about what direction of the stadium they are supposed to find. Noah, Abigail, and Samantha identify the width as the shorter distance across the stadium running in an east/west direction. However, George seems convinced I want the longer distance. I come over and remind them the front entrance to the stadium is in the middle of the longer side. As a result, they should reference the ‘width’ as the longer distance running in a north/south direction. I also
inform them I designed the project this way as I thought it would be easier for students to figure out reasonable results if they determined the longer distance. They correctly communicate this is because they know how long a football field is supposed to be.

George then begins drawing out the stadium on his markerboard in the middle of the team. As George draws the field they restate what they know about the field in yards and discuss why they need to convert it to feet. At this point they settle on knowing it must be at least 360 feet:

George: So now we have to estimate the bleachers.

Noah: I don’t think it will matter because there’s stuff under the bleachers.

Noah gestures with his hands how the rows slope up.

Noah: We just need the there to there distance.

Noah gestures a straight distance with his hands.

Noah: I think it would be

Abigail: How many rows of bleachers? (cutting Noah off)

George: There’s probably 40 yards. (answering Noah’s question)

Noah: Yeah 40 yards probably.

George: There’s got to be more than that.

Abigail: Well how many rows of bleachers do you think there are?

George: That’s not going to help us, the rows are going up.

George gestures with his hand a slant going uphill for how the rows look in the stadium.

Abigail: But if it’s going up it still
Abigail gestures with her hands a distance between seats going uphill.

Noah: They’ll stick stack on each other.

There is some inaudible ‘thinking aloud’ by Noah and George as they try to process.

Noah: There’s probably about 60 rows. You could figure out what each row is.

DB: So how many feet do you think it is?

George: 3?

Noah: Yeah 3 probably.

DB: Maybe. You’re kind of crammed in there like sardines. Have you ever sat in north or south end zones? I would never buy season tickets there! (March 22, 2016)

Based on these estimates they calculated 60 rows behind each end zone with 3 feet between each row to be an additional 360 feet of bleacher space. They then added 360 feet to their 360 foot estimate from the field, giving them a total estimate of 720 feet. They all agreed that seemed reasonable and ended up being only 80 feet under the actual width of 800 feet! Having students assess the reasonableness of the situation first allowed students to validate their own work as they continued through the project.

As students were calculating and experimenting with their ideas, they caught several of their own errors because the results did not match previous conversations about appropriate results. For example, when Team 2 was working on developing their model, they had made a minor calculation error resulting in a width that was lower than their initial guess of 720 feet. Noah explained “that seems a little short” as they were trying to
find the source of their error (March 23, 2016). As Team 2 went back through their work, they realized the mistake was simply a subtraction error, but it changed the width of the stadium by nearly 200 feet. Without assessing reasonableness first, however, students may not have realized they had made this error. In the end, their own knowledge of the situation, particularly about football and how stadiums are set up, allowed them to assess whether or not their model accomplished the task’s goal.

**Project 3.** In Project 2, I provided the opportunity for students to shift the source of validation. By Project 3: Enrollment at River Junction Senior High (see pages 152-164) students were finding creative ways to do this on their own. Analyzing enrollment data to predict River Junction High School’s population provided an abundance of opportunities for students to validate their own strategies and models.

**Example of new validation techniques employed.** As I checked in with teams in the middle of Project 3, I asked members of Team 5 how they knew they their model worked. The responded:

*Dylan*: Yeah well first we found the average difference in these graduating classes and K-3, like how much dropped from elementary to high school, like dropped out or moved schools or whatever. We found the average which was 449.

*DB*: What does that number mean?

*At this point all three boys are trying to communicate the same idea at the same time and talk over top one another.*

*Luis*: These students *(pointing to his data table at the group of K-3 students)*
dropped out or lost 449 by when they go to high school.

*DB*: All of those put together?

*Luis*: K through 3

*Dylan and Ryan*: Yeah K through 3 total.

*DB*: So you’re not saying it decreased that every class?

*Ryan*: No it’s the four classes together to get the whole high school.

*DB*: So you’re saying on average you lose 449 from these four years beginning in kindergarten to when they get to high school?

*All three*: Yeah

*DB*: That’s interesting, so did you take that into account?

*Dylan*: So we found the K-3 total in 2015-2016 because that would be the classes in 9 years, and that was 3158. Then we subtracted the 2748 we got from our model and we got 410 for the difference which is near all these (pointing to the work about losing students that they had been explaining).

That seems pretty reasonable. (March 29, 2016)

Students had devised their own strategy to validate the model they had developed by finding other patterns within the data set. Team 5’s Dylan, Ryan, and Luis noticed they could track a group of students through the data set as they went from grade to grade. As described in the proceeding dialogue, they totaled each group of kindergarten through third grade students and their corresponding high school enrollment in order to compare the K-3 enrollment to the number of students these grades ended up enrolling in high school. Specifically, these three boys determined there were 2560 students in
kindergarten, first, second, and third grade combined in the 2000-2001 school year. By the time this group got to high school and the third graders were now twelfth graders, their enrollment total dropped to 2114 students, or a loss of 446 students over those years. They subsequently did this for each group of students which could be tracked as shown in Figure 5.1:

<table>
<thead>
<tr>
<th>Year</th>
<th>K-3 Total</th>
<th>Corresponding High School Years</th>
<th>9-12 Total</th>
<th>Loss of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000-01</td>
<td>2560</td>
<td>2009-10</td>
<td>2114</td>
<td>446</td>
</tr>
<tr>
<td>2001-02</td>
<td>2581</td>
<td>2010-11</td>
<td>2135</td>
<td>446</td>
</tr>
<tr>
<td>2002-03</td>
<td>2606</td>
<td>2011-12</td>
<td>2086</td>
<td>520</td>
</tr>
<tr>
<td>2003-04</td>
<td>2687</td>
<td>2012-13</td>
<td>2225</td>
<td>462</td>
</tr>
<tr>
<td>2004-05</td>
<td>2719</td>
<td>2013-14</td>
<td>2278</td>
<td>441</td>
</tr>
<tr>
<td>2005-06</td>
<td>2770</td>
<td>2014-15</td>
<td>2365</td>
<td>405</td>
</tr>
<tr>
<td>2006-07</td>
<td>2816</td>
<td>2015-16</td>
<td>2393</td>
<td>423</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Average Loss: 449</td>
</tr>
<tr>
<td>2015-16</td>
<td>3158</td>
<td>2023-24</td>
<td>Predicted: 2748</td>
<td>410</td>
</tr>
</tbody>
</table>

*Figure 5.1:* Summary of the work done by members of Team 5 to validate their model and predictions.
They had calculated on average, a group of students will lose 449 students as that group moves from K-3 to 9-12. Dylan explained how they found the total K-3 population in the 2015-2016 school year to be 3158 and how their model predicted a population of 2748 once this group of students were all in high school during the 2023-2024 school year. This team felt the 410 students lost by this prediction was close enough to the average loss of 449 for the model to be valid.

Reflecting on the shift in validation techniques. At the time, I was impressed with how students’ needs for validation came from each other and their own analysis. In the specific case Team 5 highlights from Project 3: Enrollment at River Junction Senior High, opportunities for learning in this project were born out of their own ideas. They no longer needed my validation or approval; they had generated ways in which to provide this validation for themselves. Watson (2003) suggests mathematical tasks should provide opportunities for students to apply thinking skills students “naturally possess to construct mathematical meaning” and providing these opportunities allow the learner to alter and enhance a personal understanding of the concept (p. 38). Through my encouragement, students created their own opportunities for learning by applying skills and knowledge they already had in order to validate results of their analysis. And by doing this, they simultaneously gained a better understanding of the mathematics and task itself.

As I reflect on the conversation with Dylan, Luis and Ryan now, not only did I see a shift in mathematical outcomes as a result of this new learning, but my questioning over the course of this conversation also changed further exemplifying this shift in focus.
Throughout this dialogue, the intent of my questions was not focused on trying to gauge their own understanding or thinking about next steps. I asked questions to gauge my own understanding of the thinking these students were trying to communicate. They were clarifying questions for myself, not for students.

As students continued through the modeling tasks in the unit, skills specifically related to assessing reasonableness, validating conclusions, and identifying limitations would continue to develop as indicated in Chapter 4. However, this learning about modeling would not have occurred without a shift away from teacher validation and towards student generated ideas. Students had to relearn, or learn perhaps for the first time, how to reason through, reflect on, and justify their own work. They could no longer seek out my approval; they had to learn how to generate their own methods to verify whether or not they had accomplished the goal of the task successfully.

**Student-to-Student Discourse**

Throughout the modeling unit, students continued to create additional opportunities for learning as they developed skills enabling them to communicate effectively with each other. Wagner, Herbel-Eisenmann, and Choppin (2011) state that opportunities to learn for every student demands “a diligent and sustained focus on equity and discourse” (p. 1). Similarly, in group work, student learning has been found to depend heavily on how students interact with each other (Esmonde, 2011). As I indicated throughout Chapter 4, the student exchange of strategies, ideas, interpretations, and results had become an essential part of the modeling process as students communicated “particular perspectives or solutions that contribute to the collective negotiation of
knowledge” (Wagner, Herbel-Eisenmann, & Choppin, 2011, p. 9). However, learning to communicate and participate in discourse was not without conflict and, in fact, often was the biggest source of frustration for most students. Students were comfortable in conversations originating from me as the teacher, and not necessarily ones focused entirely on student-to-student interactions. Yet, both productive and more ‘unproductive’ student-to-student discourse provided new knowledge of each task, modeling, and student perceptions of mathematics. Throughout the unit, student discourse opened learning opportunities as it (1) emerged within teams and/or (2) transpired across teams.

**Teammates as source of learning and conflict.** Chapter 4 contains several examples of dialogue that illustrate the learning opportunities students created as they processed and completed each modeling task in their teams. I was convinced going into this unit learning would be maximized when students could work together, listen to each other, and find value in each other’s input. Students were already used to sitting in a group, and I had done team-building activities throughout the school year to help them become comfortable working with one another, but this complete reliance on each other to accomplish a common goal was not the type of learning environment they had experienced. As a result, in my planning I carefully thought out and selected students I thought could work well together but would still challenge each other’s ideas. For some students, like those in Team 2 and 5, learning how to communicate in this manner seemed to happen naturally. For others, especially in Teams 1 and 6, students encountered conflict and frustrations as they relearned how to learn in this environment.
Understanding the nature of positive student discourse. In an effort to try to narrow down the nature of conflicts which originated as a result of this new learning, I interviewed students from each of Teams 1, 2, 5, and 6 at the conclusion of the unit. Luis from Team 5 and Samantha from Team 2 both alluded to positive learning opportunities when their teammates had different ideas than their own, but each person found validation in offering up their ideas. They listened to their teammates, but also felt their own input was heard and valued. The cross-student communication was two-way. Luis put it this way:

We all gave us our input. If one of us wasn’t sure at first how to approach it, we are all good at different things so we all could help... When we weren’t sure what to do, I’d give my thinking and ideas on what we should do. (Interview, April 28, 2016)

I also noticed these kinds of interactions with students in Luis’s team. Teammates showed that they valued each other’s input and ideas and gave each person opportunities to express their own ideas; their ideas were validated by other members of the team. For example, in the dialogue transcribed on pages 205-206 of this chapter, Team 5 described their unique validation techniques for verifying the model they had developed for Project 3: Enrollment at River Junction Senior High. In this dialogue, all three students participated equally. I did not have to direct questions at any particular person in an attempt to involve them in the conversation. Dylan, Ryan, and Luis took turns responding to my questions and listened to each other’s responses. All three were able to respond to my questions, showing a mutual understanding of the work they had done.
Similarly, Samantha provided rationale for the learning opportunities her team generated in this new student-centered environment. She commented:

They had other ideas than I had so if I didn’t know what to do then someone else always had an idea about something different. And also there was a lot of different ideas so, that was nice...I usually don’t want to think out of the box and have my own reasons for things. Even talking to my group about different things helped me be more creative and be like ‘we should try this.’ (May 17, 2016)

Throughout the modeling unit, Team 2 impressed me with how well they navigated each project. This team-centered approach also often resulted in the most thorough presentations and explanations. This team did not always agree, but they found value in hearing different ideas than their own, and recognized how these different ideas would ultimately help them accomplish their goal. Unfortunately, not all teams found value in alternate approaches and minor disagreements. Sometimes student discourse, or a lack of it, led to significant conflict affecting the development of modeling abilities.

**Understanding the sources of conflict within student discourse.** I also describe in Chapter 4 several conflicts that emerged in Teams 1 and 6 as they struggled to adjust to this new learning environment. These conflicts seemed to escalate during Project 3: Enrollment at River Junction Senior High. I wrote on March 30th:

To be successful in these modeling task you need good cooperation, teamwork, and contributions from team members. Most teams are doing a great job of this, but I have noticed a few students taking the lead and ‘overpowering’ their teammates. William and Nicholas do this at Team 1. Evan and Chris do this a bit
at Team 6. But what can I do to help Brooke, Matt, and the others find success and feel validated in their own ideas? How do I empower them and change their mindsets at this point?

At the time, I was worried the self-esteem of Team 1’s Brooke and Matt and Team 6’s Monica and Megan was being negatively affected by the lack of productive discourse occurring within each team. I sensed Brooke, Matt, Monica, and Megan wanted to be involved in the modeling process, but did not know how to include themselves in the work their teammates were doing. I wanted to encourage them to share their ideas with their teammates.

After interviewing Chris, Matt, and Brooke, it seems now my asking, “How do I empower them?” in my reflection journal was not the right question. Chris described how he and Evan worked off of their own ideas without including Megan and Monica:

It was just me and Evan doing everything else and just there, you know. Even when they tried to input we weren’t kind of letting them, and even when they did we were already like two steps ahead of them and they were behind us. So I felt like they couldn’t match us and what we were thinking...He (Evan) would tell me what he was thinking and I’d see what he was thinking and then I’d go off with him. And they (Megan and Monica) were like, they didn’t know what we were doing half of the time. They’re like what are you guys doing. And we would just ignore them because we couldn’t stop our thinking and we just had to keep going until we finished our thoughts. (April 27, 2016)
Chris here describes how he and Evan understand each other quickly and did not take the time to explain their own ideas or listen to the ideas of the rest of the team. In his interview, Chris bluntly said, “we would just ignore them.” To Chris and Evan, there was no value in hearing other perspectives and Megan and Monica did not feel encouraged to share their ideas and remained silent. Reflecting on this now, students did not necessarily desire encouragement from me. They needed it from each other. During each modeling task, rather than ask, “How do I empower them?” I should have asked, “How do I help students empower each other?”

Similarly, Brooke described in her interview after the modeling unit had concluded how it felt to be on the receiving end of having ideas ignored by her peers Nicholas and William. She discussed how Nicholas would be “rude” when she or Matt were confused and would ask about the work Nicholas and William were doing. Brooke felt her and Matt tried to encourage communication, but Nicholas and William would just “go off on their own again” and “they really wouldn’t listen to our ideas” (April 26, 2016). Brooke described how frustrating it was to have ideas “shot down immediately” with visible disappointment on her face (April 26, 2016).

As I listened to Brooke describe how her teammates inadvertently diminished her feelings of self-worth, I was disappointed in myself for allowing the conflict to continue for as long as I did. I had hoped with more experience, Team 1’s Nicholas, William, Brooke, and Matt would learn to listen and value each other’s input. However, this team never figured out how to productively work together and I eventually moved Brooke and Matt to another team. In reflecting on this conflict, Brooke stated:
I guess the main source of conflict was not being able to communicate well enough... It was really hard getting ignored by them and trying to see what they were doing. You have no idea what they’re doing because they are moving so far ahead and they won’t explain anything to you. (April 26, 2016)

Unlike Samantha and Luis, Brooke did not feel her ideas and input were heard and valued by her teammates. As a result, she struggled learning how to model and develop modeling abilities. She went on to describe how these interactions impacted her learning:

I had no idea how to even start with the modeling process, well I had an idea, but it was a lot harder for me to do anything with the modeling because I had no idea what they were doing. So when we had the presentation it was like okay, I get like one slide, and I get to explain that really well and I don’t care about the rest. The rest is up to them. I don’t have to explain the rest. I don’t need to know what’s going on. (April 26, 2016)

This math struggle did not arise from apathy or a lack of mathematical knowledge, but from a student’s inability to participate in productive student-to-student discourse. When doing this type of modeling, knowing how to participate in two-way conversations is crucial. In the case of Team 1 and Team 6, rather than create opportunities for learning, they limited the new knowledge each student could have gained by listening to and valuing multiple perspectives.

Adjustments to address conflicts. Despite my efforts trying to help students in Teams 1 and 6 learn how to learn in this new environment, after Project 3: Enrollment at River Junction Senior High (see pages 152-164), I thought it would be best to switch
members of Team 1 and 6 to see if placing Brooke, Matt, Megan, and Monica at the same team would create a more inclusive environment. I described in Chapter 4 how the interest in modeling and quality of outcomes produced improved after this switch. Matt felt validated in Project 4: How Safe are We? (see pages 164-180) as his new team pursued his idea about creating separate rankings for the most dangerous small towns and cities in our state. Brooke lead her team in the development of a topic to study for the final Student Choice project (see pages 180-208). In addition, Brooke, Megan, and Monica worked together to collect and analyze the data they would need to model the growth of the Humane Society for this final project.

**Reflecting on relearning and conflict from student-to-student discourse.**

During her interview, Brooke described the improvement in her own learning because of the validation she received from her new teammates and improvements in communication. She commented:

> When I said something they listened… We all worked together on a project and we split up the work so not all was going to one person. I feel like it’s been a lot more useful because when we did it with the old team I had no idea what was going on...I was really able to understand the entire presentation and everything we were doing and it was a lot better. (April 26, 2016)

Brooke provides perhaps the most important non-mathematical learning which occurred for most students as a result of these modeling tasks in a new student centered environment: the value of listening and how one can be **empowered by simply being heard.** This is consistent with research suggesting opportunities for learning occur when
teammates employ *joint attention* with two-way communication by allowing members to both explain and listen to the explanations of others (Esmonde, 2011; Barron, 2003). As Brooke says in her interview, “when I said something they listened,” I came to realize that students simply want to be heard. They want their ideas to be validated. It became clear to me that students want their perspectives and experiences valued by their peers. Students also realized, perhaps for the first time, their voice can provide powerful learning opportunities for themselves and others. Without this student voice being heard, the development of modeling abilities seems to be limited.

**Team-to-team questioning and communication.** Student-to-student discourse primarily occurred within teams as each modeling project developed. In addition, during the communication component of the modeling cycle, teams were given an opportunity to describe their model and results to other teams. These presentations were supportive in helping students themselves see multiple solution strategies, and it forced students to think about their audience and how they might interpret the results of their analysis in terms of the situation being studied. However, I struggled encouraging students to provide feedback and questions to teams in response to these presentations. As the unit progressed, students for the most part got more comfortable communicating with their teammates, but I found that support in the form of graphic organizers encouraged more two-way communication between the presenting team and their audience. I desired more dialogue than students were providing in the first three projects. I did not want students to simply notice there were other ways to approach the task. In order to maximize the potential for the development of modeling abilities, I wanted them to analyze, question,
compare, and provide specific feedback to gain a greater understanding of and appreciation for these various approaches.

**Engaging in team-to-team communication.** In Project 4: How Safe are We? and Project 5: Student Choice specifically, students seemed to finally create additional learning opportunities for themselves by engaging in team-to-team questioning and communication. Looking back on it now, I think this occurred because of support provided by the graphic organizers forcing students to develop questions, a greater variety of models being developed, and more comfort with the modeling cycle. Two presentations for Project 4: How Safe are We? (see pages 164-180) in particular stand out because both presentations resulted in student-to-student discourse enhancing the development of modeling abilities, but they brought about new learning in different ways: one from a productive exchange of ideas while another from a more negative interaction.

**Positive team-to-team discourse in Project 4.** Recall in Project 4: How Safe are We?, students were asked to develop a model creating a ranking system for the most dangerous cities in our state. In this project, I hoped students would come to appreciate the importance of understanding the methodologies behind a mathematical analysis and value information on how a ranking system was developed. Both Team 1 and Team 2 created a methodology that weighed violent crimes more heavily, but Team 1 also took into account overall crime and created a point system to rank cities. After Team 2’s presentation, I asked Team 1 to lay their rankings side by side with the rankings developed by Team 2 so the class could compare both rankings and discuss differences in each team’s methodologies. The class noticed much of the ranking was similar but some
of the cities were flipped around. Noah from Team 2 pointed out how he thought his team weighed the number of violent crimes per capita so much Loganview would have been at the top of their list no matter the total of property crimes, whereas Team 1 weighed the location of their rank by assigning a point value--not the number of crimes--thus limiting the impact the number of violent crimes could have on the final rank.

As she compared the set of rankings and Team 1’s corresponding point totals Noah just discussed, Samantha asked members of Team 1, “How did you guys rank the last three when they are all the same number?” (see Figure 5.2) (April 6, 2016):

<table>
<thead>
<tr>
<th>Team 1 Rankings</th>
<th>Team 2 Rankings</th>
</tr>
</thead>
<tbody>
<tr>
<td>8. Platte City</td>
<td>8. Armstrong</td>
</tr>
<tr>
<td>8. Armstrong</td>
<td>10. Platte City</td>
</tr>
</tbody>
</table>

*Figure 5.2. The last three cities on the rankings for Team 1 and Team 2 showing how the cities are the same but in a different order.*

Chris and Evan with Team 1 both immediately responded with “ties” (April 6, 2016). These students had not elaborated on this idea of ‘ties’ in their presentation and no one but Samantha referred to these point totals being the same. As a result of Samantha’s analysis and questioning, Evan elaborated, “In our system because we don’t use decimals, we have two ties for 6th place and 8th place.” Samantha then goes to the board and says:

So these last three that we looked at (Figure 5.2) could have been the same here (pointing to Team 1’s rankings) but they just put them here in that order. So like
how all of you were saying these were in a different order they are actually the same. (April 6, 2016)

By engaging in a productive exchange of ideas and questions, members of Teams 1 and 2, and in particular Noah and Samantha, were able to pinpoint mathematical differences leading to explanations about why some cities appeared to be flipped around in a comparison of these rankings. Without this team-to-team questioning, students in the room and I would have missed out on additional opportunities for learning about modeling and its potential in developing alternative solution approaches. The student-to-student dialogue, questioning, and clarification here showed great improvement over past projects in their ability to analyze situations, comfort in sharing ideas, and ability to communicate their reasoning.

Conflict from team-to-team communication in Project 4. Almost immediately following the dialogue between members of Teams 1 and 2, students in class asked questions about the work Team 4 (Jason, Eve, Jacob, and Emily) had done, trying to understand why the results of their weighted ranking methodology were so different from other teams. The two lists in Figure 5.3 compare Team 4’s ranking to the ranking developed by other teams who also weighted violent crimes more than property crimes. Students in class were concerned why Team 4 had created a similar methodology but obtained drastically different results.

<table>
<thead>
<tr>
<th><strong>Team 4’s Weighted Ranking</strong></th>
<th><strong>Ranking from other Weighted Scores</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Central</td>
<td>1. Loganview</td>
</tr>
</tbody>
</table>
2. River Junction
3. Spring Hill
4. Loganview
5. Jefferson

2. Metroville
3. River Junction
4. Spring Hill
5. Cedar Valley

Figure 5.3. A comparison of Team 4’s ranking of most dangerous cities to a ranking developed by teams with a similar methodology.

In response to student questioning of their work, Team 4 got very defensive, to the point Eve left the front of the room to sit down at her team’s tables with visible emotion and frustration on her face. I suggested that the source of the problem and confusion might be the number of violent crimes they had recorded for Loganview, as they only documented 66 violent crimes while other teams recorded 130. Then Samantha from Team 2 hollered “They’re all wrong.” Eve defending herself chimed in by saying “But that’s for 2013” (April 6, 2016). As Team 4 and those involved in the heated discourse collectively went “Oh” I yelled out “Oh that’s interesting! Because then Loganview isn’t consistently the most dangerous place” (April 6, 2016). I went to the board to try to ease Team 4’s frustration by pointing this out and suggesting using data from different years would have created different rankings. I told the class there was a good lesson to learn from this. It is important to understand what data was used in an analysis. Team 4 had not communicated well their data was from 2013, nor why they decided to look at 2013 rather than the 2014 data I had given them. In reflecting on this I wrote in my journal:

This conversation also made me think about a limitation to this methodology that I had not thought about previously, the disadvantage of only using one year’s
worth of data. No teams asked for data from other years and that would have been good information to have or for me to encourage teams to look at if they had had time. (April 6, 2016)

The absence of multiple years worth of data was a major design flaw to this project and shows a limit of my own mathematical knowledge about the situation. This prevented models to be as representing of the situation as they could have been. In the design of this project, this could have been easily fixed by providing the database of reported crimes from the FBI and allowing students the freedom to decide how many years and what years in particular they wanted to include.

The next day we began class by reflecting on Team 4’s presentation and methodology as I thought it could be one of the most powerful learning opportunities for students associated with this project. In their groups, students discussed potential issues in the analysis conducted by all teams. George clarified, “I get the limitation because Loganview’s crime rate doubled (from 2013 to 2014) so I can see where they are coming from.” Abigail agreed “I think for our data we should have looked at multiple years, like the past five years or something. One year could be completely different from the next” (April 7, 2016). Megan even commented on her presentation graphic organizer (see Appendix R) how she felt Team 4 had the most effective methodology as “They showed limitations and why it’s important to explain where the data came from” (April 7, 2016).

Reflecting on relearning and conflict from team-to-team discourse. The team-to-team dialogue that occurred as a result of these Project 4 presentations looked drastically different. In a comparison of methodologies from Teams 1 and 2, students
were leading the conversation, and it was effective because both teams felt confident in their analysis and approached the situation as though no one did anything wrong. The strategies were just different and students sought to understand why. The emotion and feedback felt different than how the dialogue occurred with Team 4’s presentation when I sensed I needed to jump in and ‘rescue’ Team 4. The personalities of Team 4 do not take productive criticism and comparisons as it may be intended. They view criticism as a personal attack, and it makes them uncomfortable. I wrote in my research journal after these exchanges:

I am not sure how to change this or even if it is possible in the short amount of time we have together. It needs to be built into the culture of the room throughout the year, which only happens when you give students choice and control and the means to communicate in front of peers and at the front of the room. I apparently have not given students ample opportunity to do this and need to make it a habit in the future. (April 6, 2016)

Looking back on this situation now, it seems at the time students still wanted to provide feedback as a result of someone being right or wrong. In order to create opportunities for learning, students should, instead, value insight, different perspectives, and questions that make them reflect on their own work. I should have created more opportunities throughout the school year for students to give this kind of feedback to each other in order to potentially enhance discourse and student learning. Perhaps I did not set appropriate sociomathematical norms (Yackel & Cobb, 1996) necessary to communicate
to students “what counts as mathematical difference, sophistication, and explanation” (Stein, 2007, p. 286).

In the end, the more negative exchange ended up resulting in some of the most powerful learning students had about their work: (1) what productive discourse should look like, and (2) limitations to models when working with data. Just as Manouchehri (2007) found in her study of inquiry discourse in a mathematics classroom, these “unexpected” and sometimes “false solutions” can enhance student learning and mathematical understanding (p. 299). Again, students during a month of modeling had to relearn how to learn in a student-centered environment while discussing authentic mathematics. Communication and discourse in all forms provide powerful learning opportunities for students, and despite frustrations, students came to value and respect the effect it had on their understanding of modeling and the development of modeling abilities.

**Transitioning from Right and Wrong to Multiple Solutions**

As I was designing this modeling unit, I realized early on students would have to not only learn how to interact with each other differently, but would need to think about mathematics itself differently. This is particularly true as students identify solutions to mathematical problems. Typically, students only have been exposed to math problems whose goal was to identify a single correct answer. As Chris stated in his interview, his prior math experiences seemed to stress “there’s only one certain answer that I can solve it, you have to find it” (April 27, 2016). In addition, students were accustomed to having procedures to follow that were demonstrated by their teacher. These procedures allowed
students to find the correct answer when applied appropriately, and several students in class enjoyed mathematics because of this concreteness. Samantha, for example, said she was most successful in math class when she was memorizing steps. She stated in her interview “then I know how to do it. When I see something that’s the same I just memorized, it just helps...Give me a procedure. This is the equation, use it when you see this” (May 17, 2016).

However, modeling in its purest sense has no single correct strategy, nor follows a prescribed procedure for determining what is best to accomplish the task’s goal. As Luis observed and commented in his interview, “In the past it’s just $x$ equals that. But now we have to explain the meaning and how they correlate” (April 28, 2016). To develop modeling abilities especially related to flexible and creative thinking and reasoning, students would need to relearn, or perhaps learn for the first time, how to learn when multiple solutions are possible. Looking back now, learning in this manner seemed to require a belief change about the very nature of mathematics.

**Encouraging Belief Change through Unit Design**

Schoenfeld (1985) found aspects of the problem-solving process were heavily influenced by student’s beliefs on what is mathematics and the learning of mathematics. Anticipating this need to challenge rigid beliefs on the nature of mathematics and specifically transition perceptions from right and wrong to multiple solutions, I structured the modeling unit from concrete to abstract. The first two projects had multiple solution strategies students developed, but there was a target number in mind. There were specific recording times in Project 1: The High Definition Trade-off Dilemma and a particular
length in Project 2: Calculating the Width of Memorial Stadium. During these projects, students would constantly ask questions like “Do you know the right answer?” or “Is this correct?”

