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Mathematical Approaches in Quilt Design

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Mathematics often is thought of as the study of numbers and geometry. But mathematics is so much more. It also includes the study of patterns, enumeration, classification, problem solving, and logical reasoning. More loosely, mathematics is not just a collection of facts, but also an action: a way of doing, or a systematic way of thinking.

Figure 1. Bubb'Illusion II, 46" W by 46" L, 1999. The design of this quilt is based on a pattern from the book 'Op-Art Quilts' by Marilyn Doheny.[1] Image courtesy of GdeV.

In this paper, quilt making will be discussed in the context of the latter view of mathematics. In particular, three quilts made with a structured approach to design will be discussed: Bubb'Illusion II, Wild Flowers, and Cyclic Permutations. Bubb'Illusion II represents early work of the author, and is based on the mathematical concept of an arithmetic sequence (a sequence of numbers in which subsequent terms differ by a constant). Only one fundamental design decision needs to be made, and that is to determine the sequence. Many more design decisions are made in the two remaining quilts, Wild Flowers and Cyclic Permutations. These design decisions are deconstructed, and highlighted.

[1] Marilyn Doheny, *Op-Art Quilts: Fun, Fast, and Fabulous 3-D Illusions* (Edmonds, WA: Doheny Publications, 1996).

Bubb'Illusion II (fig. 1) is made in the 'Op-Art' style, an abstract style of art popularized by Victor Vasarely.[2] The diamond in the center of the quilt is based on an irregular grid, in which the widths of the rows and columns are determined by a sequence of numbers. The sequence of numbers has a regular structure, containing both descending and ascending subsequences, rather than a random structure. Here, the sequence of numbers is

$$\{6, 5, 4, 3, 2, 1, 2, 3, 4, 5, 6, 7, 8, 9, 8, 7, 6, 5, 4, 3, 2, 1, 2\}.$$

The construction process of the quilt is algorithmic, in the sense that there is a logical order in which to cut strips of fabric and to sew strips of fabric together.

Although most traditional quilts are based on regular grids, it is not uncommon to see the use of irregular grids in contemporary quilts. Often the irregular grid is based on arithmetic sequences (sequences in which subsequent terms differ by a constant), such as in the quilt described above, but there are many variations. One popular variation is to use the Fibonacci sequence,

$$\{0, 1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots\},$$

in which terms are the sum of the two preceding terms.

Figure 2. Wild Flowers, 47.5" W by 55.5" L, 1997.
Original design © by GdeV. Image courtesy of GdeV.

Two mathematical questions, which underlie the design for *Wild Flowers* (fig. 2), are worth highlighting. First note that the quilt is made up of twenty square units, each containing a large

[2] Magdalena Holzhey, *Vasarely* (Koln: Taschen GmbH, 2005).

‘flower’, with a square center and angular petals (the black background and the white on-point squares can be ignored for the purposes of the following discussion). Further, all the flowers are made with five different prints (striped, yellow-with-black-dots, etc.).

The first question deals with the number of ways to make pairs of different prints, one to be used for the center of a flower, and the other for the petals of the same flower. If all five fabrics are available for the center, and all four remaining fabrics are available for the petals, then there are precisely twenty (five times four) different ways in total to make uniquely colored flowers.

The second question deals with the arrangement of the twenty flowers. In particular, is it possible to arrange the flowers so that no row and no column sees a repetition in the prints used for the center, nor a repetition in the prints used for the petals? The arrangement shown is one possible solution satisfying the conditions set in the question. At first glance, the arrangement of the units may appear random, but there are in fact a number of patterns that can be observed in the final arrangement. For example, the order of the petal prints is the same in each column (in each column, the yellow-with-black-dots print appears under the striped print, etc.). A similar statement can be made about the order of the center prints in each column. Further, the center and petal prints used in the top flower in the first column are the reverse of those used in the top flower in the third column. This pattern repeats down the columns, and also in the second and fourth columns. By studying the quilt further, many other patterns can be observed.

