

University of Nebraska - Lincoln

DigitalCommons@University of Nebraska - Lincoln

Faculty Publications from the Department of
Electrical and Computer Engineering

Electrical & Computer Engineering, Department of

2013

Achievable Throughput Regions of Fading Broadcast and Interference Channels under QoS Constraints

Deli Qiao

University of Nebraska-Lincoln, dlqiao@ce.ecnu.edu.cn

Mustafa Cenk Gursoy

University of Nebraska-Lincoln

Senem Velipasalar

University of Nebraska-Lincoln

Follow this and additional works at: <http://digitalcommons.unl.edu/electricalengineeringfacpub>



Part of the [Computer Engineering Commons](#), and the [Electrical and Computer Engineering Commons](#)

Qiao, Deli; Gursoy, Mustafa Cenk; and Velipasalar, Senem, "Achievable Throughput Regions of Fading Broadcast and Interference Channels under QoS Constraints" (2013). *Faculty Publications from the Department of Electrical and Computer Engineering*. 301.
<http://digitalcommons.unl.edu/electricalengineeringfacpub/301>

This Article is brought to you for free and open access by the Electrical & Computer Engineering, Department of at DigitalCommons@University of Nebraska - Lincoln. It has been accepted for inclusion in Faculty Publications from the Department of Electrical and Computer Engineering by an authorized administrator of DigitalCommons@University of Nebraska - Lincoln.

Achievable Throughput Regions of Fading Broadcast and Interference Channels under QoS Constraints

Deli Qiao, Mustafa Cenk Gursoy, and Senem Velipasalar

Abstract—Transmission over fading broadcast and interference channels in the presence of quality of service (QoS) constraints is studied. Effective capacity, which provides the maximum constant arrival rate that a given service process can support while satisfying statistical QoS constraints, is employed as the performance metric. In the broadcast scenario, the effective capacity region achieved with superposition coding and successive interference cancellation is identified and is shown to be convex. Subsequently, optimal power control policies that achieve the boundary points of the effective capacity region are investigated, and an algorithm for the numerical computation of the optimal power adaptation schemes for the two-user case is provided. In the interference channel model, achievable throughput regions are determined for three different strategies, namely *treating interference as noise*, *time division with power control* and *simultaneous decoding*. It is demonstrated that as in Gaussian interference channels, simultaneous decoding expectedly performs better (i.e., supports higher arrival rates) when interfering links are strong, and treating interference as noise leads to improved performance when the interfering cross links are weak while time-division strategy should be preferred in between. When the QoS constraints become more stringent, it is observed that the sum-rates achieved by different schemes all diminish and approach each other, and time division with power control interestingly starts outperforming others over a wider range of cross-link strengths.

Index Terms—Buffer violation probability, effective capacity, fading broadcast channels, fading interference channels, power control, quality of service (QoS) constraints, throughput.

I. INTRODUCTION

FADING broadcast and interference channels can be seen as two of the basic building blocks of multiuser wireless systems. For instance, broadcast channels can model the downlink scenario in cellular systems. Interference channels model communication scenarios in which multiple transmitters

communicate with their individual intended receivers while inflicting interference on the other receivers. In densely deployed wireless networks, interference is very common, and the design and analysis of efficient transmission strategies in such cases are of significant interest. Therefore, due to their importance, broadcast and interference channels have been extensively studied from an information-theoretic point of view with the goal of identifying the fundamental performance limits. For instance, Li and Goldsmith in [1] characterized the ergodic capacity region of fading broadcast channels and determined the optimal resource allocation policies. They showed that the capacity region, which is achieved by superposition coding and successive decoding whose order is determined by the effective noise levels, is convex. They provided an algorithm to determine the optimal power allocation in each fading state. The same authors in [2] investigated the outage capacity.

The study of the interference channel has proven to be more challenging. The capacity region of even the two-user interference channel is known in only some special cases. For instance, the capacity region is known for *strong* Gaussian interference channels [4], [5], where the cross links, which are the links from transmitters to their unintended receivers, are stronger than the direct links. In [4], this capacity region was characterized as the intersection of the capacity regions of two multi-access channels seen by the receivers. Moreover, in the special case of *very strong* interference channels, capacity region was shown to be *not* affected by the interfering links [6]. Hence, the capacity region is rectangular-shaped, bounded by the capacities of the direct links. On the other hand, the capacity region of the *weak* interference channels, where the strength of cross links is lower than that of the corresponding direct links, is still an open problem. For the special class of *weak* interference channels in which the strength of one of the cross links is zero, the sum capacity is achieved by treating the interference as noise [8]. This strategy was also shown to be sum-capacity achieving in weak Gaussian interference channels under certain conditions [9], [10], [11]. For general cases, the best known achievable rate region is due to Han and Kobayashi [5]. Han-Kobayashi scheme splits the messages of both users into common and private parts, and common information of both users are jointly decoded at both receivers while the private information is decoded only at the intended receiver and is treated as noise at the other receiver. In [12], Etkin *et al.* showed that Han-Kobayashi type scheme can achieve the capacity region of the two-

Manuscript received July 18, 2012; revised December 31, 2012 and May 21, 2013. The editor coordinating the review of this paper and approving it for publication was A. Zanella.

D. Qiao was with the Department of Electrical Engineering, University of Nebraska-Lincoln, Lincoln, NE 68588. He is now with the Communications Technology Lab, Huawei Technologies Co., Ltd., Shanghai, China, 201206 (e-mail: qiaodeli@huawei.com).

M. C. Gursoy and S. Velipasalar are with the Department of Electrical Engineering and Computer Science, Syracuse University, Syracuse, NY, 13244 (e-mail: {mcgursoy, svelipas}@syr.edu).

This work was supported by the National Science Foundation under Grants CNS-0834753 and CCF-0917265. The material in this paper was presented in part at the 2011 IEEE International Symposium on Information Theory, St. Petersburg, Russia, and in part at the 2012 IEEE Global Communications Conference, Anaheim, CA, USA.

Digital Object Identifier 10.1109/TCOMM.2013.071213.120498

user Gaussian interference channel to within 1 bits/s/Hz. In [13], the authors considered an interference channel model with arbitrary number of users but interference experienced or caused by only one user. In this work, capacity region was characterized to again within a constant number of bits through the use of a deterministic channel model first introduced in [14]. Such a deterministic approach considers an interference-limited regime in which noise power is small compared to signal powers, and hence closely approximates the Gaussian interference channel in the limit of high signal-to-noise ratio (SNR). In [15], Zhao *et al.* studied the maximum sum-rate achieved with Gaussian superposition coding and successive decoding in two-user interference channels, employing the deterministic channel model. It is shown that maximum sum-rate achieved with successive decoding of Gaussian codewords oscillates between those achieved with Han-Kobayashi and single-message schemes. An overview of the Gaussian interference channels and general inner and outer bounds on the capacity region have been provided in [16, Chapter 6]. Recently, ergodic fading interference channels was studied in [17] where the authors have noted that capacity-achieving schemes can in general require encoding and decoding over all fading states.

Generally, information theoretic studies do not take into consideration source arrivals and buffer/queuing limitations. In this paper, we on the other hand present a study on the throughput when users operate under buffer constraints. In particular, motivated by the fact that in certain delay sensitive scenarios, such as interactive or streaming video applications, constraints on delay bound violation probability may be required (rather than limitations on the average delay), we consider statistical quality of service (QoS) constraints in the form of limitations on the buffer violation probabilities, and study the achievable source arrival rate regions under such constraints in fading broadcast and interference channels. For this analysis, we employ the concept of effective capacity [18], which can be seen as the maximum constant arrival rate that a given time-varying service process can support while satisfying statistical QoS guarantees. The effective capacity formulation uses large deviations theory and incorporates the statistical QoS constraints by capturing the rate of decay of the buffer occupancy probability for large queue lengths.

Effective capacity has recently been considered as a performance metric in multiuser scenarios for systems operating under queuing constraints. For instance, Liu *et al.* in [21] studied a model in which two users collaborate to send their data to a common destination node using frequency-division multiplexing. In this setting, effective capacity is used to identify the achievable rate regions under QoS constraints. In [22], we investigated the throughput of fading multiple-access channels in the presence of QoS limitations when superposition coding strategies are employed for transmission. While these two studies concentrated on multiple-access scenarios, references [23] and [24] considered fading broadcast channels. Du and Zhang in [23] obtained the optimal resource allocation policies in time-division-based wireless downlink transmissions. Balasubramanian and Miller in [24] derived the optimal time division strategy for each channel state but considered fixed power transmissions.

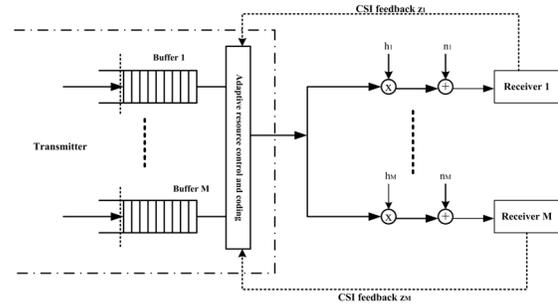


Fig. 1. The broadcast channel model.