As the unit progressed, students would encounter much more abstract tasks where no single correct answer was possible. As teams presented their analysis for each of these latter projects, students began realizing the importance of seeing multiple interpretations of the problem. For example, in reflecting on the presentations for Project 3: Enrollment at River Junction Senior High (see pages 152-164), Evan commented to his team, “With the linear graphs, I didn’t think that a linear graph would actually fit very well. Then as people showed us their examples, I found out the linear graph was pretty much just as good as an exponential graph. ‘Cause I thought an exponential graph was the only way to go” (April 1, 2016). Matt commented how he learned a lot from Team 2’s presentation as they modified their enrollment model for just the last four years. He was recapping his learning with his team as he said “They found the law with the 18 year olds; that was really smart of them” (April 1, 2016). Luis thought he learned the most from Project 3 because of the different approaches used by students. In his interview he stated “I just liked how, like some of us thought the linear regression and some of us did the exponential, and how we explained the differences between them. And how some would be more accurate now, but over time, it wasn’t” (April, 28, 2016). Because of the communication component to the modeling cycle, students gained an appreciation for understanding multiple strategies in developing their own learning about modeling and
mathematics. As they came to realize the various approaches that could be utilized, the opportunity to develop skills related to flexible and creative thinking expanded.

**Barriers Form as Beliefs are Challenged**

Being exposed to various strategies as the unit progressed seemed to help students learn how to learn in a mathematics classroom where right and wrong did not exist. Not surprisingly, similar research like that done by Op’t Eynde, De Corte and Verschaffel (2002) and Presmeg (2002) found mathematics students could recognize was contingent on their philosophical beliefs about the nature of mathematics. Presmeg (2002) also found new learning can be limiting until those beliefs are changed. During a month of modeling, frustrations arose for some students as this new environment conflicted with previously held experiences and beliefs about the teaching and learning of mathematics. These barriers became evident in the feedback they could give other students and in their individual work.

**Valuing alternate perspectives.** Some students, like George, had difficulties giving feedback to teams other than it being right or wrong. He would consistently say “Ours is more accurate” or point out how he thought his team had the right approach (April 6, 2016). When I discussed this issue with George after Project 4: How Safe are We?, he told me it bothered him to think of mathematics as anything but right or wrong. In his mind, that’s what mathematics should be and in his unit reflection even wrote “The fact that nothing is right or wrong” was his biggest source of frustration as he learned how to model (April 14, 2016). Breaking him out of this mindset in a few short weeks proved to be difficult, but in the end he recognized “everyone (sees) things differently
and has justification for their thinking and it works” (April 14, 2016). Again, as students learned how to communicate and participate in productive discourse, they also realized how different the learning process looks when various approaches are used.

**Importance of reasoning.** Another barrier students encountered as they learned how to learn when multiple solutions were possible is the importance of reasoning and explanation in one’s individual work. Several times throughout the modeling unit as described in Chapter 4, I commented how individual work submitted for each project did not seem to match the thinking and conversations which occurred in the classroom. One of these students was Chris. Chris was a straight-A student and has always excelled in mathematics. On assessments in my precalculus course, he would typically have one of the highest scores. Yet, on his individual work for projects in the modeling unit he would barely score a B on the Individual Rubric. Despite understanding the expectations from the rubric and working through the modeling cycle process successfully in class, he showed little evidence he applied modeling abilities specifically related to reasoning in his individual work. For example, in his Project 2 work he developed a model but did not elaborate on his solution strategy (see Figure 5.4):
Figure 5.4. Chris’s individual work showing his calculations for getting an answer of 753.97. (March 28, 2016)

Chris, like George, only had experiences with a right or wrong answer. His prior expectations with mathematics homework centered around showing work that resulted in an answer as shown in Figure 5.4. I asked Chris during his interview why he felt he struggled, especially in the first half of the unit, as he learned how to represent the work he had done. He blamed it on prior experiences in all courses only requiring him to “state your information and be done.” He commented, “Usually we just had to give a number and they already knew what the number meant. But now we have to elaborate what the
number actually meant and how we got the number and how it applied to the actual situation we were doing” (Interview, April 27, 2016).

This focus on acquiring an answer in Chris’s prior educational experiences prevented him from initially scoring well in the modeling unit. He had to learn the goal of modeling was not just on finding an answer but on reasoning through the development of a strategy and what his results meant in terms of the situation. By Project 4: How Safe are We? and Project 5: Student Choice, Chris began doing a better job describing his thinking which he credits to his teammates and seeing other’s work. It seems conversations and examples of the various solution strategies employed and reasoning students used to develop appropriate models helped Chris change the target of his work.

**Reflecting on the Transition from Right and Wrong to Multiple Solutions**

This shift from focusing on right and wrong to reasoning through multiple solution strategies seemed to be one of the most profound new learnings students had about mathematics. During his interview, Chris thought the most important thing he learned about math was:

- That there are different ways to actually input the math into like, a situation.
- Different models people used different types of math to get the answer. So it’s not just one specific way to do it. There are multiple ways to do it. (April 27, 2016)

Similarly, Luis was initially hesitant to participate in his team’s conversations as he thought he was wrong. Eventually, he broke out of his shell and became a vocal leader in his team. During his interview, Luis described how he credits this to there being:
No right way to do it. So any feedback I could give would help my team...Before I just thought of a problem on paper and we’d get the answer by applying whatever method but now I see there are many different ways to use it and more than one way to get the answer. (April 28, 2016)

Students like George, Chris, and Luis initially had reservations about modeling because of its abstractness. The development of their modeling abilities was limited until their beliefs about mathematics changed to valuing multiple approaches. Once students held this less rigid belief, they had greater opportunity to develop skills related to flexible and creative thinking (see Leiken, 2009) in the development of strategies and the reasoning which justified those strategies could work. Students expanded their knowledge about modeling not as a result of a strategy being right or wrong, but from what new insights about the situation and mathematics could be gained through discourse and the opportunity to see other approaches.

**Finding Perseverance Through Authentic Mathematics**

A significant concern of mine going into the modeling unit was the uncertainty of whether or not students would be willing to use significant lengths of time productively to complete each task. In my course prior to this unit, students only needed several minutes to accomplish the goals of a mathematics problem I gave them. Modeling, in the way I intended would take several class periods to complete. Would students be able to structure and use time as a resource? Would they have the perseverance necessary to see the task to the end? To my surprise, students did not give up on any of the five tasks I asked them to complete. Nothing brought me more joy throughout the unit than watching
students work through both positive and negative experiences to produce a quality product showing in-depth mathematical thinking and problem solving. Students successfully learned how to learn in an environment where perseverance was required. What helped them develop this skill?

During the unit, as I noticed students persevering despite at times uncertainty and conflict, I hypothesized perhaps students found the tasks interesting, which provided motivation and incentive to see a task to the end. Upon interviewing students, however, I learned the source of this perseverance was rooted much more deeply than in a student’s interest of a task. It seemed to come about as a result of providing an authentic experience for students. The freedom to employ their own strategies and interests in addition to the real life connections exhibited in the task became an immense source of pride for students. They wanted to persevere and see their ideas to the end.

**Freedom to Develop Own Ideas**

Strongly connected to new beliefs on the importance of multiple solution strategies, students found pride in developing their own strategies and solutions. Consistent with research done by Schukajlow and Krug (2014), interest in completing complex mathematical tasks were positively affected as students developed multiple solution strategies and gained confidence in their ability to do so. Not having set procedures to follow did at first frustrate some students, but as the unit progressed, this freedom was what many students appreciated most about modeling tasks. Chris commented in his interview how he most enjoyed “just the freedom of being able to do, do what your team wants to do...There wasn’t like a specific procedure you had to do.
You just had to use the information you had and the go off and brainstorm how to be able to solve it” (April 27, 2016). Luis agreed, citing his enjoyment from “having a chance to work with a team on real life problems without any instruction” (Interview, April 28, 2016). Going into the unit, I thought the ‘lack of instruction’ on how to proceed through each task would be a major source of conflict. I was surprised to learn that for some students, this freedom had the opposite effect.

In their interviews and end-of-unit reflections, students described their previous mathematics experiences as being very rigid and procedure driven. In her interview, Brooke commented how her previous teachers would “just kind of give me the steps and tell me how to use it” (April 26, 2016). This is contrary to an authentic experience students would encounter in real life that would require them to problem solve, be creative, develop their own strategy, and reason through how to carry out that strategy. Students learned to appreciate this freedom as ideas became their own. And as students found little successes along the way in carrying out their ideas, they learned to persevere through modeling tasks.

**Taking pride in student work.** Because students had the freedom to be creative and try out their own ideas rather than a predetermined set of steps, students found pride in their work. In her interview, Samantha commented how in previous math courses she was “using numbers to find other numbers that didn’t even make sense. Like, it didn’t really matter to me. It was just an answer for a grade” (May 17, 2016). As she completed modeling tasks, however, she began finding self-confidence in her ideas. Her work began to matter to her, causing her to persevere through difficult situations. One
such instance that stood out to Samantha was when she developed a general model for finding the width of any object using indirect measurements for Project 2: Calculating the Width of Memorial Stadium. She described how completing this model made her feel:

Noah didn’t know how to figure it out so I was like, I’ll just do it on my own.
And when I sent him it and he’s like ‘it looks good’... It was fun being able to do something on my own. Even if it was a group project I figured it out for them. It was exciting....it was fun to figure it all out and actually have it work. The other groups didn’t have that. (May 17, 2016)

The general model for this project was extremely challenging, yet Samantha persevered and even referred to it as being “fun.” The freedom to apply her own ideas was not a detriment. Rather, it gave her a source of pride and allowed her to persevere through such a difficult task.

**Motivation beyond grades.** Applying and finding success in her own ideas seemed to allow Brooke to persevere despite conflict with her team members. She described a specific occurrence from Project 1: The High Definition Trade-off Dilemma about how she initially used guess and check to narrow down recording times in 1080p and 720p for Heather’s GoPro situation. William and Nicholas thought she was wrong, but after spending a considerable amount of time developing equations, they found Brooke’s results to be correct. Brooke stated, “Even though I did use guess and check at first to get those numbers...I felt really proud of myself...It felt good” (April 26, 2016).

As Brooke and other students found success and validation in their own ideas, they became invested in the project and the mathematics required to accomplish the task’s
goals. Later in her interview Brooke would add “it just felt like a better experience than when I did everything else that was just handed to me. Because I had to work a little bit more for it, so the payoff was a lot better” (April 26, 2016). Having their voice heard and their perspectives valued motivated students to persevere. Even if the task was harder, because it was their idea, accomplishing the goal in the end brought a greater sense of pride. Rarely would students allude to motivation stemming from a desire for a ‘good grade.’ I simply do not think for most students, a good grade is intrinsically motivating enough to cause students to persevere on one problem for several days; the task needs to be more personally fulfilling for students.

**Rooted in Real Life Mathematics**

By far the most compelling aspect of the modeling unit for students was it being focused on mathematics found in real life. Not surprisingly then, this authenticity was the number one source of perseverance in each modeling task. Zbiek and Conner (2006) state modeling can be motivating for students as they realize (1) mathematics can be useful for some people as it relates to a particular real life situation and (2) mathematics can be used to understand the complexities occurring in real life situations. During a month of modeling, it was not necessarily true all students found each situation itself interesting. And I certainly could not provide motivation to students by explaining how they would encounter each particular situation in their everyday lives, although students realized quickly the general thinking skills and modeling abilities required could be utilized on a daily basis. What was important was that someone would need to know
about the situation studied; students found the task relevant if they could see its relevance to any particular person even if that person was not necessarily themselves.

**Student reflections on mathematics in real life.** Student interviews and unit reflections are covered in comments like “the most important thing I learned was math can be applied to almost any situation” (Emily Student Work, April 14, 2016), “It gave me a whole new perspective about how it is actually used in the real world...looking at different problems that would happen to real life and figure them out was helpful” (Luis Interview, April 28, 2016), or how modeling “really opened my eyes for what/how math can be interpreted in everyday life because I have always questioned how math relates to real life.” (Jason Student Work, April 14, 2016). Students found tremendous value in knowing, seemingly for the first time, how the mathematics they had learned in a classroom could be used in situations outside of a classroom. This authenticity lead to curiosity, which in turn motivated students to persevere and accomplish the task’s goals even as students became frustrated or confused.

**Brooke’s newfound perseverance.** A particular case exhibiting perseverance brought on by freedom and authenticity was Brooke and her desire to learn modeling throughout the unit. Brooke was the only student I had failing my class going into the month of modeling. She rarely completed assignments, spent her time doodling in class, and showed little interest in the content students were learning. In informal conversations with her prior mathematics teachers, it seemed Brooke had consistently shown little motivation and poor work ethic in other courses as well. She was not even recommended by her prior teacher to take my precalculus course because of below average grades.
However, her academic aptitude in mathematics is incredibly high, as indicated by the 96th percentile score she received on the MAPs test (Measures of Academic Progress) she took for me in the spring of 2016. Brooke’s failing grade in my class was not due to an inability to learn mathematics but little motivation to participate in class activities. She displayed a limited desire to put in the time, energy, and practice she needed to learn and apply the standards within our curriculum, and generally seemed to be completely uninterested in class.

**Brooke’s prior mathematics experiences.** In an interview with Brooke, I asked her about these experiences in my class and previous math courses, hoping to understand why a student with so much potential would be content with below-average grades. As I suspected, she discussed her indifference towards the material resulting in little desire to participate productively. She describes most of her previous math experiences as “boring.” The procedure-driven instruction of these courses seemed to be the root of this boredom as she claims she “would understand it so doing something repetitively like that...it’s just really boring for me and I don’t like to do it” (April 26, 2016). However, Brooke provided a reminder to me that having poor study habits does not necessarily mean a student does not want to learn. She states, “My work ethic is not great but my willingness to learn, I am always willing to learn new things. I’m just not the greatest at actually doing it” (April 26, 2016). Brooke seems to suggest here the importance of relevance and ‘a hook’ to her educational experience. Grades are one of the least contributing factors affecting her desire and willingness to learn mathematics. To put it simply, she needs to be interested to be successful.
Brooke’s new interest in mathematics. As the modeling unit progressed, however, Brooke had a shift in work ethic and participation in class activities. During class discussions, she would offer up her ideas and suggestions. She was consistently on task and meeting classroom expectations, even encouraging Matt to have a better attitude towards himself and his modeling abilities. And despite frustrations with her peers in Team 1, she desired to work on the mathematics required in each project. After I switched her to work with Megan and Monica in Team 6, she became the leader I always hoped she would be. Her perseverance to develop models to present to the class was at a level I had never seen from her before. I thought I had finally piqued her interest in mathematics.

Finding relevance for her learning. During my interview with Brooke, I wanted her to try to pinpoint where the source of this new motivation and perseverance came from. She commented:

I really liked modeling because we used it to do things you wouldn’t normally think of. When people ask how am I supposed to use this in real life? There’s a lot of real life situations that you can apply modeling to. And, you can apply it to almost any kind of situation. I think that what was really important to learn because it will help further on in life with, like, anything I guess. (April 26, 2016)

Brooke suggested here the authenticity of each task contributed to her desire to participate and complete the modeling cycle for each task. She saw, perhaps for the first time in her mathematics class, relevance to what she had learned and the expectations her
teacher placed upon her. This authenticity seemed to lead to the perseverance necessary to successfully develop models for each project.

*New learning of self.* As I continued my interview with Brooke, another interesting component emerged that seems to also have contributed to this newfound perseverance. She learned something new about herself in the modeling unit. In Chapter 4, I described the task Brooke had developed with her team for Project 5: Student Choice (see pages 180-208) about modeling the animal intake and out take levels for our local animal shelter. This was a topic Brooke had thought up and took the lead on, both in the data collection and analysis phases of the project. In her unit reflection she wrote “The last project was my absolute favorite thing I’ve ever done not only in math but in my entire school career. It was something I was able to do on my own and that I actually wanted to do” (April 14, 2016). I asked her to elaborate on why this was during her interview. She described how she was “happy” working on the project and thought it was “a lot more fun” than mathematics tasks she had done in the past. Brooke suggested the source of this happiness could be because it was a project she had picked, but more likely stemmed from her newfound love of data analysis. She commented, “I really, really, really like data. I don’t know. I’ve never really thought about it before. But when we did the River Junction project and then the animal shelter project, it was a lot more fun doing those two projects” (April 26, 2016). Brooke had never before in her math classes worked with authentic data. The data I gave students for Project 3: Enrollment at River Junction Senior High and Project 4: How Safe are we? were not made up numbers, and they were presented to students in a way that seemed to pique their curiosity. In the final
project, Brooke had the freedom to select a topic that interested her and collect data herself by working with the animal shelter’s director. Because of the authenticity of each task, Brooke learned something new about her own interests and learning that might hopefully affect her future goals and aspirations. In the short term, like the majority of students in class, this authenticity and freedom to pursue her own interests and strategies allowed Brooke to persevere through the modeling cycle.

**Reflecting on Student Perseverance**

The motivation to cycle through the modeling process was rooted more deeply than in a student’s perception of whether or not they would ever personally be affected by the situation presented. As Blumenfeld et al. (1991) cautions, interest might be enhanced when students explore situations to which they relate, but this interest may not be motivating enough to allow students to work out more long-term, complex tasks. Throughout a month of modeling, students found perseverance because they were allowed to be creative. They became invested in their work as they carried out their own ideas and strategies. They also saw relevance to what they were doing to life outside of a mathematics classroom, but not necessarily because they specifically related to the topic. Students seemed to appreciate the task when they found the mathematics necessary for some particular person to understand a complex, authentic situation.

As a result, even when they became frustrated students did not give up. After frustrations mounted in Project 2: Calculating the Width of Memorial Stadium, I reflected on what seemed to help students develop their models despite uncertainty about the
picture needed, geometric skills required, or the time frame I forced them to do it in. I wrote:

Teachers complain a lot about students giving up too easily. Perhaps we are not giving them the right tasks; they are just not interested in the result of the task and therefore give up. Perhaps well designed tasks that students are curious about will help students develop perseverance. Maybe we as teachers are simply not asking the right questions. We need to ask, will students want to know ‘the answer’ to this? Will they be naturally curious enough about its result to invest the time and energy we want them to? (March 26, 2016)

Perhaps we stifle students’ curiosity, interest, creativity, and thus perseverance, when we as math teachers focus on problems requiring students to ‘solve for x.’ Why would students persevere and use time productively when they have few opportunities to apply their own ideas in these situations? What about ‘solve for x’ will motivate students when the mathematics required is challenging for them? There is a place for procedure driven problems within a mathematics curriculum, but it cannot be all math teachers focus on or perseverance may wane as soon as ‘a grade’ no longer feels relevant to a student.

**The Case of Matt**

No student exhibited more struggles relearning how to learn during the modeling unit than Matt. Matt was a successful Hispanic student and creative thinker, and he worked hard in class to complete assignments and meet the expectations I set for him. His peers often looked up to him in class for ideas, and he was not afraid to ask questions or offer up those ideas. He had a wonderful sense of humor, was good-natured, and
always pleasant to work with. Outside of class, he excelled in the arts and felt at home on stage in front of an audience. Going into this modeling unit, I had no reservations about Matt and his openness to learn in an environment now focusing on student-driven communication, creative and flexible thinking leading to multiple solution strategies, and perseverance. The resistance, frustrations, and negative attitude he exhibited as he navigated this new environment simply caught me off guard and illustrate how hard modeling authentic tasks in this new environment can be for some students.

**Needing to Have Ideas Validated**

I decided to sit Matt with Brooke, William, and Nicholas in Team 1 as it seemed they were all creative thinkers and had similar interests. Matt, William, and Nicholas were in a specialized computer programming course together located at a different district facility. Because of their prior history and experiences together, I was not expecting the conflict that emerged. From day one, communication between members of this team was an issue. William and Nicholas came across to Brooke and Matt as very fast thinkers. They would often explore their own ideas and get so focused on those ideas, they would leave Matt and Brooke alone to fend for themselves.

Matt did not create opportunities for his learning because neither he nor his teammates seemed to know how to participate in productive student-to-student discourse. Matt, being the extrovert that he is, needed this two-way communication to help him process each task and the mathematics required. In his interview he stated “I mean I was still willing to work pretty hard, I just didn’t know what to do...I didn’t know how to include myself” (April 27, 2016). Without feeling empowered and included by his
teammates, Matt essentially gave up. He had little perseverance to complete the modeling cycle for the first three projects as it seemed he did not feel permitted to pursue his own ideas by his teammates and did not yet see the relevance within the authenticity of each task. Matt’s only motivation for working in class during the first three projects of the unit, according to him was “to just get an okay grade on this” (March 30, 2016).

**Negative Effects on Self-Esteem**

This lack of communication also negatively affected Matt’s self-esteem. When he saw how quickly William and Nicholas were able to navigate the modeling cycle and develop appropriate strategies for Projects 1, 2, and 3, he became even more frustrated, and his confidence waned. He stated, “William and Nicholas are very talented. Very very smart. They work very fast, and in their own little world. I am very slow” (April 27, 2016). At no point prior to our modeling unit had Matt exhibited negative feelings towards mathematics or his ability to learn mathematics. As he refers to William and Nicholas being more talented than him, he eludes to this lack of confidence. In particular, he seems to believe being *good* at math is a result of how *fast* one can solve mathematical problems. Because Matt’s teammates could process the task much more quickly, Matt developed the perception he was *bad* at math. This new false perception would continue to impact his ability to model as the unit progressed.

Matt had a difficult time generating his own ideas to experiment with, especially as his confidence diminished. Without the discourse he so desperately wanted, he began to shut down emotionally. Matt described how this lack of discourse with his teammates gave him “anxiety.” He commented, “It made me feel very useless. I wasn’t used to that
feeling... I realized how much more talented all these people are around me, it wasn’t the best feeling” (April 27, 2016).

It seemed for Matt, feeling “useless” was the ultimate insult to his self-worth. He prides himself on being a hard-working and passionate young man. Matt felt he tried to find different ways he could contribute to his team’s strategies, but because he never received validation for those strategies, he stopped trying. Esmonde (2011) calls this “access to identities” stating that in group work, students need “positive positional identities” that “place them as authoritative and competent members of the classroom community” (p. 54). Because Matt was not given opportunities to contribute and develop a positive identity towards his ability to work on complex tasks, his new mathematics learning suffered.

After Project 2: Calculating the Width of Memorial Stadium, I noticed Matt’s self-esteem seemed to be at an all time low. He agreed, commenting in his interview:

Geometry is one of my worst subjects. So on top of being slower than everyone else we are on the worst subject for me. So that was, that was like my breaking point. I think that was when I gave up. I stopped trying and I just let them do whatever they wanted. (April 27, 2016)

It broke my heart to see such a happy, outgoing student feel so uncomfortable and full of anxiety. I tried to give him encouragement and positive feedback on his ideas and abilities, but it was not me he needed approval from. He needed to feel valued by his peers.
Supporting Matt as Beliefs are Challenged

I thought about moving Matt after Project 2: Calculating the Width of Memorial Stadium, but had hoped because geometry would not be utilized much in the next couple projects, he and his team would be on more even ground. I was optimistic this may provide Matt opportunities to have his ideas validated. However, conflicts continued in Project 3: Enrollment at River Junction Senior High, and I decided to move Brooke and Matt for Project 4: How Safe are We? (see pages 164-180). I was curious to see if a new group would allow Matt to find more opportunities for learning about modeling through student discourse. I felt at the time moving around students in Teams 1 and 6 was good for all involved as communication between teammates increased and both Brooke and Matt had their ideas heard. Matt, in his interview, agreed:

We all worked a little bit better together...they want to talk about everything. I mean, I’m still slower than all of them. They were all picking up on it really fast and there were points I made stupid mistakes. But overall, I think the team just talked more and that helped me out a lot. (April 27, 2016)

Because communication was such an essential ingredient to a student’s success in the modeling unit, not having it proved to inhibit Matt’s opportunities for learning about modeling and the development of modeling abilities. The validation he received from Megan, Monica, and Brooke in his new team for Project 4: How Safe are We? went a long way in helping Matt find success navigating the modeling cycle. He proudly told me in his interview “I came up with the actual idea for that project. I came up with how we could present it and what information we’ll use. And, I don’t know, that was a cool
feeling to actually use my idea” (April 27, 2016). Based on his change in attitude and learning outcomes, I regretted not moving Matt and Brooke earlier in the unit.

**Reflections on Matt’s Beliefs and Learning**

Despite my best efforts at encouraging Matt, William, and Nicholas to work together, their personalities simply did not allow them to engage in positive discourse in the short time allowed during a month of modeling. During his interview after the unit, I asked Matt what else I could have done to help him find more success in the first half of the unit, and he replied simply with “it’s important that you just read the groups fairly well. Getting the right people with the right groups will really help them out and get the best experience for them” (April 27, 2016). I thought going into this unit I had carefully considered groups. For the majority of students, teams worked well together. However, I learned you never really know until you try. When students are put into unfamiliar environments where they will need to relearn how to learn, you will learn new things about your students that might create conflicts you did not predict.

**Understanding the source of Matt’s frustrations.** Matt’s experiences in a month of modeling brought out his beliefs about mathematics. These perceptions were previously unknown to me and in fact, contradicted how I thought Matt viewed mathematics and what it meant to do and be good at mathematics. At no time throughout the school year prior to a month of modeling had Matt shown so much desire to be ‘correct’ or ‘fast.’ I always considered him to be creative and flexible; he was heavily involved in the arts at school and excelled in drama. It surprised me to find out how rigid he believed mathematics should be. It seems this belief was rooted in his prior
experiences in mathematics and from his own feelings on how he best learned mathematics.

In his interview, Matt describes his prior experience as “Just steps on how to learn things. They said this is how you solve triangle equations and gave us a couple different triangle equations” (April 27, 2016). Yet, perhaps correlating to his perception that he is ‘slower’ at learning math than his peers, he felt he best learned math from “practice, lots of practice. If I could get a whiteboard and just practice equations all day I’m sure I’d be good...I like having a set of rules just give me a set of instructions I’ll do them for you” (April 27, 2016). Particularly throughout the first three projects of the unit, Matt showed increasing frustrations as he attempted to navigate a learning environment void of steps and procedures to follow.

I had never before viewed Matt as a steps guy. Looking back now, I think the lack of discourse with his first team did not empower him and provide him experiences that may have helped him change this belief about mathematics teaching and learning. Through the first few projects in particular, I tried to focus my conversations with him on changing this perception about mathematics. After Project 2: Calculating the Width of Memorial Stadium, for example, Matt, Nicholas, William, and Brooke had calculated a value for the width of the stadium that was different than several other groups. This was because their work focused on the assumption my distance and the stadium were parallel. This assumption was, in fact, not true because of slight errors in the measurements I provided. Matt, however, was still stuck on having ‘the right answer’ and expressed concern their team had done something wrong. During class, as his team worked on
Project 2, he commented “What’s the point of thinking if you’re gonna think wrong?” (March 25, 2016). We then had a conversation about realizing the messiness of authentic mathematics, which will be discussed more in Chapter 7:

DB: In real life, you might get incorrect information. It doesn’t mean that the thinking you did was incorrect, it means that things you are starting with might not match up like they should.

Matt: Right, you started off wrong.

DB: It might not have been you, it might have been me. (March 25, 2016)

Up until this point, Matt still focuses on his work and how his thinking is ‘wrong.’ He had not thought about the authenticity of the task and the fact measurements might be wrong making his assumption incorrect. I then went on to describe all the issues I had taking the measurements used in this project and how in modeling, the mathematical calculations are only half the story. I state “Real life is messy and it’s not about whether you have ‘the answer.’ I’m looking at all the thinking” (March 25, 2016).

During Project 2, I wanted to reassure Matt the thinking he had done and the calculations his team had made were correct. They had successfully navigated the modeling cycle. Matt was just focused on the end result and not enough on the entire process. The discrepancies in their final ‘answer’ were a result of an incorrect assumption based on errors outside of their control. I ended the conversation with “Math teachers have ingrained in your brain that there is a right answer and it’s all about that answer” to which Matt replies “And that’s what you are trying to break. Got it” (March 25, 2016). In the end, I think Matt broadened his views of mathematics to be less rigid
not because of anything I said, but from the opportunities for learning he engaged in after
I switched Matt into a different group, and the discourse he encountered from different
teams during project presentations. There, he was able to discuss alternative perspectives
allowing him to look at mathematics through the lens of multiple solution strategies.

**Matt’s New Learning**

Matt’s journey through a month of modeling forced him to experience
mathematics learning in a way that was uncomfortable to him. I disrupted the routines he
had mastered: following along to his teacher’s instructions and practicing until he got it
correct. He had to relearn how to learn in an environment valuing student created
opportunities through validation and discourse, multiple solutions strategies, and
perseverance brought on by authenticity and empowerment from one’s own ideas. Upon
reflecting on this relearning, Matt commented:

> It was definitely, just a big change. Just not a book question. We all got to do our
> own thing. We got to work with people who didn’t necessarily go with our ideals.
> Just lots of conflicts, lots of frustrations there, and (long pause), yeah I think that’s
> it. Since I’m more of a step guy it was just a lot harder for me. (April 27, 2016)

Matt’s issues with the freedom given, lack of steps provided, and reliance on teammates
required initially prevented him from developing the modeling abilities I targeted for the
unit. Interestingly, research done by Woods (1994) found that when traditional students
are asked to take ownership of their learning in a more student-centered environment,
such as Matt’s experiences during a month of modeling, they often exhibit the same
psychological steps associated with trauma and belief: (1) shock, (2) denial, (3) strong
emotion, (4) resistance and withdrawal, (5) struggle and exploration, (6) return of confidence, and (7) integration and success. Matt had difficulties getting past the negativity he experienced in steps 3 and 4 as he resisted these changes and withdrew emotionally from class. I felt by the end of the unit, however, Matt finally understood the goals I had for his learning and valued the experiences he had, including ones marked with frustration. He stated he learned:

Modeling is solving a problem with math and then be able to communicate that problem. I think the communication of the problem and your answer is probably one of the most important things. I mean, that’s what you are doing it for. Let’s say you have a big company or something, you’re going to be presenting that.

