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The Domain-general and Domain-specific Profiles of Computation and Problem-Solving Difficulties

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THE DOMAIN-GENERAL AND DOMAIN-SPECIFIC PROFILES OF COMPUTATION AND PROBLEM-SOLVING DIFFICULTIES

by

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The purpose of the study was to explore the domain-general and domain-specific profile of two important mathematics difficulties. Three domain-general measures (working memory, processing speed, reasoning), and three domain-specific measures (language comprehension, mathematics vocabulary, math fluency) were completed among 125 Chinese 4th grade students. Of these 125 students, 28 were classified as students with only calculation difficulties (CD), 34 were classified as having problem-solving difficulties (PD), 20 were classified as students with calculation and problem-solving difficulties (CPD), and 43 were typically developing (TD) peers. Multivariate analysis showed that, compared to TD, CD was associated with weakness in working memory and mathematics vocabulary. PD was associated with deficit in language comprehension as well as mathematics vocabulary. These findings, taken together, suggest that CD and PD represent distinct MD deficit. CD was associated with weakness in numerical working memory, whereas PD was associated with weakness in language comprehension. Both CD and PD were experiencing mathematics vocabulary deficit. Implications for understanding mathematics competence and identification of mathematics difficulties are discussed.

Keywords: computation difficulties, problem-solving difficulties, domain-general factors, domain-specific factors

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CHAPTER 1: INTRODUCTION

Developing mathematics competency is critical for school and career success (National Council of Teachers of Mathematics, 2000). However, learning mathematics is a big challenge for many children. Converging evidence shows that approximately 5-9% of the school-age population suffers from mathematics disabilities (MD) (e.g., Badian, 1983; Berch & Mazzocco, 2007; Gross-Tsur, Manor, & Sgalev, 1996). The current study used both computation and word-problem-solving as the screening measures for MD, since mathematics comprises several related branches, within which computation and word-problem-solving are core capacities for primary school students to develop (Common Core State Standards Initiative, 2010; National Mathematics Advisory Panel, 2008). According to Peng, Wang, and Namkung (2018), investigating the deficits in domain-specific factors (e.g., Geary, 1993; Gersten, Jordan, & Flojo, 2005) and domain-general cognitive factors (e.g., Johnson, Humphrey, Mellard, Woods, & Swanson, 2010; Swanson & Jerman, 2006; Peng & Fuchs, 2016) are two major approaches to understand the deficit profiles of MD. The present study aimed to investigate the domain-specific and domain-general cognitive deficit profiles among subgroups with difficulty in one, the other, both, or neither. Such investigation can provide implications for identifying math disability and designing effective intervention for MD.

Domain-specific and Domain-general Factors Associated with Computation and Word Problem Solving

Computation and word-problem-solving are two mathematics tasks that are correlated but distinct from each other. On the one hand, computation and word-problem-solving are closely related. Levine, Jordan, and Huttenlocher (1992) suggested that capacity with calculation precedes word-problem-solving and that calculation is a leading indicator of word-problem-solving. Later longitudinal study (Fuchs et al., 2006; Swanson & Beebe-Frankenberger, 2004) also confirmed that initial calculation skill predicts word-problem-solving outcomes. On the other hand, calculation also differs from word-problem-solving by the addition of linguistic information. Therefore, computation and word-problem-solving may tap some shared and distinct domain-specific and domain-general factors. In the following, we describe those factors in detail.

Domain-specific factors. Language comprehension is a critical factor influencing word-problem-solving more than computation. For word-problem-solving, Kintsch and colleagues (i.e., Kintsch & Greeno, 1985; Nathan, Kintsch & Young, 1992) presented a model on the process of solving word problems. This model involves three inter-correlated but progressive levels of representations: textbase, situation model, and problem model. Textbase is the propositional representations formed during the process of reading a problem to capture the meaning of the passage. After the formation of textbase, it is organized into a (qualitative) situation model (also termed problem sechema) and then mapped into a (quantitative) problem model capturing the algebraic problem structure (Nathan, Kintsch & Young, 1992). Regarding this model, it is obvious that textbase in word-problem-solving involves the same language comprehension process as any other form of text comprehension (e.g., reading comprehension). In addition, organizing the propositions into problem schema also requires language comprehension. Besides this model, relevant literature has confirmed the importance of language comprehension by highlighting the capacity to recall problem statements in solving word problems. For example, Commins et al. (1988) suggested that students who could correctly recall the textbases were more likely to produce correct solutions. Taken together, language comprehension is essential for the problem-solving performance of students.

It is reasonable to expect that language comprehension might differentiate PD from CD. Fuchs et al. (2008) investigated the cognitive deficit profiles of CD and PD on a wide range of measures (e.g., working memory, reasoning, processing speed, attention). They used a language factor score across Grammatic Closure, listening comprehension, and expressive vocabulary to indicate students' language capacity and their finding suggests that language is the only measure that can directly separate CD and PD. Yet, the language factor score in Fuchs et al. (2008) could not fully reflect language skills, which was also acknowledged by the author as one limitation of their study. That is, they used relatively simple comprehension and vocabulary measures. To refer to students' comprehension ability, they used listening comprehension and Grammatic Closure, in which students only need to identify the missing word in a sentence or a passage. To indicate students' vocabulary skills, their study used WASI vocabulary, which asks students to identify the object in the picture. In the present study, we used a language factor score across general vocabulary as well as reading comprehension. The general vocabulary measure not only asked students to identify the words but also asked students to use it as a phrase. The reading comprehension measure asked students to read texts and answer questions, which require a deeper understanding of those texts. Combining comprehension and vocabulary measures in our study can reflect students' language comprehension skills.

Another domain-specific factor is mathematics vocabulary, which influences both computation and word-problem-solving. According to the perspective of functional linguistics, besides general oral language and written language, which enable children to develop commonsense knowledge of the world, there is discipline language (Halliday, 2004). Mastering discipline-specific ways of using language can help students understand how a discipline organizes knowledge. Mathematics, as a discipline, also has evolved a language, which serves to construct mathematical knowledge and reasoning. Unlike other disciplines, mathematics relies on the resources of linguistic discourse, mathematical symbols, and visual display simultaneously to prove axioms, theorems, and lemmas (O'Halloran, 2004). According to recent studies, mathematics vocabulary has a strong correlation with both calculation (Powell & Nelson, 2017; Powell et al., 2017; Forsyth & Powell., 2017) and word-problem-solving (Fuchs et al., 2015).