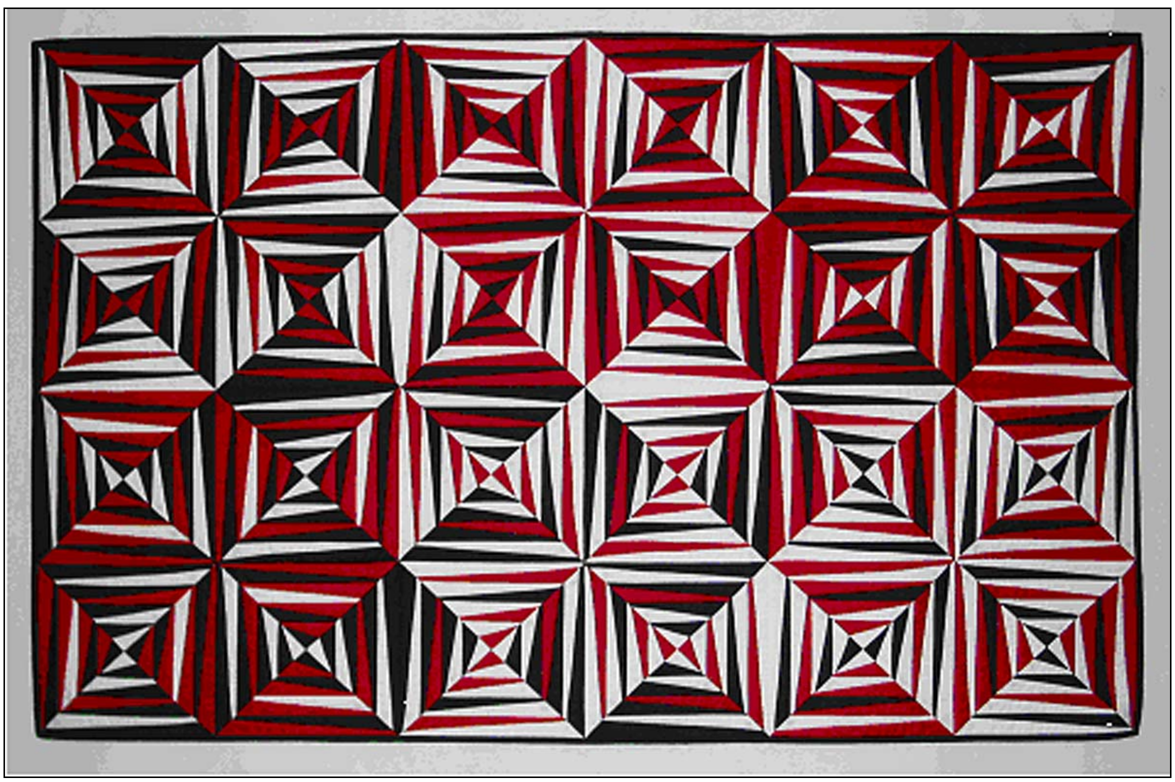


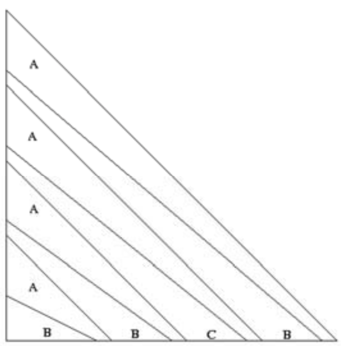
Figure 3. Cyclic Permutations, 46.5" W by 31"L, 2005.
Original design © by GdeV. Image courtesy of GdeV.

Deconstruction of the quilt *Wild Flowers* (fig. 2) illustrates the use of self-imposed rules – rules that are not necessarily obvious to the viewer, but which impose a structure on the design that often is perceived subconsciously. *Wild Flowers* is a forerunner to *Cyclic Permutations* (fig.

3) where similar sets of rules are taken to the extreme. Fundamental to the design in both quilts is the mathematical question that asks in how many ways the rules can be followed.

Cyclic Permutations (fig. 3) is a three-color quilt, with the title of the piece referring to the way in which the basic units (right-angled isosceles triangles) are colored. The apparent complexity of the quilt is the result of an asymmetric division of the triangles, the exhaustion of all possible ways of coloring the triangles according to a set of self-imposed rules, and an exploration of all the ways in which the triangles can be combined into squares according to a second set of self-imposed rules. Together, the rules are sufficiently rigid to provide structure, yet sufficiently loose to allow a large variety of squares. These squares are then combined according to a third set of self-imposed rules to yield a quilt rich with perceptual ambiguity, intended to challenge the viewer's visual sensation.