(April 27, 2016)

Matt seemed to learn to value the entire modeling process, not just strictly ‘the answer.’

Here he comments on the importance of the communication and interpretation components of modeling and authentic mathematics. After Project 3: Enrollment at River Junction Senior High and at the peak of his frustrations, I asked Matt if he thought this unit had been a beneficial use of his time and he replied simply that it was not.

During his interview after the conclusion of the unit, I asked him the same question. He changed this perception stating, “Yeah, it’s real world experience. These are problems we’ll come up with...This is stuff that we’ll be doing with math. People are constantly asking when we’ll do this, this is their answer” (April 27, 2016).

Like the majority of students in class, Matt came to see the significance of the journey he had gone through during the month of modeling as it gave him relevance to
the mathematics he had learned. The authenticity of each task and the experience students had to go through to complete each task, although frustrating at times, ended up providing students with the best opportunities to develop modeling abilities.

**Conclusion**

Modeling authentic situations in a student centered environment required students to reexamine their own beliefs about the teaching and learning of mathematics. Because their prior experiences seemed to have been so centered around following a set of steps explained by a teacher, they had to relearn how to learn mathematics to focus on the development of specific modeling abilities. In order to develop skills related to reflection in modeling, students were required to verify and assess reasonableness of outcomes by creating their own opportunities for learning and eliminating the need for teacher validation. Additional student-driven learning opportunities through student-to-student discourse allowed students to better develop the communication skills needed to navigate the entire modeling process. As students beliefs about mathematics shifted to ones focusing on multiple solution strategies, students applied creative and flexible thinking to generate those strategies and reasoning to justify why they worked. Finally, these authentic tasks required students to find sources of perseverance beyond grades brought on by the freedom to apply their own ideas and curiosity as they examined opportunities to apply mathematics in real life situations. But, as Matt’s experiences throughout the unit exemplify, relearning how to learn in this type of environment can cause intense conflict both socially and emotionally. These frustrations and belief changes, however, created profound opportunities for me as a teacher as I learned to support students
relearning how to learn. These new learnings and perspective on the teaching of modeling are analyzed further in Chapter 6.
CHAPTER 6

RELEARNING HOW TO TEACH

The goals of my research on modeling were twofold: (1) to understand student learning associated with modeling authentic tasks and (2) to understand the teacher’s role in designing and teaching these tasks. As indicated by Chapter 5: Relearning how to Learn, student learning and conflict was extensive throughout a unit on modeling. However, the role of the teacher also had to change for this learning to occur and conflicts to be addressed. To put it simply, I had to relearn how to teach mathematics both as I planned for and taught a unit on modeling. In this chapter, I address the fundamental belief changes and new learning associated with my preparation to teach as described in Chapter 3: The Prequel to a Month of Modeling. I also illustrate new knowledge and skills needed during the month of modeling instruction as I attempted to help students develop modeling abilities especially focused on mathematical and quantitative literacy and address barriers as students learned how to communicate in a student-centered environment. These new insights include understanding the importance of activating student real life knowledge, changing perceptions of what scaffolding students do and do not need, and shifting the role of teacher questioning.

Preparing to Teach

As indicated in Chapter 3: The Prequel to a Month of Modeling (see pages 66-109), I had to drastically expand my background knowledge on modeling and curriculum design in order to design a unit on modeling for my precalculus course. I spent months researching the modeling process in an attempt to understand what
modeling looks like both on paper and in the classroom. This allowed me to narrow down the modeling abilities I wished to target and the design criteria I would use to write my curriculum. In addition to this background knowledge, however, I had to change certain beliefs and routines in the way I prepared to teach mathematics. Specifically, I had to look at mathematics itself differently, relearn how to assess mathematics, and accept not being the only expert in the classroom.

**What is Mathematics?**

During the research phase of this project as I began to learn more about modeling, I found myself questioning my own prior experiences learning mathematics. I had always excelled in mathematics throughout my secondary, undergraduate, and graduate courses, but what did I actually find success doing? I picked up on procedures quickly, took diligent notes, and worked hard to complete assignments asking me to replicate learned procedures. As a teacher, I wanted a different experience for my students but also found myself dictated by a long list of standards outside of my control, asking students to again learn and apply specific procedures. If I had asked myself “what is mathematics?” prior to this study, I fear I would have answered it in the same way my students did: focusing on operations, procedures, and answers.

This study focused on exploring and changing an aspect of my instruction. However, reform could not take place without changes to my fundamental beliefs about the nature of mathematics dictated by my own previous experiences as a student (Ernest, 1989). Beliefs, especially perceptions on what is mathematics, can dictate how teachers view student learning of mathematics and how teachers should teach mathematics (Smith
III, 1996). Fortunately, designing and preparing for a unit on modeling forced me to think about the nature of mathematics differently. Developing a targeted list of modeling abilities and design criteria by dissecting the modeling cycle and current research allowed me to think about mathematics as more than a collection of procedures and right answers. For example, as described by my planning preparations in Chapter 3, it became apparent to successfully navigate the modeling cycle, students would need the opportunity to apply their own thinking and ideas and not a predetermined strategy. Therefore, design criterion two: students have the opportunity to bring in various and available mathematical experiences, knowledge, representations, and data analysis techniques to develop and refine models, needed to be emphasized in the design of each task. Similar to the learning and conflict students encountered as described in Chapter 5: Relearning How to Learn (see pages 213-267), I had to look at mathematics through a lens of multiple solution strategies, reasoning, and authentic applications in my preparation to teach. The design criteria helped by providing a framework for each task, providing students with perhaps the best opportunity to apply the type of thinking skills emphasized by targeted modeling abilities and not specific mathematics procedures. Finding authentic tasks that could be modeled was a challenge, but this new perception of mathematics leading to the five design criterion provided a common direction and cohesiveness to the overall unit.

**Assessing this New Mathematics**

During my preparation for this unit, it became apparent I would need to think about assessment differently. Because philosophically I had to shift mathematics from a
focus on procedures to general thinking skills and multiple solution strategies, preparing to assess this new mathematics and expectations also needed to be different. The use of open-ended, higher-order thinking tasks like the five projects found in the modeling unit allowed students to construct and build upon their own knowledge, and as a result, I would need to find alternative forms of assessment which more closely aligned to these new learning and instructional goals (Shepard, 2000).

As Wilson (1995) suggests, “new views of student learning demand information gathering procedures that extend beyond the traditional standardized multiple-choice tests” (p. 244). In my typical routine preparations, I would make an answer key of the mathematical outcomes students should acquire and assign specific points towards successful application of procedures in reaching those outcomes. This type of assessment was impossible in the modeling unit, however, and even seemed to contradict the overall learning goals and design criteria. How do I assess and provide feedback on more general thinking skills when multiple solution strategies were possible?

Using rubrics. The structure and intent of rubrics seemed to fit in well with the goals of this new mathematics, but I had little previous experience designing or using rubrics in my teaching. Again, I relied on current research to help build my background knowledge and change my perceptions of assessment in mathematics as described in Chapter 3. Designing both the Individual and Team Product Rubrics (see Appendices N & O) were helpful in forcing me to think about different levels of student work and solidify the expectations I had for learning outcomes. It also helped me narrow down what aspects of the modeling cycle seemed to be most important in the development of
modeling abilities, resulting in a greater weight towards a student’s overall score and more emphasis in my own teaching and questioning of students. In addition, the use of rubrics provided transparency for students so they understood these new expectations (Shepard, 2000). However, they were flexible enough to accommodate different interpretations and solutions to each task (Kouba, 1999).

**Teacher is Not the Only Expert**

Preparing for a unit on modeling allowed me to think about mathematics and assessment differently because the learning targets of this unit looked vastly different than the targets which typically dictated my planning. However, the combination of these new perceptions and knowledge forced me to overcome perhaps my biggest fear in this modeling unit: *what if I do not have the answer?* Because modeling allows students the freedom to apply their own real life and mathematical knowledge and skills thereby allowing for several solution strategies, I could not prepare for every potential outcome. This goes against my training and preparation to teach. In my daily lessons, I consistently plan and prepare for misconceptions, specific learning deficiencies, and the differentiation needed to address various learning needs. I learned as I designed and worked through each project in the unit, no matter how much I prepared ahead of time, students would always come up with ideas I had not thought of. I essentially would be attempting to teach mathematics I have not mastered (Heaton, 2000) as students and I would be sharing the expertise needed to successfully navigate the modeling process and complete each task. And I had to realize this was okay. I would be learning, too.
**Adjusting to this new role.** Preparing to share the role of ‘expert’ in my classroom was philosophically challenging. As a teacher, students ask me questions in hopes I can guide them to the correct answer or strategy. If I do not have a good response, it is embarrassing. I fear it diminishes my reputation with students because they want reassurance I know what I am talking about. As I developed the modeling unit, however, I needed to prepare for ways in which I could guide students to their own conclusions. Instruction could not be designed around my previous role as the mathematics expert. Students and myself needed to view new knowledge as being generated from the real life and mathematics expertise of each other. As Blum and Ferri (2009) suggest, teachers should plan for *independence-preserving* responses and interventions to assist students during the modeling process. I needed to prepare to assist students in ways that allowed their ideas and expertise to be the focus without being dependent on me as the teacher. By doing this, I could create an environment where students can experience mathematics as “dynamic, growing, and created by people” (Smith III, 1996, p. 357).

In my planning documents, I tried to pinpoint what aspects of the modeling cycle I thought students would have the most difficulties. I would then list out specific questions I could use to help students as they got stuck which were designed to help students elaborate on their own ideas. For example, in my preparation for Project 3: Enrollment at River Junction Senior High, I listed out the following questions I could ask students:
1. What do you think future enrollment will be based on in real life? Don’t look at the data, just think logically. What determines high school enrollment now? What will determine enrollment in the future?

2. What data do you wish to focus on? Why?

3. What are some mathematical features in the data you are using? What trends or patterns do you notice? What type of function would best fit this data do you think and why?

4. Do you think it is more useful to look at grade level growth or what happens to a class as students move from kindergarten to 12th grade? Why?

5. Do you have all the data you need to develop your model in the way you plan to? If you don’t have all the data, is it actual data or predicted data? How can you gain access or predict this data if you need it? (Planning Documents, February 20, 2016)

Most questions like the ones listed here were generated as I worked through and reflected on each task. In my planning, I tried to anticipate potential problems before they happened. But because I would not know the specifics of the problem I could only prepare for general cases by using broad questions that might help students navigate the modeling cycle. However, by anticipating potential student ideas I could structure my own responses in hopes of providing more enriching mathematical discussions in class (Stein et al., 2008).

**Over-preparing.** The need I typically feel to be the expert in the classroom caused me to potentially ‘over-prepare’ for the first few projects. In my planning, I tried
to develop as many ways to approach the task as I could think of, so I could more easily pinpoint sources of error in student’s work and guide them through their misconceptions. In reflecting on this now, I think this greatly diminished my ability to adequately teach modeling during the unit, as my questioning tended to guide students to what I thought they should do rather than allowing students opportunities to figure it out on their own. Thankfully during my planning, I gradually released this need to be the only expert in the room, realizing it was impossible to prepare for all possible outcomes. By trying to prepare too much in advance I lost the authenticity of the task, and I needed to prepare both myself and students for ambiguity they would encounter as they developed their final student choice task for Project 5.

_Shifting mindset._ In my preparation for Project 4: How Safe are We?, for example, I began to realize the significance of viewing learning and instruction through shared expertise of teacher and students. This shift in mindset also resulted in a shift in my planning. I decided not to develop specific models for this task and wrote in my planning journal:

I have not tried to dissect this task too much to try and determine as many different approaches as I can. I want to struggle through this task a bit with students and not be as much of a “teacher” but a “guide” to help students by asking questions to get them thinking about what they might be able to do. I also need to prepare for the Student Choice Project by not having all the ‘answers’ ahead of time. I cannot be prepared for what students might pick or decide to do
on the final project where they pick the situation. I will be going in blind.

(February 28, 2016)

In my preparation for a month of modeling, a lack of experience modeling myself caused me to try to prepare in the same ways I had always known: working tasks out ahead of time and thinking about all possible outcomes and misconceptions. My need to be the only expert caused me to doubt my own ability to teach mathematics in such an open-ended manner. Once this perception changed, however, my preparation could focus less on outcomes and more on the process thus forcing my teaching during the unit to do the same.

Resorting to old habits. Admitting to myself and my students I could not be viewed as the keeper of knowledge was difficult but essential for the development of modeling abilities. I would need to rely on my mathematical content knowledge in the moment to adequately assess and guide student needs and misconceptions. In addition to this mathematical awareness, I had to plan and stress to students I was not an expert in the real life field we were studying in each task. As is discussed later in this chapter, having adequate real life knowledge about each situation seems to be important for successful navigation of the modeling cycle. During my planning in the design phase of this unit, I had to do additional research on aspects of each situation in order to ensure I was providing students with accurate information.

Unfortunately, there seems to be a fine line between providing too much real life information, however, and not enough. As indicated in Chapter 4 and Chapter 5, my need to be the expert and over-prepare funneled students into particular strategies. I
unconsciously resorted to old *folkways* of teacher knowledge out of habit for being the *expert* (Buchmann, 1987). Particularly in the design of Project 4: How Safe are We? (see pages 164-180), I tried to make myself and students experts in an area I thought they had little background knowledge about, yet I did not allow them to become experts on their own. By restricting their data analysis to reported crimes for only one year, I decided what information was necessary when I should have allowed students to make the decisions. By resorting to old *folkways* (Buchmann, 1987) as the information provider, I inadvertently lessened the cognitive demand and potential for the development of problem-solving skills (Borko et al., 2000). Looking back now, and especially to prepare students for the final student choice task, I needed to have valued student expertise rather than force my own. It is more authentic for students to develop and seek out information as they realize a need for it.

**Reflections on My Preparations to Teach Modeling**

Cross (2009) found that a teacher’s beliefs about the nature of mathematics proved to be a principal source of influence on their beliefs about teaching and student learning. As a result, preparing for a unit on modeling proved to challenge fundamental beliefs and routines in the way I typically planned mathematics instruction. I had to broaden my own knowledge base on modeling, curriculum design, and assessment in order to more closely align to the new mathematics and subsequent learning goals I would be asking students to develop. I eventually realized I could not be looked upon as the expert with the answers in the classroom but a coach ready to guide students down the path they created for themselves. It was difficult to get away from this mindset and I was
not always successful in doing so, but as I got more comfortable with the modeling process as a learner, I could more adequately prepare for it as a teacher. Planning for modeling in a broad sense proved to be essential, yet the importance of quality teaching in the moment cannot be overlooked, and is the focus of the majority of this chapter.

**Changing Perception of Student Needs**

In my preparation to teach a unit on modeling, I initially anticipated students struggling with the ambiguity and openness of the mathematics required. I knew they were used to specific directions in their mathematics tasks, delineating the skills and strategies needing applied. As I designed the unit, I built in scaffolding in the form of a graphic organizer with general prompts to help students navigate the modeling cycle (see Appendix P), guiding questions specifically for Project 3: Enrollment at River Junction Senior High and Project 5: Student Choice (see Appendix Q) to help students navigate the ambiguity of those tasks, and a list of potential topics students could use for the student choice task in Project 5. Although a few students used the questions on the graphic organizer as a guide for team conversations, the majority of students did not ask for the graphic organizer, and no student needed the guiding questions for Project 3 or the additional list of topics for Project 5.

The scaffolding students ended up needing throughout the unit changed dramatically from the initial design. Looking back on this now, I am thankful instruction ended up looking different than my design, as I am afraid the guiding questions would have forced students into a particular strategy and contradicted design criterion four: *tasks encourage students to express their own thinking, ideas, and justifications and*
critique the reasoning of others through the modeling process and in the final product.

Students did not seem to need scaffolding focused on the mathematics or task itself, but rather needed support and instruction on how to function in a student-centered environment. Opportunities for meaningful mathematical conversations was heavily influenced by what I as the teacher did to structure these opportunities (Loef Franke, Kazemi, & Battey, 2007).

**Encouraging Teamwork**

As discussed in Chapter 5: Relearning How to Learn, students had to relearn how to learn in an environment focused on student-centered instruction and discourse. Because of this, I had to relearn how to teach and recognize student needs in this new environment. These new needs were not ones I had planned for, but as the unit progressed and conflicts emerged, it became apparent additional scaffolding and support would be needed to help students learn to communicate and navigate changing classroom expectations. Specifically, I ended up providing targeted support to encourage communication, discourse, and successful completion of goals within each team.

**Providing opportunities to communicate.** Because students would be working entirely with each other for several days at a time, a few days into the unit I decided to have teams answer specific discussion questions activating their prior knowledge, discuss current ideas, or reflect on their learning at the beginning of each class period as opposed to letting them jump into their work immediately at the start of class. Sometimes this discussion would simply be a share-out of the progress each student was making or a deliberate opportunity for students to ask questions of each other at their team.
For example, on the first day of Project 3: Enrollment at River Junction Senior High, each student first had to share out to their team (1) “what’s one thing I learned” (from Project 2: Calculating the Width of Memorial Stadium) and (2) “what adjustments do I need to make for Project 3?” (March 28, 2016). During these structured discussions, each student would be required to talk as students rotated around their groups, thereby allowing each student to participate, listen, and give feedback. These discussions also provided me a way to listen and gauge student progress towards successful navigation of the modeling cycle and further anticipate sources of student conflict.

**Daily goals and expectations.** Also to encourage teamwork, each day I wrote and discussed team goals for that particular class period and stressed a timeline of daily events needing to occur if presentations for each task were to be completed on time. For example, at the beginning of Project 3: Enrollment at River Junction Senior High, I wrote out daily goals leading up to the day administrators would listen to presentations. On the first day I told students their goal was to play around and make sense of the data. By the second day they should be developing a usable, mathematical model and enrollment prediction, and on the third day they would work on interpreting their results and creating their presentation which would be conducted on the fourth day (March 28, 2016). I wrote similar timelines on the board to discuss throughout each project. In addition, when teams began preparing for presentations and communication of their results, I wrote out and discussed a more detailed outline of expectations.
With so many new learning goals, I realized students needed constant visual and auditory reminders about expectations for learning outcomes. For example, as teams prepared Project 2: Calculating the Width of Memorial Stadium presentations I wrote on the board students should:

1. Explain the relationships you used
2. Communicate what your results mean about the situation (what the distances and angles represent in real life)
3. Describe how you know your answer makes sense
4. State any assumptions you made or things you tried that didn’t work like you thought they should. (March 25, 2016)

Up until a month of modeling, students were comfortable working with their peers on tasks requiring only several minutes to complete while only trying to obtain a single correct answer. Now I was asking them to work on a task for several days at a time, while completely relying on each other to develop an accurate mathematical outcome in addition to communicating and justifying their results. Providing daily goals and presentation outlines seem to be basic, but were necessary to address student needs associated with reaching a common long-term goal. During her interview, Samantha commented how helpful this support was in keeping her team focused on the overall goals of each task. She stated:

I liked how you had a lay out of ‘you should probably get this done today and this done today.’...Like in English we have to get all this stuff done but we didn’t know if we should do this today or should everyone work on something different
today and it never works out like that...Then we knew if we needed to do more things outside of class or in class. (May 17, 2016)

Samantha and Team 2 found the daily goals helpful in making sure her team stayed on task and had enough time to get everything done that was required. During the unit, I realized quickly I could not simply give teams a modeling task with three class periods to work and tell them ‘here you go.’ I needed to provide additional structure and scaffolding to help students communicate amongst themselves and stick to the expectations and timeframe I had set for each project.

**Student-to-Student Discourse**

Additional unplanned scaffolding and support was needed as students engaged in discourse with other team members. Wagner, Herbel-Eisenmann, and Choppin (2011) stress the importance of providing opportunities for students to communicate mathematical ideas, but caution that teachers should structure equitable discourse so students use language with a purpose of deepening knowledge of mathematics content. Somehow, as the authors suggest, I needed to provide opportunities for students to see discourse as essential to the learning of mathematics.

As discussed in Chapter 5, students had to learn how to critique and provide feedback to other teams, particularly during the presentation phase of each project. After the first two projects, I was frustrated by the lack of quality feedback and student generated-questions being provided. During presentations for these first two projects, most questions and feedback were teacher-driven. As we progressed through the unit, I tried to shift this communication to be more student-driven by including the use of sticky
note questions in Project 3: Enrollment at River Junction Senior High and more specific graphic organizers for Project 4: How Safe are We? and Project 5: Student Choice (see Appendices R and S).

**Use of graphic organizers to encourage student discourse.** After each presentation for Project 5: Student Choice (see pages 180-208), I gave one minute for students to discuss with their team the analysis just described to them before participating in a question and answer session with the presenting team members. Both during the presentation and this team discussion, students had to write down on their graphic organizers responses to: “What other knowledge or factors might have been good to consider in this analysis?” and “1 Question you have” (April 13, 2016). Compared to the first few projects, students showed exceptional growth towards their understanding of the modeling process in this final project as they were critiquing and trying to clarify their own interpretations of the work being done. During presentations for Project 2: Calculating the Width of Memorial Stadium, student feedback primarily consisted of positive statements like “Good job” or “nice work.” For Project 5, however, as described in Chapter 4, student questions were focused towards understanding where the data used came from and why the team made certain decisions regarding the analysis. In this final student choice project, I think the questions overall were also improved from previous projects because students were seeing the situation and analysis for the first time. Because they themselves had little knowledge of the situation, data used, or models being developed, students picked up on missing information or unclear remarks because they had to pay closer attention and internalize and reflect on the methods teams were
presenting. Although these communication improvements are probably credited to many factors, the graphic organizer seemed to provide a structured platform for students to reflect on each presentation and engage in student-to-student discourse.

**Reflections on Support Needed**

Scaffolding and support needed throughout a unit on modeling was primarily focused on helping students learn how to function in a student-centered environment. From a mathematical or task-specific component, students did not seem to need the scaffolding I had planned. As the unit unfolded, however, I needed to plan for and develop support based on changing student needs related to positive and productive teamwork and discourse. Apparently throughout the school year, I had not established the environment necessary to truly allow for the sharing of student ideas and needed to now create these new classroom spaces and norms (Walshaw & Anthony, 2008). I had to focus energy on the “management of ideas within classroom discourse” (Shulman, 1987, p. 1) so student explanations could “stimulate, challenge, and extend other students' thinking” (Walshaw & Anthony, 2008, p. 539). It seems the new learning environment, and not the new mathematics, warranted specific scaffolding. These needs related to our new student-centered environment were not ones I had initially designed for or anticipated.

**Activating Students’ Real Life Knowledge**

Baranes, Perry, and Stigler (1989) argue, “In its most successful moments, school enables children to access the wealth of real-world knowledge that they have accumulated outside of school” (p. 317). I personally have had few opportunities in my
previous teaching experience to activate students’ real life knowledge. Applying prior math knowledge happens on a daily basis, but rarely have I needed to focus conversations in math class on students’ physical intuition (Hestenes, 2013). I understood from my own research on modeling, this knowledge from the real world domain was an important component to the modeling cycle, and thus warranted inclusion as the first design criterion for my curriculum: *task generates a need for students to translate and connect knowledge of real life situations and experiences to knowledge of mathematics.* However, during the month of modeling it became much more obvious teaching to this real life knowledge is crucial for the development of modeling abilities. And if students do not have this knowledge to draw upon, they need opportunities to research and build this knowledge base (Fisher, Fray, & Lapp, 2012). Without adequate information from the real world domain, students struggled developing mathematical literacy skills as they recognized authentic material to mathematize and quantitative literacy and reflection skills as students assessed the reasonableness of their outcomes. As the unit progressed and I began to see its significance for successful completion of the modeling cycle, I had to relearn how to teach as I attempted to activate students’ real life knowledge in order to develop these skills.

**To Develop Mathematical Literacy**

Consistent with results of PISA studies (see Fleishman et al.; OECD, 2013a) and as indicated in Chapter 4, throughout the modeling unit students seemed to struggle initially applying skills related to mathematical literacy as they attempted to recognize appropriate mathematics and numerical relationships present in an authentic situations.
However, I quickly learned teaching to mathematical literacy would require students to activate knowledge in the real domain and take a considerable amount of time. That is, I could not rush the process. Originally I planned on both the first two projects (The High-Definition Trade-Off Dilemma and Calculating the Width of Memorial Stadium) taking only three days: two days to develop the model and one day to communicate results to other teams. As students were trying to make sense of both projects it became clear an additional day would be needed. In fact, for all projects I ended up building in an entire class period just for students to apply their real life knowledge and transfer this into mathematical knowledge through the use of mathematical literacy skills. Allowing students the time to engage in productive struggle by discussing, researching, and questioning their own real life knowledge seemed to be immensely helpful in mathematizing authentic situations for the development of models and the variety of strategies students employed. This finding is consistent with research literature suggesting that allotting appropriate amounts of time effectively is shown to influence student engagement in and successful completion of high-level cognitive tasks like those used in the modeling unit (Doyle, 1986; Henningsen & Stein, 1997).

**Project 2: Calculating the Width of Memorial Stadium.** In Project 2: Calculating the Width of Memorial Stadium (see pages 139-152), I was surprised by how frustrated students initially were trying to mathematize and create a reasonable drawing from the video and measurements I had shown. However, I tried not to rush the process and allowed students the time to adequately assess their background knowledge of the situation and bring in their own ideas to process the information. I found that even
though it took most students one class period to feel comfortable with their drawings, there ended up being a lot of variety to what students brought in and I did not want to restrict students into a particular strategy.

For example, Team 1 gathered around a mobile markerboard I had brought in specifically for their use and were standing around it drawing on each other’s work. George at Team 2 had drawn a picture of Memorial Stadium on a small markerboard while Noah pulled up the video I had originally shown of me taking measurements. George then held the iPad with the video in front of the drawn stadium to help him and his team properly situate where I was in relation to the stadium. Chris at Team 6 drew a scale drawing of the situation using a protractor and ruler to help his team visualize the location of measurements taken. Dylan at Team 5 got out a computer and pulled up the city the stadium is located in on Google Maps to orientate where I was at in relation to the stadium. His team then printed off a picture of Memorial Stadium and drew lines to mimic my location on the digital map they were referencing. Students developed all these ideas on their own without me prompting any recommendations. I gave ample time to let them struggle through, determine what clarifying questions they needed to ask me, and develop the picture themselves. The time was well worth it, as I think it gave students a better grasp of the situation and relationships they could use within the picture. Students did not seem to have any trouble communicating to me why the picture would look that way once they settled on a drawing and what geometric relationships they would need to apply.
**Lack of real life knowledge as a barrier.** When students lacked background knowledge about the situation, they had a difficult time mathematizing the situation. Some students, for example, lacked common knowledge about where the parking lot I used in the video was in relation to the stadium. Most of the clarifying questions students asked were in reference to where I was standing. This surprised me, as you could see the front of the stadium in the video, and I assumed students would know this location as it is a popular landmark in our state. For those teams I used the interstate as a reference, which is a landmark they would recognize, and showed a line driving from River Junction east to the stadium’s location. I then showed students the location of the lot on the west side of the stadium. In reflecting on these conversations I wrote in my journal:

I learned that when students have to overcome barriers related to mathematical literacy it might be due to a lack of knowledge in the ‘real domain’ - their real life knowledge did not allow them experiences from which to draw upon to help them understand features of the authentic situation. (March 26, 2016)

I incorrectly assumed that the background knowledge students would need to visualize the task was common knowledge. Students who had no experiences with the stadium to draw upon had a difficult time visualizing and translating the video into geometric representations. In my conversations and instruction with students, I needed to provide additional background information or reference aspects of the situation students might have common knowledge about in order for them to apply mathematical literacy skills.

**Project 3: Enrollment at River Junction Senior High.** When I designed Project 3: Enrollment at River Junction Senior High, I was initially worried about the size of the
data set students would need to process in order to mathematize patterns and create models to predict future River Junction Senior High enrollment. However, because the design of the task was rooted in a situation students had prior knowledge about, they could bring in this real life knowledge to explain and enhance relationships within the data. For example, at Team 6 students immediately began noticing trends and patterns in the data. Monica exclaimed “That’s a lot of kindergarteners!” (March 28, 2016). Monica also noticed a drop-off between 10th and 11th grade, but had not yet thought about why that might be. Evan was not reading the data set correctly as he was trying to follow one particular group horizontally along the data table. He was looking at all the grades in a particular year, and noticed the classes decreased in enrollment as the grade level went up, but he was incorrectly describing it as the same group of kids as they progressed through school. He was trying to explain this to his team when I walked by and asked:

*DB*: Do you agree with Evan?

*Several Students*: No

*Chris*: No you go this way. (*pointing diagonally*)

*Evan*: Oh that’s right.

*Megan*: We don’t stay in 1st grade. (*laughing*)

*Evan*: It follows the same pattern though.

*Megan*: But the kindergarten class every year grows. (March 28, 2016)

This team then tries to determine why the class size would be decreasing. Evan suggests some could be because of student dropouts. Monica comments that some students might move out of the district. Then Evan describes how River Junction would lose some
eventually to other private schools in town that do not offer elementary programs. Chris points out the drop in numbers from 8th to 9th grade with Evan and Megan correctly asserting that some of that is due to students transferring to a rival high school in our town that is not considered a part of our school district.

**Real life knowledge leads to shift from teacher to student.** These students, like the majority in the room, were bringing in their own knowledge of the community to explain trends in the data they were noticing. I wrote in my reflection journal:

Students were choosing to dig into the data before I was even done with the directions! They wanted to get going, and they didn’t feel like they needed me to get started. Someone once told me they knew they were a successful teacher when their students no longer needed them. I didn’t really feel needed much through this task like I thought I would be, and I couldn’t be happier about it.