Fuchs et al. (2008) revealed that language deficit is unique to PD. However, their study did not differentiate the mathematics vocabulary from general language skills while investigating the domain-specific factors. In another study, Forsyth and Powell (2017) investigated the role of mathematics vocabulary for MD, and they found that compared to typical developing peers, MD was associated with weaknesses in mathematics vocabulary. However, their study cannot clearly support this claim for two reasons. First off, their study used only computation as the screening measure for MD and did not control their word-problem-solving performance. Therefore, it is still not clear whether mathematics vocabulary is a deficit for CD or both CD and PD are experiencing deficient mathematics vocabulary. Secondly, although their finding was largely supportive of the importance of mathematics vocabulary for MD, it is possible that mathematics vocabulary is a proxy measure for other general language and cognitive measures because their study did not include other factors crucial for computation or word-problem-solving. The present study has included general language skill and other critical cognitive skills together into analysis with the aim of investigating the role of mathematics vocabulary for both CD and PD.

Besides linguistic skills, math fluency (e.g., 3 + 4 = 7) is another important domain-specific factor influencing computation but not so much for word-problem-solving. Math fluency refers to simple arithmetic problems that can be solved via counting or be automatically retrieved from long-term memory. Prior studies revealed that the difficulties to use retrieval-based processes to solve simple computation and word problems is the most consistent finding in the MD literature (e.g., Barrouillet et al., 1997; Geary, 1990, 1993; Garnett & Fleischer). Students who demonstrate difficulty with counting (Geary, Bow-Thomas, & Yao, 1992), using immature backup strategies (Geary et al., 2007), and unable to make the shift to memory-based retrieval of answers (Geary et al., 1987; Goldman et al., 1988) would waste their working memory resources and struggle with computation difficulties.

However, in their profiling study on CD and PD, Fuchs et al. (2008) did not find a semantic retrieval deficit for students with mathematics learning difficulties. One possible explanation is that it should be more specific numerical facts retrieval rather than semantic retrieval that can distinguish CD and PD. Another reason is, as indicated by a recent review on MD (Peng, Wang, & Namkung, 2018), most prior studies investigating

MD profiles used calculation as the screening measure, which highly relied on math fluency (Fuchs et al., 2006). Therefore, the present study further investigated math fluency for MD by comparing its importance for CD and PD.

Domain-general factors. Besides domain-specific factors, domain-general factors are also related to mathematics difficulties (Peng et al., 2018). Working memory is a critical factor influencing both computation and word-problem-solving. For computation, prior work provides the basis for hypothesizing that working memory or the capacity to maintain target memory items while processing an additional task (Daneman & Carpenter, 1980) is associated with computation (Fuchs et al., 2005; Geary et al., 1991; Hitch & McAuley, 1991; Siegel & Linder, 1984; Webster, 1979; Wilson & Swanson, 2001). To solve number combinations, students must hold the numerals and operators in working memory while using various counting strategies to arrive at the correct answer. Through numerous successful counting opportunities, repeated associations of the problem with its answer have been established in long-term memory, which may, in turn, facilitate procedural computation by direct retrieval of math facts. Therefore, it is highly possible to detect a working memory deficit among students with calculation deficits.

For word-problem-solving, according to Kintsch and Greeno (1985), working memory is involved in the construction of a problem model. In other words, working memory represents the process through which students translate the linguistic information to a mathematical equation. During the process of building a problem model, multiple pieces of information are being stored and manipulated in memory. Hence, working memory represents the attentional and representational systems needed for effective execution of procedures to reach a correct solution (Ackerman, 1988). However, there are mixed findings regarding the importance of working memory for word-problem-solving. In line with theoretical model, Passolunghi and Siegel (2001) found that compared with good problem solvers, children who were poor in arithmetic problem-solving were experiencing a general deficit in working memory. There are other studies (e.g., LeBlanc & Weber-Russell, 1996; Passolunghi & Siegel, 2004; Swanson & Sachse-Lee, 2001) corroborating the finding that good and poor problem solvers differed on working memory tasks. Meanwhile, the robustness of the relation has been questioned by other studies. For instance, Swanson, Cooney, and Brock (1993) found only a weak relation between working memory and problem-solving accuracy among typically developing third and fourth graders. And this relation disappeared after reading comprehension ability has been included into consideration. Fuchs et al. (2008) did not find working memory, verbal as well as numerical, a deficit for CD and PD. The current study further investigated the role of working memory between Chinese CD and PD.

Processing speed is another critical domain-specific factor, which influences computation more than word-problem-solving. Processing speed, the efficiency with which information is processed, may indicate the speed that numbers can be counted, therefore, underlies simple arithmetic task (Salthouse, 1996). With slower processing, the interval for deriving counted answers and for pairing a problem stem with its answer in working memory increases; this creates the possibility that “decay” sets in before completing the computational sequence (Fuchs et al., 2008). Fuchs et al. (2008) revealed that processing speed is a distinguishing factor for CD and PD. That is, students with computation difficulties (CD and CPD) have deficient processing speed.

IQ is the last domain-general factor that may be critical for both computation and problem-solving. In terms of computation, IQ, the capacity to complete visual patterns, may support arithmetic development (Geary, Hoard, Nugent, & Bailey, 2012; Fuchs et al., 2013), considering its role in understanding arithmetic relations and principles (Geary et al., 2012). However, IQ may not be an important predictor of computation in later grades when students mostly rely on facts retrieval or working memory in computations (Locuniak & Jordan, 2008). In contrast, IQ may be a constant important variable for word-problem-solving. Specifically, previous literature has identified IQ as a unique predictor in the development of problem-solving skills (Agness & McLone, 1987; Fuchs et al., 2005). As Cooper and Sweller (1987) states, students need to grasp problem-solution strategies, categorize problems into problem-types or schemas, and generalize taught problems to novel problem situations to solve word problems. The processes of categorizing a WP as a specific problem type, or schema, and adopting appropriate solution-strategy makes strong demands on reasoning ability. However, Fuchs et al. (2008) did not find that IQ distinguish CD from PD. Therefore, the role of IQ (reasoning) for CD and PD is still not clear.