The above-mentioned works on broadcast channels basically studied the performance achieved with orthogonal signaling through time-division or frequency-division multiplexing. In this paper, similar to our approach in [22] for multiple access channels, we consider the more general and information-theoretically optimal schemes of superposition coding and successive interference cancellation (or equivalently successive decoding) in the analysis of broadcast channels. We determine the effective capacity region and explore in the two-user case the optimal power control policies that achieve the points on the boundary.

In the interference channel case, we concentrate on a two-user model, and consider three different strategies, namely treating interference as noise, time division with power control, and simultaneous decoding. Different from the successive decoding considered in broadcast channels, simultaneous decoding is a joint decoding method [16, Chapter 6] and can be seen as a special case of the Han-Kobayashi scheme with no private information. We determine the effective capacity regions achieved with these schemes and hence characterize the achievable throughput when users operate under QoS limitations.

The rest of this paper is organized as follows. Section II introduces the fading broadcast and interference channel models. In Section III, effective capacity is briefly described as the performance metric. Our main characterizations of the throughput in fading broadcast and interference channels are provided in Sections IV and V, respectively. Finally, we conclude in Section VI.

II. SYSTEM MODEL

A. Fading Broadcast Channels

As shown in Figure 1, we consider a broadcast channel model in which the transmitter wishes to send messages to M users with individual QoS constraints under a total average power limitation. It is assumed that the data sequences generated for each user are initially stored in individual buffers before they are transmitted over the wireless channel. The discrete-time signal at the j^{th} receiver in the i^{th} symbol duration is given by

$$y_j[i] = h_j[i]x[i] + n_j[i], \quad j = 1, \dots, M \text{ and } i = 1, 2, \dots$$

where M is the number of users, $x[i]$, composed of M independent information signals, denotes the complex-valued channel input, and $h_j[i]$ represents the fading coefficient of

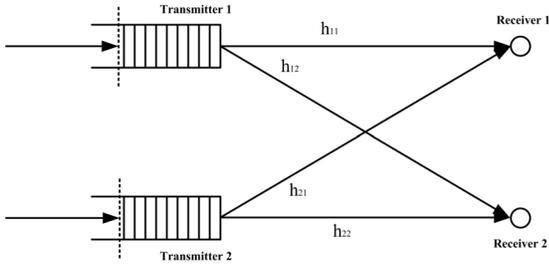


Fig. 2. The interference channel model.

the j th user. We denote the magnitude-square of the fading coefficients by $z_j[i] = |h_j[i]|^2$. The channel input is subject to the average energy constraint $\mathbb{E}\{|x[i]|^2\} \leq \bar{P}/B$ where B is the bandwidth available in the system and hence \bar{P} is the average power constraint (assuming that the symbol rate is B complex symbols per second). $y_j[i]$ is the channel output at the j th receiver. Above, $n_j[i]$ for $j = 1, 2, \dots, M$ is a zero-mean, circularly symmetric, complex Gaussian random variable with variance $\mathbb{E}\{|n_j[i]|^2\} = N_j$. The additive Gaussian noise samples $\{n_j[i]\}$ are assumed to form an independent and identically distributed (i.i.d.) sequence.

We denote the average transmitted signal to noise ratio with respect to receiver 1 as $\text{SNR} = \frac{\bar{P}}{N_1 B}$. Also, we denote the instantaneous transmit power for user j as $P_j[i]$. Now, the instantaneous transmitted SNR level for receiver j becomes $\mu_j[i] = \frac{P_j[i]}{N_j B}$. Then, the average power constraint at the transmitter is equivalent to

$$\mathbb{E} \left\{ \sum_{j=1}^M \gamma_j \mu_j[i] \right\} \leq \text{SNR}, \quad (1)$$

where we have defined the ratio $\gamma_j = \frac{N_j}{N_1}$.

B. Fading Interference Channels

As depicted in Fig. 2, in the case of interference channels, we consider a scenario where two transmitters with individual power and buffer constraints (i.e., QoS constraints) communicate with their intended receivers while causing interference to the other receiver. Again, it is assumed that data is initially stored in the buffers before they are transmitted over the wireless channel. The input-output relationships in the i th symbol duration are given by

$$y_1[i] = h_{11}[i]x_1[i] + h_{21}[i]x_2[i] + n_1[i] \quad (2)$$

$$y_2[i] = h_{12}[i]x_1[i] + h_{22}[i]x_2[i] + n_2[i] \quad (3)$$

where $x_j[i]$ for $j \in \{1, 2\}$ denotes the complex-valued channel input of transmitter j , h_{jk} for $j, k \in \{1, 2\}$ is the fading coefficient of the link between the j th transmitter and k th receiver. We denote the magnitude-square of the fading coefficients by $z_{jk}[i] = |h_{jk}[i]|^2$. Similarly as in the broadcast channel model, $n_1[i]$ and $n_2[i]$ are zero-mean, circularly symmetric, complex Gaussian random variables with variances given by $\mathbb{E}\{|n_j[i]|^2\} = N_j$, and they form an i.i.d. sequence over time. Finally, $y_j[i]$ for $j = 1, 2$ denotes the received signals.

The channel input of user j is subject to an average energy constraint $\mathbb{E}\{|x_j[i]|^2\} \leq \bar{P}_j/B$ for all j , where B is the

bandwidth available in the system. With these definitions, the average transmitted signal to noise ratio of user j is $\text{SNR}_j = \frac{\bar{P}_j}{N_j B}$. We denote the ratio of the noise variances as $\gamma = \frac{N_2}{N_1}$. Then, the instantaneous interference level of transmitter 2 at receiver 1 becomes $\gamma \text{SNR}_2 z_{21}$. Similarly, the instantaneous interference level of transmitter 1 at receiver 2 becomes $\frac{\text{SNR}_1 z_{12}}{\gamma}$.

III. PRELIMINARIES

In [18], Wu and Negi defined the effective capacity as the maximum constant arrival rate¹ that a given service process can support in order to guarantee a statistical QoS requirement specified by the QoS exponent θ . If we define Q as the stationary queue length, then θ is the decay rate of the tail distribution of the queue length Q :

$$\lim_{q \rightarrow \infty} \frac{\log P(Q \geq q)}{q} = -\theta. \quad (4)$$

Therefore, for large q_{\max} , we have the following approximation for the buffer violation probability: $P(Q \geq q_{\max}) \approx e^{-\theta q_{\max}}$. Hence, while larger θ corresponds to stricter QoS constraints, smaller θ implies looser QoS guarantees. Similarly, if D denotes the steady-state delay experienced in the buffer, then $P(D \geq d_{\max}) \approx e^{-\theta \xi d_{\max}}$ for large d_{\max} , where ξ is determined by the arrival and service processes [20]. Since the average arrival rate is equal to the average departure rate when the queue is in steady-state, effective capacity can also be seen as the maximum throughput in the presence of such constraints. The effective capacity is given by

$$C(\theta) = - \lim_{t \rightarrow \infty} \frac{1}{\theta t} \log_e \mathbb{E} \{ e^{-\theta S[t]} \} \quad \text{bits/s}, \quad (5)$$

where the expectation is with respect to $S[t] = \sum_{i=1}^t s[i]$, which is the time-accumulated service process. $\{s[i], i = 1, 2, \dots\}$ denotes the discrete-time stationary and ergodic stochastic service process.

In this paper, we assume that the fading coefficients stay constant over a frame duration of T and vary independently for each frame. In this scenario, $s[i] = TR[i]$, where $R[i]$ is the instantaneous service rate in the i th frame duration $[iT; (i+1)T)$. Then, (5) can be written as

$$C(\theta) = - \frac{1}{\theta T} \log_e \mathbb{E}_{\mathbf{z}} \{ e^{-\theta TR[i]} \} \quad \text{bits/s}, \quad (6)$$

which is obtained using the fact that the instantaneous rates $\{R[i]\}$, which are in general functions of \mathbf{z} , vary independently from one frame to another. Note that the service rates will be specified by the instantaneous channel capacities or the instantaneous achievable rates of the corresponding links in fading broadcast and interference channels.

Throughout the rest of the paper, we use the effective capacity normalized by bandwidth B , which is denoted by

$$C(\theta) = \frac{C(\theta)}{B} \quad \text{bits/s/Hz}. \quad (7)$$

¹For time-varying arrival rates, effective capacity specifies the effective bandwidth of the arrival process that can be supported by the channel.

IV. THROUGHPUT OF FADING BROADCAST CHANNELS UNDER QOS CONSTRAINTS

In this section, we initially review the ergodic capacity region of the fading broadcast channels with perfect channel side information (CSI) at both the transmitter and the receivers, and identify the instantaneous transmission rates. Subsequently, we formulate the effective capacity region and characterize the maximum constant arrival rates that can be supported over the broadcast channel with statistical QoS constraints. We also investigate in the two-user case the optimal power control schemes.

