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# Thyristor-Controlled Switch Capacitor Placement in Large-Scale Power Systems via Mixed Integer Linear Programming and Taylor Series Expansion

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**Abstract— Allocation of flexible alternating current transmission system (FACTS) devices to an electric power transmission network may be formulated as a nonlinear mathematical program. Solving such a nonlinear program for a large transmission network is computationally very expensive, and obtaining the optimal solution may be impossible. We present a Taylor series expansion approximation of the nonlinearities of the problem and propose a mixed integer linear program (MILP) for finding the optimum location and proper settings of a Thyristor-Controlled Series Capacitor (TCSC) in an electric power network. The objective of this problem is to minimize total generation cost based on the DC load flow model. The proposed method is implemented for the 118-bus IEEE test case and the results are discussed.**

***Index Terms*-- Flexible AC Transmission System, DC optimal power flow, mixed integer linear programming, Taylor series expansion, generation cost.**

**Nomenclature**

| | |
|---|---|
| $Diag(.)$ | Diagonal matrix of vector (.) with nondiagonal members equal to zero. |
| $\underline{c}^T$ | Transposed vector of generator marginal cost |
| $\underline{P}_g$ | Vector of bus generator output |
| $\underline{P}_g^{\max}$, $\underline{P}_g^{\min}$ | Vector of max and min capacity of generators |
| $\Pi$ | Set of all generators |
| $\underline{P}_d$ | Vector of real power load |
| $[B]$ | Susceptance matrix of the system |
| $\underline{\theta}$ | Vector of voltage angles |
| $\underline{\theta}^{\min}$, $\underline{\theta}^{\max}$ | Vector of min and max voltage angles |
| $[A]$ | Network node incidence matrix |
| $[D]$ | Diagonal matrix of line susceptances ( $Diag(\underline{D})$ ) |
| $\underline{P}_l$ | Vector of real power flow conveyed by transmission lines |
| $\underline{P}_l^{\max}$ | Vector of max capacity of transmission lines |
| $\Omega$ | Set of all transmission lines |
| $\underline{\Delta}$ | Vector of compensation level for transmission lines |
| $\underline{\Delta}^{\max}$ | Vector of max compensation level |
| $[\Delta]$ | Diagonal matrix of line compensation level ( $Diag(\underline{\Delta})$ ) |
| $\xi$ | Min acceptable level of compensation |
| $\eta$ | Number of installed FACTS devices in the system |
| $n_l$ | Total number of transmission lines |
| $y_k$ | Binary variable for transmission element (k=1: FACTS device installed on the line k; k=0: line k is not selected for a FACTS installation.) |
| $\underline{C}_R$ | Congestion rent factor |
| $TC_R$ | Total congestion rent for the system |
| $\underline{LMP}$ | Vector of Locational Marginal Prices |
| $\Delta_{ii}\ \forall i \in \Omega$ | Elements on the main diagonal of matrix $[\Delta]$ |

## I. INTRODUCTION

Power system restructuring and the availability of renewable energy reinforces the insufficient capabilities of transmission networks [1]. Open access by the market to the power pool, availability of bilateral contracts for power delivery, and dispersed locations of renewable energy have impacted the power flow on the grid and have created flow bottlenecks. Any flow bottlenecks result in increased consumer costs.

One way to remedy the flow congestion is to expand the network in the congested corridors. Expanding a network by building a new transmission line requires a long lead time, and it may take as long as a decade to clear regulatory requirements for building a new transmission line. In addition, building a new line is very costly and will result in a substantial increase in consumer costs [2]. Another option is to install power flow control devices. Although installing power flow control devices requires capital investment and installation costs, their total cost is much less than the cost of building a new transmission line. In addition, the timeframe for completing a power flow control project is much less than that of a network expansion project [3].

The capacity constraints of transmission lines change the optimal dispatch point of the generating units. This change worsens the optimal solution and increases the overall generation cost of the system [4]. Power flow control devices, known as flexible alternating current transmission systems (FACTS), improve the capability of existing transmission systems. Besides, FACTS devices also have key roles in improving technical aspects of power systems, which is discussed in [5].

There are two types of FACTS devices. These two types are installed either in a series or in a shunt in the system [5]. Series FACTS devices are used to hedge against the transmission congestion and to improve the capability of an existing network. A Thyristor-Controlled Switch Capacitor (TCSC) is a series type of FACTS. In this paper, we are interested in determining the optimal location of a TCSC in a network while minimizing the total generation cost of the system.

The solution provided by a FACTS allocation model should identify the optimal placement and the optimal setting of the device. The optimal setting identifies the level of compensation needed for the line [1]. Two approaches have been used for FACTS allocation in the literature. The first approach uses metaheuristic algorithms, e.g., genetic algorithms or particle swarm optimization, with the objective of optimizing either the total generation cost or the system loadability [6-9]. The heuristic methods do not always provide the optimal solution [10]. The second approach uses optimization techniques such as mixed integer linear programming (MILP) [10], Locational Marginal Price (LMP) analysis [11], or sensitivity factors analysis [12-13].