Chinese Sample

Most previous profiling research of MD focused on children from English-speaking countries (e.g., US). The current study selected Chinese sample to further investigate deficit profiles of MD. Chinese students were superior in tasks tapping computational skills, whereas their U.S. peers performed as well as or better on more creative problem-solving tasks (Cai, 1997, 1998, 2000). Contributing factor identified for Chinese students’ superior computation skills was Chinese numerical language characteristics such as the regularity of a Chinese number-naming system which enhances cognitive representation of numbers and understanding of place value concepts (e.g., Ho & Cheng, 1997; Miura & Okamoto, 2003). Specifically, Chinese number words above 10 are generated by consistent rules, while English number names above 10 have a few irregular modifications. For instance, in Chinese, the literal translation of 11 from Chinese into English is “ten-one”, similarly, 12 is “ten-two”. Due to this language advantage, mastering numerical knowledge (e.g., counting, number names, and simple arithmetic calculation) is easier for Chinese students than for their US peers. In addition, mathematics instruction difference between China and America may be another explanation for this cross-cultural difference. Chinese’s math classroom teaching often starts with the teaching of procedure, followed by repeated practices (Leung & Park, 2002). Many classroom teachers in China ask students to memorize the specific procedure for solving different problems so that students can recognize the problem types immediately and solve them (Cai & Nie, 2007). Taken together, the language and instruction characteristics of Chinese students may explain their superior performance over their U.S. peers in early mathematics and to simple calculation task selectively. If so, the automatic retrieval of number facts may not be a problem for Chinese students. Hence, we might hypothesize that Chinese students with computation difficulties have a unique cognitive deficit profile. Their deficiency in computation might result from a deficit in conceptual understanding regarding numbers and operations. That is, Chinese CD may demonstrate deficit in the conceptual understanding of numbers of operators rather than deficit in numerical facts retrieval.

Contribution of the present study

In this study, we further investigate the deficit profiles of CD and PD by including both domain-specific (general language, math vocabulary, and math fluency) and domain-general factors (working memory, processing speed, and IQ). So far, no study has examined whether and how mathematics vocabulary emphasized in the curriculum would affect mathematics outcomes, especially among older children. The present study was designed to extend the literature in three ways. First off, we included factors proven more critical for computation and word-problem-solving into the multivariate analysis. Second, the current study focused on fourth graders. According to prior linguistic studies (Biancarosa & Snow, 2006; Graham & Perin, 2007) starting from this age level, students start to develop their discipline language capacity and 70% of students are experiencing discipline language learning deficit. Third, the present study focused on Chinese fourth grade students with MD. Because no study, to our current knowledge, has investigated the cognitive profile of Chinese students with specific difficulty in word-problem-solving. The findings of the current study would provide insights into manifestations and characteristics of CD against PD among Chinese children.

CHAPTER 2: METHOD

Participants

Participants were 237 fourth graders (125 boys) from a typical elementary school in a southern city of China, who were the basis for the present report. We chose this school because according to the records of the county education bureau, this school ranks in the middle regarding the fourth graders' academic performance in that county. The mean age of the sample was 10.19 (*SD* = .43) years old. All students were typically developing children. No students were identified with any disabilities. Our research received appropriate institutional review board approval from all appropriate agencies and participants.

Difficulty Status Group Formation. To identify participants involved in the current study, we screened 237 fourth-grade students. We administered three screening measures including non-verbal IQ (RAVEN), computation (WRAT), and word problem-solving (factor score across WISC and WJ Applied problem) to identify participants in this study. We excluded students (n=1) with standard scores below the 25th percentile on the Chinese version of the Raven's Progressive Matrices Test (Zhang & Wang, 1985) because children with learning difficulties were defined as having normal intellectual ability. Then, we used the commonly used cutoff score 25th percentile (e.g., Fletcher et al., 1989; Swanson et al., 2008; Peng et al., 2012) to identify students at risk, and the 35th percentile as a cut-point in the designation of lack of difficulty. We excluded students in the buffer zone (i.e., scoring between the 26th and 34th percentiles on either or both math outcome. Finally, we matched the word-problem-solving performance of TD and CD, and the calculation performance of TD and PD. We identified 28 children who were at risk for CD (<25th percentile on WRAT-Computation and >35th percentile on problem-solving factor score), 34 children who were at risk for PD (<25th percentile on problem-solving factor score and >35th percentile on WRAT-Computation), 20 children who were at risk for CPD (<25th percentile on WRAT-Computation and <25th percentile on problem-solving factor score), and 43 TD children (>35th percentile on WRAT-Computation and >35th percentile on problem-solving factor score). Therefore, our examination of CD and PD could reflect the differentiating characteristics of each deficit group. Table 1 shows the demographic and screening data of the CD, PD, CPD, and TD group. All groups were comparable in terms of age, $F(3,121) = 1.04$, $p = .38$, and gender, $\chi^2(3) = 4.27$, $p = .23$. With respect to computation tasks, the CD and CPD groups were comparable, and both groups showed statistically significantly poorer performance than the TD and PD groups. Considering word problem-solving tasks, TD, and CD groups were comparable, and both groups showed statistically significantly greater performance compared to PD and CPD groups.

Table 1 Performance by Difficulty Status

	CD(n=28)			PD(n=34)			CPD(n=20)			TD(n=43)		
	n	M	SD	n	M	SD	n	M	SD	n	M	SD
Age(months)		121.57	4.39		123.5	5.64		121.1	5.35		122.7	6.31
Girls	14			20			6			22		
1.WRAT_Computation		-1.20	0.50		0.06	0.51		-1.51	0.73		0.18	0.26
2.Problem solving		0.13	0.36		-0.82	0.43		-1.05	0.38		0.15	0.33
WP_WISC		0.66	0.75		-0.53	0.86		-0.98	0.87		0.45	0.70
WP_WJ		0.37	0.68		-0.73	0.88		-0.82	0.85		0.71	0.60
3.Working memory		-0.61	0.70		-0.33	1.18		-0.73	0.92		0.09	0.75
4. Processing speed		-0.22	0.77		-0.2	0.87		-0.22	1.27		0.14	1.07
5. IQ		-0.23	0.94		-0.10	0.97		-1.14	1.20		0.05	0.88
6. Language comprehension		0.03	0.88		-0.45	0.94		-1.26	0.81		0.12	0.73
Reading comprehension		0.02	0.80		-0.50	1.12		-1.15	0.61		0.08	0.75
General vocabulary		0.01	1.01		-0.28	0.88		-1.11	1.05		0.15	0.77
7. Math vocabulary		-0.4	0.87		-0.41	0.89		-0.95	0.77		0.14	0.79
MVNO		-0.07	1.03		-0.07	1.04		-0.44	0.99		0.31	0.89
MV_G		-0.14	1.06		-0.12	1.01		-0.60	0.75		0.47	0.88
MV_MD		-0.04	1.02		-0.08	0.91		-0.7	0.95		0.42	0.90
8. Math fluency		-0.30	1.06		-0.40	0.94		-0.52	0.97		-0.08	0.95