A. Ergodic Capacity Region

The capacity region of the fading broadcast channel with perfect CSI, achieved by superposition coding at the transmitter and successive interference cancellation at the receivers, is [1]

$$\mathcal{R}_{\text{BC}} = \left\{ (R_1, \dots, R_M) : R_j \leq B \mathbb{E}_{\mathbf{z}} \left\{ \log_2 \left(1 + \frac{\mu_j(\mathbf{z}) z_j}{1 + \sum_{k=1}^M \mu_k(\mathbf{z}) \frac{\gamma_k z_j}{\gamma_j} \mathbf{1}[\gamma_j/z_j > \gamma_k/z_k]} \right) \right\} \right\} \quad (8)$$

where $\mathbf{1}[\cdot]$ is an indicator function (i.e., $\mathbf{1}[x] = 1$ if x is true, and zero otherwise), $\mu_k(\mathbf{z}) \frac{\gamma_k z_j}{\gamma_j} = \frac{P_k[i] z_j}{N_j B}$ denotes the interference level caused by the message for user k , $\mu = (\mu_1(\mathbf{z}), \dots, \mu_M(\mathbf{z}))$ can be regarded as the power allocation policies that need to satisfy the average power constraint (1), and $\mathbf{z} = (z_1, \dots, z_M)$ is a random vector comprised of the magnitude squares of the channel coefficients. Also, note that $\gamma_j/z_j > \gamma_k/z_k$ is equivalent to $\frac{N_j}{z_j} > \frac{N_k}{z_k}$, which is used in determining the decoding order for the users. More specifically, the permutation order for successful decoding is given by $\frac{N_{\pi(1)}}{z_{\pi(1)}} > \frac{N_{\pi(2)}}{z_{\pi(2)}} > \dots > \frac{N_{\pi(M)}}{z_{\pi(M)}}$. If the noise normalized by the channel gain, i.e., $\frac{N_k}{z_k}$, is regarded as the effective noise level, the decoding order above is seen to be in the decreasing order of the effective noise levels. For instance, the user experiencing the smallest effective noise and hence the best channel decodes its own signal last after decoding all the other users' signals and subtracting their interference. Hence, this user experiences no interference.

The ergodic capacity region \mathcal{R}_{BC} is convex [1]. For any channel state vector \mathbf{z} , the decoding order is fixed and the maximum instantaneous rate for user j is

$$R_j = B \log_2 \left(1 + \frac{\mu_j(\mathbf{z}) z_j}{1 + \sum_{k=1}^M \mu_k(\mathbf{z}) \frac{\gamma_k z_j}{\gamma_j} \mathbf{1}[\gamma_j/z_j > \gamma_k/z_k]} \right) \text{ bits/s.} \quad (9)$$

B. Effective Capacity Region

In this subsection, we first identify the effective capacity region when superposition coding with successive decoding is employed. Then, we concentrate on the two-user case and investigate the optimal power adaptation strategies.

Suppose that $\Theta = (\theta_1, \dots, \theta_M)$ is a vector composed of the QoS constraints of M users. Let $\mathbf{C}(\Theta) =$

$(C_1(\theta_1), \dots, C_M(\theta_M))$ denote the vector of the normalized effective capacities. Following mainly the approach in [22], we first have the following characterization.

Definition 1: The *effective capacity region* is described as

$$\mathcal{C}_{\text{BC}}(\Theta) = \bigcup_{\substack{\mathbf{R} \\ \text{s.t. } \mathbb{E}\{\mathbf{R}\} \in \mathcal{R}_{\text{BC}}}} \left\{ \mathbf{C}(\Theta) \geq \mathbf{0} : C_j(\theta_j) \leq -\frac{1}{\theta_j T B} \log_e \mathbb{E}_{\mathbf{z}} \left\{ e^{-\theta_j T R_j} \right\} \right\} \quad (10)$$

where $\mathbf{R} = \{R_1, R_2, \dots, R_M\}$ represents the vector composed of the instantaneous transmission (or equivalently service) rates of M users. Note that the union is over the distributions of the vector \mathbf{R} such that the expected value $\mathbb{E}\{\mathbf{R}\}$ lies in the BC ergodic capacity region \mathcal{R}_{BC} given in (8) and power allocation policies satisfy the average power constraint (1).

The *effective capacity region* given in Definition 1 represents the set of all vectors of constant arrival rates $\mathbf{C}(\Theta)$ that can be supported in the fading broadcast channel in the presence of QoS constraints specified by $\Theta = (\theta_1, \dots, \theta_M)$. Since reliable communication is considered, arrival rates are supported by instantaneous service rates whose expected values are in the BC capacity region.

Using the convexity of the BC ergodic capacity region \mathcal{R}_{BC} , we obtain the following preliminary result on the effective capacity region defined in (10).

Proposition 1: The *effective capacity region* $\mathcal{C}_{\text{BC}}(\Theta)$ is convex.

Proof: Let the vectors $\mathbf{C}(\Theta)$ and $\mathbf{C}'(\Theta)$ belong to $\mathcal{C}_{\text{BC}}(\Theta)$. Then, there exist some rate vectors \mathbf{R} and \mathbf{R}' for $\mathbf{C}(\Theta)$ and $\mathbf{C}'(\Theta)$, respectively, such that $\mathbb{E}\{\mathbf{R}\}$ and $\mathbb{E}\{\mathbf{R}'\}$ are in the BC ergodic capacity region. By a time sharing strategy, for any $\alpha \in (0, 1)$, we know from the convexity of the BC ergodic capacity region that $\mathbb{E}\{\alpha \mathbf{R} + (1 - \alpha) \mathbf{R}'\} \in \mathcal{R}_{\text{BC}}$. Now, we can write (11) through (13), given on the next page.

In (11) through (13), all operations, including the logarithm and exponential functions and expectations, are component-wise operations. For instance, the expression in (11) denotes a vector whose components are $\left\{ \frac{1}{\theta_j T B} \log_e \left(\mathbb{E} \left\{ e^{-\theta_j T R_j} \right\} \right)^\alpha \left(\mathbb{E} \left\{ e^{-\theta T R'_j} \right\} \right)^{1-\alpha} \right\}_{j=1}^M$. Similarly, the inequalities in (11) and (13) are component-wise inequalities. The inequality in (11) follows from the definition in (10). (13) follows from Hölder's inequality and leads to the conclusion that $\alpha \mathbf{C} + (1 - \alpha) \mathbf{C}'$ still lies in the *effective capacity region*, proving the convexity result. \square

We are interested in the boundary of the region $\mathcal{C}_{\text{BC}}(\Theta)$. Now that $\mathcal{C}_{\text{BC}}(\Theta)$ is convex, we can characterize the boundary surface by considering the following optimization problem [1]:

$$\max \lambda \cdot \mathbf{C}(\Theta) \quad \text{subject to: } \mathbf{C}(\Theta) \in \mathcal{C}_{\text{BC}}(\Theta) \quad (14)$$

for all priority vectors $\lambda = (\lambda_1, \dots, \lambda_M)$ in \mathfrak{R}_+^M with $\sum_{j=1}^M \lambda_j = 1$.

1) *Two-user Case:* We know that the decoding order is given by the effective noise level. More specifically, only the user with the better channel can decode the information intended for the other user [26, Section 14.5], and, as a result,

$$\alpha C(\Theta) + (1 - \alpha)C'(\Theta) \leq -\frac{1}{\Theta TB} \log_e \left(\mathbb{E} \left\{ e^{-\Theta T \mathbf{R}} \right\} \right)^\alpha \left(\mathbb{E} \left\{ e^{-\Theta T \mathbf{R}'} \right\} \right)^{1-\alpha} \quad (11)$$

$$= -\frac{1}{\Theta TB} \log_e \left(\mathbb{E} \left\{ \left(e^{-\Theta T \alpha \mathbf{R}} \right)^{\frac{1}{\alpha}} \right\} \right)^\alpha \left(\mathbb{E} \left\{ \left(e^{-\Theta T (1-\alpha) \mathbf{R}'} \right)^{\frac{1}{1-\alpha}} \right\} \right)^{1-\alpha} \quad (12)$$

$$\leq -\frac{1}{\Theta TB} \log_e \mathbb{E} \left\{ e^{-\Theta T (\alpha \mathbf{R} + (1-\alpha) \mathbf{R}')} \right\}. \quad (13)$$

experiences no interference. We define $\gamma = \frac{N_2}{N_1}$ as the ratio of the variances of the background Gaussian noise components, and assume that the value of γ is fixed in the remainder of the analysis. Let $\mathcal{Z} = \{\mathbf{z} : \gamma z_1 > z_2\}$ be the region in which user 1 has less effective noise level and hence, it can first decode and eliminate the message intended for user 2, and hence sees no interference. Then, $\mathcal{Z}^c = \{\mathbf{z} : \gamma z_1 < z_2\}$ represents the region in which user 1 decodes its signal in the presence of interference from user 2's signal. In this scenario, we can write the maximum instantaneous rate for user 1 as

$$R_1 = \begin{cases} B \log_2(1 + \mu_1(\mathbf{z})z_1), & \mathbf{z} \in \mathcal{Z} \\ B \log_2\left(1 + \frac{\mu_1(\mathbf{z})z_1}{1 + \gamma \mu_2(\mathbf{z})z_1}\right), & \mathbf{z} \in \mathcal{Z}^c \end{cases}, \quad (15)$$

and the instantaneous rate for user 2 as

$$R_2 = \begin{cases} B \log_2(1 + \mu_2(\mathbf{z})z_2), & \mathbf{z} \in \mathcal{Z}^c \\ B \log_2\left(1 + \frac{\mu_2(\mathbf{z})z_2}{1 + \mu_1(\mathbf{z})z_2/\gamma}\right), & \mathbf{z} \in \mathcal{Z} \end{cases}. \quad (16)$$