(April 2, 2016)

As students analyzed the data for Project 3, I felt the focus shift from a teacher-centered environment to one run by students. In the previous projects and conversations, students were generating ideas but it took a lot of prompting and questioning on my part. In this third project, I got the sense from students they were beginning to feel more confident in their own ideas and willing to dig into those ideas without my validation or approval. Perhaps this is also because of their own real life knowledge about the situation. No students requested the additional guiding questions I had created nor did I feel any team needed them to get started with their analysis, possibly because mathematizing the
situation became easier once students were able to bring in their own knowledge of the situation.

**Project 4: How Safe are We?** I again asked students to analyze a large data set in Project 4: How Safe are We? (see pages 164-180) to create a methodology which would assign a safety score and set of rankings for cities in our state. Similar to Project 3: Enrollment at River Junction Senior High, students were using their own real life experiences to make sense of the data on the FBI’s database. By using those experiences, some students were beginning to think about how they might apply mathematical literacy skills in order to find trends in the situation and create a model. For example, Noah was wanting to weigh and consider violent crimes more. He said “If you want to look at how dangerous an area is the first thing I look at is how many murders are there? How many rapes are there? How many aggravated assaults are there?” (April 1, 2016). The new members of Team 1 now with Evan and Chris immediately began exploring the data to determine why Loganview was so high on the list of most dangerous cities I began class with. Evan noticed Loganview had 130 violent crimes, compared it to our city of River Junction which had 131 violent crimes, and exclaimed “Loganview is a dangerous city! They have like way less than half our population and only 1 less dangerous crime.” Chris replied with “Dude, that’s crazy” (April 1, 2016).

As with all teams, as students dug into the data more they began generating ideas which would lead to their methodology. For example, Evan suggested to his team they may want to consider effects of populations or crimes over time. Chris offered it may be
important to take into account the severity of the crime in their final model, which is an idea members of Teams 1, 2 and 4 decided to pursue further.

**Students expand real life knowledge.** As students continued to analyze information from the FBI database, some teams were doing additional research to expand their own background knowledge on crime reporting. For example, Luis from Team 5 called me over to ask about the reasoning behind taking the number of crimes, dividing it by the total population, but then multiplying it by 100,000. I asked him why he thought they multiplied by 100,000 and he didn’t have an answer. I then took his iPad and asked what would happen if we took the number of violent crimes for River Junction (131) and divided it by our population (50,999) and asked, “what would the number look like?” Elijah stated “It’s going to be really small” (April 1, 2016). But I wanted to know how small, trying to reference the actual appearance of this number. Luis grabbed his calculator and calculated the value of this and got 0.002568. I then grabbed his calculator to hold it up to his team and asked:

\[ DB: \text{What does this number tell you?} \]

\[ \text{Luis: The number of violent crimes per population?} \]

\[ DB: \text{Per what then, the number of crimes per?} \]

\[ \text{Luis: Per person} \]

\[ DB: \text{They literally times it by 100,000 for convenience. You are going to have write out this number all the time. You are going to have to write out 0.002568. You can choose to do it that way, there is nothing wrong with that. The reason they multiply by 100,000 is because you get a number} \]
that’s just easier to write. Because you are saying here, you have 256.8 violent crimes per what?

_Luis:_ 100,000

_Ryan:_ Oh, okay.

,DB: So the way you communicate that information is different.

_Luis:_ Should we round it?

,DB: You can decide however you want. It is totally up to you. This is really honing in on, how are you going to persuade us? You just have to be able to back up the decisions you make.

_Luis:_ We were just wondering because River Junction doesn’t have 100,000.

,DB: You’re right. It’s scaling it up and creating an equal playing field for everyone. (April 1, 2016)

We then get into a conversation about how when we do this, we assume crime will continue at the same rate, which is discussed further in Chapter 7. As these students processed this conversation later, however, they started to adapt this new background knowledge to their own needs and understanding. They felt more comfortable scaling the major cities down, since it bothered them to report crimes that essentially did not exist by nearly doubling River Junction’s population and crimes reported. As a result, when reporting crime rates they decided to multiply by 10,000 and report crime as the amount per 10,000 as opposed to 100,000 people. The did this because the smallest city they analyzed had 12,000 people, so by scaling all cities down to 10,000 they were not creating crimes that had not been reported.
Reflections on using real life knowledge for mathematical literacy. As in previous projects, this productive struggle and application of real life knowledge took a considerable amount of class time. In relearning how to teach, I had to provide the time, tools, resources, and opportunities for students to bring in available and research unavailable information in the real domain. Students do not necessarily need to be experts in the field, but without access to accurate physical intuitions, starting the modeling process may have been out of reach for some students. For Project 5: Student Choice, when students had to develop their own situation to model, students spent two to three class periods simply narrowing down and researching a topic to acquire enough background knowledge where a model could be possible. Without me realizing this step as important to the modeling process and teaching to this type of knowledge, the student outcomes of each task would most likely have looked vastly different.

The use of real life knowledge towards the development of mathematical literacy cannot be understated during the modeling unit, but I could find no research literature which also emphasizes this role. During a month of modeling, I had to learn to let the progression of these skills play out naturally from a student perspective. Students had to decide what real life information they needed and if there were any gaps in their background knowledge requiring additional clarity in order to answer questions related to mathematical literacy: where do I see math here and what are the essential features of the situation? As Chris stated in his interview, the aspects of modeling that most stuck out to him were “having to find out what exactly you need to do. The information you need prior to finding the model. Being able to sort the information you don’t need or you do
need” (April 27, 2016). Despite my desire to keep to my original schedule and timeline, I realized the significance of giving adequate time to process real life knowledge towards successful completion of the modeling cycle and thus needed to provide opportunities for this knowledge to be utilized both in the design of the task and in my instruction in the classroom.

To Develop Quantitative Literacy Skills and Reflection

In Chapter 5, I discuss new student learning associated with validating results of a modeling analysis. Research has found that focusing on the importance of real world knowledge and sense making in modeling and interpretations of word problems can help students develop tendencies towards realistic problem solving (Verschaffel & De Corte, 1997). As students transitioned from needing my approval to creating their own opportunities for learning during a unit on modeling, students had to apply their own knowledge to develop skills related to quantitative literacy and reflection, particularly in order to assess the reasonableness of their mathematical results. In order to avoid “mindless calculations” which do not take into account “real life aspects of the situations described in the tasks” (Palm, 2008, p. 38) my design and instruction had to provide opportunities for students to apply real life knowledge about each situation. Students needed to ask: are the results reasonable based on what I know about the situation, and to do this students needed to know something about the situation. Recognizing this need during my planning, I also included this aspect of modeling in design criterion three: Students are able to use the task’s information and goal(s) to assess reasonableness of analysis (model) and if necessary refine the model. Thus, having real life knowledge
about the modeling task helped students initially as they applied skills related to mathematical literacy and then later in the process as they translated the mathematical results back into real life information through quantitative literacy and reflection.

**Determining what’s reasonable.** In Chapter 5, for example, I describe how I asked students to predict a reasonable width for Memorial Stadium in Project 2: Calculating the Width of Memorial Stadium (see pages 139-152) before students had conducted any sort of mathematical analysis. This instructional tactic forced students to apply only their real life knowledge, allowing them to generate a reasonable range they could later use to validate their model. The real life knowledge students had about football stadiums, and my instruction providing opportunities for students to deliberately think about and bring in this knowledge, helped students as they assessed the reasonableness of their results. In many cases, students found mistakes in either my measurements, which is discussed in Chapter 7, or their own work, causing them to modify their models. Based on the success of this instructional tactic, I would activate this prior knowledge at the beginning of Project 3: Enrollment at River Junction Senior High and Project 4: How Safe are We? by asking students to predict or explain what was reasonable before a mathematical analysis had even occurred.

**Lack of real life knowledge makes assessing reasonableness difficult.** When students lacked adequate background information about the authentic situation, it was hard for them to assess the reasonableness of their situation (see Palm, 2008). In his interview, Luis stated this as the aspect of modeling he enjoyed the least. He commented:

*Luis:* Probably the part I liked the least was being sure what you had done was
right. Because we didn’t have, there was no set way to it. So after we got
done using the model and we’re not exactly sure if it is right or not.

*DB:* So not having some perspective maybe or knowledge of the situation to
know if it is reasonable?

*Luis:* Yeah. (April 28, 2016)

Luis here eludes to a desire for more information when he states “we’re not exactly sure
if it is right or not.” Perhaps having some real life knowledge about the situation is
helpful as students complete the modeling cycle and reflect on their work. It allows them
to assess reasonableness and reflect on the potential inaccuracies of their analysis.

*Needing to expand real life knowledge.* As students transitioned from viewing
mathematics as right or wrong to one with multiple solution strategies as discussed in
Chapter 5, I had to be careful to provide opportunities through the activation of real life
knowledge for students to validate their own work. This proved to be especially
challenging in the final student choice project as the situations students selected were
outside of my control and my own real life knowledge. For example, in Chapter 4 I
discuss the mistakes Team 2 had made in the development of their model to estimate the
blades of artificial turf on the football field (see pages 191-194). I thought at the time
students had no background knowledge about turf grass to understand if their results had
been reasonable or not, which may have allowed them to notice their mistakes prior to
communicating their model to the class. During her interview, I asked Samantha about
her perception of what happened in Project 5. She stated she wished she “had more
information. I didn’t even know if I was right with the turf....Especially because we gave
you all those (limitations), well it could be torn up or this could happen” (May 17, 2016).

Through the design and instruction of the first four projects, Samantha realized the importance of having background knowledge to draw upon as she applied skills related to quantitative literacy and reflection. Samantha recognized the lack of knowledge she and her teammates had about the situation was a disadvantage as they closed out the modeling cycle and were unable to adequately validate the results of their analysis.

Looking back now, I wish I had stressed the activation of this real life knowledge at the beginning of Project 5: Student Choice like I had done for the previous three tasks. As I had expected, trying to provide support for seven groups analyzing seven different situations proved to test my classroom and time management abilities, causing me to perhaps overlook this essential component to the modeling cycle. In my instruction, I should have provided time for students to research their topic and set a concrete goal for their analysis, then come back and have them discuss what would be reasonable before allowing them to complete a mathematical analysis. This aspect of the modeling cycle was overlooked in my instruction; without adequate time to develop intuition about each situation, students did not have the real life knowledge needed to properly assess and validate their conclusions. This may have caused students like those in Team 2 to make mathematical errors they otherwise might not have.

**Reflections on the Need for Real Life Knowledge**

De Lange (2003) suggests that in order to develop mathematical and quantitative literacy skills, mathematics “should not only be connected to other mathematics but also be embedded in the real world of the student” (p. 87). Realizing the need for students to
apply real life knowledge throughout the modeling process in both the design of authentic tasks and daily instruction was significant learning for me in my preparation and teaching during the month of modeling. This real life knowledge provided additional opportunities for students to apply skills related to mathematical and quantitative literacy as students answered: where do I see math here, what are the essential features of the situation, are the results reasonable based on what I know about the situation, and what are potential inaccuracies of my analysis? Relearning how to teach when real life knowledge seems to be just as important as mathematical knowledge may not necessarily be difficult, but can easily be overlooked and understated. When students had adequate time and opportunity to apply known physical intuitions and research unknown information in the real domain, their mathematical outcomes seemed to be more complex, accurate, and interesting. When my instruction or design limited the opportunities for students to apply this everyday knowledge, completing the modeling process successfully seemed to be substantially harder.

**The Role of Teacher Questioning**

I hypothesized going into a month of modeling, the role of teacher questioning would be crucial for instruction. During my planning phase for each task, I wrote out general questions to guide students through the modeling process in places I thought they might have difficulties navigating. Going into this unit, I thought I had excellent questioning skills and would teach daily lessons through constant questioning of students’ ideas and prior knowledge. However, as I analyzed recordings of my teaching on a daily basis during the modeling unit, I realized my questioning seemed to funnel
students into particular ideas as opposed to focusing on their ideas (Herbal-Eisenmann & Breyfogle, 2005). The realization I perhaps was not as good at questioning as I believed was a tremendous disappointment after the first week of instruction. It quickly became apparent to me I would need to relearn how to teach by redirecting the intent of questions during the modeling unit.

From Funneling to Focusing

Herbal-Eisenmann and Breyfogle (2005) describe common questions mathematics teachers use to elicit responses from students. They encourage teachers to transition from funneling questions designed to “guide students through a procedure or desired end” to focusing questions aimed at listening and guiding students through what they are thinking rather than what the teacher thinks they should do (Herbal-Eisenmann & Breyfogle, 2005, p. 485). As I watched recordings and reflected daily on my teaching during the first project of the modeling unit in particular, I realized I funneled students into particular ideas and strategies too quickly. I inadvertently was forcing my own ideas onto students about their models when I should have been focusing conversations on student ideas. In their studies of teacher responses to modeling tasks, Blum and Ferri (2009) also found that teachers, sometimes even unknowingly, imposed their own preferred solution onto students during conversations when they should instead find an optimal balance between “maximal independence of students’ and minimal guidance by the teacher” (p. 54). In contrast to how my instruction played out in the first project, I desired to ask probing questions which might help students discover potential solutions and answers on their own (Kersaint & Chappell, 2001). During a month of modeling, by
transitioning from funneling to focusing questions I hoped to better provide students opportunities to develop modeling abilities related to flexible and creative thinking and reasoning as they developed, carried out, and justified their own, independent strategies.

Questioning in Project 1: The High Definition Trade-Off Dilemma. At the beginning of Project 1: The High Definition Trade-Off Dilemma (see pages 121-139), the majority of students initially processed mathematical relationships between recording times and resolutions on Heather’s GoPro by guessing and checking specific values. I was pleased most students thought to do this, but I too quickly wanted them to develop specific equations from these relationships. I funneled students into a particular strategy rather than allowing them to develop these relationships for themselves. For example, on the first day of Project 1, I was working with Samantha, Jacob, Jason, and George asking them to explain the relationships they were using to narrow down recording times.

Through the course of this conversation, they explained to me they started by determining how many minutes one could get of each resolution per gigabyte of space. They then used these rates to calculate the number of minutes one could record at each resolution and continued to do this until they got as close to the total 300 minutes as they could. I encouraged them to use these relationships to develop equations:

\[ DB: \text{Now generalize this, don’t say this specifically...think of it in general. You’re taking 4.09 times the number of gigs at 1080p. That is your first variable.} \]

\[ \text{If you are continually taking this times the number of gigabytes at 1080p (pointing to each line of their guess and check table), and this times the number of gigabytes at} \]

\[ \text{1080p, and this times the number of gigabytes at} \]
1080p. Tell me in general what you are doing?

*Jacob*: 4.09 times the number of gigabytes at 1080p

*DB*: So what can you define x to be?

*Jacob*: the number of gigabytes at 1080p

*DB*: And then what did you do?

*Samantha*: Added it to

*Jacob*: add it to the number

*DB*: 5.25 times

*Samantha*: y

*DB*: And y will be

*Samantha*: 720

*DB*: And when you added these together you should have gotten

*Jacob*: 300

*DB*: You just found your first equation people. Nice work. (March 16, 2016)

In this conversation, I practically tell students what they should be thinking and doing. My questioning ended up being statements where students simply needed to ‘fill in the blank.’ Thinking about this now, it seems like I was so excited about the relationships they had discovered, I rushed the modeling process. In reflecting on this conversation later that day I wrote:

I do however need to be careful that I don’t divulge too much information or try to pigeonhole students into a specific strategy. Sometimes I am too quick to get students on a specific path or try to generalize too quickly when maybe I should
let them “play” a little longer...George at Team 2 stated once “We are supposed to have an equation?” and that made me cringe because he was right. For the first part of this task nowhere did I say they must have an equation/model. They simply needed to tell Heather how long to record in each mode. I wanted them to develop this model when I should have let it occur more naturally as they tried to generalize their work for an unknown time or an unknown GB size. (March 16, 2016)

I was disappointed in myself for the questioning tactics I utilized initially for this first project, but I am thankful I noticed how much I dominated conversations early on in the unit. By funneling students into the strategy or model I thought best exhibited their work, I restricted the development of flexible and creative thinking and reasoning. Students did not have to answer: what mathematical relationships can I use which fit the information or data? I practically answered this question for them.

As we progressed through Project 1, I continued to reflect on my own lack of productive questioning skills, pushing myself to relearn how to teach. The next day as I reflected on conversations I had with teams I wrote:

Instructionally I need to make sure I listen more! I am still too quick to interject, point out work that specific students are doing to gear students towards a specific route...I also summarize information for students that I should have them summarize themselves, whether that be restating their work, variables, or equations. Sometimes I get into too big of a hurry as I try to work with six different teams that I don’t stop to just listen. (March 17, 2016)
At this point, I began realizing in order to change the intent of questions from a funneling to focusing approach, I needed to listen more. In order for me to guide students through their own thinking and the modeling process, I would need to stop and really listen to the student voice. My questions needed to adapt to what students were thinking and communicating and not be prearranged to channel students into a particular method.

**My continued efforts to focus questions.** Throughout the unit, I continued transitioning from a funneling to a focusing questioning approach in my daily instruction. Focusing questions tended to be particularly important at the beginning of the modeling process as students developed their ideas and potential models to pursue. I needed to be sure students were pursuing relationships, and thus models they noticed, and not ones I encouraged them to develop. For example, on the first day of Project 3: Enrollment at River Junction Senior High (see pages 152-164) as students worked with enrollment data for the River Junction school district, I was checking in on the ideas each team was generating when I approached students in Team 5:

**DB:** So what are we thinking?

**Ryan:** Solve for these right here *pointing to a kindergarten through third grade block of students he had sectioned off on his data table*. Like here, then go down here where they graduated *pointing to their corresponding ninth through twelfth grade enrollment*.

**Dylan:** Like we did their four classes to their four classes *point to the same elementary to high school blocks of students*. And we’ll compare these four classes to that four classes.
DB: And how are you going to compare them?

Dylan: Like how much it dropped?

Ryan: How much it started with.

DB: Like each specific class or the total?

Dylan: The total of these four classes together (pointing again to the kindergarten through third grade block of students) to the total of these four classes (pointing again to the corresponding ninth through twelfth grade classes). Then we can predict more down here (pointing to later kindergarten through third grade blocks of students), like how much this number can drop when they get to high school.

DB: I like this, the way you guys are organizing this. It is helping you I think (pointing to their highlights and structure attempting to organize the data set and correlate the chunks of classes they are referring to). That’s a great place to start. So how can we, is everyone on board with that?

All students nod in agreement.

DB: So what’s your next steps then?

Dylan: I’m going to find out these totals for these classes here (pointing to the kindergarten through third grade classes).

DB: And then what are you doing? (pointing to Ryan and Luis) What could each of us do to collectively accomplish this goal?

Ryan and Luis are silent and seem to not know how to respond.

DB: So you are going to find the totals of each of these (pointing to Dylan), what
other information are we going to need? (March 28, 2016)

These students then go on to describe how totaling the high school grades and finding out how many students on average were lost from the elementary grades to their corresponding high school classes was additional information they needed for their analysis. In this example conversation, students applied flexible and creative thinking as they attempted to mathematize the various relationships they noticed within the enrollment data set. In Chapter 5, I described how these students then used reflection and reasoning skills to validate the model they developed using the relationships initially communicated in this particular conversation. Because I did not funnel students into a particular strategy, but rather attempted to focus the conversation on ways in which they could clarify and extend their own ideas, these students were able to create an appropriate model and unique validation method no other group thought to try.

**Reflections on focusing questions.** The freedom to pursue their own ideas, as described in Chapter 5, was a powerful motivator for students as they found the perseverance to complete the entire modeling cycle. However, this freedom would not have been possible had I not realized early on and attempted to reconcile issues I had engaging in focusing questioning techniques. The student voice in a student-centered environment while engaging in authentic tasks is crucial and is consistent with research that has found the use of more open-ended, focusing questions as having positive influences on student knowledge construction (Erdogan & Campbell, 2008). As the teacher, I needed to be sure it was not my voice but rather student ideas dominating conversations and model development.
Questions to Develop Quantitative Literacy Skills

Teacher questioning throughout the modeling unit seemed to be particularly important during the latter part of the modeling process, as students struggled to interpret the mathematical results of their analysis back into real life knowledge through the application of quantitative literacy skills. During the research phase of the modeling unit, it became apparent this aspect of modeling was crucial, and perhaps the most difficult part for students. In the fifth design criterion I developed for my curriculum, I state: *tasks require students to communicate the results of their mathematical analysis in context of the original real life situation.* Yet, designing a task to allow for this opportunity, and helping students understand how to do this, is an entirely different story.

**Project 1: The High Definition Trade-Off Dilemma.** To help students develop quantitative literacy skills, I knew I would need to ask questions like “What do your variables mean?” or “What information does your model communicate to someone?” However, there were two particular aspects of Project 1: The High Definition Trade-Off Dilemma in relation to quantitative literacy that frustrated students: understanding recording rates would remain constant and recognizing when models actually communicated gigabytes used as opposed to time.

**Constant recording rates.** First, in reflecting on their mathematical results students could accurately communicate how the memory card size or recording time affected their equations, but they had trouble describing why the rates would not change as the memory card changed. The following conversation occurred with students in Team 2 and highlights the struggles students were having:
DB: Now why didn’t the 4.09 and 5.52 change?

Samantha: It was from the total time from GoPro.

George: It was per gigabyte. It told us how many minutes per gigabyte you could record.

Noah: In all actuality it could change.

DB: What would it change?

Noah: If you change the gigabytes you had on your card, say you had a 32 gigabyte, you would divide the total time by that. You’d get a different number.

George: It shouldn’t change.

DB: Did you try it?

Noah shakes his head ‘no.’

DB: Okay grab your calculator.

Abigail and Noah grab their calculators.

George: It shouldn’t change though.

DB: George says it isn’t going to change, Noah says it is going to change so let’s see. So if I now have a 32 gigabyte card, what’s the number on top going to be? (referencing the work they had done to determine their minutes per gigabyte rate)

Noah: The number on top is the time converted from hours.

DB: But, if I have a 32 gigabyte card

George: You couldn’t record as long.
Noah: Oh you’d have different durations.

DB: You couldn’t hold 262 minutes anymore. So if I have a 32 gigabyte card how many minutes could you hold (at 1080p)?

Noah: Half that, 131.

DB: So you’d take 131 and do what?

Noah: Divide it by 32. It wouldn’t change.

DB: So how do I explain that to somebody without using technical math?

Samantha: Because you are dividing both by two.

DB: You could divide both by two.

Abigail: When you have a bigger gigabyte you have more time on it so if you have a smaller gigabyte you have less time on it.

George: No you don’t.

Samantha: Yes huh

George: The gigabyte is the same size.

DB: She was talking about the total memory card.

George: Well you have to be more specific.

DB: But George is hitting on what you would communicate though. The average person is not going to want to know ‘well you could repeatedly divide by two’ and use proportions; that’s too complicated. So George is saying that one gigabyte (pointing to George to respond)

George: Is the same. One gigabyte on a 64 is the same as one gigabyte on a 32.

DB: Exactly. Good thinking George! (March 18, 2016)
George’s background knowledge here about the way the technology worked help him correctly assess the size of the memory card would not matter. One gigabyte is one gigabyte and would get the same amount of time recorded no matter the overall size of the memory card. In order to help students come to this conclusion, I questioned and encouraged students to think about their analysis from an outsider’s perspective. Using questions like “how do I explain that to somebody without using technical math” encouraged students to think about how their analysis could be communicated to non-mathematicians especially as they prepared to write their letters to customers of GoPro.

**Recognizing when models communicated gigabytes used.** Another issue with quantitative literacy arose in teams who had defined their variables to be gigabytes used at each resolution. Because of the way these teams had set up their model, the model ended up communicating the number of gigabytes that would be used for each resolution for a particular time and memory card. Yet, some students were communicating to me their model was calculating the number of minutes, as that is the goal of the task, and they felt confident their model would work. In order to help them realize they were missing something in their model (they needed directions to convert from gigabytes used to number of minutes used) I asked them if they had validated their model. Noah, in Team 2, had apparently plugged numbers back into the model to verify it had worked and could communicate to me and his team the numbers this work along with the fact that their variables $x$ and $y$ represented gigabytes used at 1080p and 720p. Despite correct
mathematical calculations and representations, however, his team still incorrectly transferred this back into real life knowledge and implications:

DB: Now, what are those numbers that you are plugging in? (pointing to Noah’s work)

Samantha: We explained that. (referring to their letter to customers at GoPro)

Noah: $x$ is the total for the 1080p and the $y$ is the total for 720p and that’s like the total you should use to get the best quality.

George: for the 64 gig

DB: Okay so when I’m a customer at GoPro and I use your model to solve and I solve that system of equations. The information I get from your model is what information?

George: The amount of minutes.

Samantha: The max amount of time you can record at 1080p.

DB: Is it giving you time?

George & Samantha: Yes

DB: What did you define $x$ and $y$ to be? When you did that (pointing to Noah), when you solved your system, what did you define $x$ to be Noah? You said it.

Noah: $x$ is the gigabytes of 1080p

DB: And $y$ is?

Noah: the gigabytes of 720p

DB: So when you solve your system, and you get $x$ equals and $y$ equals, what do
you get from your model?

*I pause as students look at each other and look at their work. They are not sure how to answer this.*

*DB*: Like I get $x$ is 10, what does that 10 mean?

*Samantha*: The time you can record.

*DB*: So I can record for 10 minutes?

*Abigail*: 10 minutes at

*George*: 1080p.

*Samantha*: No

*Noah*: It can’t. Because the other equation equals to ‘G’ and $G$ is the total number of gigabytes you get in the thing (memory card). *(referring to the equation $x + y = G$)*

*DB*: So when I get an answer of 10 for $x$, what does the 10 mean?

*Samantha*: You have to multiply it by 4.09. *(the minutes per gigabyte rate they had come up with)*

*DB*: Because what’s the 10 mean?

*George*: It’s the number of gigabytes.

*DB*: It’s the gigabytes.

*Samantha*: Ah, then you times the time *(meaning rate)* and that’s your answer.

*DB*: Exactly, so in your letter to GoPro, you’re going to explain that the results of your model, the $x$ and $y$ you get are what again?

*Noah*: The gigabytes.
DB: The gigabytes. Now my goal as a customer, I don’t want to know gigabytes.

So I use up 16 gigabytes on my card, I want to know what piece of information?

Noah: Time

DB: So in your letter, after you explain your model, you’re also going to have to include what information for them?

Samantha: When you solve our model \( x \) will be the number of gigabytes so you’ll need to times it by (pauses)

Noah: by the time

Samantha: the times for 1080p and 720p. (referencing their minutes per gigabyte rate) (March 18, 2016)

Students who had generated a model where they had defined \( x \) and \( y \) to be the number of gigabytes used as opposed to the amount of time needed at each resolution had a more difficult time translating and interpreting mathematics back into real life. This is because they knew their goal was to be able to communicate the amount of time needed at each resolution and they had verified their model worked by going back and checking their previous results. However, because they knew their model ‘worked’ and they knew their goal was time, they incorrectly assumed their system of equations would lead to someone being able to determine the amount of time to record in each mode. The questioning I exhibited in this conversation with members of Team 2 was more specific and funneling as I was trying to get students to reflect on how they began their model: defining variables to mean something in particular. But rather than simply telling what they had
done wrong, I asked questions which forced students to make this realization on their own. Because quantitative literacy seemed to be so difficult for students, I needed to focus questions and conversation on particular misconceptions and errors students were making.

**Project 3: Enrollment at River Junction Senior High.** Issues with quantitative literacy continued to be present throughout the unit. In Project 3, some students struggled correctly interpreting their models and mathematical analysis in terms of River Junction Senior High’s enrollment (see pages 152-164). For example, as I was working with students in Team 5, I asked several questions to get them to think more deeply about the interpretation and validation piece to modeling and how that related to their work. Initially they had developed the equation $y=43.17x+84,678$ and I was worried how they would interpret and describe this to administrators coming in to view presentations the next day:

*Dylan:* Yeah so in our linear regression the $x$ is the year of the graduating class and the $y$ is the high school population.

*Ryan:* Yeah

*DB:* So Luis is this what you came up with right here? (*Pointing to the equation $y=43.17x-84678$*)

*Luis:* Yeah

*DB:* I’m an administrator and I’m looking at this. How are you going to communicate this?

*Dylan:* Show it in our calculator (laughing).
*DB:* I’m not a mathematician. I don’t know what this means.

*The three boys look around silently at one another as they try to figure out how to answer. When no one does I clarify.*

*DB:* So what’s the 43.17 mean?

*Dylan:* That’s like what you are multiplying the year by I guess.

*DB:* But what does it mean in terms of the situation? I’m an administrator interested in this, what are you going to communicate? What do I want to know from that number? What does that tell me?

*Ryan:* I want to say *(trails off followed by a few moments of silence).*

*DB:* Mathematically what does that number mean? So in a line, what does that number represent?

*All three:* Slope

*Ryan:* Your rate

*Dylan:* That’s the average growth rate of the high school population.

*DB:* Which means what?

*Dylan:* How many students it goes up by each year?

*DB:* Do we agree with that?

*Ryan & Luis:* Yeah

*DB:* So when I’m communicating this I’m saying “We found based on this model we are increasing about”

*All three:* 43 students

*DB:* Every
All three: year

DB: Now is it constant? Is it perfectly like that?

Dylan: No

DB: So you are truly communicating that this is what?

Luis: Average

DB: Now what does this number say according to your model? What does negative 84,678 mean?

All three look at each other and start laughing awkwardly.

Dylan: That’s what the calculator came up with (laughing).

Ryan: It works.

DB: How do you know it works? (March 30, 2016)

Students then describe to me how they validated their model, which was described in Chapter 5: Relearning How to Learn. Once we got back to the major issue at hand, however, students still could not accurately describe what the -84,678 meant about the situation:

Dylan: I don’t know.

Luis: It’s the number of students.

DB: At when?

Dylan: River Junction’s population (laughing)

DB: What does the output of this tell you? You’re saying x is the year. So mathematically if I think about a line, what does this say?