Note. Performance is expressed as z scores in relation to the representative sample of 237. CD = computational difficulty; PD = problem-solving difficulty; CPD = computational and problem-solving difficulty; TD = typical developing. IQ = Raven's Progressive Matrices Test; WRAT_Computation = Computation subtest of Wide Range Achievement Test-4; Problem solving = A Factor score across Woodcock Johnson IV Tests of Achievement - Applied Problems and word problem subtest of Chinese version of the Wechsler Intelligence Scale for Children-Fourth Edition.

Measures

Calculation. We adapted and used the calculation subtest of the Wide Range Achievement Test-4 (WRAT-4, Wilkinson & Robertson, 2006). For this test, the child had 30 minutes to solve 80 calculation problems of increasing difficulty. The number of items solved correctly was the total score. Cronbach's alpha for the current sample was .83.

Word problems. We calculated the factor score of two tests of word problems. Using principle axis component analysis, we combined the scores of these word problem tests to indicate word problem-solving ability. The first word problem test was a paper-and-pencil test adapted from the Chinese version of the Wechsler Intelligence Scale for Children-Fourth Edition (Zhang, 2008) (WISC-word problems). For this test, the child was asked to solve 31 mathematical word problems of increasing difficulty presented on paper. The examiner read each word problem to the child to avoid difficulties in reading these word problems. The child was given 40 minutes to finish this test. The total number of word problems solved correctly was the score of this test. Cronbach's alpha for the current sample was .71.

The second word problem test was a paper-and-pencil test adapted from Woodcock Johnson IV Tests of Achievement - Applied Problems (Schrank et al., 2014). We translated these word problems into Chinese, with minor changes on some words to make them more appropriate for Chinese children (e.g., we used "meter" to replace "mile", because "mile" is not used to indicate length/distance in China). The child was given 40 minutes to solve 45 mathematical word problems for this test, with the examiner reading story problems to the child, if necessary. The total number of word problems solved correctly was the score of this test. Cronbach's alpha for the current sample was .84.

IQ. We used non-verbal reasoning to indicate IQ. Specifically, we used the Chinese version of the Raven's Progressive Matrices Test (Zhang & Wang, 1985). For this test, the child was required to circle the replacement piece that best completed a pattern presented in an item. There are 60 items of increasing difficulty in the test. The total number of problems solved correctly was the final score. Cronbach's alpha for the current sample was .85.

Processing speed. We used the character coding speed, which is adapted from the coding speed test of Chinese version of Wechsler Intelligence Scale for Children-Fourth Edition (CWISC-4, Zhang, 2008). This test consisted of 9 character-symbol pairs (又/┤, 个/┴, 上/ (, 口/V, 王/┐, 了/├, 广/┼, 工/), 大/└) followed by a list of the same 126 characters. The child was required to write down the corresponding symbol for each character as fast and accurately as possible (e.g., if it is 又, write ┤ under it). The score was the number of symbols written correctly within 2 minutes. The reported test-retest reliability of this test was .70.

Working memory. This test was adapted from Backward Digit Recall from the Working Memory Test Battery for Children (Pickering & Gathercole, 2001). For this test, the child listened to a string of random numbers presented by an audio player at the speed of one digit per second and then said the series backward. There were 21 series, with difficulty increasing as more numbers are added to the series. We gave feedback on the first three test series to lower the floor of this assessment. The score was the total number of series recalled correctly. Cronbach's alpha for the current sample was .76.

Language comprehension. We used a principal component factor analysis to create a weighted composite variable of language comprehension using two measures of language skills. First, the *Chinese Character Recognition Measure and Assessment Scale for Primary School Children* (Wang & Tao, 1993) was used to measure students' general vocabulary capacity. In this test, the child is required to identify 194 characters by using each character in a phrase/word. The score is the total number of characters used correctly in a phrase/word. Cronbach's alpha for the current sample was .96.

The second measure, *reading comprehension*, was a researcher-developed measure that consisted of eight passages (four narrative passages, three expository passages, and one poem) with 40 questions (multiple-choice and short-response). Questions were designed to tap the understanding of the main idea of the passage, inferencing, and understanding vocabulary in the content. Cronbach's alpha for the current sample was .76.

Mathematics vocabulary. To measure students' understanding of mathematics vocabulary, we selected all mathematics vocabulary terms that appeared in the elementary mathematics textbook published by People's Education Press (PEP) for the 3^{rd}, 4^{th}, and 5^{th} grade. The PEP edition was selected because it fully reflects the curriculum standards of math in China (Ministry of Education of China, 2011), and it is the most wildly used mathematics textbook which is also used by the students in the current study. We did not include mathematics vocabulary introduced before 3^{rd} grade because we found ceiling effects on those early vocabulary during our pilot study. There was a total of 91 target vocabulary in those categories in 3^{rd} through 5^{th} grade (see Appendix A). Since the Chinese mathematics curriculum in elementary grades has categorized mathematics knowledge into three types: numerical operations, geometry, and measurement. Therefore, in our study, our selected mathematics vocabulary naturally grouped into three categories. To investigate the importance of mathematics vocabulary for students' mathematics performance. We used a principal component factor analysis to create a weighted composite variable of mathematics vocabulary based on three types of mathematics vocabulary (i.e., measurement, geometry, and numerical operation).

On this basis, two testing formats (i.e., multiple choices and oral question) were adopted to measure students' understanding of vocabulary specific to mathematics. Multiple choice test, taking the paper format, was suitable for large-scale assessment. Hence, our multiple-choice questions covered all target vocabulary from 3^{rd} to 5^{th} grade. Due to the fact that mathematics vocabulary is correlated in some ways and the purpose to better assess students' conceptual understanding, a portion of our testing items required students to understand the relations of a group of mathematics vocabulary. For example, to test students' understanding of "kilometer", "decimeter", "meter" "millimeter", we asked students to choose the right ranking order from the options (A. kilometer>decimeter>meter> millimeter; B. centimeter >meter> decimeter > millimeter; C. meter >decimeter>centimeter>millimeter; D. meter> millimeter >decimeter>centimeter). For the multiple-choice test, students were given 40 minutes to answer 58 questions.