Now, using these instantaneous service rates R_1 and R_2 in the effective capacity expressions and recalling the average SNR constraint in (1), we can express the Lagrangian of the convex optimization problem in (14) as (17) given on the next page, where $\beta_j = \frac{\theta_j TB}{\log_e 2}$ for $j = 1, 2$, $p_{\mathbf{z}}(z_1, z_2)$ is the joint distribution function of the fading states $\mathbf{z} = (z_1, z_2)$, and $\kappa \geq 0$ is the Lagrange multiplier. Next, we define (ϕ_1, ϕ_2) as

$$\begin{aligned} \phi_1 &= \int_{\mathbf{z} \in \mathcal{Z}} (1 + \mu_1(\mathbf{z})z_1)^{-\beta_1} p_{\mathbf{z}}(z_1, z_2) d\mathbf{z} \\ &+ \int_{\mathbf{z} \in \mathcal{Z}^c} \left(1 + \frac{\mu_1(\mathbf{z})z_1}{1 + \gamma \mu_2(\mathbf{z})z_1}\right)^{-\beta_1} p_{\mathbf{z}}(z_1, z_2) d\mathbf{z}, \\ \phi_2 &= \int_{\mathbf{z} \in \mathcal{Z}^c} (1 + \mu_2(\mathbf{z})z_2)^{-\beta_2} p_{\mathbf{z}}(z_1, z_2) d\mathbf{z} \\ &+ \int_{\mathbf{z} \in \mathcal{Z}} \left(1 + \frac{\mu_2(\mathbf{z})z_2}{1 + \mu_1(\mathbf{z})z_2/\gamma}\right)^{-\beta_2} p_{\mathbf{z}}(z_1, z_2) d\mathbf{z}. \end{aligned}$$

From these definitions, we see that ϕ_1 and ϕ_2 depend on the power allocation policies μ_1 and μ_2 .

Next, we derive the optimality conditions (that the optimal power control policies should satisfy) by differentiating the Lagrangian with respect to μ_1, μ_2 in regions \mathcal{Z}^c and \mathcal{Z} , and making the derivatives equal to zero. These optimality conditions are given in (18)–(21) on the next page, which are obtained by evaluating the derivative of the Lagrangian \mathcal{J} with respect to μ_1 when $\mathbf{z} \in \mathcal{Z}$, μ_2 when $\mathbf{z} \in \mathcal{Z}^c$, μ_1 when $\mathbf{z} \in \mathcal{Z}^c$, and μ_2 when $\mathbf{z} \in \mathcal{Z}$, respectively².

²Essentially, we assume that there are four power allocation policies $\mu_{1,\mathcal{Z}}, \mu_{1,\mathcal{Z}^c}, \mu_{2,\mathcal{Z}}$, and μ_{2,\mathcal{Z}^c} , where $\mu_{i,\mathcal{Z}}$ and μ_{i,\mathcal{Z}^c} represent the power adaptation for user i 's stream when $\mathbf{z} \in \mathcal{Z}$ and $\mathbf{z} \in \mathcal{Z}^c$, respectively. Hence, the expressions in (18)–(21) can be seen to be obtained through partial differentiation of the Lagrangian with respect to $\mu_{1,\mathcal{Z}}, \mu_{2,\mathcal{Z}}, \mu_{1,\mathcal{Z}^c}$ and μ_{2,\mathcal{Z}^c} .

We immediately note from (19) that when $\mathbf{z} \in \mathcal{Z}$, the optimal power control policy for receiver 2 can be expressed as

$$\mu_2 = \left[\frac{1}{\alpha_2^{\frac{1}{\beta_2+1}} \left(\frac{z_2}{1 + \mu_1 z_2/\gamma} \right)^{\frac{\beta_2}{\beta_2+1}}} - \frac{1 + \mu_1 z_2/\gamma}{z_2} \right]^+ \quad \forall \mathbf{z} \in \mathcal{Z} \quad (22)$$

where $\alpha_2 = \frac{\gamma \kappa \phi_2 \log_e 2}{\lambda_2}$. We can see from (22) that when $\mu_1 > \gamma(\frac{1}{\alpha_2} - \frac{1}{z_2})$, we have $\mu_2 = 0$. Also, when $\mathbf{z} \in \mathcal{Z}$, if no power is allocated for user 2 and hence $\mu_2 = 0$, then we see from (18) that we can write the optimal μ_1 as

$$\mu_1 = \left[\frac{1}{\alpha_1^{\frac{1}{\beta_1+1}} z_1^{\frac{\beta_1}{\beta_1+1}}} - \frac{1}{z_1} \right]^+ \quad \forall \mathbf{z} \in \mathcal{Z} \quad (23)$$

where $\alpha_1 = \frac{\kappa \phi_1 \log_e 2}{\lambda_1}$. Note also that when messages for both receivers are being transmitted simultaneously, μ_1 and μ_2 are the positive solutions to (18) and (19). This, after straightforward derivations, implies that the equation

$$\frac{z_1}{\alpha_1} (1 + \mu_1 z_1)^{-\beta_1-1} - \left(\frac{z_2}{\alpha_2 (1 + \mu_1 z_2/\gamma)} \right)^{\frac{1}{\beta_2+1}} = 0 \quad (24)$$

has a positive solution that is less than $\gamma(\frac{1}{\alpha_2} - \frac{1}{z_2})$. Although we cannot prove the uniqueness of the solution, if it exists, there is always one non-negative solution to the above equation in our numerical evaluations.

Following similar steps, we can characterize the power control policies for the case in which $\mathbf{z} \in \mathcal{Z}^c$. In this case, it is worth noting that the condition for having $\mu_1 > 0$ and $\mu_2 > 0$ is now that the equation

$$\frac{z_2}{\alpha_2} (1 + \mu_2 z_2)^{-\beta_2-1} - \left(\frac{z_1}{\alpha_1 (1 + \gamma \mu_2 z_1)} \right)^{\frac{1}{\beta_1+1}} = 0 \quad (25)$$

has a positive solution that is less than $\frac{1}{\gamma}(\frac{1}{\alpha_1} - \frac{1}{z_1})$. This can be regarded as the symmetric case of (24).

As seen in the above discussion, we have no closed-form expressions for the optimal power control policies which are in general interdependent on each other. The optimal power control policies can be determined through numerical computations. Thus, we propose the algorithm below to obtain the optimal power adaptation policies. This algorithm is used in the numerical result presented next. The main approach of the algorithm is to first initialize the values of ϕ_1 and ϕ_2 , and keep updating the two values with the μ_1 and μ_2 generated through the last iteration. Finally, when the conditions stated in (18)–(21) are satisfied, we find the final values of ϕ_1 and

$$\begin{aligned} \mathcal{J} = & -\frac{\lambda_1}{\beta_1 \log_e 2} \log_e \left(\int_{\mathbf{z} \in \mathcal{Z}} (1 + \mu_1(\mathbf{z})z_1)^{-\beta_1} p_{\mathbf{z}}(z_1, z_2) d\mathbf{z} + \int_{\mathbf{z} \in \mathcal{Z}^c} \left(1 + \frac{\mu_1(\mathbf{z})z_1}{1 + \gamma\mu_2(\mathbf{z})z_1} \right)^{-\beta_1} p_{\mathbf{z}}(z_1, z_2) d\mathbf{z} \right) \\ & -\frac{\lambda_2}{\beta_2 \log_e 2} \log_e \left(\int_{\mathbf{z} \in \mathcal{Z}^c} (1 + \mu_2(\mathbf{z})z_2)^{-\beta_2} p_{\mathbf{z}}(z_1, z_2) d\mathbf{z} + \int_{\mathbf{z} \in \mathcal{Z}} \left(1 + \frac{\mu_2(\mathbf{z})z_2}{1 + \mu_1(\mathbf{z})z_2/\gamma} \right)^{-\beta_2} p_{\mathbf{z}}(z_1, z_2) d\mathbf{z} \right) \\ & -\kappa (\mathbb{E}_{\mathbf{z}}\{\mu_1 + \gamma\mu_2\} - \text{SNR}) \end{aligned} \quad (17)$$

$$1) \frac{\lambda_1}{\phi_1 \log_e 2} (1 + \mu_1 z_1)^{-\beta_1 - 1} z_1 - \frac{\lambda_2}{\phi_2 \log_e 2} \left(1 + \frac{\mu_2 z_2}{1 + \mu_1 z_2/\gamma} \right)^{-\beta_2 - 1} \frac{\mu_2 z_2^2/\gamma}{(1 + \mu_1 z_2/\gamma)^2} - \kappa = 0 \quad \forall \mathbf{z} \in \mathcal{Z} \quad (18)$$

$$2) \frac{\lambda_2}{\phi_2 \log_e 2} \left(1 + \frac{\mu_2 z_2}{1 + \mu_1 z_2/\gamma} \right)^{-\beta_2 - 1} \frac{z_2}{1 + \mu_1 z_2/\gamma} - \gamma\kappa = 0 \quad \forall \mathbf{z} \in \mathcal{Z} \quad (19)$$

$$3) \frac{\lambda_1}{\phi_1 \log_e 2} \left(1 + \frac{\mu_1 z_1}{1 + \gamma\mu_2 z_1} \right)^{-\beta_1 - 1} \frac{z_1}{1 + \gamma\mu_2 z_1} - \kappa = 0 \quad \forall \mathbf{z} \in \mathcal{Z}^c \quad (20)$$

$$4) -\frac{\lambda_1}{\phi_1 \log_e 2} \left(1 + \frac{\mu_1 z_1}{1 + \gamma\mu_2 z_1} \right)^{-\beta_1 - 1} \frac{\mu_1 \gamma z_1^2}{(1 + \gamma\mu_2 z_1)^2} + \frac{\lambda_2}{\phi_2 \log_e 2} (1 + \mu_2 z_2)^{-\beta_2 - 1} z_2 - \gamma\kappa = 0 \quad \forall \mathbf{z} \in \mathcal{Z}^c. \quad (21)$$

ϕ_2 together with the associated optimal power control policies μ_1 and μ_2 .