Luis: That’s the starting point. I mean the y-intercept.
DB: Which is my y value when x is what?

Luis & Dylan: Zero

DB: So I plug in zero for x what are you saying?

Dylan & Ryan mumble some inaudible thoughts as they process.

DB: You’re saying its year

All three: Zero

DB: Not in zero years, you’re saying its year

All three: Zero

Dylan: Like B.C.

DB: So this number means what? *(pointing back to the -84,678)*

Ryan: The population in year zero.

Dylan: *(laughing)* Nobody was educated.

Ryan: They just did not go to school. *(all three laughing)*

DB: So if you use this, you’re communicating that at year zero the population of

the high school is what?

Ryan: -84,678

DB: And does that seem realistic?

Ryan: No that’s not possible.

DB: So if you decide to stick with this, which is fine, when you communicate that

you need to say that this is a what about your model?

Dylan: It’s an assumption we made? *(as he searches for the words he wants to use)*
DB: More like there’s a limitation to it.

All three: yeah (March 30, 2016)

We then discuss ways in which they might change their model so it is not so difficult to interpret:

DB: So to modify this model a little bit, how could you change the definition of your variable? (no response) Rather than saying x is the year

Dylan: x is the year past 2001 (referencing the 2000-2001 school year)

DB: That would make this (grabbing their data table to reference) not year 2000-2001 but year what?

Luis & Dylan: Zero

DB: So when you see this number, this should represent the number of kids when?

Dylan & Luis: 2001

DB: Then you can kind of compare these two models. (March 30, 2016)

This lengthy conversation illustrates the difficulties students have in quantitative literacy. That is, this team had a basic linear model. Students have been working with linear functions mathematically since around the 8th grade. However, it took a lot of questioning to get them to accurately communicate what the slope and y-intercept meant in terms of the situation in order to get them to then communicate this to their audience the next day and realize the unreasonableness of the -84,678 initial value. The had checked their model using separate trends and relationships within the data table so they were confident it was producing a reasonable estimate in the end. As a result, it did not seem they were questioning the validity of having a y-intercept of -84,678. I think the
oddity of this number also made it more difficult to interpret. Students knew the output of their model was the River Junction population, so the -84,678 did not make sense to them, making it difficult to communicate how it related to their work, analysis, and data.

**Reflections on my questioning in Project 3.** In my questioning of students in this example from Project 3: Enrollment at River Junction Senior High, I tried asking the more general questions “what does it mean in terms of the situation?” or “I’m an administrator and I’m looking at this. How are you going to communicate this?” Having administrators listen to presentations at the conclusion of this task proved to be a useful way to provide context and a need for interpreting mathematical information in terms of the situation. This outside audience was an instructional tactic I wish I had used more throughout the unit because of the need for interpretation and communication it provided. But, students did not seem to initially know how to respond to these more general questions. As a result, my questioning had to become more specific to particular mathematical features of their model so students could connect these mathematical features to conclusions about student enrollment. Nearly every statement they made I responded to with another question encouraging them to think more deeply about the mathematics they had done and the limitations to the way in which they set it up.

**Project 4: How Safe are We?**. As the modeling unit progressed, students recognized the importance of interpreting their model and analysis in terms of the situation but continued to have difficulties in doing it accurately. Project 4: How Safe are We? (see pages 164-180), in particular, caused issues for some students as crime scores coming as a result of weighing violent crimes could not be interpreted as crimes per
capita. It was useless without other scores to compare. In my efforts to help students recognize the mistakes they were making, unfortunately, my relearning how to teach with questioning and quantitative literacy in mind fell short of these new expectations. Team 2, for example, had weighed the violent crime totals by two, but were incorrectly referring to it as the crime rate per capita. The following conversation illustrates both the issues students continued to have with quantitative literacy and my own failure at relearning this new role of questioning:

**DB:** Walk me through what you did here.

**Samantha:** The data I used was from the website you gave us. It was important for our model we created. We needed population, violent crimes, and property crimes.

**George:** Then the model we used was people think violent crimes are more important so we multiplied it by two, added it to the property crimes, divided it by the population, then multiplied by a percent [by 100 to get a percent]. Crime rate per capita.

**DB:** Be careful about this issue with percentages. A lot of people have had this same issue. When you get this number here, and you say it is 4.873%, what does that mean?

**Noah:** Crime per capita. Crime rate per person, based on the population, the percentage of… *(pause)* How much crime for 100 people.

**George:** Like that is 1 in 25.

**DB:** There is a slight problem with saying that because of this *(pointing to their*
Because you weighed the violent crimes times two. There isn’t really 262 violent crimes reported, there is really 131 violent crimes reported. So, when you take it times 2, this percent is not communicating the percent of people that could be affected by a crime because you doubled it. And it is not really doubled in real life. You’re saying it is twice as important to consider when determining how dangerous a city is. I like this idea of weighing it, what it’s actually communicating to you though isn’t quite this thing.

George: It’s how safe a city is.

DB: Yeah, it’s just a weighted score.

Abigail: That we can use for comparison.

DB: Yeah you’re getting a number by itself, like if I give you 4.8 something for River Junction, is that score by itself all that useful?

George: No

DB: Because of the weighting aspect to it.

George: Basically all it does is give us something to compare to other cities.

DB: Exactly, so it is only useful when you have

George: A list. (April 5, 2016)

In this example, I resorted back to previous habits of divulging too much information to students. Rather than pointing out the specific problems in interpreting weighted scores as crimes per capita, I should have continued questioning students to come to their own conclusion about this misconception. Unlike the examples I discussed from Project 2:
Calculating the Width of Memorial Stadium and Project 3: Enrollment at River Junction Senior High, the issues students had with quantitative literacy in Project 4 were resolved by my thinking and not student thinking. I should have asked questions like (1) why did you multiply violent crimes by two, (2) how many crimes did the city actually have, or (3) by doubling it, how have you changed the perception of crime and danger within the city? This conversation illustrates not only student issues with quantitative literacy, but the continuing role, difficulties, and importance of teacher questioning in helping students develop these skills.

**Significance of quantitative literacy.** By the end of the unit, students continued to struggle accurately applying quantitative literacy skills. In Project 5: Student Choice (see pages 180-208), students spent so much time researching and developing the background knowledge necessary to begin a modeling analysis they did not have adequate time to properly interpret and communicate what the results of this analysis meant about the situation they studied. Fortunately, however, students seemed to appreciate its importance in the modeling process when working with authentic situations.

For example, Chris felt the importance of interpreting his analysis in a way others could understand was one of the most significant things he learned while modeling. He stated “Like when we had the system of equations for the GoPro, we had a number $x$ minus another number $y$ and if you just gave that to any average joe they wouldn’t know what to do with that. Having to put into words so that anyone could understand” (April 27, 2016). Similarly, Luis felt the most important thing he learned was how important it
was to be able to explain mathematical language in a way non-mathematicians could understand. In his interview he declared, “Since we were looking at word problems and situations, the numbers, you had to be able to explain what they meant, so you could explain to someone what the answer was like you were teaching” (April 28, 2016). It seems students continue to need opportunities to apply skills related to quantitative literacy so fewer mistakes are made and teacher intervention is needed. I am pleased, however, students at least valued this part of the modeling cycle and understood its importance when working with authentic mathematics.

**Reflections on the Role of Questioning**

Despite my sometimes less than stellar use of questioning skills, I realized how important my questioning was in helping students navigate the modeling cycle, generate their own ideas, and correctly apply mathematical and quantitative literacy skills. It seems students, too, valued the new role of questioning in the development of modeling abilities, as several students credited it as the instructional tactic I used that most helped their learning. In response to me asking “What did I do during the unit that you found helpful as you learned how to model mathematical situations?” Chris commented “Coming around to each group and asking good questions to help stimulate us. To go to the next step we need to get to” (April 27, 2016). And even though at times I felt I divulged too much information too quickly, students did not seem to think I focused conversations on my ideas or a particular strategy. For example, Luis commented “You didn’t give us the answer. You helped us, we thought of something and you helped us I don’t know, you knew what we were trying to do so you just helped us a little bit more.
But you didn’t give us the answer, so that was pretty cool” (April 28, 2016). Similarly, Brooke appreciated how I:

didn’t necessarily tell us what we were doing was right or wrong. You told us like how it would work or how it maybe wouldn’t work in the end for the final solution. Usually you told us how to elaborate on our idea. You never told us our idea was wrong. And I really liked that. (April 26, 2016)

Finally despite Matt’s frustrations, he, too, appreciated the new role I took through questioning and acting as a guide for students through the modeling process. He stated:

You let us solve it ourself and you weren’t looking for a real answer. You were looking for us to just try and if we failed we failed, but that wasn’t a big deal. You weren’t looking for the right answer. I think that was the biggest deal. It kind of showed us one mistakes are okay, and two, this wasn’t about the right answer. It was about getting real world experience and that was probably the best thing you did. (April 27, 2016)

It seems even though I was not perfect and did not always meet the expectations I had for myself, the emphasis I put on developing questioning skills that would focus conversations on student ideas and misconceptions helped students learn how to model. Acting as a guide through the modeling process and using questions to stimulate student feedback, clarification, and new ideas helped transition the classroom from teacher to student-centered, and provided greater opportunities for the development of modeling abilities.
Conclusion

Preparing for and teaching a month of modeling, as expected, challenged my beliefs, routines, and instructional practices. During the research and planning phase of a unit on modeling, I had to reassess my perceptions on what mathematics looks like and how mathematics can be assessed while I came to realize modeling would force me to approach learning through a shared expertise between teacher and students. While the modeling unit unfolded, I relearned how to teach by activating students’ real life knowledge, addressing new student needs associated with communication and functioning in a student-centered environment, and shifting the role teaching questioning should play in the development of modeling abilities. These new teacher learnings seemed to be especially important in helping students realize the significance of and correctly apply skills related to mathematical and quantitative literacy. Perhaps most significantly, I learned both in my curriculum design and instructional strategies how mindful I need to be about funneling students towards a particular strategy or model to develop. At all phases of this project I realized how much I inadvertently reflected my own ideas onto students as opposed to guiding students towards development and clarification of their own. Embarking on such an in-depth analysis and reflection on an aspect of my practice I knew was under-developed brought about powerful learning opportunities for me which will continue to affect all aspects of my teaching practice.
CHAPTER 7

RELEARNING MATHEMATICS WITHIN AUTHENTIC TASKS

Throughout this unit on modeling, both my students and I had to relearn aspects of the teaching and learning process in order to provide better opportunities for the development of modeling abilities. Modeling, however, also would not have been possible without reshaping perceptions of mathematics itself. In particular, modeling embedded within authentic tasks forced my students and me to think beyond traditional mathematics focusing on replicating procedures that had shaped our prior experiences and knowledge. We learned quickly that mathematics applied in a month of modeling would not be the clean and clear mathematics from our past. Authentic mathematics was going to be obscure and messy. As we analyzed situations and data, we realized certain assumptions might need to be made in order to create a reasonable model. In addition, stating different assumptions would lead to different models being developed. Finally, limitations to each model often came about as a result of these assumptions and other real life aspects of the authentic situation.

Authentic Situations

Yerushalmy (1997) suggests the most worthwhile modeling experiences may happen as students mathematize authentic situations. However, in order to analyze the mathematical implications stated in this chapter, it is important to clarify first what is meant by authentic. As indicated in the Introduction section, some researchers suggests authenticity is rooted within real-life situations. Palm (2008) describes an authentic task as one which describes a situation that has occurred or may occur in real life. Similarly,
Forman and Steen (2002) suggest authentic tasks are ones embedded in real life contexts often containing realistic yet sometimes incomplete data and allow for multiple strategies. Tran and Dougherty (2014) recommends criteria for an authentic task and suggests the “purpose of the task needs to be as clear to the students as it would be if they actually encountered it in context” (p. 674). Another, slightly modified view of authenticity falls into the discipline of mathematics itself. Lajoie (1995) suggests that by studying the underlying implications of NCTM standards, authenticity might simply describe what mathematicians do in their domain and be rooted in the skills of problem solving, communicating, reasoning, and making connections. In this chapter, I adopt a definition of authenticity which connects these two viewpoints and suggests that an authentic task is embedded in \textit{real life}, both in the language of the situation itself and the thinking skills students need to apply for successful completion of the task.

\textbf{The Need for Identifying and Stating Assumptions}

I had little experience as a student doing the kind of mathematics I call “authentic” in general and modeling tasks in particular. I was confused after my review of current literature about the purported need for stating assumptions throughout the modeling process. The Common Core Standards for Mathematical Practices online (2014b) states that students should be “comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later” and using tools like technology to help “visualize the results of varying assumptions” (para. 5 & 6). Similarly, Zbiek and Conner (2006) suggest recognizing and stating assumptions about the situation and mathematics employed may be essential to
some modeling tasks. Teachers often struggle admitting their own ignorance or knowledge limitations, and I am ashamed to admit during my planning that I had no idea what research and standards meant by stating assumptions. I included this criteria in the Individual Rubric (see Appendix N) under “Formulate Model” since the research had stressed it, and I had seen it in emphasized in rubrics and research done by Anhalt and Cortez (2015). I honestly hoped I would figure it out when I saw it in class, and thankfully this ‘lightbulb moment’ happened before we got too far into the modeling unit. Once I was confronted with this new learning about authentic mathematics, namely the importance of recognizing assumptions and the mathematical implications these assumptions have on a potential solution, I could then help guide students to identify and realize the importance of stating assumptions as they formulated a model.

**Assumptions in Project 1**

As students were working on Project 1: The High-Definition Trade-Off Dilemma (see pages 121-139), I was nervous about helping students navigate the modeling process when I was not sure what an essential component to it meant. How was I supposed to assess students on “provides complete work including assumptions used” as the Individual Rubric (see Appendix N) states if I did not understand the importance of this myself? In conversations with students, we discussed the assumption each recording resolution would take up the same amount of space on a memory card regardless of its size. As George stated in class, “one gigabyte on a 64 is the same as one gigabyte on a 32” (March 18, 2016), but I was not convinced this was mathematically significant to their models. Like George, to me it seemed more like a fact of the technology, not an
assumption we needed to make about the situation. Thankfully, this first project was not actually assessed and I only provided feedback. It gave both myself and students time to adjust and familiarize ourselves with the process. In Project 2: Calculating the Width of Memorial Stadium (see pages 139-152), however, the need for recognizing and stating assumptions emerged.

**Assumptions in Project 2**

When I designed and worked through Project 2: Calculating the Width of Memorial Stadium, I was worried there were not enough solution strategies students could employ. That is, I was concerned it was too concrete. Fortunately, this concern was negated as students began noticing issues with the measurements and their work.

**Measurement errors in authentic situations.** Chris in Team 5 needed to draw a scale drawing of the measurements in order to help him visualize the situation and relationships he might use. His team liked where he was going with his drawing and also found it helpful, but when he was done he came up with the image in Figure 7.1:
Figure 7.1. Chris’s scale drawing of the measurements given for the Width of Memorial Stadium Project. (March 22, 2016)

In Figure 7.1, the blue line is the distance I walked in the video used to introduce Project 2 and the red line indicates the width of the stadium. Chris’s team was a little confused and thought he did something wrong because the angle line of sights intersected at a distance that did not match their everyday knowledge of the situation. The red line where the stadium fell in his picture did not seem to be where the stadium should fall. The stadium was angled too steeply. Students expected the stadium to fall in a line nearly due north and south. Chris went to his protractor and double checked his angles again and did not notice any problems:

_Evan_: Look Chris you are not right. I just looked at a Google map and the stadium runs north-south.

_Chris_: But why don’t they meet up at the same place? It’s not right?
Chris appears frustrated.

Evan: Chris look at theirs. It lines up.

Chris: Where though?

Chris and Evan move to Team 3 to look at the picture Dylan had taken of the stadium on Google Maps.

Monica: Maybe it’s Buhrman’s fault.

Chris: It doesn’t make sense!

Chris asks me to come over to check in on them at this point and I see Chris’s picture.

DB: Ooohhh. So what are we checking up on?

Chris: It doesn’t intersect at the same spot.

Chris references an imaginary line where the stadium should be in his picture indicated by a red line in Figure 7.1.

Evan: I looked at a Google map and it shows that it does run north south. So Chris’s work is very confusing to me because his doesn’t show it.

DB: Okay, you are getting at this idea that trying to stand 2000 feet away and trying to use a compass, there might be a little bit of what?
Megan & Monica: Error!

Monica: I told you she might be wrong! (March 22, 2016)

Chris’s frustrations here came as a result of the authenticity of the situation he was working with. He and Evan did not think any errors could be occurring within the situation, so Chris had to have done something wrong. As it turns out, and I was pleased to see, Chris had discovered mistakes in the measurements I had taken. The error was not his but mine and came about because of the difficulties of working in ‘real life.’ So the first assumption we had to make was that no human error occurred, or at least the errors that did occur were minor enough to still provide a credible estimate for the width of the stadium. We believed finding an estimate for the width was possible with the measurements provided.

My ‘lightbulb’ moment. After reconciling the measurement errors students at Team 5 noticed, Evan goes on to describe how he looked at Google maps and thought the stadium runs due north and south. He felt my line and the stadium would be close enough to being parallel that it would provide a decent estimate for the width and a credible model. He stated, “so say I made the assumption, I think it’s safe to make the assumption after looking at the Google Map... they are parallel” (March 22, 2016). As soon as Evan said this out loud, my perspective on authentic mathematics changed. Evan’s assumption about my distance and the stadium being parallel was an aspect of this problem I had not thought about previously. When I worked on this problem myself, I did not even use this idea. Theoretically these distances should be parallel but I did not need to use that fact to find the width and develop a model. He provided my ‘lightbulb’
moment. I finally understood the need for stating assumptions in modeling authentic situations, and it came as a result of a student’s perspective on the authentic task. The task’s ambiguity allowed for a different strategy to be used than my own. Using the assumption my distance and the width were parallel provided them with an estimate of 753.8 feet which is very reasonable in comparison to the actual width of 800 feet. The complete oversight on my part for the need to clarify and state assumptions was substantial learning for me throughout this unit. And once I finally understood its value, I could help students understand its importance as well.

Applying new learning to Project 2. Once Evan provided me with the realization of why CCSS-M emphasized assumptions in the modeling process, conversations with other teams could be focused on mathematical and real life implications based on assumptions made, as a few students did not realize they had even used this parallel assumption in their work. These students did not explicitly identify they made an assumption or think about how the mathematics they applied was affected by that assumption. For example, Team 6 and Team 1 both assumed my walking distance was parallel to the width of the stadium. Team 1, however, did not realize they should state this assumption. When I went through their work initially in class, I could not figure out why their answer was different than everyone else’s. They had gotten a width of about 628 feet which was quite a bit lower than the 750 feet to 800 feet estimates other teams were generating. Matt, as described in Chapter 5, was frustrated his team’s answer was ‘wrong’ as he had not yet transitioned to viewing mathematics through the lens of multiple solution strategies. I happened to notice they had used Law
of Sines in the end and not Law of Cosines, which meant they were able to find an additional angle within a triangle containing the stadium’s width as a side. I located this angle in their picture and asked members of Team 1 how they got it:

*Nicholas*: These are parallel. *(pointing to my distance and the width)*

*DB*: So there is your

*Nicholas*: Assumption

*DB*: So in your model you will need to state that you assumed these were

*Nicholas & William*: Parallel

*DB*: If you get a different answer than everyone else tomorrow, it does not necessarily mean you did anything wrong, what does it mean?

*Matt*: That we are wrong.

*DB*: No it doesn’t. This whole model is based on the fact that what?

*William*: We assumed that angle.

*DB*: We assumed that angle because we assumed what?

*Nicholas*: They were parallel. *(March 24, 2016)*

I knew Matt kept focusing on their strategy being wrong, so I wanted to reassure them they had not made any mathematical errors here. The assumption which drove their work might not be correct. I asked them why that could be, and they could not come up with a valid reason. I had to offer up the idea that maybe I did something wrong to which everyone laughed but Matt. William replied with “you’re evil” *(March 24, 2016)*. In reflecting later on the work done by Team 1 and Team 6 and the significance of recognizing assumptions used, I wrote:
Team 1 and Team 6 did end up using parallel relationships so their models ended up looking different than other teams as they found additional angles therefore not needing to apply the Law of Cosines. They only applied Law of Sines in their model. Both teams had widths of the stadium that were less than teams who hadn’t used this assumption. And interestingly, they got different answers. I think this is because they used two different triangles for their final result but will need to go back through their work to be sure. (March 25, 2016)

Both Team 1 and Team 6 applied correct strategies and calculations, yet they ended up with drastically different results based on the mathematical implications from the assumption they made. Theoretically, their assumption was reasonable so their model would have worked to accomplish the goal and resulted in the same width. No student at either of these teams did anything ‘wrong;’ they were simply exposed to the ambiguity associated with authentic mathematics and specifically its potential for human error.

**Measurement errors create issues with assumptions.** Other students also began noticing the significance of and perhaps limitations to using the assumption my distance and the stadium are parallel, and now I could adequately guide and provide feedback on their ideas. Like Chris and Evan in Team 6, students in Team 5 began having issues with this assumption of the lines being parallel. On day two of Project 2, I went over to check on their progress, and Dylan flipped his paper in frustration exclaiming “We’re getting two different answers!” (March 23, 2016). I then asked his teammates to clarify what it was I was supposed to be looking at on their drawings that caused them frustration. They pointed out the green and purple triangles in Figure 7.2 and explained they did Law of
Sines on both triangles:

![Diagram](image)

*Figure 7.2. Dylan’s picture referencing the triangles Team 5 was using for their calculations. (March 23, 2016)*

Because the missing width is overlapping in both these triangles, they argued, it should be the same distance, but they were getting two different results: 753 feet and 628 feet. Quickly looking at their work I thought I noticed the cause of these discrepancies. I realized they had found angles other teams had not noted in their pictures and began asking how they calculated those angles:

*DB:* Where did you get that angle? *(pointing to one of the adjacent angles to the unknown width of the stadium Stadium)*

*Dylan:* That angle? Well it should be the same because if you put it on parallel lines. *(pointing to the distance I walked and the width of the stadium)*

*DB:* So you are making an assumption and you’re assuming what?

*Dylan:* That these angles are the same. *(pointing to two alternate interior angles in his picture)*
**DB:** Because you are assuming these are? *(pointing to my distance and the width)*

**Ryan:** Parallel

**Dylan:** Because you walked in a straight north south line.

**DB:** Okay, so because you are getting two different answers here, what is that saying?

**Dylan:** You did not walk in a straight line.

**Ryan:** That they’re not parallel.

**DB:** And the assumption is wrong.

**Ryan:** So we’ll have to change the angles.

**Dylan:** And your compass is messed up!

**DB:** Maybe I didn’t walk in a straight line. They noticed something kind of similar *(pointing to Team 6)* to that. There might be some human error aspect to this which is definitely a possibility.

**Luis:** Do you know the right answer?

**DB:** I do, but I’m not telling you yet. *(March 23, 2016)*

Like members of Team 1, Dylan, Ryan, and Luis had not explicitly stated their assumption these distances were parallel or thought about how it might affect their mathematical results. We then discuss what we could do if we removed the assumption of the lines being parallel. Dylan immediately recognizes that they could still complete the task, they would just need to apply Law of Cosines because they would only know one angle and not two.
The thinking demonstrated by this team, and in particular Dylan, is a good example of how students were using flexible thinking, reflection, and reasoning to assess that something was wrong with their model. They could not pinpoint the cause of the discrepancies in their calculations but I appreciate how these students thought to look at two triangles to verify their work as opposed to only working with one. They initially thought these discrepancies were a result of their own miscalculations which was causing frustrations. Because of the ‘lightbulb’ moment that occurred with Evan and later with Team 1, I was quickly able to pinpoint what I thought was the cause of these discrepancies and proceeded to ask questions of this team to get them to realize where the ‘mistake’ actually occurred. They were relieved that the mistake was not on their end, but is a result of dealing with real life situations. Authentic mathematics is often filled with uncertainty and susceptible to human error! Interestingly, the distances Team 5 had calculated for the width (753 feet and 628 feet) were the same results calculated by Team 6 and Team 1 respectively. I did not notice it at the time, since Team 5 ended up changing their approach and not presenting these particularly values, but had I made this connection in class we could have further explored the mathematical relationships brought on by this theoretically correct assumption.

**Understanding the mathematics behind Project 2 models.** In an effort to further understand the mathematical implications of assumptions made in the modeling process, I later went on to analyze and compare the general models created by Evan and William. Evan (Team 6) and William (Team 1) had replicated the steps they made to find the width of the stadium to create a general model which could be applied to similar
situations, yet the models were different. I was curious if mathematically these models were in fact the same, and had the assumption been accurate, would have resulted in the same result. I wondered how these models compared to the general model I had developed without using the parallel assumption. Were the discrepancies that occurred brought about because of the ambiguity and opportunity for multiple solutions within authentic mathematics and not because of some mathematical error one of us had committed? To make comparisons easier, I changed all models to refer to the same variables as indicated in Figure 7.3:

Figure 7.3. Picture showing variables used in the models developed by Evan, William, and myself.
To use these models in a similar situation, a person would need to take measurements of angles A, B, C, and D and length W to find the unknown width \( X \). In my descriptions of each model, I refer to distances, triangles, and angles within Figure 7.3.

**William’s model.** William and Team 1 developed a model by first finding the length \( \overline{ac} \) using \( \Delta abc \) in Figure 7.3. They did not need to use their assumption yet, as all angles in this triangle could be calculated only in reference to the original measurements. But once William knew the length of \( \overline{ac} \), he used the assumption line segments \( X \) and \( W \) were parallel to find \(<bcd\> \) and \(<cda\>\), as they were alternate interior to angles \((180 - B)\) and \(C\). They then applied Law of Sines to \( \Delta acd \) to calculate the length the stadium’s width \( x \) resulting in the model:

\[
x = \frac{W \sin C \sin(B - A)}{\sin(A - C)} \cdot \frac{\sin(180 - B)}{\sin(180 - B)}.
\]

They did not recognize an application of the Difference Identity for Sine which would have allowed them to simplify the denominator, but I was impressed with how fairly clear and concise this model was. William had also plugged in the known values from Project 2 and validated their model worked to get 628 feet.

**Evan’s model.** Evan and Team 6 also developed a model based on line segments \( X \) and \( W \) being parallel and could also find the measures of \(<bcd\> \) and \(<cda\> \) in Figure 7.3 (see page 316) as a result. However, I initially thought the differences in the answers and models between Teams 1 and 6 was because they used two separate triangles in the end to calculate \( X \) like Dylan and Team 5 had done. Once I dug into Evan’s work, however, I realized both William and Evan actually ended up applying Law of Sines to the same \( \Delta \)
acd, but they used different triangles to find length $\overline{ac}$. Evan had found missing angles in this triangle by applying the parallel assumption and alternate interior angles, then using Law of Sines to $\Delta aec$, making his result for $\overline{ac}$ slightly different. Evan applied the assumption in two different calculations for his model while William only needed to use it once. As a result, Evan calculated his model to be:

$$x = \frac{W \sin C \sin (180 + C - B) \sin (B - A)}{\sin (B - C) \sin (A - C) \sin (180 - B)}.$$

Evan also validated his model with the known measurements getting back his width of 753.8 feet. This model seemed to be slightly more complicated than William’s, yet they both used the same assumption. Even though their specific calculations were different, these general models should work so long as the assumption is true. Why then are they different? Did one of them make an error?

**Comparing William’s and Evan’s models.** In an effort to answer these questions, I placed the equations equal to each other and attempted to narrow down the specific differences:

$$x = \frac{W \sin C \sin (B - A)}{\sin (A - C) \sin (180 - B)} = \frac{W \sin C \sin (180 + C - B) \sin (B - A)}{\sin (B - C) \sin (A - C) \sin (180 - B)}.$$

I noticed Evan’s model contained the additional fraction:

$$\frac{\sin (180 + C - B)}{\sin (B - C)}.$$

However, by applying the Sum Identity for Sine, I simplified the numerator to get the equivalent fraction:
Finally, I realized because sine is an odd function where \( \sin(-x) = -\sin(x) \), \( \sin(B - C) \) was in fact equal to \( -\sin(-(C - B)) = -\sin(C - B) \). The numerator and denominator in the fraction under question was the same, allowing me to simplify it down to one. I was surprised and a little relieved by this discovery. It meant both William and Evan created valid models based on the assumption they had made. Evan’s model would simplify down to William’s model, creating a general model that could be used in similar situations when \( W \) and \( X \) are parallel.

**Comparing to my model.** When I initially worked out Project 2 for myself, I determined a model by eventually applying Law of Cosines to \( \triangle cbd \) in Figure 7.3 (see page 316) which allowed me to not need the angles \(<bcd \) and \(<cda \) William and Evan used as a result of their assumption. No student in class who had applied Law of Cosines ended up creating a general equation to model the width, perhaps because it is so much more complicated than the ones developed by William and Evan. I ended up finding the stadiums width \( x \) with the model:

\[
x = \sqrt{\left( \frac{W \sin A}{\sin(A - C)} \right)^2 + \left( \frac{W \sin B}{\sin(B - D)} \right)^2 - \frac{2W^2 \sin A \sin B \cos(D - C)}{\sin(A - C) \sin(B - D)}}.
\]

Obviously, this is substantially different than the models created by William and Evan. To my knowledge, there is no efficient way to manipulate and simplify this equation to more closely align to these other models. However, in order to more thoroughly understand the mathematical implications of the parallel assumption, I wanted to be sure all these models would work had the assumption been true. To test this, I decided to use
Geogebra and begin a diagram with parallel lines so the assumption was guaranteed. I then replicated the measurements taken in Project 2 to a specific case unrelated to it to get the following picture (see Figure 7.4):

![Diagram with parallel lines and angle and distance measurements](image)

**Figure 7.4.** Specific case where X and W are known to be parallel.

Within Figure 7.4, the angle and distance measurements are irrelevant, as I could move them within the Geogebra file to get any measurements I wanted, so long as W and X continued to be parallel. I had Geogebra calculate the width X so I knew the ‘correct’ answer (in this specific case 4.31), then I plugged in the measurements for A, B, C, D, and W into William’s and my models. To my relief, both models calculated a width of 4.31 feet. This meant despite the models looking drastically different, they would both be valid as long as the assumptions both were based on held true in the authentic situation. In the end, all three models were accurate and would have calculated the actual width had
the measurements I had taken been more accurate! Their mathematical differences were simply as a result of different assumptions made because of mathematical features within authentic situations.