However, some attributes of the target vocabulary were not covered in the multiple-choice formats, and those attributes may be more appropriately assessed by the oral testing format. Therefore, in the oral question test, we included vocabulary with multiple characteristics, while we only test one characteristic in the multiple-choice test. For example, the curriculum lists several characteristics in understanding the vocabulary "parallelogram". We tested one characteristic (altitude of the parallelogram) in the multiple-choice test and tested the other characteristic (shape and structure) in the oral test by asking "what is parallelogram by providing the definition of the shape?" (see Appendix B for examples of multiple choice items and oral question items). For the oral question test, the tester read 35 questions to the child who had 30 seconds to answer each question. The whole process for each student was audiotaped and later transcribed to text for scoring. The first and second author independently scored the oral question test and the inter-rater agreement (the number of questions scored the same divided by the total number of questions) was .96, and the inconsistency was solved through discussion. Cronbach's alpha for the current alpha was 0.87.

Math fluency. The Woodcock Johnson IV Tests of Achievement - Calculation Fluency subtest (WJ-Calculation Fluency, Schrank, McGrew, & Mather, 2014) comprised 160 addition and subtraction number combinations. The child was given three minutes to solve the problems as fast and accurately as possible. The number of items solved correctly was the total score. Cronbach's alpha for the current sample was .98.

Procedure

Tests were administered in several sessions in the spring semester of 4^{th} grade during April and June (towards the end of 4^{th} grade). General vocabulary, reading comprehension, multiple-choice mathematics vocabulary, IQ, calculation, and word problem tests were administered to students in several whole-class sessions. Processing speed, working memory, and mathematics vocabulary oral question tests were administered to students individually in the quietest place available at their schools. At the end of each testing session, all students were given a small present as a memento of their participation.

CHAPTER 3: RESULTS

In Table 2, we display means, standard deviations, and correlations for our sample on computation, problem-solving, three domain-general dimensions (working memory, processing speed, IQ), and three domain-specific dimensions (language comprehension, mathematics vocabulary, and math fluency). There were significant and positive correlations among all domain-general and domain-specific measures, ranging from medium (r = .18) to high (r = .64). The strongest relationship was between word-problem-solving and language comprehension. Considering the correlations among included domain-general and domain-specific measures (see Table 2), we conducted a MANOVA analysis to evaluate whether groups differed on those factors using IBM SPSS version 24.

Table 2 Means, Standard Deviations, and Correlations

Variable	*N*	*M*	*SD*	1	2	3	4	5	6	7	8
1. Cal_WRAT	237	38.62	6.18	—							
2. WP	234	0.00	0.80	.54	—						
3. WM	234	8.24	3.54	.40	.40	—					
4. PS	237	44.62	6.30	.19	.20	.21	—				
5. IQ	236	40.58	7.26	.43	.43	.38	.18	—			
6. LC	231	0.00	1.00	.50	.64	.39	.27	.47	—		
7. MV	233	0.00	1.00	.49	.60	.49	.24	.46	.60	—	
8. MF	237	104.25	21.92	.38	.35	.18	.27	.25	.41	.39	—

Note. Cal_WRAT = calculation subtests of Wide Range Achievement Test-4; WP = A factor score across the Wechsler Intelligence Scale for Children-Fourth Edition-word problems and Woodcock Johnson IV-word problems; WM = working memory; PS = processing speed; IQ = non-verbal reasoning; LC= A factor score acrss reading comprehension and general vocabulary; MV = A factor score across numerical operation vocabulary, measurement & data vocabulary, and geometry vocabulary; MF = Woodcock Johnson calculation fluency;

Overall analysis

In our multivariate analysis, the between-subjects factor was math difficulty status (TD vs. CD vs. PD vs. CPD); the within-subjects factor was domain-general and domain-specific measures (working memory vs. processing speed vs. IQ vs. language comprehension vs. mathematics vocabulary vs. math fluency). The interaction between math difficulty status and included measures was significant, Wilk's $\Lambda = 0.58$, $F(21, 305) = 3.54$, $p < .001$. In addition, the elevation effect was also significant, Hotelling's $t = 0.63$, $F(21, 314) = 3.68$, $p < .001$.

To help interpret the interaction between math difficulty status and included measures, we plotted z scores on the six dimensions for each of the four difficulty status groups (see Figure 1). Means and standard errors are based on z scores. As shown, TD performed at a higher level than did the CD and PD groups than did the CPD groups; CD and PD performed similarly on most dimensions. In addition, the profile shape varied across the six dimensions as a function of the difficulty status group, therefore, the difficulty status group by cognitive dimensions interaction appeared to be evident.