POWER CONTROL ALGORITHM

- 1 Given λ_0, λ_1 , initialize ϕ_1, ϕ_2 ;
- 2 Initialize κ ;
- 3 Determine $\alpha_1 = \frac{\kappa \phi_1 \log_e 2}{\lambda_1}$, $\alpha_2 = \frac{\gamma \kappa \phi_2 \log_e 2}{\lambda_2}$;
- 4 **if** $\gamma z_1 > z_2$
- 5 **then if** $z_1 > \alpha_1$
- 6 **then** Compute μ_1 from (23);
- 7 **if** $\mu_1 > \gamma \left(\frac{1}{\alpha_2} - \frac{1}{z_2} \right)$ or $z_2 < \alpha_2$
- 8 **then** $\mu_2 = 0$;
- 9 **else if** (24) returns positive solution
- 10 **then** Compute μ_1 and μ_2 from
- 11 (18) and (19);
- 12 **else** $\mu_1 = 0, \mu_2 = \left[\frac{1}{\frac{1}{\alpha_2^{\beta_2+1}} - \frac{\beta_2}{z_2^{\beta_2+1}}} - \frac{1}{z_2} \right]^+$;
- 13 **else**
- 14 **if** $z_2 > \alpha_2$
- 15 **then** $\mu_2 = \frac{1}{\frac{1}{\alpha_2^{\beta_2+1}} - \frac{\beta_2}{z_2^{\beta_2+1}}} - \frac{1}{z_2}$;
- 16 **if** $\mu_2 > \frac{1}{\gamma} \left(\frac{1}{\alpha_1} - \frac{1}{z_1} \right)$ or $z_1 < \alpha_1$
- 17 **then** $\mu_1 = 0$;
- 18 **else if** (25) returns positive solution
- 19 **then** Compute μ_1 and μ_2 from
- 20 (21) and (20);
- 21 **else** $\mu_2 = 0, \mu_1 = \left[\frac{1}{\frac{1}{\alpha_1^{\beta_1+1}} - \frac{\beta_1}{z_1^{\beta_1+1}}} - \frac{1}{z_1} \right]^+$;
- 22 Check if the obtained μ_1 and μ_2 satisfy the average power constraint with equality;
- 23 **if not satisfied with equality**
- 24 **then** update the value of κ and return to Step 3;
- 25 **else** move to Step 26;
- 26 Evaluate ϕ_1 and ϕ_2 with the obtained power control policies;
- 27 Check if the new values of ϕ_1 and ϕ_2 agree (up to a certain margin) with those used in Step 3;
- 28 **if do not agree**
- 29 **then** update the values of ϕ_1 and ϕ_2 and return to Step 2;
- 30 **else** declare the obtained power allocation policies
- 31 μ_1 and μ_2 as the optimal ones.

Note that in the algorithm description, we have not specified

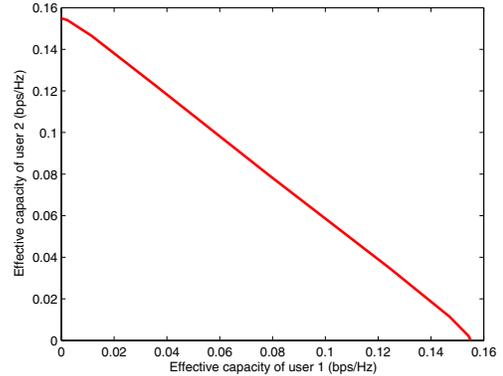


Fig. 3. The effective capacity region of the two-user fading broadcast channel, achieved with superposition coding and successive decoding.

how the values of κ, ϕ_1 and ϕ_2 are updated for each iteration in order to keep the numerical search algorithm generic. In our numerical computations, we have updated κ using the bisection search algorithm. The values of ϕ_1 and ϕ_2 are updated in Step 29 of the algorithm by assigning them the values evaluated in Step 26. Hence, the most recent values are carried over to the new iteration.

In Fig. 3, we provide the effective capacity region achieved with superposition coding in a Rayleigh fading environment in which z_1 and z_2 are independent exponential random variables with $\mathbb{E}\{z_1\} = \mathbb{E}\{z_2\} = 1$. We assume $\gamma = 1$, $\text{SNR} = -10$ dB, and $\theta_1 = \theta_2 = 0.01$. $T = 2$ ms, $B = 10^5$ Hz. The solid line provides the boundary region of the superposition scheme with optimal power control policies, and characterizes for the given values of the parameters the pairs of maximum arrival rates that can be supported through the fading broadcast channel.

V. THROUGHPUT OF FADING INTERFERENCE CHANNELS UNDER QOS CONSTRAINTS

In this section, we study the arrival rates that can be supported when transmitting over interference channels in the

presence of QoS requirements. We concentrate on the two-user fading Gaussian interference channels. In our setting, service rate R in (6) is the instantaneous achievable rate in fading interference channels in each frame duration of T in which the fading stays constant. Hence, we assume that coding is performed separately and independently on each fading state³. In each frame duration, fading Gaussian interference channel can be regarded as a Gaussian interference channel with fixed channel gains $\{z_{jk}\}$. With this perspective, we identify the instantaneous achievable rates R by considering simple inner bounds for non-fading Gaussian interference channels.

We also note that unlike our analysis of broadcast channels in Section IV, we do not consider power adaptation with respect to fading in interference channels. Hence, the throughput regions are characterized under the assumption of fixed transmission power levels for all channel states. Power control over fading can further improve the throughput regions characterized in this section.

A. Classification of Two-User Gaussian Interference Channels

In the two-user case, Gaussian interference channel can be classified as *very strong*, *strong*, *weak*, or *mixed*, depending on the channel gains.

A *very strong* Gaussian interference channel [6], [4], [16, Chapter 6] is the one in which the sum-rate constraint becomes equal to the sum of the individual rate constraints or more explicitly in the Gaussian setting it is the one in which we have

$$\begin{aligned} \log_2 \left(1 + \frac{\text{SNR}_1 z_{12}/\gamma}{1 + \text{SNR}_2 z_{22}} \right) &> \log_2(1 + \text{SNR}_1 z_{11}), \\ \log_2 \left(1 + \frac{\gamma \text{SNR}_2 z_{21}}{1 + \text{SNR}_1 z_{11}} \right) &> \log_2(1 + \text{SNR}_2 z_{22}) \end{aligned} \quad (26)$$

which reduces to the following conditions in terms of the channel gains

$$\begin{aligned} z_{12} &> \gamma z_{11}(1 + \text{SNR}_2 z_{22}), \\ z_{21} &> z_{22}(1 + \text{SNR}_1 z_{11})/\gamma. \end{aligned} \quad (27)$$

Therefore, in this case, interfering links have very strong channel gains, and consequently interference is very strong.

A *strong* Gaussian interference channel [4] [16] is the one in which the cross-link channel gains are higher than the direct-link channel gains, i.e.,

$$z_{12} > \gamma z_{11} \text{ and } z_{21} > z_{22}/\gamma. \quad (28)$$

A Gaussian interference channel is *weak* if the direct-link channel gains are larger than the cross-link channel gains, i.e.,

$$z_{12} < \gamma z_{11} \text{ and } z_{21} < z_{22}/\gamma. \quad (29)$$

A Gaussian interference channel is *mixed* if

$$z_{12} > \gamma z_{11} \text{ and } z_{21} < z_{22}/\gamma, \quad (30)$$

³As discussed in [17], separable coding on each fading state in ergodic fading interference channels is in general suboptimal. In certain scenarios, sum-capacity is shown in [17] to be achieved by encoding and decoding jointly across all fading states. However, note that such a scheme can be prohibitive in delay/QoS limited settings.

or

$$z_{12} < \gamma z_{11} \text{ and } z_{21} > z_{22}/\gamma. \quad (31)$$

B. Throughput Regions under QoS Constraints

We consider three different transmission and reception strategies in two-user fading interference channels, namely *treating interference as noise*, *time division with power control* and *simultaneous decoding*. Initially, we note the instantaneous achievable rate regions of these schemes as functions of the realizations of the fading coefficients and then we identify the corresponding throughput regions in the presence of QoS constraints by formulating the effective capacity regions.