The basic unit in *Cyclic Permutations* is a right-angled isosceles triangle, divided into eight thin strips, as shown in figure 4. Each triangle unit contains all three colors, namely red, white, and black, according to a predetermined set of rules, as follows. Each strip is labeled with an A, B, or C, also as shown in figure 4. All strips labeled with an A are given one of the three colors. All strips labeled with a B are given one of the two remaining colors. Finally, the last strip, labeled C, is given the remaining color.



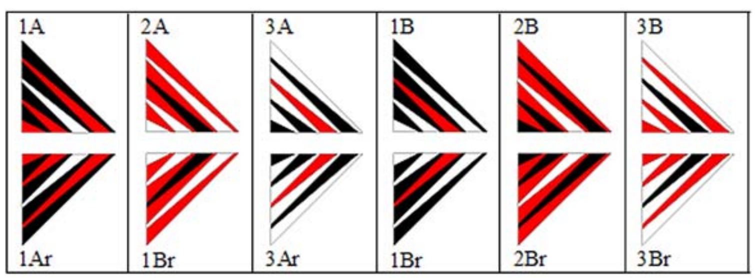


Figure 4 (left). Diagram of the basic unit in the quilt Cyclic Permutations (fig. 3). The unit consists of a right-angled isosceles triangle, divided into eight thin strips. The four strips labeled A are one color, the three strips labeled B are a second color, and the remaining strip is a third color.

Figure 5 (right). Using three colors, there are precisely six ways to color the triangle unit shown in fig. 4, as shown in the top row. After reflection of the original triangle units about their bottom edge, another six triangles are obtained, as shown in the bottom row.

For example, if A corresponds to black, then B can correspond to either red or white. If B corresponds to red, then C must correspond to white. This choice of coloring results in the finished triangle unit as shown at the top left in figure 5 (referred to as triangle 1A). Since there are three color choices for the strips labeled A, two color choices for the strips labeled B, and one color choice for the strip labeled C, there are in total six (three times two times one) ways to color the triangle unit. All six ways are shown in the top row of figure 5. Note that the division of the triangle unit into strips is asymmetric. That is, another six triangles can be created by reflection, for example about the bottom edge (reflection about the left edge gives the same result). The six reflected triangles are shown in the bottom row of figure 5.

The twelve colored triangle units can be combined in a myriad of ways, ranging from random placement to placement according to a set of rules. In the quilt Cyclic Permutations, four triangle units are fit together to form squares, as shown in figure 6. Triangles diagonally opposite of each other are identical, that is, each square consists of two pairs of identical triangles. One pair is chosen from the six original triangles (top row, fig. 5), and the other pair is chosen from the six reflected triangles (bottom row, fig. 5). Combinations are such that corresponding strips in neighboring triangles have different colors. With these restrictions, each triangle in the top row of figure 5 can be combined with two triangles from the bottom row in figure 5, resulting in a total of twelve possible ways to combine the triangles into squares. For example, triangle 1A can be combined with triangles 1Br and 3Ar, but not with triangles 1Ar and 1Br (since strips A are black in both the original and reflected triangles), nor with triangle 2Br (strips C are white in both triangles), nor with triangle 3Br (strips B are red in both triangles). The combination of triangles 1A and 1Br results in the second square from the left in the middle row in figure 6, while the combination of triangles 1A and 3Ar results in the leftmost square in the top row.

The squares in figure 6 have been grouped into six pairs so that the two squares within each pair are mirror images of each other. Last but not least, rules are determined for combining squares. Again, this can be done randomly, or according to a set of rules. In the quilt Cyclic Permutations, there are six sets of four identical squares. Each set corresponds to one of the squares from the two middle columns in figure 6. The four identical squares form a larger square, and orientation of the small squares within the large square is such that touching edges do not have the same color.