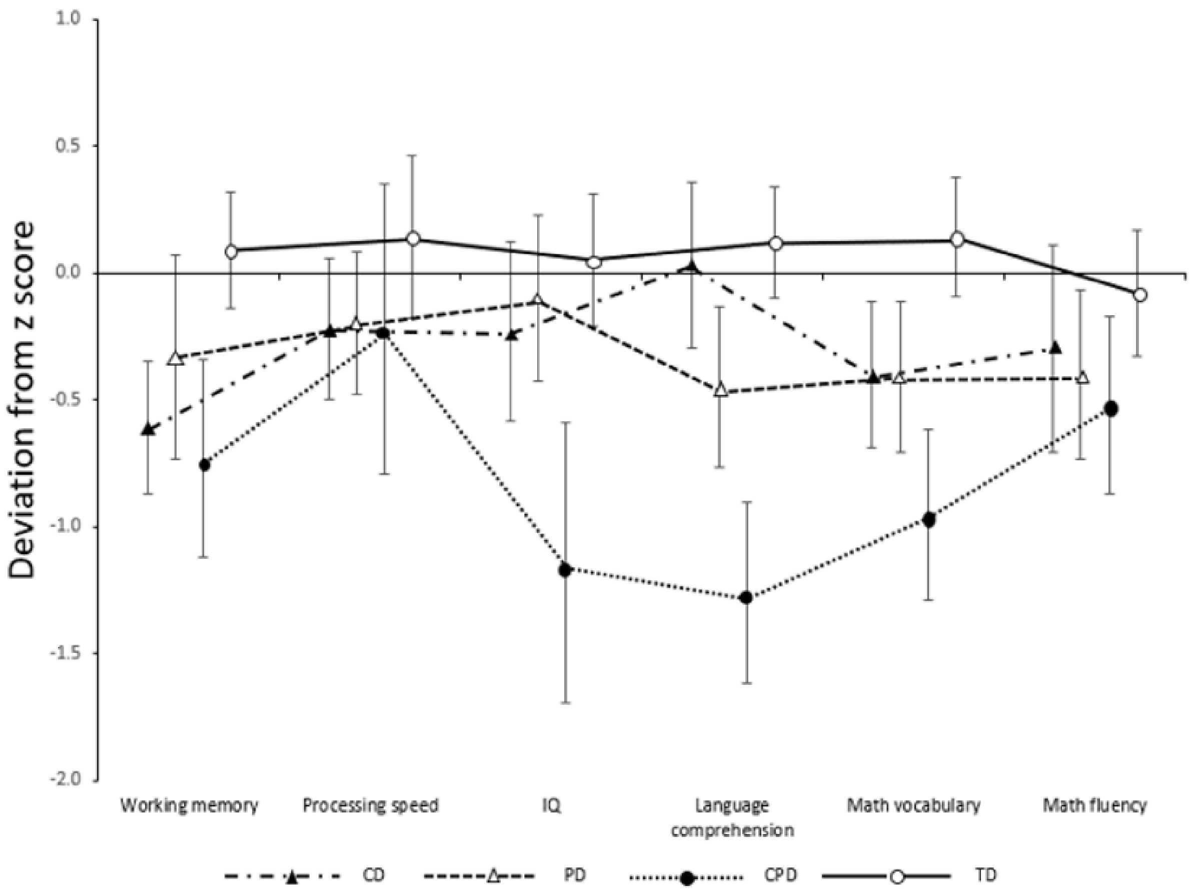

Figure 1. z score on domain-general and domain-specific dimensions by difficulty status. *Note*. Symbols for the four groups (see key) appear under each cognitive dimension. Groups with overlapping error bars on one dimension were not significantly different from each other.

Univariate post-hoc analysis

Since the interaction and elevation effects were significant in MANOVA analysis, we conduct six contrasts (TD vs. CD, TD vs. PD, TD vs. CPD, CD vs. PD, CD vs. CPD, PD vs. CPD) as the follow-ups test. Because we do multiple comparisons, we used Tukey's HSD correction for post hoc comparisons to adjust the *p*-value. The selection of the post hoc correction method is due to unequal sample sizes across groups. Due to our small and unequal sample sizes, we used Hedge's g to calculate the effect sizes (Hedges & Olkin, 1985). We compared TD to each mathematics learning difficulty groups and then compared difficulty groups to one another. Table 3 displays effect sizes for each comparison between difficulty status categories.

Table 3 Effect Sizes as a Function of Difficulty Status

	Contrasts						
	TD vs.			CD vs.		PD vs.	
Variable	CD	PD	CPD	PD	CPD	CPD	Statistical outcomes
Computation	3.57**	0.49	3.68**	-2.39**	0.43	2.54**	TD = PD > CD = CPD
Problem solving	0.17	3.08**	4.51**	2.34**	3.15**	0.52	TD = CD > PD = CPD
Working memory	0.95*	0.43	1.00*	-0.28	0.15	0.36	TD = PD > CD = CPD
Processing speed	0.37	0.34	0.31	-0.02	0.00	0.02	TD = CD = PD = CPD
IQ	0.31	0.16	1.19**	-0.13	0.85*	0.97**	TD = CD = PD > CPD
Language comprehension	0.11	0.68*	1.83**	0.52	1.51**	0.89**	TD = CD > PD > CPD
Math vocabulary	0.65*	0.65*	1.80**	0.01	0.65	0.63	TD > CD = PD = CPD
Math fluency	0.22	0.34	0.45	0.10	0.21	0.12	TD = CD = PD = CPD

Note. See Table 1 for means, standard deviations, and sample sizes. CD = computational difficulty; PD = problem-solving difficulty; CPD = computational and problem-solving difficulty; TD = typical developing.
*Significant differences in means as determined using the Tukey HSD post hoc correction.
* $p < .05$; ** $p < .01$

Domain-general measures. For processing speed, there was no significant group difference among TD group and other disability groups. The MANOVA revealed significant large group differences on both working memory, $F(3,121) = 4.89$, $p = 0.003$, and IQ, $F(3,121) = 6.65$, $p < 0.001$. Post hoc for working memory revealed that TD performed significantly better than CD and CPD. For IQ, post hoc testing showed that there were no significant differences among TD, CD, and PD, but those groups performed significantly better than CPD.

Domain-specific measures. The MANOVA yielded a significant group effect of large effect size on language comprehension, $F(3,121) = 13.07$, $p < 0.001$. Post hoc testing showed that there was no significant difference between TD and CD groups and that the TD group scored significantly higher than PD and CPD group. The MANOVA revealed significant group differences on mathematics vocabulary, $F(3, 121) = 8.56$, $p < 0.001$. Post hoc testing revealed that the TD group scored significantly higher than all difficulty groups. In addition, there was no significant group difference among all groups on math fluency task.

CHAPTER 4: DISCUSSION

The goal of the current study was to explore the deficit profiles of CD and PD based on Chinese sample. Regarding CD, the current finding revealed that CD performed significantly lower than TD on working memory and mathematics vocabulary. In terms of PD, language comprehension and mathematics vocabulary are two factors that can separate PD from TD. As for comorbid difficulties (CPD), the cognitive deficiencies associated with single mathematics difficulties are also apparent: for computation, working memory (as revealed for CD and for CPD); for problem solving, language comprehension (as revealed for PD and for CPD). Taken together, current findings suggest that CD and PD may represent distinct MD deficit. That is, compared to TD, CD and PD demonstrate relatively different profiles. CD is related to numerical WM deficits, whereas PD was related to language comprehension deficits. Both CD and PD are related to mathematics vocabulary deficits.