This realization took a lot of perseverance and curiosity on my part. I needed knowledge of geometric relationships and trigonometric identities to make this conclusion, but in the end this analysis came about because I did not settle on the obvious. I did not simply make judgements based on students achieving the ‘wrong answer.’ I sought to dig deeper into their thinking in order to explain and compare the mathematical relationships they employed. I had the patience to realize working with authentic mathematics was messy. Providing feedback and grading student work could not simply be based on right or wrong.

**Reflections on assumptions in Project 2.** These models and comparisons illustrate the complexities that exist within authentic situations, especially ones susceptible to human error. Both William and Evan demonstrated strong mathematical skills and the development of modeling abilities through the creation of valid models. Without closer inspection, however, one might have thought discrepancies in their original values and general models were a result of errors on their part when in fact they had applied their strategies correctly. To put it simply, my model works in any situation one might take four angle measurements from two locations a known distance apart to calculate an unknown width. The known and unknown distances do not need to be parallel, thus making the model more complicated. If one knows or assumes to know these distances are parallel, the models created by William and Evan would be
substantially easier to work with and just as valid. The importance of stating assumptions and recognizing the mathematical implications of those assumptions could not be overlooked and brought about new learning for my students and especially myself.

Experiencing mathematical features of authentic mathematics in Project 2: Calculating the Width of Memorial Stadium changed the development of modeling abilities in the remaining projects, particularly in relation to data analysis.

**Assumptions About Data in Projects 3, 4, and 5**

During the modeling unit, I learned to focus attention on the awareness of *assumptions* as a teaching principle connecting the real world to the mathematical world (Seino, 2005). The ‘lightbulb’ moment that occurred in Project 2: Calculating the Width of Memorial Stadium forced both students and myself to think about the importance of assumptions and their mathematical implications in the development of models. Particularly because of the presentations in Project 2 and discourse which ensued between teams as students realized the impact assumptions had on models developed, recognizing assumptions in subsequent projects did not seem to cause the confusion and frustration it had previously. Once students bought in, and I understood the impact assumptions had on the modeling cycle, it ended up having significant effects on modeling abilities related to data analysis.

Because of the ambiguity of authentic data, students would need to make assumptions about trends in data in order to create a credible model. And as a result of the mathematics in Project 2, students began to recognize that stating different assumptions would result in different mathematical models. For example, in Project 3:
Enrollment at River Junction Senior High (see pages 152-164), some students assumed enrollment for River Junction Senior High would continue growing at a constant rate. If students made this assumption, their resulting model was linear. On the other hand, if students assumed enrollment would continue at a constant percentage growth, they developed an exponential model. In Project 4: How Safe Are We? (see pages 164-180), although less obviously, students had to assume crime would occur at the same rate when finding the number of crimes per capita. Finally throughout Project 5: Student Choice (see pages 180-208), students had to make assumptions about the data they collected in order to track trends and develop a usable model.

In textbook problems students typically came across in my course, they did not need to have a conversation about assumptions because either the directions were so explicitly stated or the data they encountered was indicative of specific trends. Traditional word problems, as Tran and Dougherty (2014) describe, includes contexts that are not an essential part of the solution. Students do not need to take into account real life features of the situation in order to develop a solution. The uncertainty and messiness occurring within authentic tasks, however, forced my students and I to rethink the information and data we encountered in order to assign specific mathematical relationships that could be modeled. The ways in which students perceived and applied these mathematical structures reflected their underlying assumptions (Zbiek & Conner, 2006). Once students progressed to the final project, they did not seem to have difficulties recognizing the assumptions they made in their analysis or the mathematical and real life implications of those assumptions. However, this new learning may not have occurred
without Evan and Team 6, William and Team 1, and the members of Team 5 first communicating their assumptions in Project 2. In other words, this new insight about mathematics was not brought about by teacher knowledge. It seemed, on the contrary, to be enhanced by a lack of it, and would only have occurred because of student perspectives, insights, and ideas.

**Recognizing Limitations to Models Developed**

Another aspect to modeling that brought about new learning of mathematics was the significance of recognizing limitations to models developed. By applying modeling abilities related to reflection and reasoning, students had to ask themselves: *what are potential inaccuracies of my analysis and will this always work?* Often times, these limitations were brought about because of the assumptions students made in the creation of their models. Other limitations occurred because of the nature of the real life situation being modeled. In a month of modeling, students learned fairly quickly limitations exist in authentic mathematics because real life rarely mimics the clear and concise mathematics they learned within the classroom. Patterns are harder to find and even harder to predict because of the uncertainty and ambiguity present within authentic situations. The new learning associated with recognizing limitations was not as substantial as the insights provided by identifying assumptions as students were able to bring in real life knowledge of the situation to generate potential limitations. However, this combination of mathematics and real life knowledge provided a nice platform for students to further develop modeling abilities.
Limitations and Its Relationship to Assumptions

Recognizing and stating assumptions generally occurred towards the beginning of the modeling process as students attempted to formulate a model. Students would apply everyday knowledge to understand the task then translate everyday knowledge into mathematical relationships by analyzing data and information within the task. As they analyzed this data and information, students would realize certain assumptions might need to be made in order to adequately mathematize the situation. Later as students applied skills related to quantitative literacy in attempts to translate the results of their mathematical analysis back into real life knowledge, students would need to employ skills related to reflection and reasoning to identify potential limitations to their models that occurred because of the assumptions the model is based on. For example, the models created by William and Evan for Project 2: Calculating the Width of Memorial Stadium were based on the assumption my distance and the stadium’s width were parallel. Therefore, a limitation to their model is it would only work in other situations where two distances are known to be parallel. If this information is unknown or known to not be parallel, either another assumption would need to be made or a different model used entirely. Their models only worked in specific cases and not in more general ones.

Similar relationships would exist between assumptions and limitations for Projects 3, 4, and 5. In Project 3: Enrollment at River Junction Senior High, Emily and Team 4 were able to develop a linear regression to model the growth of River Junction Senior High, since they assumed a constant growth rate and a new high school would not yet be built, perhaps affecting enrollment (April 1, 2016). As a result of this assumption,
Emily stated “our equation won’t be exact because years don’t consistently increase/decrease. The equation is only an estimate, not exact” (April 1, 2016). Emily recognized in order to develop a linear model, they had to assume enrollment growth would continue to mimic current patterns and growth rates, and no unforeseen outside factors would change enrollment. As a result of the authenticity of the task, however, Emily and Team 4 realized this assumption would end up being a limitation to their model since the given data did not follow a specific pattern, making trends hard to determine. Similarly, Alicia and Team 5 recognized they “assumed a constant growth” in the development of their exponential model yet realized “populations are not constant” so their assumption would probably not hold forever (April 1, 2016). As students became more comfortable recognizing assumptions they made in the development of their model, they seemed to simultaneously get better at suggesting potential limitations.

Limitations as a Result of Real Life Implications

Once students realized an assumption they made would cause the model to not always work, they generally brought in real life knowledge to explain these limitations to their models. For example, in Project 3: Enrollment at River Junction Senior High, Samantha stated “We assumed River Junction’s population will grow every year and there will be no obstacles to limit our growth” (April 1, 2016). Yet, Samantha and her team recognized in their presentation “One major limitation to our model is the fact that students always drop out or move schools. In our model, it would be difficult to take that into account” (March 31, 2016). Samantha realized the assumption her team made to mathematize the situation was not realistic, and was able to explain why using her own
real life knowledge of the situation. In addition, Alicia in Team 5 also realized her exponential model for River Junction Senior High’s enrollment will eventually increase quite rapidly making it no longer valid (April 1, 2016). This conclusion came about because of Alicia’s real life knowledge and understanding of realistic versus unrealistic changes in population.

Similarly in Team 6’s presentation for Project 3, Monica, Megan, Evan, and Chris carefully documented mathematical limitations to their model based on the assumptions they made about it increasing at a constant percentage rate (see Figure 7.5):

![Figure 7.5. A slide from Team 6 demonstrating the ways in which students identified potential limitations to their model. (March 31, 2016)](image)

Like Alicia, Team 6 concluded the exponential equation would eventually be inaccurate, as it would show growth substantially more than what is realistic for our city and school district. And because they had assumed a constant percentage growth rate, their model was also limited, as data trends did not show this constant growth. These three examples illustrate how students had to relearn mathematics within authentic situations. Authentic
mathematics might have limitations textbook mathematics did not, adding to the emphasis on thinking skills within the modeling cycle as opposed to just ‘answers.’

In addition to mathematical limitations, students could identify specific real life factors that might cause their predictions to be inaccurate in Project 3. In the question and answer portion of Team 5’s presentation, Jacob accurately assessed changes in the economy could make models less valid because people may move away or move to River Junction as jobs become available (March 31, 2016). Team 4 suggested changes to other high schools in the area might cause their model to be inaccurate 50 years from now (March 31, 2016). Students seemed to realize the assumptions they made about enrollment increasing at a constant rate or percentage also would cause limitations to their model, as outside factors would most likely cause these rates to fluctuate.

**Difficulties with limitations in Project 4.** The situation in Project 4: How Safe are We? (see pages 164-180) made identifying assumptions and their corresponding limitations slightly less obvious. For example, students in Team 5 could not communicate what multiplying crimes per capita by 100,000 was assuming about crimes in that city. To help students, I described a hypothetical situation about my home town with a population of 400 people having a murder this year. I described how if we scaled this up to 100,000, all of a sudden it becomes the most dangerous place in the state because it will theoretically have 250 murders per 100,000 people. Students knew for comparison, Metroville, the largest city in our state with nearly 450,000 people only had 32 murders. As a result of this conversation, students recognized they assumed crime will continue at the same rate, but students in general could not make this conclusion on
their own. However, eventually students realized that by calculating crime per capita in their models, they assumed “crime continues at the same rate” (Samantha, April 7, 2016), and as a result created a limitation since they “did not take into account different areas in larger cities” which may have higher or lower crime rates (Noah, April 7, 2016). Most students realized a limitation as a result of this assumption, because crime does not occur consistently across cities, nor as cities become larger or smaller. Again, students brought in their own experiences and real life knowledge to reflect and reason through their mathematical results.

Recognizing limitations in Project 5. Finally, in Project 5: Student Choice (see pages 180-208), students seemed to truly merge their knowledge of mathematics and real life to explain limitations to their model and analysis. Despite my frustrations from their lack of focus on quantitative literacy and interpreting results in this project, identifying limitations in presentations and individual work was consistent and clear. For example, in Noah’s individual work on Team 2’s turfgrass model he stated “We are assuming that there is an equal amount of turf per square over the whole field. This makes our model slightly inaccurate,” leading to the limitation that “every square will have different amounts of turf. Pieces of turf are torn off everyday” (April 14, 2016). Noah clearly connected his initial assumption in the development of their model to limitations to that assumption based on his knowledge of football and how the turf gets worn down.

In addition, Dylan in Team 3 elaborated on his team’s assumptions and limitations from their immigrant growth models concluding:

Some of our assumptions were that we assumed that the annual growth would be
constant...Our limitations were the linear graph(s), it wasn’t a constant growth.

We didn’t know where the migrants were coming from and we only could find 5 years of data, which limits some data. (April 14, 2016)

Here Dylan alludes to limitations of the data itself, such as not having enough of it to accurately predict trends or how a lack of constant growth contradicts the assumption of constant growth within their linear models. Again, navigating the entire modeling process allowed for unique opportunities for students to simultaneously apply knowledge of both the mathematical and real domains, further creating opportunities for the development of modeling abilities.

**Reflections on Limitations Within Authentic Situations**

Limitations in the modeling process are brought on because of the authenticity of the situation being modeled. I could find no research in the current literature discussing limitations in the modeling cycle, but it seems to be implied as essential to the process as students interpret their models in terms of the authentic situation. Although not necessarily difficult, during the modeling unit students were not used to thinking about mathematics as having limits and constraints. As a result, this new learning had a profound impact on students. In fact, Samantha in her unit reflection said the one idea from modeling which will stand out to her the most is “that every model has a limitation” (April 14, 2016). Because they were forced to identify limitations, students inadvertently had to apply modeling abilities related to reflection and reasoning as they determined under what situations their model would and would not work. Unlike other aspects of the modeling cycle, identifying limitations did not seem to cause frustrations
and conflict. Looking back on it now, this is perhaps due to students needing to primarily apply real life knowledge in order to understand these limitations. I think asking students to apply knowledge of the world around them was a lot less stressful and intimidating than asking them to apply mathematical knowledge. Students simply had to get used to thinking about mathematics as being ‘limiting’ in the first place.

**Conclusion**

Relearning mathematics within authentic tasks came about essentially because of the ambiguity associated with real life situations. Unlike the typical textbook problems students had encountered before, these situations forced students and myself to think about assumptions one would need to make in order to even conduct a mathematical analysis. As Chris stated, he learned “not everything was going to be perfect” like his textbook experiences had made mathematics out to be (April 27, 2016). As students began mathematizing a real life situation, they might have to fill in holes or gaps in knowledge with assumptions in order to create a credible model. Without stating this assumption, one can not adequately apply or understand the mathematical implications of that model. Then as students finished their analysis, they realized from these assumptions and real life knowledge the mathematics they applied might not always work. There would be limitations that need to be understood and stated to provide clarity about their models. Because of these modeling components within authentic situations, students further developed modeling abilities related to data analysis as they assumed trends within the situation and reflection and reasoning as they identified limitations restricting the credibility of their model. And as I witnessed my students encountering
authentic mathematics, I learned the importance of not making conclusions too quickly. Like their students, a teacher analyzing mathematical models driven from authentic situations needs perseverance, time, and curiosity. From the authentic situations William and Evan specifically provided, it became clear to me models that might initially seem incorrect end up providing some of the most powerful, valid, and interesting mathematics I could hope to see from my high school students.
CHAPTER 8

CONCLUSION

What we all hope for, of course, is that as the formal, institutional part of education is finished, its most conspicuous and valuable product will be seen to be the child’s ability to educate himself.

~David Hawkins (2003, p. 56)

Every year on the first day of school, I think about what each child will look like at the end of the year when they leave my room for the last time. What will they have learned? How will they have grown? What life experiences will they have this year that changes their beliefs or future goals? As a teacher in a large high school, I rarely will have a student more than one school year. I only get nine months. Nine months to change their perspectives on mathematics. Nine months to break bad habits. In some cases, I have nine months to get that child to believe they can do mathematics. Nine months to make an impact I hope students will remember for much longer. Nine months to become irrelevant. For if I have done what Hawkins (2003) suggests, I will have provided tools that enable my students to educate themselves and I should no longer be needed at each step of the way.

At the end of the day, most secondary teachers want students to leave our classroom with the skills they will need to find success in whatever career or academic endeavors they hope to pursue. We will not always be there to guide them. Outside of our classroom they will need to develop their own ideas, identify appropriate steps and goals, gather additional information, communicate at all stages of the process, and seek
out help on their own. Why then do we seem to provide so few opportunities in the mathematics classroom for students to develop and apply these skills? I, like Hawkins, want to see my students leave for the last time with them believing they can go the rest of the way without me. And I want to know that what I have taught them goes beyond a list of standards they may never need to use. I know I have done my job well when I can look at that child, say “have a great summer,” and feel confident in that child’s ability to navigate their own learning as the lessons they have encountered in room 531B transcend my classroom walls.

**Reflections on My Study**

Desiring to create opportunities for student-centered and authentic instruction that may provide these transcendent learning opportunities, I sought out opportunities to develop in my students broad modeling abilities. This study is an analysis of my efforts to promote these modeling abilities through the design and enactment of a modeling unit in my precalculus courses. My goal was not aimed at finding a need for particular mathematical skills, but rather to develop and apply more over-arching reasoning, communication, and problem-solving skills particularly as students encountered challenging and authentic tasks. I also sought to understand modeling itself, both in a mathematical and instructional sense, so that I might better provide opportunities for the development of modeling abilities. I do not claim that what I did was right or wrong nor is what should be done in other classrooms. I simply analyzed what I did throughout the design and enactment of this modeling unit and the complex relationships that evolved between students, teacher, and curriculum. In this final chapter, I reflect on and
synthesize new learning that occurred in preparation for and during the enactment of a modeling unit. I then offer up next steps in my role as a researcher and teacher leader and for the overall research community.

**Development of Modeling Abilities**

As I think about my experiences in this research study, the development of these modeling abilities stand at the forefront. *Everyday knowledge* was needed as students initially encountered and attempted to make sense of each task. The design of each task was crucial in allowing students to activate their everyday knowledge, and if students did not possess this knowledge naturally, I needed to question, structure, and provide opportunities for students to expand this knowledge base. Then as students began to mathematize the authentic situation, skills related to *mathematical literacy* were needed in order to identify appropriate mathematical relationships. Consistent with 2009 and 2012 PISA assessment data (see Fleishman et al., 2010; OECD, 2013a), students tended to struggle initially recognizing these relationships. The role of teacher questioning during this unit became significant to the development of mathematical literacy as I attempted to activate student real-life knowledge and act as a guide to encourage students to generate ideas as opposed to funneling them into predetermined strategies.

As students developed models, *flexible and creative thinking* was needed to identify ways to mathematical represent each authentic task. Again, the design of each task was important to allow for students the freedom to develop several different yet appropriate models. Because this design needed to account for multiple solution strategies, I had to relearn how to plan and assess mathematics tasks when I could not
account for all potential student outcomes. In my questioning of students, I also had to be careful not to funnel students into a particular strategy but instead focus questions on student ideas. Perhaps most significantly, I had to relearn how to provide instruction when I would not be the only expert in the room. As opposed to students looking at me as the ‘keeper of knowledge,’ I had to provide the environment and support necessary for students to feel as though we were all generators of knowledge. Because of this, students came to value and view mathematics through a lens of multiple solution strategies. As students worked through the modeling process, they realized there would be no ‘right’ and ‘wrong,’ but rather effective versus less-effective techniques. Additional conflicts, however, arose as students beliefs about the nature of ‘doing mathematics’ were challenged by these new experiences. As evident by Matt’s experiences illustrated in Chapter 5, when students began experiencing mathematics beyond a series of procedures to follow, some students experienced increasing frustrations and self-doubt.

Additional skills related to data analysis and collection were employed as students gathered evidence to more accurately understand and develop models, especially as assumptions were challenged and the uncertainty of each task due to its authenticity warranted further information. Students also needed to develop perseverance, especially during the development stage, in order to overcome frustrations caused by the ambiguity of each task. The desire to persevere, however, began to stretch beyond a acquiring a ‘grade,’ being instead influenced heavily by the authenticity of each task and the freedom students were given to pursue their own ideas.
After models were developed, *quantitative literacy* and *reflection skills* were needed to assess reasonableness and translate the results of each mathematical analysis back in terms of the authentic situation. Consistent with research done by Silver, Shapiro, & Deutsch (1993), Palm (2008), and Greer (1993), students struggled reflecting on the plausibility of their outcomes. To combat this, students had to create their own opportunities to learn by determining ways to validate the results of their analysis based on knowledge of the situation as opposed to seeking out my approval for validation. I also had to structure opportunities and ask probing questions to encourage students to bring back in and apply real life knowledge to these latter stages of the modeling cycle. These teacher questioning skills were also significant in helping students appreciate the importance of interpreting results of a modeling analysis in a way those not familiar with the analysis could understand.

Modeling abilities related to *reasoning* were employed as both students and teacher relearned mathematics within authentic tasks in order to determine if and under what conditions a model worked. Particularly, we had to learn the role assumptions played in the development of models. If different assumptions were made about the situation and data being analyzed, different models were produced. As a teacher in particular, I found I needed to dig into student ideas in order to understand student reasoning rather than assume different models might be wrong. These assumptions, including real life implications, also produced limitations to models developed. These limitations to reality did not seem to be difficult for students to identify, but they came to value it as an important part of the modeling cycle.
Finally communication became an essential component to the entire modeling process. Students created additional opportunities for their own learning as they found ways to engage in student-to-student discourse within teams in order to develop reasonable models and from team-to-team as students communicated the results of their analysis. Major conflicts arose because of this discourse, or more significantly, a lack of discourse when students did not feel their ideas and work was validated by their peers. As a result, the supports students ended up needing were not what I had anticipated going into the unit. My instruction did not need to focus on providing mathematics support, but rather I needed to provide structure and scaffolding as I helped students learn to communicate and interact in this new environment focused on student-generated ideas and dialogue.

**Lessons Learned**

In order to provide opportunities for the development of these modeling abilities, I elevated the rigor in my curriculum, expanded my knowledge base, and challenged student knowledge of mathematics. If I am correct in my findings, the relearning my students and I experienced as we engaged in authentic tasks can have further influence on the design and enactment of modeling tasks for subsequent teachers and researchers.

**The It is Important**

_Maybe we as teachers are simply not asking the right questions. We need to ask, will students want to know ‘the answer’ to this? Will they be naturally curious enough about its result to invest the time and energy we want them to?_  

~DB (Reflection Journal, March 26, 2016)
Learning to model simply must start with a good question to build from. In the complex interactions between students, teachers, and curriculum as emphasized in Hawkins “I, Thou, and It” (2003), the ‘It,’ is just as important as the other two components. Hawkins (2003) stated, “without an It, there is no context for the content” (p. 55). The time commitment, risk, depth of knowledge needed, and perseverance required to successfully complete the modeling cycle simply forces teachers to ask questions students want to know answers to. Leading up to the enactment of a unit on modeling, I spent months learning more about modeling and designing each of the five tasks used in the unit, carefully considering topics warranting further exploration. However, quality tasks may simply be generated by “making small changes to familiar application or story problems” (Bliss et al., 2016, p. 8). I knew little about modeling going into this study and as a result, needed to drastically expand my own expertise on the topic. Going forwards, now that I have developed a more thorough understanding of what modeling in the classroom looks like, these small changes to already existing curriculum materials as Bliss et al. (2016) suggest may be easier to recognize, develop, and manage.

But what are the right questions to ask? Modeling tasks need to be developed and designed in such a way as students can draw upon knowledge from the real domain. This real-world knowledge seemed to be essential both as students began the modeling process and mathematized each situation, and then again as students interpreted their results and reflected on the reasonableness of their analysis. These tasks must also provide for multiple interpretations for appropriate models. Similar work done by Stein and Lane
(1996) in their analysis of the QUASAR project, found the highest learning gains in mathematics were related to the extent to which tasks made use of multiple solution strategies, representations, and explanations. Without the opportunity for multiple methods, engagement in each task may wane and student gains in relation to high levels of mathematical thinking and reasoning may be small (Stein and Lane, 1996). The mere presence of high quality tasks does not guarantee quality outcomes (Henningsen and Stein, 1997), but the importance of good questions centered within an authentic situation cannot be overlooked when designing modeling instruction.

**Modeling is Not Quick and Easy**

_I like having a set of rules, just give me a set of instruction I'll do them for you...I realized how much more talented all these people are around me, it wasn't the best feeling...so I stopped trying and I just let them (my team) do whatever they wanted._

~Matt (Student Interview, April 27, 2016)

Each of the five modeling tasks students interacted with during the modeling unit were some of the most time and labor intensive mathematical problems my students had ever encountered. As Matt eludes to in the previous excerpt, a few students experienced so much frustration because of this that they began to doubt their own mathematical knowledge and abilities and wanted to give up. The freedom and lack of explicit directions quality modeling tasks provides contradicts most students’ previous mathematics experiences in the classroom. Teachers need to be prepared for this resistance and provide support to help students overcome resulting fears and doubts.
Because modeling tasks can be so labor intensive and frustrating for students, time needs to be structured and well spent. In reflecting on each task, all projects ended up taking 4 days to complete and followed the same structure. This structure allowed me to provide additional supports, questioning, and organization for students while they navigated the entire process as I could predict what parts of the modeling cycle students would be encountering next. On day one of each task, students made sense of the task, applied everyday knowledge, and collected and analyzed data. In other words, students simply played and began testing ideas. Students needed this time to understand their goal and the mathematical relationships they were noticing that might be useful. By the second day, students began honing in on specific mathematical skills that could be used for the transference of everyday knowledge to mathematical relationships. On these days students began to flex their creativity and reflect on how to validate their conclusions. As they validated, they may have noticed places to make modifications to the mathematical relationships they were using. The second day of each analysis was where most of the pure and straightforward mathematical skills were applied.

By the third day of the each project, students began transferring mathematical knowledge back into real life knowledge using quantitative literacy, reflection and reasoning as students tried to determine what their mathematical analysis meant in terms of the situation and why they felt it worked to accomplish their goals. Sometimes students made modifications to their models on this day based on the realization that they may have interpreted something incorrectly, but for the most part students focused on how results may be reported. Finally on the fourth day, students primarily focused on the
communication aspect to modeling by presenting, explaining, and validating the work they had done from the previous three days. The only deviation from this timeline occurred for the final student choice project when an extra day was needed to develop a worthwhile situation to model and engage in research in order to expand student’s everyday knowledge of their situation.

As evident throughout this study, modeling simply is not easy and takes time to properly develop. Bliss et al. (2016) in their recent report *Guidelines for Assessment and Instruction in Mathematical Modeling Education* provides several teaching principles for implementing mathematical modeling in the classroom and suggests modeling cannot occur in isolation. Because modeling is challenging, Bliss et al. (2016) suggest students must know they have time and support to seek out answers whether by working in teams, sharing ideas, and doing additional research and data collection or by the teacher providing support through organization and scaffolding, encouragement, and guidance. Because of the complexities of each task, just giving students time is not enough. Understanding how students should best use this time and providing supports to use this time appropriately also needs to be embedded within teacher instruction.

**Authenticity is Essential**

*Looking at whether or not the Humane Society would need more kennels is not a math problem. That's a real life problem. And we used math to answer that. I didn’t think about how that would apply before. But now math applies to pretty much any situation you can think of.*
In my experiences as a teacher, students consistently ask “where am I ever going to use this in real life?” The issue may be rooted in how we as teachers focus too much on where students might use specific content when we really should simply be teaching kids how to think authentically. This study came about because I wanted to help students learn how to model authentic situations. Through the process, I inadvertently provided students relevance to learning mathematics. This relevance was not a result of specific content or even the particular real life situations students had to analyze. It came about because students had to approach a problem like one they would encounter in real life: an unfamiliar situation without examples or a guide to follow but only a question needing answered. The five modeling tasks were not content-free, but the focus also was not on learning or applying a specific mathematical skill. As students realized, the modeling abilities which were the focus of the design of this unit were consistent with the authentic thinking students would need to use outside of a mathematics classroom.

Throughout a month of modeling, students were asked to think in ways they had little experience with: deeply analyzing a situation written in non-mathematical language, mathematizing and problem solving solve their way through this situation, and developing their own strategies towards resolving the question needing answered. Because of this focus on thinking and not necessarily mathematics content, as Brooke stated in the previous quote, students began to realize mathematics could be applied to nearly any situation they may come across.
In addition, students found each task motivating not simply because they thought they themselves might come across that particular situation someday, but because the situation was *authentic to someone*. That is, *someone* might find this task relevant and the resulting mathematics useful. In fact, we as teachers might inadvertently detract students from wanting to learn mathematics by creating superficial situations we think students might encounter in their own daily lives. Never once throughout the modeling unit did students say they could see how a particular task related to their own everyday life. They simply found it interesting that each situation was worded and presented authentically so they could see how it might be useful for *someone, somewhere*.

As a result, teachers and curriculums may not want to separate classroom mathematics from what is real. As Bliss et al. (2016) states:

Mathematicians are in the habit of dividing the world into two parts: mathematics and everything else, sometimes called the ‘real world’. People often tend to see these two as independent of one another—nothing could be further from the truth. When you use mathematics to understand a situation in the real world, and then perhaps use it to take action or even to predict the future, both the real world situation and the ensuing mathematics are taken seriously. (p. 95)

In the end, students took each of the five tasks seriously in the modeling unit because each was authentic, both in the situation itself and in the thinking, decisions, and communication that was required from each student.

**Mathematical Power and How One Develops It**

*Before I just thought of a problem on paper and we’d get the answer by applying*
whatever method but now I see there are many different ways to use it and more than one way to get the answer.

~Luis (Student Interview, April 28, 2016)

Going into a month of modeling, student experiences in mathematics seemed to be, as Luis suggests, focused on application of procedures and acquisition of correct answers. Bliss et al., (2016) state the “emphasis on single right answers to questions, being taught a fixed body of specific skills, and reliance on mechanistic algorithms has given students an erroneous view of mathematics” (p. 45). This limited focus creates a very singular and narrow-minded perception of what is mathematics and how one learns it. In stark contrast, according to NCTM (1989), a mathematically powerful individual has the capabilities necessary to explore, conjecture and reason logically, as well as the ability to use a variety of mathematical methods effectively to solve nonroutine problems. This notion is based on the fact that mathematics is more than a collection of concepts and skill to be mastered. It includes methods of investigating and reasoning, means of communication, and notions of context. (p. 5)

The use of modeling tasks in my classroom provided opportunities to experience mathematics through a lens of multiple solutions aimed at developing appropriate solution strategies. By engaging in the modeling cycle, students began to view mathematics not as a collection of procedures, but as a means to understand a problem. Even Matt came to realize “one, mistakes are okay, and two, this wasn’t about the right
Modeling provided opportunities to create \textit{mathematically powerful students} that can engage in mathematical problem solving and reasoning, connect mathematics to other disciplines and daily life, communicate mathematical outcomes, gain confidence in one’s mathematical ability, and appreciate the value of mathematics (NCTM, 1989, p. 5). There was no longer value in being ‘right’ or ‘wrong,’ but a focus on reasoning through possibilities of what might help us understand the task at hand.