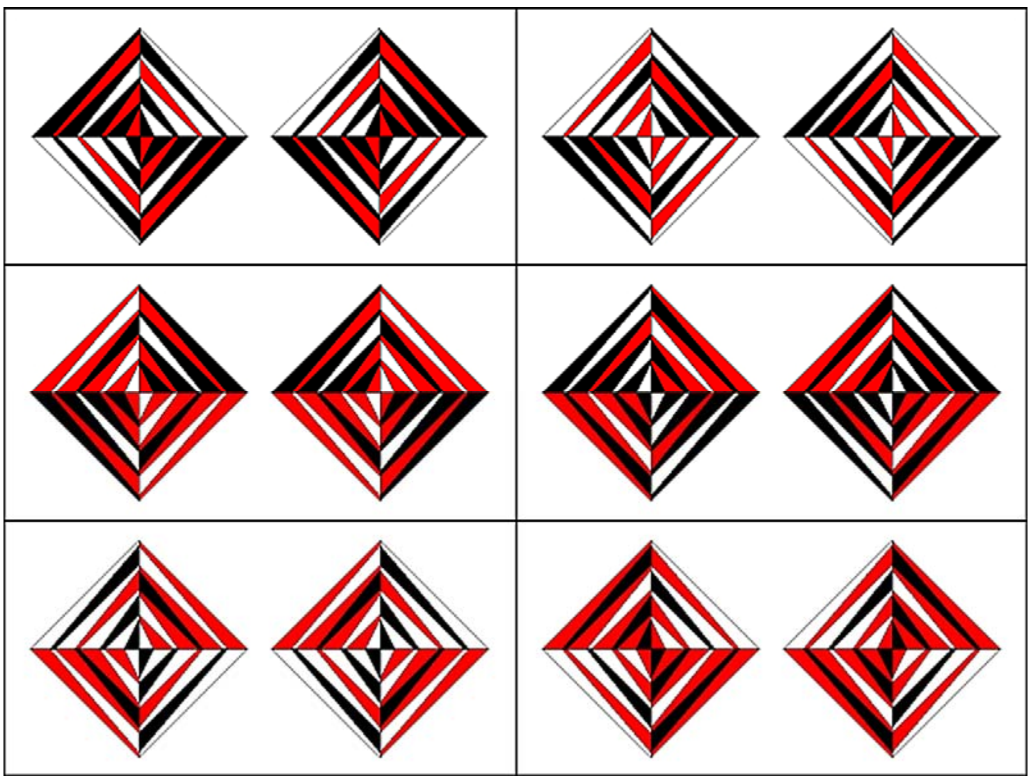


Figure 6. Using the restrictions as described for the quilt Cyclic Permutations (fig. 3), there are twelve ways to combine triangle units into squares. The twelve combinations can be grouped into six pairs. The two squares in each pair are mirror images of each other.

The rules described above are self-imposed, and can be changed. Different rules lead to different designs, for example those shown in figure 7. Each of the six designs in figure 7 is made up of eight identical squares (the rules for combining triangles into these squares are left to the reader to decode). The orientation of the four squares in both the top and bottom halves of each design is such that touching edges have the same color, but the color of the touching edges in the top half differs from the color of the touching edges in the bottom half. In a sense, the top and bottom halves of each design are inverses of each other. Further, comparing the designs in the top and bottom rows of figure 7, the roles of original triangles and reflected triangles are reversed, with serendipitous effects. In the top row of figure 7, the secondary on-point square shapes that are formed in the middle of both the top and bottom halves of the designs appear blurry (it may help to squint while looking at the figure), while in the bottom row, the secondary on-point square shapes appear sharp.