Domain-specific factors

In terms of language comprehension, the ability to make sense of language, it is necessary to consider the major distinction between mathematical computation and problem-solving. Whereas a computation problem is already set up for solution, a word problem requires students to capture the meaning of the text, identify the problem type, and construct the calculation operation to find the missing information. This obvious difference may largely alter the nature of the task. This corroborates previous work (Fuchs et al. 2015, 2018) about word-problem-solving as a form of text comprehension.

As for mathematics vocabulary, the only measure could distinguish TD from CD, PD, and CPD. It is worth noting that, the current study has included measures believed to be crucial for computation and word-problem-solving. Using multivariate analysis, we have controlled the correlation among those measures, which suggested that mathematics vocabulary is not a proxy measure for other cognitive and linguistic measures. Mathematics vocabulary is a distinct construct, which students with mathematics learning difficulties widely suffered. At least two explanations seem possible for the role of mathematics vocabulary in MD. Theoretically, from the linguistic perspective, language helps people transform their experience about the world into knowledge. After students enter into the third crucial stage of language development (Halliday, 2004), mastering discipline language becomes the focus of their academic study. Therefore, it is reasonable that mathematics vocabulary, the basic material that consists mathematics discipline language, is necessary for students to grasp the math knowledge. Correspondingly, students experiencing mathematics-learning difficulties would manifest deficit in mathematics vocabulary. Practically, language is embedded in mathematics formal schooling. Without sufficient mathematics vocabulary knowledge, students would allocate their limited working memory resources to hold the vocabulary in their mind while learning the new math knowledge, which would interfere with their learning process.

With respect to math fluency, the capacity to retrieve simple arithmetic facts from long-term memory, cannot serve to distinguish CD and PD. There are two possible explanations. Although the screening measure for CD is WRAT-computation, a measure highly associated with arithmetic facts retrieval (Fuchs et al., 2006), only seven items out of a total of forty items are simple arithmetic facts. Therefore, students demonstrated computational difficulties might not suffer from arithmetic facts retrieval deficits, but struggle with other more complicated computational problems. Second, the Chinese instruction characteristic, which highlights the practice of procedural skills, might be another reason. After three years of formal schooling, Chinese students have gained enough exposure to simple arithmetic facts, which enables the direct retrieval from long-term memory. Therefore, this revealed difference in cognitive deficit may be a result of Chinese mathematics instruction characteristics. That is, for Chinese CD, the deficit in arithmetic facts retrieval might be remediated due to practice.

Taken together, Chinese students with computational difficulties are associated with weaknesses in mathematics vocabulary rather than math facts retrieval. It is reasonable to infer that they are struggling with more complicated computation problems rather than simple facts retrieval. Mathematics vocabulary difficulties might be a reason for their weaknesses in more complicated computation problems, which suggest that drill and practice might not be able to solve their weaknesses in computation. This finding is critical for Chinese mathematics instruction, which overestimates procedural practice in classroom instruction.

Domain-general factors

As for working memory, the multivariate results of the present study corroborate its role in both CD and CPD, but not in PD. This is surprising. One possible explanation may lie in the domain of working memory. Usually, working memory is considered as a domain-general cognitive factor (Baddeley, 2002), but learning research shows that working memory may also show domain-specificity (Peng & Fuchs, 2015; Peng, Namkung, Barnes, &Wang, 2016). For example, previous work has suggested that types of MD may affect the specificity of working memory deficits profiles of MD. That is, computation skill closely correlates with numerical working memory while word-problem-solving is more associated with verbal working memory (e.g., Raghubar, Barnes, & Hecht, 2010; Peng et al., 2016). Since the current study used digit span backward, whereby participants are required to manipulate numerical information. CD is more likely to demonstrate severe numerical working memory deficits, whereas PD may show more severe verbal working memory deficits. Another explanation may be the assessment of word problems. In our study, students had a written copy of the word problems rather than those studies (e.g. Swanson & Beebe-Frankenberger, 2004) in which word problems were read aloud to participants. The oral presentation format of word problems may increase the working memory load required for the test itself. Students have to maintain the oral information in their memory while comprehending the problems. Finally, according to Pasolunghi, Cornoldi, and De Liberto (1999), working memory is related to word-problem-solving processes by the inhibition of irrelevant information. Unlike Fuchs et al. (2008) that used complex word problems, irrelevant information is not included in our word problems, which tap working memory to a less extent to suppress irrelevant information in word problems. Therefore, it is reasonable that current findings did not reveal the working memory deficit for PD.

In terms of processing speed, our data do not support the suggestion that difficulties in processing speed is a cognitive deficit of CD (Fuchs et al., 2008; Fuchs et al., 2006). Previous investigations (Stevenson, 1992; Stevenson, Chuansheng, & Lee, 1993; Stevenson & Stigler, 1992) suggested that international differences in students' mathematical performance might be associated with the amount of exposure to mathematics instruction, rather than real differences in capacities. That is, rather than having a different deficit profile, Chinese CD's deficit in processing speed may be covered up by their math instruction character highlighting practice. Specifically, in early school age, students spend a lot of time and working memory resources with slow counting and inefficient counting strategies (Geary et al., 1991), therefore, processing speed is critical for CD, since enables the development of direct retrieval by reducing the time young children spend on pairing problems with its answer. However, the repetitive practice can also enhance the connection between the problem and its answer in memory, which enables the direct retrieval of math facts. However, as for complex arithmetic problems, the simple practice of procedures may become less efficient since students can no longer rely on those practices to memorize a set of facts than reach the correct answer. In contrast to the role of processing speed and math fluency, which can be remediated through repetitive practice, the role of working memory cannot be remediated by practice; rather, it becomes even more important for more complex computation task. Because the capacity to choose appropriate operations, manipulate numerals and operators, as well as keep temporal answers in mind is essential for solving complex computation problems.

As for IQ, the present study found that compared to TD, students with a single deficit in either computation or word-problem-solving were not associated with weaknesses in reasoning. Only CPD were associated with lower IQ compared to TD, CD, and PD, considering that the current study selected students with normal IQ. It is possible since the mastering of mathematics skills, both foundational and complex skills requires reasoning (Fuchs et al., 2006; Fuchs & Fuchs, 2002; Resnick & Resnick, 1992). As for foundational skills, students learn to master numerical symbols and the rules in calculation (Fuchs et al., 2006). As for more complex mathematics skills, solving complex word problems relies heavily on students' reasoning skills. According to Cooper and Sweller (1987), students need to grasp problem-solution strategies, categorize problems into problem-types or schemas, and generalize taught problems to novel problem situations to solve WPs. The processes of categorizing a WP as a specific problem type, or schema, and adopting appropriate solution-strategy makes strong demands on reasoning ability. Taken together, there is a strong relation between IQ and mathematics learning. Since CPD have demonstrated more pervasive mathematics difficulties in both computation and word-problem-solving, it is possible they were associated with lower IQ compared to other students.