In [14], allocating TCSC based on MILP is discussed. However, to deal with the nonlinear characteristic of the problem, the authors simplified nonlinear equality constraints to inequalities. In [15], in order to linearize the allocation problem, the author assumes that the voltage angles of two adjacent buses would not change before and after TCSC installation. In this paper, we formulate the allocation problem as an MILP; and the first order Taylor series expansion is employed to linearize the problem. To verify the capability of this model, we apply it to the 118-bus IEEE test case.

The rest of this paper is organized as follows. Section II describes the DC Optimal Power Flow (DCOPF) model, and a modified DCOPF with the presence of TCSC is presented. Section III explains the steps needed to carry out the linearization of the problem. Section IV discusses the test case and the results of the DCOPF model and modified DCOPF model for allocating TCSC. Finally, Section V presents some concluding remarks.

## II. MODIFIED DCOPF FORMULATION WITH PRESENCE OF TCSC

The basic DCOPF formulation is represented as follows:

$$Min \quad \underline{c}^T \underline{P}_g$$

$$Subject\ to: \quad \underline{P}_g - \underline{P}_d = [B]\underline{\theta} \quad where: [B] = [A][D][A]^T \quad (1)$$

$$\underline{P}_l = [D]\,[A]\underline{\theta} \quad (2)$$

$$-\underline{P}_l^{max} \le \underline{P}_l \le \underline{P}_l^{max} \quad ,\forall l \in \Omega \quad (3)$$

$$\underline{P}_g^{min} \le \underline{P}_g \le \underline{P}_g^{max} \quad ,\forall g \in \Pi \quad (4)$$

$$\underline{\theta}^{min} \le \underline{\theta} \le \underline{\theta}^{max} \quad (5)$$

Constraints (1) and (2) induce power balance at each node and Kirchhoff's law, respectively. Constraints (3) and (4) enforce physical operating limits on the power flow through each line and a generation limit for each unit. Constraint (5) represents voltage angle limits at each bus.

TCSC can be modeled by adding a compensation amount [Δ] to the original [D] matrix [14]:

$$[D]^{new} = [D]^{basic} + [\Delta] \quad (6)$$

Here, $\Delta_{ii}\ \forall i \in \Omega$ denotes a desired change for the selected line *i*. A selected line is identified by the binary variable $y_k$. This variable depicts whether the line is selected for compensation ($y_k = 1$) or not ($y_k = 0$). As stated before, if line k is selected for compensation during optimization, its respective element in the vector $\underline{\Delta}$ ($\Delta_k$) is greater than zero; and hence $y_k$ should be equal to 1. For zero elements in vector $\underline{\Delta}$ , which means the line is not selected to compensate during optimization, the assigned $y_k$ is zero. By considering these modifications, the basic DCOPF formulation is modified as follows:

$$Min \quad \underline{C}^T \underline{P}_g$$

$$Subject\ to: \ \underline{P}_g - \underline{P}_d = [A]([D]+[\Delta])[A]^T \underline{\theta}$$

$$= [A][D][A]^T \underline{\theta} + [A][\Delta][A]^T \underline{\theta} \quad (7)$$

$$\underline{P}_l = ([D]+[\Delta])[A]^T \underline{\theta}$$

$$= [D][A]^T \underline{\theta} + [\Delta][A]^T \underline{\theta} \quad (8)$$

$$-\underline{P}_l^{max} \le \underline{P}_l \le \underline{P}_l^{max} \quad ,\forall l \in \Omega \quad (9)$$

$$-\underline{P}_g^{min} \le \underline{P}_g \le \underline{P}_g^{min} \quad ,\forall g \in \Pi \quad (10)$$

$$\underline{\theta}^{min} \le \underline{\theta} \le \underline{\theta}^{max} \quad (11)$$

$$0 \le \underline{\Delta} \le \underline{\Delta}^{max} \quad (12)$$

$$\underline{\Delta} + (1-\underline{Y})M - \underline{\xi} \ge 0 \quad (13)$$

$$\underline{\Delta} \le M\underline{Y} \quad (14)$$

$$\sum_{k=1}^{n_l} y_k = \eta \quad (15)$$

Here, $\underline{\xi}$ is a vector showing the minimum acceptable compensation level in the system. M is a large number greater than or equal to $\Delta^{max} - \xi$. Constraints (13) and (14) ensure that if $\Delta_{ii} = 0\ (\forall i \in \Omega)$, then $y_i = 0$; and if $\Delta_i > 0\ (\forall i \in \Omega)$, then $y_i = 1$. $\underline{\Delta}$ and $\underline{\theta}$ are both calculated by solving the optimization problem. Hence, the second order term $[\Delta]\underline{\theta}$ in equality constraints makes the feasible solution of this optimization problem nonconvex.