1) *Treating Interference as Noise*: In this case, both receivers treat the signals from the unintended transmitters as noise. Treating interference as noise is a common approach in practical communication systems. Though a relatively simple strategy, it has been shown to achieve the sum-capacity in very weak Gaussian interference channels [9], [10], [11]. The instantaneous achievable rates for this scheme are

$$\begin{aligned} R_1 &\leq B \log_2 \left(1 + \frac{\text{SNR}_1 z_{11}}{1 + \gamma \text{SNR}_2 z_{21}} \right), \text{ and} \\ R_2 &\leq B \log_2 \left(1 + \frac{\text{SNR}_2 z_{22}}{1 + \text{SNR}_1 z_{12}/\gamma} \right). \end{aligned} \quad (32)$$

Plugging the instantaneous rate expressions in (32) into (6) and normalizing with the bandwidth, we can easily see that the effective capacity region of treating interference as noise is a rectangular region described by

$$C_{1,\text{TIN}} \leq -\frac{1}{\theta_1 T B} \log \mathbb{E} \left\{ \left(1 + \frac{\text{SNR}_1 z_{11}}{1 + \gamma \text{SNR}_2 z_{21}} \right)^{-\beta_1} \right\} \quad (33)$$

$$C_{2,\text{TIN}} \leq -\frac{1}{\theta_2 T B} \log \mathbb{E} \left\{ \left(1 + \frac{\text{SNR}_2 z_{22}}{1 + \text{SNR}_1 z_{12}/\gamma} \right)^{-\beta_2} \right\} \quad (34)$$

where $\beta_i = \frac{\theta_i T B}{\log_e 2}$.

2) *Time Division with Power Control*: Another simple scheme is time division with power control. In this case, users transmit in non-overlapping intervals. Suppose a fraction of time $\tau \in [0, 1]$ is allocated to user 1 for transmission and its power level is adjusted to \bar{P}_1/τ . Therefore, user 2 transmits in the remaining $(1 - \tau)$ fraction of the time with power $\bar{P}_2/(1 - \tau)$. Then, the instantaneous achievable rates are

$$\begin{aligned} R_1 &\leq \tau B \log_2 \left(1 + \frac{\text{SNR}_1}{\tau} z_{11} \right) \text{ and} \\ R_2 &\leq (1 - \tau) B \log_2 \left(1 + \frac{\text{SNR}_2}{1 - \tau} z_{22} \right). \end{aligned} \quad (35)$$

Now, the effective capacity region achieved with time division with power control is

$$C_{1,\text{TD}} \leq -\frac{1}{\theta_1 T B} \log \mathbb{E} \left\{ \left(1 + \frac{\text{SNR}_1 z_{11}}{\tau} \right)^{-\tau \beta_1} \right\} \quad (36)$$

$$C_{2,\text{TD}} \leq -\frac{1}{\theta_2 T B} \log \mathbb{E} \left\{ \left(1 + \frac{\text{SNR}_2 z_{22}}{1 - \tau} \right)^{-(1-\tau)\beta_2} \right\} \quad (37)$$

where again $\beta_i = \frac{\theta_i T B}{\log_e 2}$ and $\tau \in [0, 1]$.

3) *Simultaneous Decoding*: Finally, we consider the scheme in which each receiver jointly decodes messages from

both transmitters. For given channel gains, the instantaneous rate region achieved with simultaneous decoding is [16, Section 6.4.1]

$$\begin{aligned} R_1 &\leq B \log_2(1 + \text{SNR}_1 z_{11}) \\ R_2 &\leq B \log_2(1 + \text{SNR}_2 z_{22}) \\ R_1 + R_2 &\leq B \min \left\{ \log_2(1 + \text{SNR}_1 z_{11} + \gamma \text{SNR}_2 z_{21}), \right. \\ &\quad \left. \log_2(1 + \text{SNR}_1 z_{12}/\gamma + \text{SNR}_2 z_{22}) \right\}. \end{aligned} \quad (38)$$

The above rate region is attained by joint decoding of the messages and cannot be achieved in general through successive decoding. At the same time, joint decoding can have high complexity, especially when compared with the simple scheme of treating interference as noise. In [15], the authors imposed a successive decoding constraint that bridges the complexity gap between joint decoding and treating interference as noise.

Simultaneous decoding inner bound given in (38) becomes the capacity region in *very strong* and *strong* interference channels, indicating the optimality of this strategy in these special cases. Indeed, in *very strong* interference channels in which (27) is satisfied, the instantaneous capacity region simplifies to

$$R_1 \leq B \log_2(1 + \text{SNR}_1 z_{11}), \quad \text{and} \quad R_2 \leq B \log_2(1 + \text{SNR}_2 z_{22}). \quad (39)$$

In *weak* and *mixed* interference channels, the region in (38) defines an achievable rate region. In *mixed* interference, sum-rate constraint in (38) can be further simplified. For instance, if $z_{12} > \gamma z_{11}$ and $z_{21} < z_{22}/\gamma$, sum-rate constraint becomes

$$R_1 + R_2 \leq B \log_2(1 + \text{SNR}_1 z_{11} + \gamma \text{SNR}_2 z_{21}). \quad (40)$$

If, on the other hand, $z_{12} < \gamma z_{11}$ and $z_{21} > z_{22}/\gamma$, we have

$$R_1 + R_2 \leq B \log_2(1 + \text{SNR}_1 z_{12}/\gamma + \text{SNR}_2 z_{22}). \quad (41)$$

As seen above, in simultaneous decoding, the instantaneous achievable rate region varies with the set of channel gains of direct and cross links.

We denote the average rate region achieved by performing simultaneous decoding in each fading state by $\mathcal{R}_{\text{IF-SD}}$ and define this region as

$$\begin{aligned} R_1 &\leq B \mathbb{E}\{\log_2(1 + \text{SNR}_1 z_{11})\} \\ R_2 &\leq B \mathbb{E}\{\log_2(1 + \text{SNR}_2 z_{22})\} \\ R_1 + R_2 &\leq B \mathbb{E}\left\{ \min\{\log_2(1 + \text{SNR}_1 z_{11} + \gamma \text{SNR}_2 z_{21}), \right. \\ &\quad \left. \log_2(1 + \text{SNR}_1 z_{12}/\gamma + \text{SNR}_2 z_{22})\} \right\}. \end{aligned} \quad (42)$$

Note that the authors have considered in [15] the sum-rate maximization problem of successive decoding for the two-user interference channel. Here, the throughput region is investigated with fixed power levels at the transmitters with QoS constraints.

Next, we seek to determine the throughput region achieved with simultaneous decoding at the receivers under statistical QoS constraints. For this purpose, we define the effective

capacity region as the collection of the pairs of constant arrival rates at transmitters 1 and 2 that can be supported under buffer constraints at the transmitters when simultaneous decoding is employed at both receivers. Suppose that $\Theta = (\theta_1, \theta_2)$ is a vector composed of the QoS constraints of two transmitters. Let $\mathbf{C}(\Theta) = (\mathbf{C}_1(\theta_1), \mathbf{C}_2(\theta_2))$ denote the vector of the normalized effective capacities. We first provide the following definition:

Definition 2: The effective capacity region of the fading interference channel achieved with the simultaneous decoding is defined as

$$\begin{aligned} \mathcal{C}_{\text{SD}}(\Theta, \text{SNR}) = & \bigcup_{\substack{\mathbf{R} \\ \text{s.t. } \mathbb{E}\{\mathbf{R}\} \in \mathcal{R}_{\text{IF-SD}}} } \left\{ \mathbf{C}_{\text{SD}}(\Theta) \geq \mathbf{0} : \mathbf{C}_{j,\text{SD}}(\theta_j) \leq \right. \\ & \left. - \frac{1}{\theta_j T B} \log_e \mathbb{E}_{\mathbf{z}} \{ e^{-\theta_j T R_j} \} \right\} \end{aligned} \quad (43)$$

where $\mathbf{R} = (R_1, R_2)$ represents the vector composed of the instantaneous transmission (or equivalently service) rates from two transmitters. Note that the union is over the distributions of the vector \mathbf{R} such that the expected value $\mathbb{E}\{\mathbf{R}\}$ lies in the region $\mathcal{R}_{\text{IF-SD}}$ defined in (42).