To develop mathematical power while modeling students had to engage in authentic tasks with multiple solution strategies, but they also had to change perceptions of who owns mathematical knowledge. Students typically look to teachers as the keeper of knowledge who enables students to learn by sharing their expertise. In modeling, however, expertise is shared between teachers and students while knowledge is generated and constructed by students themselves. Because modeling tasks are open-ended and allow for multiple interpretations, students have access and freedom to think for themselves and construct new knowledge from previous knowledge and experiences.

Hawkins (2003) descriptions of “I” and “Thou” are often perceived as representing complex relationships between a teacher and student, but we might also interpret it as the complex interactions from student-to-student as knowledge is expanded and shared from peer to peer. These student-centered interactions became essential to successful navigation of the modeling cycle and shifted student perceptions of how one develops mathematical knowledge. In her interview, Jesse stated “I think it was good for us to be able to do things on our own and figure things out by ourselves. It was helpful to
get ideas but at the same it was helpful to not because we could think through things. We got through them, we just helped each other out.” (May 17, 2016). Through the use of modeling tasks, students came to realize they had the power to develop thorough and complex mathematical ideas by sharing expertise with each other. I as the teacher was no longer the ‘keeper of knowledge,’ but the support system as students generated this knowledge for themselves.

**Communication Needs to be Two-way**

_They had their own ideas and they really wouldn’t listen to our ideas. They just kind of shot them down immediately. So, it was really frustrating to have that happen. I guess the main source of conflict was not being able to communicate well enough._

~Brooke (Student Interview, April 26, 2016)

The importance of communication in the mathematics classroom is stressed throughout NCTM and CCSS-M standards documents, yet what became abundantly clear through mounting frustrations in a unit on modeling was the importance of this communication being two-way. That is, not only did students need to discuss their own ideas and strategies, but they needed these ideas to be heard by their peers. They needed their peers to listen. Conflicts arose in teams, particularly with students like Brooke and Matt, when students felt like their ideas were not heard and valued by their peers. In research on group work, Esmonde (2011) found students simply want to be perceived as “authoritative and competent members of the classroom community” (p. 54). As a result, they seek approval and validation from their peers. So much emphasis is placed on the
need to have students discuss and talk about mathematics that we may forget to stress what productive communication should look. Talking for the sake of talking is not enough. Teachers need to structure and support classroom interactions that does not just value what students say, but also emphasizes the power in what is *heard*.

**Teaching is Research**

When I initially set out on this dissertation journey, I was nervous about the size and scope my data collection and analysis would need to be to provide an in-depth understanding of the teaching and learning associated with mathematical modeling. After all, my training is in teaching, not in research. What I came to realize, however, was the process of teaching is like the process of research. When teachers embark on a serious inquiry into their teaching, learning, and curriculum, they naturally employ research methods. We collect data on students without referring to a methodology. We talk to students about prior experiences, strengths, weaknesses, and perceptions but do not call it an interview. We analyze student work and assessment data without referring to them as artifacts. We reflect on conversations and classroom events without painstakingly documenting every interaction. Research, although more systematic and intentional, is simply what good teachers do on a daily basis without calling it ‘research.’

In this study, I deliberately and meticulously collected data not just for research, however, but to understand my teaching. What I learned about myself in reflecting on my teaching practices extends beyond modeling instruction and encompasses interactions I have with students on a daily basis. For example, I always viewed myself as a good ‘questioner.’ I teach through constant questioning and rarely answer a question posed to
me without asking a question in return. However, because of the extensive data collection methods and analysis employed in this study, I came to recognize that perhaps I am not as good at questioning as I previously thought. I realized early on in the modeling unit that I “divulge too much information or try to pigeonhole students into a specific strategy. Sometimes I am too quick to get students on a specific path or try to generalize too quickly when maybe I should let them play a little longer” (March 16, 2016). I also found that because I try to reach so many students in one class period I sometimes “get into too big of a hurry” and “I don’t stop to just listen” (March 17, 2016). These realizations were initially disappointing to me but has allowed me to alter the intent and direction of daily conversations with students in all my courses. Now when conversing with students, I am careful not to answer questions too quickly and focus my own questions after conscientiously listening to and taking into account student ideas. Thankfully, reconciling the processes of teaching and research has provided me opportunities to improve the quality of my teaching beyond the time-frame of this particular research study.

**How Modeling has Evolved in My Classroom**

The events that were laid out in this study occurred during the 2015-2016 school year. During the 2016-2017 school year, I was able to convince another precalculus teacher at River Junction Senior High to incorporate the modeling tasks used in a month of modeling in both our classrooms. However, we decided to spread out these tasks and embed them within the content of our curriculum that seemed the most natural fit. After one semester of the 2016-2017 school year, we were able to use slightly modified
versions of The High-Definition Trade-Off Dilemma and Enrollment at River Junction tasks. As with most new teaching experiences, the lessons learned the first time around generally allow for more quality outcomes the second time.

In the fall of 2016 when students encountered the High-Definition Trade-Off Dilemma task, for example, I was able to encourage students to develop more precise and usable models for consumers to use to determine recording times. Students did not leave their models as a system of equations, but rather combined and simplified these equations into one manageable formula where a customer would simply plug in known values for memory card size and time needing recorded and the formula computed recording times at each resolution. In contrast to the first time I used this task in my classroom, more students were also able to identify limitations to these models as the size of the memory card would restrict the total number of minutes to record. Several groups, for instance, developed a table of specifications that allowed customers to find the maximum allowable recording times in each resolution and stated the model would only be usable if the desired recording times fit within what the memory card allowed. During the month of modeling, students and myself had not considered this limitation. But, because my own knowledge of the situation expanded, I could more easily encourage students to think about situations in which their model might not work.

Teaching mathematics with reform approaches is not easy and requires teachers to think about instruction and student learning differently. My instruction during a month of modeling was not perfect nor will it ever be, but as I continually practice, develop, and
refine my modeling instruction, I provide additional and more powerful opportunities for students to develop modeling abilities.

**Expanding Outside My Walls**

My natural next steps as a teacher-researcher is to replicate what I have learned through modeling opportunities in precalculus to other curriculums. What does modeling look like in an algebra classroom? How will it work in geometry? As a school district, we have adopted the Common Core State Standards in Mathematics and Language Arts even though our state has their own set of standards. Our new high school mathematics curriculum aligned to these standards was enacted beginning in the 2016-2017 school year. As a result, learning how to develop modeling abilities both in students and teachers has never been more important at a district level. As a teacher leader, others look up to me to provide insight, training, and feedback on how to instructionally implement and support student learning of these standards. Since modeling is a crucial aspect to CCSS-M, it will need to be an essential part of our students’ mathematics experience.

Finally, placing the results of this study within a larger theoretical context may provide knowledge to others wanting to support student learning in mathematical modeling (Cobb & Gravemeijer, 2008). By providing domain-specific instructional theories consisting of “a substantiated learning process that culminates with the achievements of significant learning goals as well as the demonstrated means of support that learning process” (Cobb & Gravemeijer, 2008, p. 77), I have provided useful research and instructional strategies to others wanting to modify or build on my results.
both within my district as we transition to CCSS-M standards and those outside my
district wanting to improve their own modeling design and teaching abilities.

Future Research

The work detailed in this study suggests future research may need to focus on the
development of modeling abilities with students who typically struggle in mathematics.
The precalculus students who participated in this study are typically perceived to be
“good” at math. With a few exceptions, these students were ones who had found
previous success in their mathematics courses and most generally liked learning
mathematics. Do modeling abilities develop differently in students who consistently
perform below grade level? And what supports are needed for them to successfully
navigate the modeling process?

In this study, my goal was to develop general modeling abilities not particular
mathematical skills. Yet, “models can be used as motivation for learning new techniques
and new content” (Bliss et al., 2016, p. 45). As a result, additional research on the design
and enactment of modeling tasks may be necessary when the learning of specific content
is the goal. Does the design and instruction of modeling change when the intent changes
to the learning of specific mathematical skills? And if it changes, how does it change?

Finally, the development of modeling abilities over a prolonged period of time
needs to be better studied and understood. This study was only a small snapshot of five
weeks of modeling instruction. The students in this study were primarily juniors in high
school who had encountered modeling for the first time. What happens to modeling
abilities when modeling is introduced periodically and continuously over several years
and throughout several curriculums? How do student perceptions of mathematics evolve as a result?

**In Conclusion**

Looking back on my experiences over the last 15 months of this dissertation study, the lesson to be learned is simply “Start big. Start small. Just start” (Bliss et al., 2016, p. 92). Teaching in new ways, with unfamiliar curriculum, towards unknown student outcomes can feel daunting. This uncertainty turns some teachers away from reform efforts and back into more comfortable and familiar environments. I began this study with the desire to create opportunities for students to educate themselves (Hawkins, 2003) by simply asking (1) how do I expand my own learning in order to help students develop mathematics and thinking skills needed to be productive citizens and (2) on what do I focus this learning? These questions developed into a more comprehensive and thorough analysis the magnitude of which may well be intimidating for the average teacher, yet insight and understanding can be gained by simply starting somewhere. We need to continue to encourage teachers to reflect on teaching and learning in order to identify manageable ways practices may evolve. For the only thing worse than giving up is never trying at all.
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Appendix A
Modeling Unit Student Reflection

Please be open and honest about your experiences these last few weeks. Mrs. Buhrman is very interested in knowing if you learned anything over the last few weeks and if you feel modifications need to be made for the future. Your input is very valuable!

1. What is one moment or idea that will stand out to you most from this modeling unit?

2. What do you feel is the most important thing you learned in this modeling unit?

3. Describe one thing you accomplished that made you proud of yourself.

4. What factors or support helped you when you found moments of success in this unit? In other words, when were you individually most successful?

5. What did you learn from your peers during the modeling unit?

6. When was your team most successful? When was your team least successful?

7. How much did you accomplish on your own after class as you tried to individually work towards your team’s goals? What success did you find? What hindered your success if you didn’t accomplish much at night for homework?

8. What moments caused you frustration throughout the unit? Can you recall them specifically? What helped you get through the frustrations? (Why didn’t you give up?)

9. Have you ever had any experiences like this in your previous math classes? If so, can you describe what you did?

10. Do you think projects like the ones you participated in are valuable for a student’s learning experience and should be incorporated more throughout high school? Why or why not?
Appendix B
Samantha Interview Protocol
IRB#14999

Project: The Design and Enactment of Modeling Tasks: A self-study on the development of modeling abilities in a secondary mathematics course

Time of Interview: 1:56 P.M.
Date: May 17, 2016
Place: Room 531B, River Junction Senior High
Interviewer: Mrs. Danielle Buhrman
Interviewee: Samantha
Position of interviewee: precalculus student

Relating to ‘prior history’ and general learning
1. How would you describe your previous math experiences? Thinking back to earlier in precalc, algebra 2, geometry, middle school, etc., what do you feel like you learned?
   b) What did you struggle with?

2. How do you feel you best learn mathematics? What helps you be successful in math class?
   b) How would you describe your work ethic and willingness to learn mathematics in a typical mathematics classroom environment? What typically motivates you?

Related to the unit itself
3. (Show students the modeling cycle.) In general, what do you feel is the most important thing you learned about mathematics in the unit?
   b) What did you learn about the modeling cycle in this unit? What about modeling itself sticks out to you the most?

4. What did you like most about the unit?
   b) What did you like least?

5. What did I do during the unit that you found helpful as you learned how to model mathematical situations?
   b) What supports or actions did I do that you didn’t find helpful in learning how to model?

6. How did your interactions with your peers help you during the unit?
   b) What interactions with peers hindered your success you during the unit?

Specific to Samantha’s experience
7. In the beginning in the Intro to Modeling section, you noticed that modeling tasks seemed to not just have one right way to do it. Did you notice that as you participated in the unit?
8. During the Memorial Stadium Project, I remember you being particularly excited and devoting a lot of time to your work. You even showed me a picture of a teammate giving you a thumbs up to your work as you collaborated through your group messaging outside of class. What made you so excited?

9. In the River Junction enrollment project, you also showed a lot of perseverance. You and your team developed two models based on trends you were seeing in the enrollment data. What about this project motivated you to work so hard?

10. Were there any other times you felt especially successful in the unit that you can recall? Can you describe to me what happened and why you felt successful?

11. In the last project where your team found the number of blades of grass on the football turf, you didn’t seem as ‘into it.’ Why?

12. (Grab Samantha’s project 5 work.)
Your team made a mistake in your model for the turf project. Can you clarify to me what you and your team were thinking there?

13. I don’t really recall a time I saw you especially frustrated or struggling, but can you think about a time you struggled during the unit. Can you describe to me what happened? What did you struggle with? Did anything help you overcome this struggle?

Overall reflections….
14. (Show unit outline with project list)
What modeling task did you feel you learned the most from? What did you learn?

15. Which project was the most interesting to you? Why?

16. (Show list of modeling abilities)
At the beginning of the unit I gave you a list of ‘modeling abilities’ that I wanted to try and help you develop over the course of the unit. Looking at this list, what modeling ability do you feel like you developed or improved upon the MOST throughout this unit? Why do you feel that way?

17. What modeling ability do you feel like you developed or improved upon the LEAST throughout this unit? Why do you feel that way? What do you think would have helped you develop these better?

18. Looking at your before and after statements on what is mathematics, has your perceptions of math changed? If so, in what ways?

19. What could I do to help students next year in the modeling unit? Do you have any suggestions to make it better?
Appendix C
Chris Interview Protocol
IRB#14999

Project: The Design and Enactment of Modeling Tasks: A study on the development of modeling abilities in a secondary mathematics course

Time of Interview: 2:00 p.m.
Date: April 27, 2016
Place: Media Center, River Junction Senior High
Interviewer: Mrs. Danielle Buhrman
Interviewee: Chris
Position of interviewee: precalculus student

Relating to ‘prior history’ and general learning
1. How would you describe your previous math experiences? Thinking back to earlier in precalc, algebra 2, geometry, middle school, etc., what do you feel like you learned?
   b) What did you struggle with?

2. How do you feel you best learn mathematics? What helps you be successful in math class?
   b) How would you describe your work ethic and willingness to learn mathematics in a typical mathematics classroom environment? What typically motivates you?

Related to the unit itself
3. (Show students the modeling cycle.) In general, what do you feel is the most important thing you learned about mathematics in the unit?
   b) What did you learn about the modeling cycle in this unit? What about modeling itself sticks out to you the most?

4. What did you like most about the unit? What did you like least?

5. What did I do during the unit that you found helpful as you learned how to model mathematical situations?
   b) What supports or actions did I do that you didn’t find helpful in learning how to model?

Specific to Chris’s experience
6. A moment for me that sticks out about your work is in the Memorial Stadium project when you were drawing a scale model and noticed some issues with the measurements I gave you. You were also getting very frustrated. What did you learn from this?
   b) You showed a lot of perseverance in particular with this project and the work/frustrations you had. What about this project motivated you to work so hard
   c) (Show work for project 2) Yet in your individual work for this project you didn’t elaborate on all the observations you made and work you had done, why is that?
7. In class you shared insightful ideas and seemed to make several connections between the real life situation and the mathematics you were conducting, yet in your individual work you struggled a bit. What you submitted for a grade didn’t seem to reflect all the thinking you had done. Why do you think that is?
b) You began elaborating more in the crime safety project and the final student choice project. Why is that? What helped?

8. In your original team with Megan and Monica, how do you feel it went? What was the source of conflict that arose?

9. How do you feel it went after I switched your group around? Were you more or less successful?

10. Were there any other times you felt especially successful in the unit that you can recall? Can you describe to me what happened and why you felt successful?

11. Other than the Memorial Stadium project, I cannot recall another time I saw you particularly frustrated or struggling. Can you recall any other time you were especially frustrated? Can you describe to me what happened? What did you struggle with? Did anything help you overcome this struggle?

Overall reflections….
12. (Show unit outline with project list) What modeling task did you feel you learned the most from? What did you learn?

13. Which project was the most interesting to you? Why?

14. (Show list of modeling abilities) At the beginning of the unit I gave you a list of ‘modeling abilities’ that I wanted to try and help you develop over the course of the unit. Looking at this list, what modeling ability do you feel like you developed or improved upon the MOST throughout this unit? Why do you feel that way?

15. What modeling ability do you feel like you developed or improved upon the LEAST throughout this unit? Why do you feel that way? What do you think would have helped you develop these better?

16. Looking at your before and after statements on what is mathematics, has your perceptions of math changed? If so, in what ways?

17. What could I do to help students next year in the modeling unit? Do you have any suggestions to make it better?
Appendix D
Brooke Interview Protocol
IRB#14999

Project: The Design and Enactment of Modeling Tasks: A study on the development of modeling abilities in a secondary mathematics course

Time of Interview: 2:35 p.m.
Date: April 26, 2016
Place: Media Center, River Junction Senior High
Interviewer: Mrs. Danielle Buhrman
Interviewee: Brooke
Position of interviewee: precalculus student

Relating to ‘prior history’ and general learning
1. How would you describe your previous math experiences? Thinking back to earlier in precalc, algebra 2, geometry, middle school, etc., what do you feel like you learned?
   b) What did you struggle with?

2. How do you feel you best learn mathematics? What helps you be successful in math class?

3. How would you describe your work ethic and willingness to learn mathematics in a typical mathematics classroom environment? What typically motivates you?

Related to the unit itself
4. In general, what do you feel is the most important thing you learned about mathematics in the unit?
   b) What did you learn about the modeling cycle in this unit? What about modeling itself sticks out to you the most?

5. What did you like most about the unit?
   b) What did you like least?

6. What did I do during the unit that you found helpful as you learned how to model mathematical situations?
   b) What supports or actions did I do that you didn’t find helpful in learning how to model?

Specific to Brooke’s experience
7. Did you feel more motivated to participate and complete tasks in the modeling unit than in prior mathematics experiences? You commented in your reflection that you were “much more willing to do my homework from this unit than anything else. Doing it made me happy and I felt accomplished.” Why do you think that is? What helped?
b) Despite this you still had difficulty submitting tasks on time (which has been a problem all year). Why is that? What could have helped you complete individual work more effectively and in a more timely manner?

8. I noticed and you commented on the last project about the animal shelter being especially interesting to you. Why was it your “absolute favorite” thing you’ve ever done?
   b) What did you learn about mathematics from this project?
   c) What did you learn about yourself during this project?
   d) Yet in your individual work for this project you didn’t elaborate on all the observations you made and work you had done, why is that?
   e) I’d like to understand more about your goal for the animal shelter project. In your presentation and individual work feedback I commented on how you didn’t seem to do anything with the models you had developed. Now looking back on it, what mathematically based questions might you have wanted to ask about your model and the situation?

9. In you original team with William and Nicholas how do you feel it went? What was the source of conflict that arose?

10. I can recall you and your team getting particularly frustrated in the Memorial Stadium project. Can you recall what happened and why you and your team struggled?

11. How do you feel it went after I switched your group around? Were you more or less successful? Why?

12. After the River Junction enrollment project with your old team I asked you if you thought this modeling experience has been useful for you and you replied with a “no.” Do you still feel that way? Why or why not?

13. Were there any other times you felt especially successful in the unit that you can recall? Can you describe to me what happened and why you felt successful?

14. You said in your final reflection in response to “5. What did you learn from your peers during the modeling unit?” that “Just because we are equally smart doesn’t make us equally matched.” What did you mean by that? Can you elaborate?

15. I also noticed throughout the unit you trying to get Matt more involved and confident in his ideas. I was very proud of you for this. Why did you feel compelled to encourage him?

Overall reflections…. 
16. (Show unit outline with project list) What modeling task did you feel you learned the most from? What did you learn?

17. (Show list of modeling abilities) At the beginning of the unit I gave you a list of ‘modeling abilities’ that I wanted to try and help you develop over the course of the unit. Looking at this list, what modeling ability do you feel like you developed or improved upon the MOST throughout this unit? Why do you feel that way?

18. What modeling ability do you feel like you developed or improved upon the LEAST throughout this unit? Why do you feel that way? What do you think would have helped you develop these better?

19. Looking at your before and after statements on what is mathematics, has your perceptions of math changed? If so, in what ways?

20. What could I do to help students next year in the modeling unit? Do you have any suggestions to make it better?
Appendix E
Matt Interview Protocol
IRB#14999

Project: The Design and Enactment of Modeling Tasks: A study on the development of modeling abilities in a secondary mathematics course

Time of Interview: 7:00 a.m.
Date: April 27, 2016
Place: Room 531B, River Junction Senior High
Interviewer: Mrs. Danielle Buhrman
Interviewee: Matt
Position of interviewee: precalculus student

Relating to ‘prior history’ and general learning
1. How would you describe your previous math experiences? Thinking back to earlier in precalc, algebra 2, geometry, middle school, etc., what do you feel like you learned?
   b) What did you struggle with?

2. How do you feel you best learn mathematics? What helps you be successful in math class? (ask more questions based on his responses)

3. How would you describe your work ethic and willingness to learn mathematics in a typical mathematics classroom environment? What typically motivates you? (ask more questions based on his responses...I want to understand what motivates him)

Related to the unit itself
4. In general, what do you feel is the most important thing you learned about mathematics in the unit?
   b) (Show students the modeling cycle.) What did you learn about the modeling cycle in this unit? What about modeling itself sticks out to you the most?

5. What did you like most about the unit?
   b) What did you like least?

6. What did I do during the unit that you found helpful as you learned how to model mathematical situations?
   b) What supports or actions did I do that you didn’t find helpful in learning how to model?

Specific to Matt’s experience
7. In your original team with Nicholas and William how do you feel it went? What was the source of conflict that arose?
8. I can recall you and your team getting particularly frustrated in the Memorial Stadium project. In your final reflection you also made the comment that this is when you felt least successful. Can you recall what happened and why you and your team struggled?

9. How do you feel it went after I switched your group around for Project 4? You mentioned in your final reflection that you felt your team was most successful for this project. Why?

10. Due to your FBLA convention, we had to modify your group again for the final project. Can you describe to me how it went? Did you find any success or progress towards understanding how to model? Why or why not?

11. Throughout the unit, you seemed to lack a lot of confidence in your ideas. Why is that?
   b) Why, mathematically speaking, do you think modeling was difficult for you?
   c) You made several comments in your final reflection how you learned you were ‘bad at math.’ Why do you feel like you were bad at this? Did anything help? What do you think would have helped you feel more successful?
   d) All year long you seemed to be very successful in class and were willing to help those around you and share your ideas. What changed during the modeling unit that made you doubt yourself?

12. After the River Junction enrollment project with your old team I asked you if you thought this modeling experience has been useful for you and you replied with a “no.” Do you still feel that way? Why or why not?

13. Were there any other times you felt especially successful in the unit that you can recall? Can you describe to me what happened and why you felt successful?

Overall reflections….

14. (Show unit outline with project list) What modeling task did you feel you learned the most from? What did you learn?

15. Which project was the most interesting to you? Why?

16. (Show list of modeling abilities) At the beginning of the unit I gave you a list of ‘modeling abilities’ that I wanted to try and help you develop over the course of the unit. Looking at this list, what modeling ability do you feel like you developed or improved upon the MOST throughout this unit? Why do you feel that way?

17. What modeling ability do you feel like you developed or improved upon the LEAST throughout this unit? Why do you feel that way? What do you think would have helped you develop these better?
18. Looking at your before and after statements on what is mathematics, has your perceptions of math changed? If so, in what ways
19. What could I do to help students next year in the modeling unit? Do you have any suggestions to make it better?
Appendix F
Luis Interview Protocol
IRB#14999

Project: The Design and Enactment of Modeling Tasks: A study on the development of modeling abilities in a secondary mathematics course

Time of Interview: 1:40 p.m.
Date: April 28, 2016
Place: Media Center, River Junction Senior High
Interviewer: Mrs. Danielle Buhrman
Interviewee: Luis
Position of interviewee: precalculus student

Relating to ‘prior history’ and general learning
1. How would you describe your previous math experiences? Thinking back to earlier in precalc, algebra 2, geometry, middle school, etc., what do you feel like you learned?
b) What did you struggle with?

2. How do you feel you best learn mathematics? What helps you be successful in math class?
b) How would you describe your work ethic and willingness to learn mathematics in a typical mathematics classroom environment? What typically motivates you?

Related to the unit itself
3. (Show students the modeling cycle.) In general, what do you feel is the most important thing you learned about mathematics in the unit?
b) What did you learn about the modeling cycle in this unit? What about modeling itself sticks out to you the most?

4. What did you like most about the unit?
b) What did you like least?

5. What did I do during the unit that you found helpful as you learned how to model mathematical situations?
b) What supports or actions did I do that you didn’t find helpful in learning how to model?

6. How did your interactions with your peers help you during the unit?
b) What interactions with peers hindered your success during the unit?

Specific to Luis’s experience
7. At the beginning of the unit, you seemed quiet and more hesitant to share your ideas. Was that actually the case? And if so, why do you think that happened?
b) Did you become more vocal and comfortable as the unit progressed? If so, why do you think that happened? If not, what would have helped you become more comfortable?

8. Your teammates made comments to me that they felt they learned a lot from you during the unit. How do you feel you contributed to your team’s success?

9. You said in your reflection that you felt least successful presenting. Why do you think that was?

10. What was your goal for your analysis on immigration growth (project 5)? I couldn’t quite follow what your team was trying to accomplish.
   b) Why do you feel you failed when you created your model for project 5?

11. Were there any other times you felt especially successful in the unit that you can recall? Can you describe to me what happened and why you felt successful?

12. I don’t really recall a time I saw you especially frustrated or struggling, but can you think about a time you struggled during the unit. Can you describe to me what happened? What did you struggle with? Did anything help you overcome this struggle?

Overall reflections….
13. (Show unit outline with project list) What modeling task did you feel you learned the most from? What did you learn?

14. Which project was the most interesting to you? Why?

15. (Show list of modeling abilities) At the beginning of the unit I gave you a list of ‘modeling abilities’ that I wanted to try and help you develop over the course of the unit. Looking at this list, what modeling ability do you feel like you developed or improved upon the MOST throughout this unit? Why do you feel that way?

16. What modeling ability do you feel like you developed or improved upon the LEAST throughout this unit? Why do you feel that way? What do you think would have helped you develop these better?

17. Looking at your before and after statements on what is mathematics, has your perceptions of math changed? If so, in what ways?

18. What could I do to help students next year in the modeling unit? Do you have any suggestions to make it better?
Appendix G
Intro to Modeling Classroom Document

**Intro to Modeling**

Precalculus

Name

Team Members:

What is Modeling?

The next four weeks of class are going to be devoted to the study of mathematical modeling. You probably don’t know a lot about modeling now, but hopefully you will after you complete five projects over the course of this unit. What you need to know now is that your team is going to be an essential resource for you. All projects throughout the unit will be completed as a team; and in order to accomplish the goals of each project you will need to communicate and work together. If you are ever gone, YOU need to make sure to touch base with Mrs. Buhrman and members of your team regarding the day of your absence. If you were in charge of doing something, that still needs to be sent to your team preferably prior to our class time (which are similar expectations to a job in ‘real life’). You also need to find out prior to the next class what was discussed in the period you missed and what you need to do to help your team! Be responsible and be someone your team can count on!

**Part 1: Why should you care about modeling?**

Now that that’s out of the way...lets get started! So what is modeling anyway? Modeling is something almost all professions will find useful in one form or another, but to answer this question, let’s first take a look at someone who does mathematical modeling for a living. As a team using a round table format (each person takes turns reading a paragraph out loud), read the article Career Interview: Mathematical Modeling Consultant. Then discuss and answer the following questions with your team. **Person D** summarize and submit your team’s responses to these three questions on the Google Form within this assignment’s section of the website.

1. **Based on this article, what do you think modeling is?**

2. **Did anything surprise you about what Nira was able to do with modeling skills?**

3. **Nira’s speciality is mathematics, but plenty of other professions use mathematical modeling to help ‘answer questions.’ Can you think of other professions that might find this skill useful?**

**Part 2: What is modeling?**

Modeling is **the process of representing a real life situation mathematically in order to better understand the situation and make decisions or predictions related to it.** Sometimes models use equations or formulas, other times it is a description. When you analyze a situation in which modeling may be useful, you are not answering a ‘mathematical question’ but rather analyzing a ‘real life’ problem which mathematics might be useful in helping you try to understand or accomplish some goal.
Here is an example of a modeling task:

**Space Shuttle Problem: No More Space Shuttles**

On July 21, 2011, the 135th and final US Space Shuttle landed in Florida after its 13-day mission into orbit, complete with a docking at the International Space Station (ISS). NASA will now have to rely on other nations or commercial endeavors to travel into space until a replacement vehicle is developed and constructed. Develop a comprehensive ten-year plan complete with costs, payloads, and flight schedules to maintain the ISS.

Some interesting facts possibly worthy of your consideration:

- The ISS is at full capacity with 6 astronauts, but can surge during shuttle docks to as high as 13.
- The ISS is scheduled to remain in service until at least the year 2020.
- Historically, transport to the ISS using US Shuttles has cost between $5000-10,000 per pound. Shuttle missions have lasted approximately 10-14 days in orbit. Missions on board the ISS typically last around six months.
- Recently, progress has been made within private industry to launch unmanned rockets into space.
- Russia is willing to launch US astronauts into space at a cost of about $60 million each.


With your team, discuss the following. **Person B** summarize and submit your team’s responses to these three questions on the Google Form within this assignment’s section of the website.