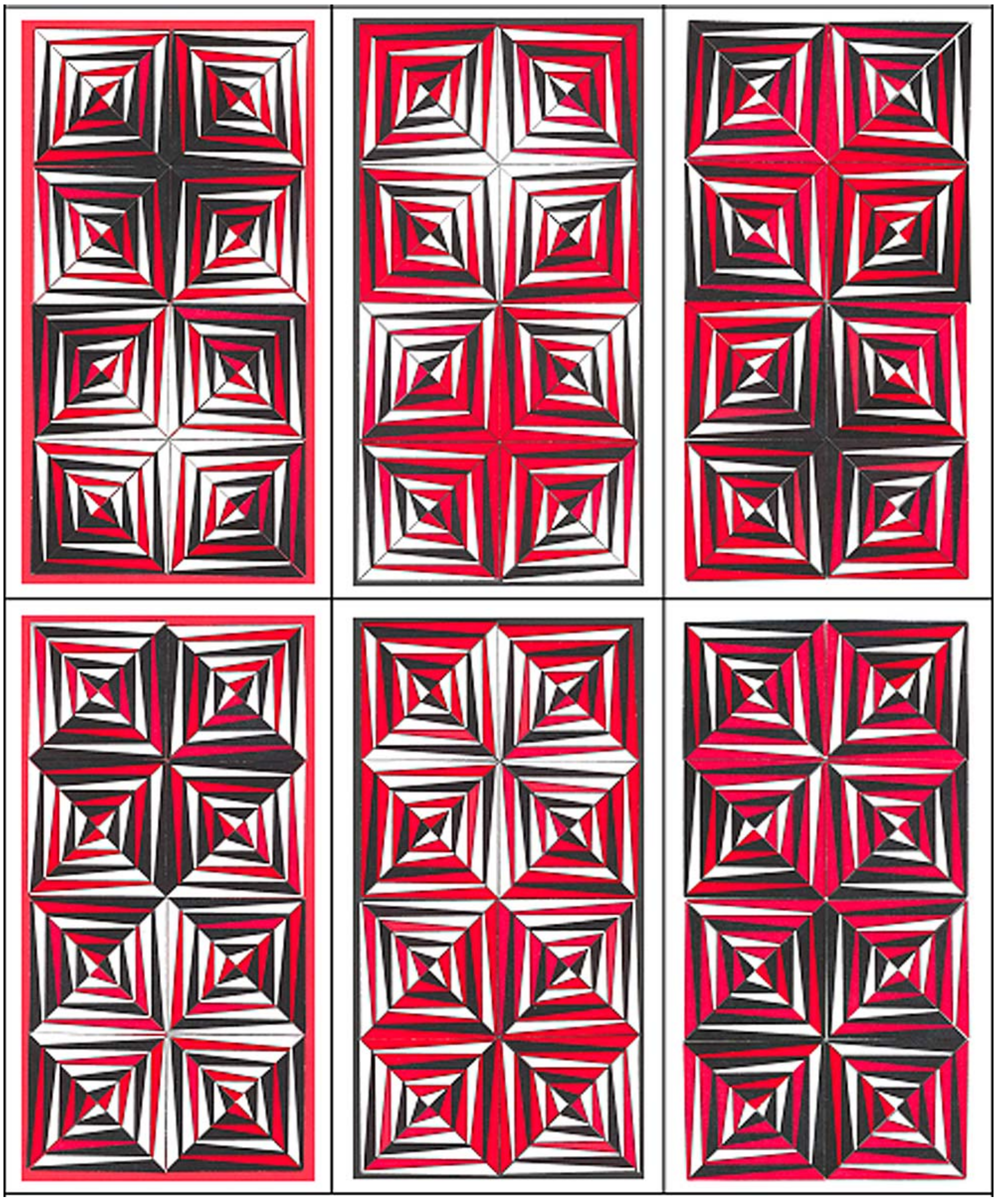


Figure 7: Mockup of new designs with the triangle units from Cyclic Permutations. Original designs © by GdeV.

Concluding Statement

The quilts discussed here reflect a modular, systematic approach to design. In a sense, this design process involves the creation of puzzles. These puzzles originate from setting rules; solutions to the puzzles originate from exhausting and enumerating all the possibilities in satisfying the rules. Rules are set so that there is more than one solution to each puzzle (rules are sufficiently loose to allow variety), but not too many (rules are sufficiently rigid to provide structure).

Starting off with a set of rules does not necessarily mean that all the rules always need to be followed. In fact, breaking a rule occasionally may lead to other attractive designs. In the context of the quilts shown, an interesting question that can be asked is how often a particular rule may be broken before the viewer's mind stops perceiving structure. Similarly, how many rules can be broken before structure no longer can be perceived?

Recognizing the mathematics in questions encountered during the design of textiles renders the possibilities for textiles infinite, that is, textiles plus math is infinity.