With the above definition, we can characterize the throughput region by considering the specific realizations of the channel states. We employ the following approach. First, we identify the instantaneous rate region for any given set of channel states. Then, choosing instantaneous service rates, i.e., $\mathbf{R} = (R_1, R_2)$, that lie on the boundary of this instantaneous rate region, we determine the boundary of the effective capacity region. For instance, assume that the channel gains and the signal-to-noise ratios are such that $\text{SNR}_1 z_{11} + \gamma \text{SNR}_2 z_{21} \leq \text{SNR}_1 z_{12}/\gamma + \text{SNR}_2 z_{22}$, then the instantaneous rate region in (38) becomes

$$\begin{aligned} R_1 &\leq B \log_2(1 + \text{SNR}_1 z_{11}) \\ R_2 &\leq B \log_2(1 + \text{SNR}_2 z_{22}) \\ R_1 + R_2 &\leq B \log_2(1 + \text{SNR}_1 z_{11} + \gamma \text{SNR}_2 z_{21}). \end{aligned} \quad (44)$$

If $\log_2(1 + \text{SNR}_2 z_{22}) < \log_2(1 + \text{SNR}_1 z_{11} + \gamma \text{SNR}_2 z_{21})$, then the region in (44) is a pentagon. By time-sharing between the edges of the dominant face of this pentagon, the users can achieve the following maximum rate pairs:

$$\begin{aligned} R_1 &= \alpha B \log_2(1 + \text{SNR}_1 z_{11}) \\ &+ (1 - \alpha) B \log_2 \left(\frac{1 + \text{SNR}_1 z_{11} + \gamma \text{SNR}_2 z_{21}}{1 + \text{SNR}_2 z_{22}} \right) \end{aligned} \quad (45)$$

$$\begin{aligned} R_2 &= (1 - \alpha) B \log_2(1 + \text{SNR}_2 z_{22}) \\ &+ \alpha B \log_2 \left(1 + \frac{\gamma \text{SNR}_2 z_{21}}{1 + \text{SNR}_1 z_{11}} \right) \end{aligned} \quad (46)$$

for some $\alpha \in [0, 1]$, which is the time-sharing parameter. Maximum instantaneous service rate pairs for other realizations of the channel gains can be found similarly. The boundary of the effective capacity region can be obtained by inserting these instantaneous rate pairs into the effective capacity formulation in (43) and varying the time-sharing parameter α .

4) Numerical Results: In this part, we present numerical analysis for independent Rayleigh fading channels with $\mathbb{E}\{z_{11}\} = \mathbb{E}\{z_{22}\} = 1$. Note that under Rayleigh fading as

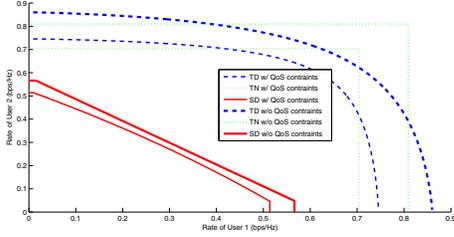


Fig. 4. The effective capacity region. $\beta_1 = \beta_2 = 1$. $\sigma_{12} = \sigma_{21} = 0.1$.

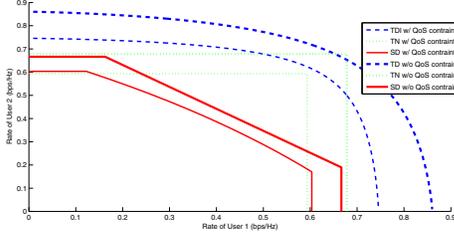


Fig. 5. The effective capacity region. $\beta_1 = \beta_2 = 1$. $\sigma_{12} = \sigma_{21} = 0.5$.

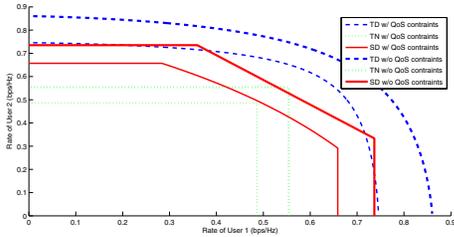


Fig. 6. The effective capacity region. $\beta_1 = \beta_2 = 1$. $\sigma_{12} = \sigma_{21} = 1.2$.

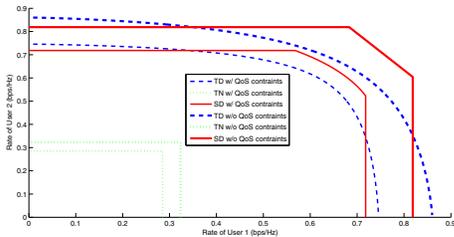


Fig. 7. The effective capacity region. $\beta_1 = \beta_2 = 1$. $\sigma_{12} = \sigma_{21} = 5$.

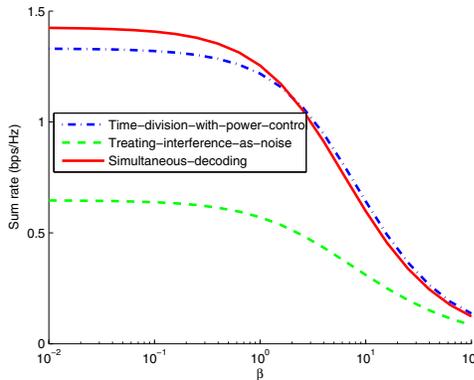


Fig. 8. The sum rate vs. β . $\sigma_{12} = \sigma_{21} = 5$.

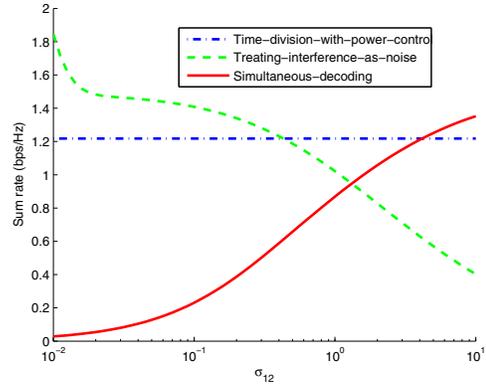


Fig. 9. The sum rate vs. σ . $\beta_1 = \beta_2 = 1$.

sumption, fading magnitude-squares z_{11} and z_{22} are exponentially distributed. We assume that fading magnitude-squares z_{12} and z_{21} in the cross links are exponentially distributed as well with variances σ_{12}^2 and σ_{21}^2 . We let $\text{SNR}_1 = \text{SNR}_2 = 1$. In Figs. 4 - 7, we plot the effective capacity regions (i.e., the collection of source arrival rates at the two transmitters, which can be supported by the interference channel) for different values of $\sigma_{12} = \sigma_{21} = \sigma$ when $\beta_1 = \beta_2 = 1$. In the figures, the solid, dotted, dashed lines represent the regions achieved by simultaneous decoding, treating interference as noise, and time division with power control, respectively. TD refers to the strategy of time-division-with-power-control, and TN represents treating-interference-as-noise, while SD stands for the simultaneous-decoding. The thick lines represent the corresponding regions achieved when there is no such QoS constraints, i.e., $\theta_1 = \theta_2 = 0$. Expectedly, we have larger arrival rate regions in the absence of QoS limitations. As we can see from the figures, treating interference as noise gives us the best sum rate performance when the channels of the cross links are weak on average, i.e., the probability of weak interference is high. As the values of σ_{21} and σ_{12} increase and hence the cross links grow stronger, the performance of simultaneous decoding improves while the performance of treating-interference-as-noise degrades expectedly. It is worth noting that time division with power control always achieves larger values on the axes i.e., when either effective capacity becomes 0. This is due to the fact that in the presence of fading, the interference channel with each set of channel states can be either *weak* or *mixed*.

In Fig. 8, we plot the sum rate of different transmission schemes as $\beta_1 = \beta_2 = \beta = \frac{\theta TB}{\log_e 2}$ varies. We can see that as β increases, the sum rates of the different strategies decrease and converge. Essentially, larger β (or equivalently larger value of the QoS exponent θ) implies more stringent QoS constraints, which results in smaller effective capacity values. Also, we notice that time division with power control becomes optimal at large β values. In Fig. 9, we plot the sum rate as a function of $\sigma_{12} = \sigma_{21} = \sigma$ when $\beta_1 = \beta_2 = 1$. We can see that as σ increases, or equivalently the fading conditions at the cross links improve, the sum rate of treating interference as noise decreases while the sum rate of simultaneous decoding increases. Therefore, there are in general two

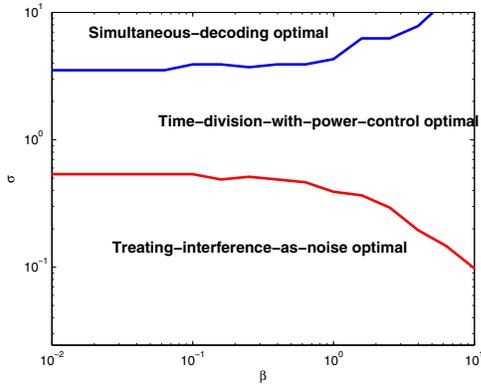


Fig. 10. The optimal division of region (β, σ) .

points at which the performances of treating-interference-as-noise and simultaneous decoding intersect with that of time division with power control. These points can be determined numerically. To see which strategy is better in a broader sense, we plot these intersection points as a function of β (or equivalently the strictness of the QoS constraints) in Fig. 10. The two lines partition the set of (β, σ) values into three regions in each of which one strategy is optimal. We can see that simultaneous decoding is optimal for large values of σ , treating-noise-as-interference is the best strategy for small values of σ , and time division with power control is optimal in between. The interesting observation is that as β increases, the region corresponding to time-division with power control expands. Therefore, time-division with power control becomes the sum-rate maximizing strategy for a wider set of cross-link variance, σ^2 , values.