4. Based on the definition of modeling, why do you think this is a good example of a modeling task?

5. What knowledge or skills do you think you will need to work on this or similar tasks?

6. How is this task different than other tasks you have been given in your mathematics courses?

**Part 3: Goals and Grading**

Your goal for this unit is:

To learn how to model authentic situations with mathematics using the Common Core State Standards modeling process so that you develop specific modeling abilities.

These modeling abilities include:

**Modeling Abilities Developed during Modeling Unit**
1. Applying everyday knowledge to understand where mathematics may occur
2. Translating everyday knowledge into mathematical knowledge (mathematical literacy)
3. Flexible & creative thinking
4. Data analysis & collection
5. Perseverance
6. Interpreting what the mathematics means in real life (quantitative literacy)
7. Reflection
8. Reasoning
9. Communication

Think of this as your target list for this unit!

With your team, discuss the following. Person A summarize and submit your team’s responses to this question on the Google Form within this assignment’s section of the website.

7. How is this list of skills different than the targets typically given to you for a unit in a mathematics course?

As we progress through this unit I hope you begin to understand that the rationale and thinking behind the mathematical decisions you make is just as, if not more, important than the mathematical procedures and operations you use. As your teacher, my goal is to help you navigate through the modeling process (see below) so that you have the best chance at developing these skills. This process is what you and your team should use to explore each task! Think of this as your ‘steps’ to follow.

Modeling Cycle - Common Core State Standards

(1) identify variables in the situation and select those that represent essential features
(2) formulate a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables
(3) analyze and perform operations on these relationships to draw conclusions,
(4) interpret the results of the mathematics in terms of the original situation
(5) validate the conclusions by comparing them with the situation, and then either improving
   the model or, if it is acceptable,
(6) report on the conclusions and the reasoning behind them.

You will see as we develop models throughout this unit that you may need to work through this process multiple times in one task. In other words, you might not succeed right away. Your strategy and the mathematics you need to use will not always be ‘obvious’ making your interpretations, reasoning, and thinking that much more important. There is no ‘right’ or ‘wrong’ answer, only appropriate and inappropriate rationale.

Unit Outline & Grades
Over the course of this 4 week unit your team will participate in the following 6 tasks:

*What is modeling?* (aka...what you are doing now)

*Project 1* - The High Definition Trade-Off Dilemma
*Project 2* - Calculating Width Indirectly: What if I can’t measure it?
*Project 3* - Enrollment at Grand Island Senior High: How big will we get?
*Project 4* - How safe are we: Developing a safety rating for Grand Island
*Project 5* - Student Choice: Your team will pick a situation you would like to know more about that can be better understood through mathematical modeling.

You might be asking yourself, if there is no ‘right’ or ‘wrong’ how will I know I have succeeded? How will you be able to assign me a grade?

To answer the first question, you will need to assess ‘reasonableness.’ That is, have you accomplished the goal of the task and are your results reasonable based on the information you have been given?

To answer the second question, I will score both the individual work you submit for each task and the final product that your team develops to accomplish the task’s goals. As a result, you will get two scores for each task with feedback. No scores or grade will be assigned for the first project to give you a chance to become familiar with the modeling cycle and expectations from the rubric. The Student Choice modeling task (project 5) is the culminating project and therefore will be scored, graded, and recorded in the 80% part of our gradebook. YOU may choose which of the remaining three modeling tasks will also be scored, graded, and recorded in the 80% part our gradebook based on which tasks best show your growth and achievement. The two projects you do not choose will still be scored and graded, but will be recorded in the 20% or formative side of the gradebook.

**Individual Score:**
Since our goal is to learn ‘how to model’ ultimately developing ‘modeling abilities,’ your individual scores for each task will be based on how well you successfully navigated the modeling cycle previously described. It is important that you include all your work, ideas, incorrect strategies, final model/assessment of the situation, and anything else you used to analyze the situation and accomplish the task’s goals. (In other words, don’t erase anything.) This certainly does not mean that you will be working by yourself. On the contrary, your team will be invaluable to you as they question your work, share ideas, make sense of the situation, and assist you in navigating through the modeling cycle. However, the work you submit must be easy to follow so that I can ‘see your thinking’ as you progressed through the task. These individual scores will be based on the Individual Modeling Cycle Rubric (see website).

**Team Score:**
Each task will require your team to develop a ‘final product’ that accomplishes the tasks’ goals. In each product your team will be asked to explain your model, exhibit how you
developed your model, and communicate the results of your analysis to the audience mentioned in the task. The score your team receives for this final product will be based on the Team Product Rubric (see website).
Appendix H
Recording Quality Project 1 Classroom Document

Modeling Unit Project #1                           Name______________________________
Precalculus                                         Team Members:

The High-Definition Trade-Off Dilemma

A friend of mine, Heather, was on a family vacation and realized her GoPro wasn’t going
to be able to record the entire 5 hour scuba dive they had scheduled. From their boat she
was able to post a message to Facebook asking for some guidance. Her original post reads:

“Help me nerdy friends! We are going for a scuba dive in the reef later today
(super excited!) and realized my GoPro Hero4 isn’t going to record the entire dive in
high-def. Obviously, I want as much of it in 1080p high-def as possible to catch all the
awesome colors and wildlife but I want to record the entire 5 hour dive so I don’t miss
anything cool...like a reef shark. I have a 64gb memory card in my GoPro, how long
should I shoot in 1080p before switching to 720p to maximize the amount of high-def
footage I get?”

Being curious, I looked up the information about my GoPro and found that a 64gb
memory card can record 4 hours and 22 minutes at 1080p (high-definition) or 5 hours and
53 minutes at 720p.

And in case you were wondering….

1080p displays about 2,073,600 pixels per image. 720p displays about 921,600 pixels
per image. So obviously, Heather would prefer to capture the awesomeness of the reef in
nearly 2.1 million pixels!

Your Task:

1. Answer Heather’s question. What is the maximum amount of time Heather
should record in 1080p mode before switching to 720p mode and still be able to
record all 5 hours of her dive? Respond to her Facebook post with your
recommendation and a thorough but brief explanation as to how you came to that
number.

2. Generalize it: Develop a model that anyone could use if they know the total time
\( t \) they need to record and would like to know how long they should record in
each mode with a 64gb memory card. Provide justification that your model
makes sense based on mathematical features of the situation.

3. How would your model change if the memory card were only 16gb? 32gb? Can
you create a model that works with any size memory card? That is, can you
create a model that takes into account the size of the memory card and the total
time needed to record, and calculate the amount of time someone should record in
each mode? (After all...this model would be very handy for people!)
4. **Team task:** Write a letter to GoPro’s corporate office in order to provide information to consumers. Explain your model from #3 and how you developed it so they may include it on the specs on their website. Are there assumptions you made or limitations to your model? At the conclusion of this project, you will read letters produced by other teams and provide feedback to them about their model and rationale.

5. This is a situation presenting a trade-off: you compromise quality for the length of the recording. Can you brainstorm other trade-off situations that may benefit from a modeling analysis? Include this in your individual write-up. (Don’t be afraid to ask around!)

**Timeline:**

You will be given two class periods to work on each of the components you need for your task. Individually you need to submit all your work for questions 1-3 & 5. Make sure you have featured all of the components of modeling in your work and that it is clear and easy to follow (see Individual Modeling Cycle Rubric and graphic organizer). You will not get additional homework to work on; this is your only goal. Your team will need to write and submit **one letter** for the team task (#4). This needs to be ready to share with the class on the third day. Please have at least 2 copies of the letter printed for class.

Inspired by the task “Can you Save Tom’s Marriage?” from *Using Mathematics to Understand Our World*. Developed by the Math in the Middle Institute Partnership, University of Nebraska, Lincoln. Copyright 2007.
Appendix I
Width of Memorial Stadium Project 2 Classroom Document

Modeling Unit Project #2  Name _______________________________
Precalculus  Team Members: ________________________________

Calculating the Width of Memorial Stadium

Make sure you have watched the video of Mrs. Buhrman describing the measurements she has taken of Memorial Stadium before starting on this project. This video is also available in this project’s folder on the website.

Finding the length or width of longer objects is difficult to do without having access to sophisticated equipment. This is especially true when it is impossible for you to physically measure an object directly using rulers or electronic measuring tools as is the case when wanting to know the width of Memorial Stadium. However, we can take indirect measurements from an unknown distance away from that particular object in order to calculate the width. In order to find the width of Memorial Stadium, using a compass I measured the due north angle measurements to North Stadium at 94 degrees and South Stadium at 112 degrees. I then walked a distance of 200 feet straight north in the Haymarket Park parking lot using the GPS tracking on my phone. Finally I measured the angles again from this new location and found that to North Stadium it was now 100 degrees and to South Stadium was 120 degrees.

Your goal is to (1) calculate the width of Memorial Stadium given the measurements provided by Mrs. Buhrman. Then, using this work as a guide, (2) create a general model that will enable you to calculate the width of any object indirectly by using similar measurements (that is, two sets of two angle measurements taken from a known distance away from each other).

Team Task: You and your team will create a poster or posters that will showcase the work you have done to accomplish both parts of your goal. Your team will also need to explain the decisions you made, your strategies, and why your model works during a Carousel Walk of these posters. All members of your team must contribute to this ‘mini-presentation’ of your poster(s); you will get approximately 4-5 minutes to present. You will get two class periods to work on your calculations, model, and posters. Be ready to present with your posters on the third day. These posters must include:

1. The work, calculations, and pictures used to find the width of Memorial Stadium.
   Be sure to show all work and pictures clearly and accurately.
2. Your general model including all work, descriptions, and pictures used. Again, be sure your work is clear, your pictures are easy to understand, and all
variables/constants are clearly indicated so that we (your audience) can follow along with your verbal explanations.

**Individually:** You will hand in any of the work you developed as part of this task’s goal. Remember your work will also be assessed using the Individual Modeling Cycle Rubric, so be sure that you include evidence of all components of the modeling cycle and that your work is clear and easy to follow. (See rubric for expectations.)
Appendix J  
Data for Project 3 Classroom Document

River Junction School District Enrollment by Grade Level 2000-2016

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Appendix K  
River Junction Enrollment Project 3 Classroom Document  

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<td>Team Members:</td>
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Enrollment at River Junction Senior High: How big will we get?  

River Junction Public Schools looks a lot different than it did 15 years ago. As evident by the 69.5 million dollar bond passed last year, River Junction continues to look for space and resources to accommodate its growing student population. However, little attention has been paid to River Junction Senior High’s building and resources. Many of you have younger siblings. Some of you may find yourselves back here in a few years starting a family. What will River Junction Public Schools look like for them? What will River Junction Senior High look like?  

Mrs. Buhrman has given you data for beginning of the year enrollment for each grade level dating back to the 2000-2001 school year. Your team will analyze this data to better understand what is happening with RJPS enrollment, why it is happening, and to predict what River Junction Senior High may look like in 9 years (when this year’s Kindergarteners enter high school) and 15 years from now, so that you may provide actionable recommendations. You will get 3 class periods to complete the analysis and prepare your presentation. Be ready to share your team’s ideas on how to predict RJSH’s population in 9 years & 15 years on the 4th day with administrators who will be visiting the class.

**Presentation of work:**  
**Team task:** You and your team will design a series of posters, a presentation, or other multi-media of your choice to explain your analysis to RJSH administration (who will actually be coming in to look at this). What do you predict Senior High will look like in 9 years when this year’s Kindergarteners enter high school? In 15 years? Your project must include:  
1. A brief explanation of your analysis, models (how you made your predictions), and why your models are valid. Some of this may be done verbally to administrators so be prepared to speak about your work.  
2. Your predictions for the total RJSH enrollment in 9 years and 15 years.  
3. Your recommendations for solutions & why based on your analysis.  
This is your time to tell people in charge what you think should happen and why based on the evidence you are presenting. Be prepared for administration and the class to ask you questions about your analysis, predictions, and recommendations. You will have about 5 minutes to present so make sure you are short but effective!  
**Individually:** You will hand in any of the work you developed as part of this task’s goal including your responses to the guiding questions you found useful. Remember your work will also be assessed using the Individual Modeling Cycle Rubric, so be sure that you include evidence of all components of the modeling cycle and that your work is clear and easy to follow. (See rubric for expectations.)
Appendix L
Crime Rankings Project 4 Classroom Document

Modeling Unit Project #4
Precalculus
Name
Team Members:

How Safe Are We?

In the last year, River Junction has been recognized as one of the most dangerous cities in our state. Scores were developed to guide residents on where to live and not live by Motovo Real Estate Blog and RoadSnacks. They then used these scores to rank cities; these sites have River Junction as the 3rd and 2nd most dangerous cities in the state, ahead of the much larger cities. I don’t know about you, but as a resident of this town that alarms me!

Your task is to create a model to determine a safety score for River Junction and other major cities in our state in order to assess how ‘safe’ we are. Your team will have 2 ½ class periods to conduct your analysis. Because class is a short period Wednesday, we will begin presentations half-way through the period on Tuesday.

Team Task: You do NOT need to create a separate presentation for this task. Rather you will project your work to the class using Mrs. Buhrman’s document camera and your team will present using your actual work as the visual/talking points. You will present this to the entire class, however, not a small group as we have done in the past. We will go in order by Team number so Team 1 be prepared to go first. :)

Your team must be able to show and explain:

1. The information/data you chose to consider in your analysis and why you decided that data was important to consider. You may want to conduct research on crime ratings or ask adults what information they would want you to consider in your safety score.
2. A description of your model (i.e., what you are calculating).
3. A thorough description of your methodology and rationale. That is, how did you take into account your data in your model and why you made the decisions you did regarding this data.
4. Use your model to find a safety score for River Junction and other major cities in Nebraska (at minimum you must include Metroville, Jefferson, Central, and Loganview). How do these scores compare to River Junction? Rank these cities from safest to most dangerous. Do you agree with the rankings done by Motovo and RoadSnacks?
5. A description of how someone might interpret your data and what the score your model generated for River Junction means. In other words, how safe are we?

Our class will critique and vote on each presentation with a prize being awarded to the most effective methodology and model developed.
**Individually:** You will submit and be scored on your individual work that contributed to your team’s presentation. After the conclusion of this project you will also be given a couple reflection questions you must answer and submit with your individual work.

Crime reporting and demographic information has to be made public. The FBI compiles a database of all reported crimes across the United States. We do not yet have access to last year’s data so we will look at the data provided for 2014 for the number of murders, rapes, robberies, aggravated assaults, burglary, theft, motor vehicle theft, and arsons reported. Please note that the FBI classifies these crimes into two types: Violent and Property Crime. All major cities in Nebraska can be found on the class website including their populations.

And FYI because I didn’t know:

- **Larceny/Theft** means taking someone else’s property without their permission and with the intent of permanently removing it from their possession.
- **Robbery** means taking something from a *person* by force or threat of force.
- **Burglary** means you enter a building with the intent of committing a crime. That crime does not have to be a theft and it does not actually have to occur for you to be convicted.

Appendix M
Student Choice Project 5 Classroom Document

Modeling Unit Project #5
Precalculus
Name ________________________________
Team Members: _______________________

Student Choice Project

Description: Over the last several weeks you have developed models for various real-life situations where a mathematical analysis was beneficial in order to better understand that situation. However, these were all tasks that Mrs. Buhrman found interesting that she thought you might benefit from knowing more about. She may have failed miserably in her assumptions of what would interest a group of teenagers! Well now it is your turn!

Your team will pick and design your own project based on a real-life situation you think could benefit from being matematized. That is, you and your team will pick an event or situation you wish to know more about. You need to be able to identify the goals of your analysis (i.e. what is it that you want to know or understand) and those goals must be accomplished through a mathematical analysis and subsequent model.

Where to look for projects if you can’t come up with one:

1. Look at the ideas you listed from other projects. You have already given yourself a decent list.
2. Read the news! There are a lot of problems and situations that would benefit from a mathematical analysis in the news.
3. Ask around...ask your parents, adults, neighbors, etc. what situations they come across in their jobs that may benefit from a mathematical model.
4. Be observant. What things do you notice in a day? What are you interested in? What questions do you have?

Criteria for your Team’s Final Product:
Your team will summarize your situation and analysis in any format you wish (Powerpoint/keynote, posters, prepared handouts that you will teach us about, a video, letter to an interested audience, etc.). However, your product must include the following:

1. Description of your task that communicates the overall situation, the problem/questions you want answered, the goals for your analysis/model, and who your primary audience is and why (i.e. who would be interested in knowing the answer to your questions).
2. The information/data you chose to consider in your analysis, what assumptions you made about this information/data, where you found that information/data, and why you decided it was important to consider. (i.e., what knowledge or information did you need to bring in to better understand and analyze your situation)
3. A description of your model (i.e., what you are calculating), limitations to your model, and what revisions you made throughout the process.
4. A thorough description of your methodology and rationale. That is, how did you take into account your data or information in your model, why you made the
decisions you did regarding this information/data, and why your model works to accomplish your goals. If your model didn’t work and you did not have time to revise it that’s okay too. Include a description of why it does not work and what you would have tried next next to revise it.

5. A description of what your model communicates to your primary audience and why it is useful for them.

**Individually:** You will submit and be scored on your individual work that contributed to your team’s presentation in addition to your team’s overall analysis and presentation.
### Appendix N
Individual Modeling Cycle Rubric

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<th>Individual Modeling Cycle Rubric</th>
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<td><strong>Identify Essential Variables</strong></td>
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<td></td>
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<td>Score X 1</td>
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<tr>
<td>Identifies &amp; justifies the choice of variables in context of the situation and Identified variables take into account essential features of situation</td>
<td>Identifies but does NOT justify the choice of variables in context of situation and Identified variables do NOT take into account essential features</td>
<td>Identifies but does NOT justify the choice of variables in context of situation and Identified variables do NOT take into account essential features (i.e., variables identified will not result in a mathematically correct model to accomplish task goal)</td>
<td>Identifies but does NOT justify the choice of variables in context of situation and Identified variables do NOT take into account essential features</td>
<td>Did not identify or justify variables used</td>
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<td><strong>Formulate Model</strong></td>
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<td>Score X 2</td>
</tr>
<tr>
<td>Formulates an appropriate model which represents the situation accurately and Presents data &amp; model in an effective and organized manner and Provides complete work including assumptions used</td>
<td>Formulates an appropriate model which represents the situation accurately and Presents data &amp; model with limited effectiveness or organization and Work is missing essential info</td>
<td>Formulates a model which represents the situation but with minor errors</td>
<td>Formulates a model but it does not accomplish the task’s goals and contains major errors</td>
<td>Does not formulate a model</td>
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<tr>
<td><strong>Analyze &amp; Perform Operations</strong></td>
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</tr>
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<td>Analyzes model to draw conclusions about the situation and Procedures &amp; calculations are correct and Appropriate math language and notation are used</td>
<td>Analyzes model to draw conclusions about the situation and Procedures, calculations, math language, or notation contain minor errors</td>
<td>Analyzes model to draw conclusions about the situation and Procedures, calculations, math language, or notation contain major errors</td>
<td>Analysis of model is incomplete</td>
<td>Does not try to analyze model to draw conclusions about the situation</td>
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<td><strong>Interpret Results</strong></td>
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<td>Interprets results in context of the original situation and Clearly communicates process, solution, &amp; results in context of the original situation and</td>
<td>Interprets results in context of the original situation and Communicates process, solution, and results with limited clarity and Shows little evidence connecting analysis to</td>
<td>Attempts to interpret the results in context of the original situation but contains minor errors and Communicates process, solution, and results with limited clarity and Shows little evidence connecting analysis to</td>
<td>Attempts to interpret the result in context of the original situation but interpretation is incorrect and incomplete</td>
<td>Does not interpret results in context of the original situation</td>
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<td>Validate Conclusions</td>
<td>Shows thorough evidence connecting analysis to situation and Show some evidence connecting analysis to situation</td>
<td>situation</td>
<td>attempts to justify the reasonableness of the model in context of the situation but contains minor errors and Some evidence of reflection was done to monitor or no attempts were made to modify model if adjustments were necessary and Attempts to identify potential inaccuracies but is incomplete</td>
<td>attempts to justify the reasonableness of the model in context of the situation but contains minor errors and Some evidence of reflection was done to monitor or no attempts were made to modify model if adjustments were necessary and Attempts to identify potential inaccuracies but is incomplete</td>
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<p>| Individual Modeling Cycle Rubric Score to Grade Conversions |
|---|---|---|
| Rubric Score | Percentage Grade | Category |
| 36 | 100 | Excellent |
| 33-35 | 95 | Excellent |
| 30-32 | 90 | Excellent |
| 27-29 | 85 | Good |
| 24-26 | 80 | Good |
| 21-23 | 75 | Fair |
| 18-20 | 70 | Fair |
| 15-17 | 65 | Unsatisfactory - consider redoing project |</p>
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<tr>
<td>13 or below</td>
<td>No Grade Given</td>
<td>Redo Project and Correct Errors</td>
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*Based on Table 3: Sample Grades and Categories, Mertler (2002)*
### Appendix O
#### Team Product Rubric

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<td><strong>Mathematical Knowledge</strong></td>
<td>Shows understanding of the problem’s mathematical concepts &amp; principles and&lt;br&gt;Uses appropriate mathematical terminology &amp; notation and&lt;br&gt;Executes algorithms completely &amp; correctly and&lt;br&gt;Uses correct data collection &amp; analysis techniques (if applicable)</td>
<td>Shows nearly complete understanding of the problem’s mathematical concepts &amp; principles and&lt;br&gt;Uses appropriate mathematical terminology, notation, &amp; algorithms with only minor errors and&lt;br&gt;Uses data collection and analysis techniques with only minor errors (if applicable)</td>
<td>Shows understanding of some of the problem’s mathematical concepts &amp; principles and&lt;br&gt;May contain serious errors in computations, notation, data collection, or analysis</td>
<td>Shows very limited understanding of the problem’s mathematical concepts and principles and&lt;br&gt;Misuses or fails to use mathematical terms and&lt;br&gt;Makes major computation, notation, data collection, or analysis errors</td>
<td>Shows no understandin of the problem’s mathematical concepts and principles</td>
<td>Score X 1</td>
</tr>
<tr>
<td><strong>Strategic Knowledge</strong></td>
<td>Uses relevant outside information of a formal or informal nature correctly and&lt;br&gt;Identifies all the important elements of the problem and shows understanding of the relationships between them and&lt;br&gt;Develops a clear &amp; appropriate model for solving the problem or develops an inappropriate model but includes rationale for why it does not work and&lt;br&gt;Accomplishes task’s goals</td>
<td>Attempts to use relevant outside information of a formal or informal nature but contains minor errors or is incomplete and&lt;br&gt;Identifies the most important elements of the problem and shows general understanding of the relationships between them and&lt;br&gt;Develops an appropriate model for solving the problem that may contain minor errors or develops an inappropriate model but includes rationale for why it does not work and&lt;br&gt;Accomplishes most of task’s goals</td>
<td>Attempts to use relevant outside information of a formal or informal nature but contains serious errors or is incomplete and&lt;br&gt;Identifies some important elements of the problem but shows only limited understanding of the relationships between them and&lt;br&gt;Attempts to develop appropriate model but is incomplete or unclear and&lt;br&gt;Accomplishes most task’s goals</td>
<td>May attempt to use irrelevant outside information and&lt;br&gt;Fails to identify important elements or places too much emphasis on unimportant elements and&lt;br&gt;Develops an inappropriate model with no description or rationale for why it does not work</td>
<td>Uses irrelevant outside information and&lt;br&gt;Fails to identify which elements of the problem are appropriate and&lt;br&gt;Does not develop a model</td>
<td>Score X 2</td>
</tr>
<tr>
<td>Communication</td>
<td>Rubric Score</td>
<td>Percentage Grade</td>
<td>Category</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------------</td>
<td>--------------</td>
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<tr>
<td>Clearly interprets results in context of the situation and Communicates effectively to the identified audience and Presents strong supporting arguments which are logically sound &amp; complete to justify how model is effective in accomplishing task’s goals</td>
<td>20</td>
<td>100</td>
<td>Excellent</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Interprets results in context of the situation and Communicates fairly effectively to identified audience and Presents supporting arguments which are logically sound but may contain some minor gaps</td>
<td>18-19</td>
<td>95</td>
<td>Excellent</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Attempts to interpret results in context of the situation but contains minor errors and Communication is somewhat vague or difficult to interpret and Arguments are incomplete or contain serious errors</td>
<td>17</td>
<td>90</td>
<td>Excellent</td>
<td></td>
<td></td>
<td></td>
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<td>Attempts to interpret results in context of the situation by contains major errors and Communicatio</td>
<td>15-16</td>
<td>85</td>
<td>Good</td>
<td></td>
<td></td>
<td></td>
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<td>80</td>
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<td>12-13</td>
<td>75</td>
<td>Fair</td>
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<tr>
<td>10-11</td>
<td>70</td>
<td>Fair</td>
<td></td>
<td></td>
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<tr>
<td>9</td>
<td>65</td>
<td>Unsatisfactory - consider redoing project</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>8</td>
<td>60</td>
<td>Unsatisfactory - consider redoing project</td>
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<tr>
<td>7 or below</td>
<td>No Grade Given</td>
<td>Redo Project and Correct Errors</td>
<td></td>
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</tbody>
</table>

*Based on QUASAR General Rubric, Lane (1993)

Team Product Rubric Score to Grade Conversions

*Based on Table 3: Sample Grades and Categories, Mertler (2002)*
## Appendix P

Modeling Cycle Graphic Organizer

<table>
<thead>
<tr>
<th>Modeling Cycle</th>
<th>Essential Questions to guide your work. Do not submit this as your individual work. You should have more than can fit here! However, you may use this to help you navigate the modeling cycle.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. Identify Essential Variables</strong></td>
<td>What are the essential features of the situation which may identify appropriate variables?</td>
</tr>
<tr>
<td><strong>2. Formulate Model</strong></td>
<td>What assumptions did you make as you analyzed the situation and conducted your analysis?</td>
</tr>
<tr>
<td><strong>3. Analyze &amp; Perform Operations</strong></td>
<td>Double check that all procedures, calculations, notation, and mathematical language is correct. What does your model conclude about the situation?</td>
</tr>
<tr>
<td><strong>4. Interpret Results</strong></td>
<td>What do the results of your analysis mean in terms of the situation? What did you find out?</td>
</tr>
<tr>
<td><strong>5. Validate Conclusions</strong></td>
<td>What are your potential inaccuracies or sources of error?</td>
</tr>
</tbody>
</table>
Appendix Q
Project 5 Guiding Questions Classroom Document

Student Choice Guiding Questions

Getting Started…..

Name___________________

Team Members:

1. Describe the general situation your team would like to learn about.

2. What is the question you would like answered? That is, what is your team’s goal? Remember that you want this to be manageable. Don’t make it so complicated that you don’t have time to finish your analysis.

3. Who would be interested in the answer to your question and why? Who is your ‘primary audience?’

4. What additional information will you need to consider about your situation and why? That is, what do you currently not know about your situation that you need to know to answer your question?

5. How will you gain access to that information? Is there research you must do or data you need collect?

6. What is your plan for gathering this information or data? Do you need access to tools that you currently do not have? What can Mrs. Buhrman do to help you? What do each of your team members need to do?
Appendix R
Crime Safety Ranking Presentation Graphic Organizer Classroom Document

Name_________________________

Remember that you will vote on the team you feel presented the most effective methodology with a prize being awarded to the winning team. You are not allowed to vote for your own team! Use this organizer to keep track of ideas from each team’s presentations so in the end you know who to vote for.

Team with Most Effective Methodology:________________________________________

Why do you feel it was effective:____________________________________________

<table>
<thead>
<tr>
<th>Team 1</th>
<th>Team 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>What stuck out to you?</td>
<td>What stuck out to you?</td>
</tr>
<tr>
<td>Was it effective? Why/why not?</td>
<td>Was it effective? Why/why not?</td>
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<table>
<thead>
<tr>
<th>Team 3</th>
<th>Team 4</th>
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<tbody>
<tr>
<td>What stuck out to you?</td>
<td>What stuck out to you?</td>
</tr>
<tr>
<td>Was it effective? Why/why not?</td>
<td>Was it effective? Why/why not?</td>
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</table>

<table>
<thead>
<tr>
<th>Team 5</th>
<th>Team 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>What stuck out to you?</td>
<td>What stuck out to you?</td>
</tr>
<tr>
<td>Was it effective? Why/why not?</td>
<td>Was it effective? Why/why not?</td>
</tr>
</tbody>
</table>
Appendix S
Student Choice Presentations Graphic Organizer Classroom Document

Name_______________________________ Project 5 Feedback

Fill this out with your ideas, input, and questions during the Project 5: Student Choice Presentations so that you can provide comments and feedback during the Q & A portion of each presentation.

**Please also make sure your team’s presentations are shared with Mrs. B!**

<table>
<thead>
<tr>
<th>Team 6</th>
<th>Team 5</th>
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</thead>
<tbody>
<tr>
<td>What other knowledge or factors might have been good to consider in this analysis?</td>
<td>What other knowledge or factors might have been good to consider in this analysis?</td>
</tr>
<tr>
<td>1 Question you have:</td>
<td>1 Question you have:</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Team 4</th>
<th>Team 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>What other knowledge or factors might have been good to consider in this analysis?</td>
<td>What other knowledge or factors might have been good to consider in this analysis?</td>
</tr>
<tr>
<td>1 Question you have:</td>
<td>1 Question you have:</td>
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</table>

<table>
<thead>
<tr>
<th>Team 2</th>
<th>Team 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>What other knowledge or factors might have been good to consider in this analysis?</td>
<td>What other knowledge or factors might have been good to consider in this analysis?</td>
</tr>
<tr>
<td>1 Question you have:</td>
<td>1 Question you have:</td>
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</table>

<table>
<thead>
<tr>
<th>Tech Group</th>
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<tbody>
<tr>
<td>What other knowledge or factors might have been good to consider in this analysis?</td>
</tr>
<tr>
<td>1 Question you have:</td>
</tr>
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