Limitations and Implications

We noted several limitations when interpreting our findings. The first is we used one-time measures to determine difficulty-status for grouping our students. According to prior longitudinal studies (Geary, 1990; Geary, Brown, & Samaranayake, 1991), the cognitive deficit profile of students with low mathematics achievement across consecutive grades was found to differ from children with low mathematics achievement scores in one grade. Future studies that use consecutive computation and problem-solving scores to identify CD and PD is needed to evaluate whether CD and PD have similar cognitive deficit profile as revealed by the current study. Another limitation was the fact that we did not classify mathematics vocabulary into subgroups. Considering mathematics vocabulary is a complex construct, which can potentially be categorized into different categories depending on different standards. Since the nature of mathematics vocabulary is still unclear, and the correlation between different mathematics domain and mathematics vocabulary also requires investigation. The current study only used the factor score of mathematics vocabulary across three categories (e.g., numerical and operation; geometry, and measurement and data), which are naturally developed from the curriculum standards' classification on mathematical knowledge. Taken together, the results of the current study ought to be treated as preliminary evidence for future studies. Future studies on the further classification of mathematics vocabulary are needed and whether different mathematics difficulties are experiencing different mathematics vocabulary deficiency also needs to be explored. A third limitation of the current study is that it cannot reveal causal or consequence of the specific MD (Buttner & Hasselhorn, 2011). Hence, future studies using other methods (e.g., experimental) can reveal whether the potential markers of CD and PD identified by the current study can be manipulated to improve performance. Finally, the small sample size and unequal extreme groups also limited this study, which might influence the generalizability of the current finding. Also, due to the small sample size, the current study used a less stringent cutoff point (e.g., 25th percentile for disability) for MD compared to previous studies (e.g., Fuchs et al., 2008). Murphy et al. (2007) indicated that different cutoff points lead to different degrees of severity in MD, which might influence the cognitive profiles of MD. Thus, future studies based on a larger sample size and a more stringent cutoff points for identifying MD are needed.

Our findings also have implications for practice and future research. First, mathematics vocabulary should gain more attention in mathematics classroom instruction. Our study found that students with mathematics learning difficulties (e.g., CD and PD) are experiencing mathematics vocabulary deficit. A recent study (Powell et al., 2017) investigating the accuracy rate of mathematics vocabulary on 1^{st} graders revealed that the average accuracy rate is below half for vocabulary introduced in 1^{st} grades. Moreover, mathematics vocabulary is becoming even challenging as students entering into later grades. Therefore, future study ought to explore how to enhance the instruction of mathematics vocabulary in classroom settings. As for intervention, current findings provide insights to view the role of mathematics vocabulary differently in mathematics intervention. That is, instead of viewing teaching mathematics vocabulary as a method to help students understand procedural steps. Development an intervention focused on enhancing students' mathematics vocabulary ability may be a more convenient and efficient way of improving students' overall mathematics performance.

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APPENDIX A.
Target Mathematics Vocabulary in Third through Fifth Grade

	Measurement	Geometry	Numerical operations
Third Grade	weight unit; kilogram; 24-hour clock; square kilometer; hectare; encoding; hour; minute; second; length unit; kilometer; meter; decimeter; centimeter; millimeter; circumference; year; quarter; month; day; small month (Chinese saying of months with 30 days or less); big month (Chinese saying of months with 31 days);	orientation; southeast; northwest; northeast; southwest; circumference of rectangle;	estimation; multiple (digit); fraction; numerator; denominator;
Fourth Grade	area unit; milliliter; square meter; square decimeter;	altitude of parallelogram; straight angle; parallel lines; interior angle of a triangle; perpendicular; angle; rectangle; vertical view; intersect; altitude of triangle; isosceles trapezoid; line segment; round angle; axis symmetric figure; acute triangle; equilateral triangle; parallelogram; obtuse angle; right angle; acute angle;	mean; quotient; distributive law of multiplication; order of operation; round; place value; thousandth; associative law of addition; associative law of multiplication; factor; ten-thousands (digit); units (digit); dividend;
Fifth Grade	liter; volume unit;	rotation; folding; face; edge;	hundredths place; algebraic equation; improper fraction; size of a fraction number; proper fraction; repeating decimal; prime number; odd number; common factor; factorization of integer; short division; least common multiple; repetend; composite number; even number;

APPENDIX B.

Examples of Mathematics Vocabulary from the Multiple-choice and Oral Question Tests

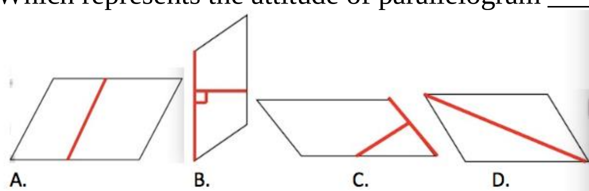
- 10 square kilometers equals to ____ hectares.

A. 1 B. 10 C. 100 D. 1000

- Ranking the following length units ____

A. kilometer>decimeter>meter> millimeter
B. centimeter >meter> decimeter > millimeter
C. meter >decimeter>centimeter>millimeter
D. meter> millimeter >decimeter>centimeter

- Which represents the attitude of parallelogram ____?



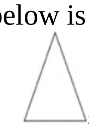
- Two rays with a common endpoint can form a ____

A. vertex B. side C. angle D. line segment

- A quadrilateral with one pair of parallel sides is ____

A. Rectangle B. parallelogram C. square D. trapezoid

- The figure below is ____



A. equilateral triangle B. right-angled triangle
C. isosceles triangle D. obtuse-angled triangle

- The known product in the division is called ____

A. divisor B. factor C. product D. dividend

- Which represent an algebraic equation ____?

A. $X \div 7$ B. $13 + 3X < 25$ C. $X=0$ D. $3X + 1.2$

- Please describe the commutative laws of multiplication.
- Considering an expression with addition, subtraction, multiplication and equation in a parenthesis. What is the order of operation?
- What is rotation?
- Please define repetend.
- What is parallelogram with respect to the shape?