VI. CONCLUSION

In this paper, we have studied the throughput regions of fading broadcast and interference channels in the presence of statistical QoS constraints. We have employed effective capacity as a measure of the maximum constant arrival rates that can be supported under buffer constraints. In the broadcast model, we have first defined the effective capacity region when superposition coding with successive interference cancellation is used and proved that this region is convex. Then, we have obtained the optimal power control policies that achieve the points on the boundary region in the two-user case. In particular, we have identified an algorithm for computing the optimal power allocated to each fading state using the optimality conditions.

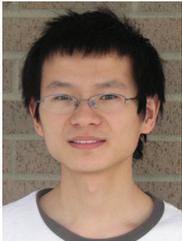
In the case of interference channels, we have considered three different strategies: treating interference as noise, time division with power control and simultaneous decoding. We have identified the instantaneous service rates and characterized the effective capacity regions for these strategies. Through numerical results, we have observed that similarly as in Gaussian interference channels, when the strengths of the interfering cross-links increase, the region achieved with treating interference as noise shrinks and that of simultaneous decoding expands. We have also seen that the sum rates of different strategies decrease and become close to each other

as QoS constraints become stricter. Moreover, we have noted that time division with power control starts being optimal over a larger set of cross-link variance values under increasingly more stringent queuing constraints.

REFERENCES

- [1] L. Li and A. Goldsmith, "Capacity and optimal resource allocation for fading broadcast channels—part I: ergodic capacity," *IEEE Trans. Inf. Theory*, vol. 47, no. 3, pp. 1083–1102, Mar. 2001.
- [2] L. Li and A. Goldsmith, "Capacity and optimal resource allocation for fading broadcast channels—part II: outage capacity," *IEEE Trans. Inf. Theory*, vol. 47, no. 3, pp. 1103–1127, Mar. 2001.
- [3] N. Jindal and A. Goldsmith, "Capacity and optimal power allocation for fading broadcast channels with minimum rates," *IEEE Trans. Inf. Theory*, vol. 49, no. 11, pp. 2895–2909, 2003.
- [4] H. Sato, "The capacity of the Gaussian interference channel under strong interference," *IEEE Trans. Inf. Theory*, vol. 27, no. 6, pp. 786–788, Nov. 1981.
- [5] T. S. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. 27, no. 1, pp. 49–60, Jan. 1981.
- [6] A. B. Carleial, "A case where interference does not reduce capacity," *IEEE Trans. Inf. Theory*, vol. 21, no. 5, pp. 569–570, Sept. 1975.
- [7] H. Sato, "Two user communication channels," *IEEE Trans. Inf. Theory*, vol. 23, no. 3, pp. 295–304, May 1977.
- [8] M. Costa, "On the Gaussian interference channel," *IEEE Trans. Inf. Theory*, vol. 31, no. 5, pp. 607–615, Sep. 1985.
- [9] X. Shang, G. Kramer, and B. Chen, "A new outer bound and the noisy interference sum-rate capacity for Gaussian interference channels," *IEEE Trans. Inf. Theory*, vol. 55, no. 2, pp. 689–699, Feb. 2009.
- [10] A. Motahari and A. Khandani, "Capacity bounds for the Gaussian interference channel," *IEEE Trans. Inf. Theory*, vol. 55, no. 2, pp. 620–643, Feb. 2009.
- [11] S. Annappureddy and V. Veeravalli, "Gaussian interference networks: sum capacity in the low interference regime and new outer bounds on the capacity region," *IEEE Trans. Inf. Theory*, vol. 55, no. 7, pp. 3032–3050, Jul. 2009.
- [12] R. Etkin, D. N. C. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inf. Theory*, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.
- [13] G. Bresler, A. Parekh, and D. N. C. Tse, "The approximate capacity of the many-to-one and one-to-many Gaussian interference channels," *IEEE Trans. Inf. Theory*, vol. 56, no. 9, pp. 4566–4592, Sept. 2010.
- [14] A. S. Avestimehr, S. N. Diggavi, and D. N. C. Tse, "Wireless network information flow: a deterministic approach," *IEEE Trans. Inf. Theory*, vol. 57, no. 4, pp. 1872–1905, Apr. 2011.
- [15] Y. Zhao, C. W. Tan, A. S. Avestimehr, S. N. Diggavi, and G. J. Pottie, "On the maximum achievable sum-rate with successive decoding in interference channels," *IEEE Trans. Inf. Theory*, vol. 58, no. 6, pp. 3798–3820, June 2012.
- [16] A. El Gamal and Y.-H. Kim, *Network Information Theory*. Cambridge University Press, 2011.
- [17] L. Sankar, X. Shang, E. Erkip, and H. V. Poor, "Ergodic fading interference channels: sum-capacity and separability," *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 2605–2626, May 2011.
- [18] D. Wu and R. Negi, "Effective capacity: a wireless link model for support of quality of service," *IEEE Trans. Wireless Commun.*, vol. 2, no. 4, pp. 630–643, July 2003.
- [19] J. Tang and X. Zhang, "Quality-of-service driven power and rate adaptation over wireless links," *IEEE Trans. Wireless Commun.*, vol. 6, no. 8, pp. 3058–3068, Aug. 2007.
- [20] J. Tang and X. Zhang, "Cross-layer-model based adaptive resource allocation for statistical QoS guarantees in mobile wireless networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 6, pp. 2318–2328, June 2008.
- [21] L. Liu, P. Parag, and J.-F. Chamberland, "Quality of service analysis for wireless user-cooperation networks," *IEEE Trans. Inf. Theory*, vol. 53, no. 10, pp. 3833–3842, Oct. 2007.
- [22] D. Qiao, M. C. Gursoy, and S. Velipasalar, "Transmission strategies in multiple access fading channels with statistical QoS constraints," *IEEE Trans. Inf. Theory*, vol. 58, no. 3, pp. 1578–1593, Mar. 2012.
- [23] Q. Du and X. Zhang, "Resource allocation for downlink statistical multiuser QoS provisionings in cellular wireless networks," *Proc. 2008 IEEE INFOCOM*.
- [24] A. Balasubramanian and S. L. Miller, "Effective capacity of a time division downlink scheduling system," *IEEE Trans. Commun.*, vol. 58, no. 1, pp. 73–78, Jan. 2010.

- [25] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.
- [26] A. Goldsmith, *Wireless Communications*, 1st ed. Cambridge University Press, 2005.



Deli Qiao received the Ph.D. degree in electrical engineering from the University of Nebraska-Lincoln, Lincoln, NE, USA, in 2012. He is currently a research engineer in Communication Technology Lab, Huawei Technologies Co., Ltd., Shanghai, China. His research interests include information theory and wireless communications, with an emphasis on MIMO communications. In 2011, he received the Maude Hammond Fling Fellowship from the University of Nebraska-Lincoln.



Mustafa Cenk Gursoy received the Ph.D. degree in electrical engineering from Princeton University, Princeton, NJ, in 2004, and the B.S. degree in electrical and electronics engineering from Bogazici University, Istanbul, Turkey, in 1999 with high distinction. He was a recipient of the Gordon Wu Graduate Fellowship from Princeton University between 1999 and 2003. In the summer of 2000, he worked at Lucent Technologies, Holmdel, NJ, where he conducted performance analysis of DSL modems. Between 2004 and 2011, he was a faculty member

in the Department of Electrical Engineering at the University of Nebraska-Lincoln (UNL). He is currently an Associate Professor in the Department of Electrical Engineering and Computer Science at Syracuse University. His research interests are in the general areas of wireless communications, information theory, communication networks, and signal processing. He is currently a member of the editorial boards of *IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS*, *IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY*, *IEEE COMMUNICATIONS LETTERS*, and *Physical Communication* (Elsevier). He received an NSF CAREER Award in 2006. More recently, he received the *EURASIP Journal of Wireless Communications and Networking* Best Paper Award, the UNL College Distinguished Teaching Award, and the Maude Hammond Fling Faculty Research Fellowship.



Senem Velipasalar received the Ph.D. and M.A. degrees in electrical engineering from Princeton University, Princeton, NJ, in 2007 and 2004, respectively, the M.S. degree in electrical sciences and computer engineering from Brown University, Providence, RI, in 2001 and the B.S. degree in electrical and electronic engineering with high honors from Bogazici University, Istanbul, Turkey, in 1999. During the summers of 2001 to 2005, she was with the Exploratory Computer Vision Group at IBM T. J. Watson Research Center, NY. Between 2007

and 2011, she was an Assistant Professor in the Department of Electrical Engineering at the University of Nebraska-Lincoln. Currently, she is an Assistant Professor in the Department of Electrical Engineering and Computer Science at Syracuse University, Syracuse, NY.

The focus of her research has been on wireless embedded smart cameras, multi-camera tracking and surveillance systems, and automatic event detection from videos. Her current research interests include embedded computer vision, video/image processing, embedded smart camera systems, distributed multicamera systems, pattern recognition, and signal processing.

Dr. Velipasalar received an NSF CAREER Award in 2011. She received the Best Student Paper Award at the IEEE International Conference on Multimedia and Expo in 2006. She is the recipient of the EPSCoR First Award, two Layman Awards, the IBM Patent Application Award, and the Princeton and Brown University Graduate Fellowships.