## III. LINEARIZING CONSTRAINTS

In order to solve the modified DCOPF model as an MILP program, it is essential that the nonlinear constraints be linearized. Here, we use first order Taylor series expansion to linearize nonlinear constraints (7) and (8). Higher order components of the Taylor series cannot be used since those components are nonlinear. In general, the first order Taylor series approximation for a function $f$ with $n$ variables near specified vector $x_0$ is denoted by (16):

$$\underline{f}(\underline{x}) = \underline{f}(\underline{x}_0) + \mathrm{J}\,\underline{f}(\underline{x}_0)(\underline{x} - \underline{x}_0) \qquad (16)$$

In (16), $\mathrm{J}\,\underline{f}(\underline{x}_0)$ represents the Jacobin matrix of vector $\underline{f}$. Therefore, using the notion of (16) to linearize constraints (7) and (8) results in (17) and (18):

$$\underline{P}_g - \underline{P}_d - [A][D][A]^T\underline{\theta}^0 - [A]\left[\Delta^0\right][A]^T\underline{\theta}^0 - \left[[A][D][A]^T + [A]\left[\Delta^0\right][A]^T \quad [A]\times Diag\left([A]\underline{\theta}^0\right)\right]\begin{bmatrix}\underline{\theta}-\underline{\theta}^0\\ \underline{\Delta}-\underline{\Delta}^0\end{bmatrix} = 0 \qquad (17)$$

$$\underline{P}_l - [D][A]^T\underline{\theta}^0 - \left[\Delta^0\right][A]^T\underline{\theta}^0 - \left[[D][A]^T + \left[\Delta^0\right][A]^T \quad Diag\left([A]\underline{\theta}^0\right)\right]\begin{bmatrix}\underline{\theta}-\underline{\theta}^0\\ \underline{\Delta}-\underline{\Delta}^0\end{bmatrix} = 0 \qquad (18)$$

In (17), and (18), $\underline{\theta}^0$ is the vector of bus angles obtained from solving the DCOPF model without any TCSC device; and $\underline{\Delta}^0 = 0$ is the amount of compensation before any FACTS devices are added. Substituting $\underline{\Delta}^0 = 0$ into Equations (17) and (18) results in:

$$\underline{P}_g - \left[[A][D][A]^T \quad [A]\times Diag\left([A]\underline{\theta}^0\right)\right]\begin{bmatrix}\underline{\theta}\\ \underline{\Delta}\end{bmatrix} = \underline{P}_d \qquad (19)$$

$$\underline{P}_l - \left[[D][A]^T \quad Diag\left([A]\underline{\theta}^0\right)\right]\begin{bmatrix}\underline{\theta}\\ \underline{\Delta}\end{bmatrix} = 0 \qquad (20)$$

Solving the proposed modified DCOPF with new constraints identifies the optimal placement of FACTS devices.

The following five steps summarize the calculation process:

**Step 1:** Run the base case DCOPF, and obtain $\underline{\theta}^0$ at all buses.

**Step 2:** Run the linearized modified DCOPF with $\underline{\theta}^0$ obtained from Step 1, and $\underline{\Delta}^0 = 0$ to identify the vector $\underline{\Delta}^{max}$. The results of this step identify the optimal placement of the TCSC in the network.

**Step 3:** Run a DCOPF model with $[D]^{new} = [D]^{old} + [\Delta]$, and store total generation cost.

**Step 4:** Repeat Step 2 with $\underline{\Delta}^{max} = \underline{\Delta}^{max} + (n\%\,change)$ until $\underline{\Delta}^{max}$ reaches it maximum limit.

**Step 5:** Select the least cost solution.

The value of $n\%$ for each iteration of Step 4 is set at 5%, and the maximum level of compensation allowed is generally 70% of the reactance of the line [11].

## IV. RESULTS OF PROPOSED METHOD FOR 118-BUS TEST CASE

The IEEE 118-bus test case was used to demonstrate the capability of the proposed approach. Data was downloaded from the University of Washington Power System Test Case Archive [17]. Generator variable costs and transmission line data were taken from [18]. The proposed TCSC allocation problem was written in MATLAB and was implemented on a 2.66-GHz personal computer using CPLEX version 12.5 [16].

The test case is comprised of 118 buses, 186 transmission lines, 19 committed generators, 99 load buses, 4519 MW load, and 5859 MW generation capacity. The minimum operating capacity of each generator is set to zero. Generation marginal cost varies between \$0.19/MWh for the generator at Bus 69, to \$10/MWh for the generator at Bus 92. DCOPF carried out for the base case results in the total generation cost of \$2054/hour. Lines 134 between Buses 82 and 77, and Line 154 between Buses 92 and 89 are congested in the base case DCOPF results.

In [19], congestion rent factor is defined as the LMP difference multiplied by the power flows through the line, divided by the total congestion cost. The matrix notation for the vector of congestion rent is shown in (21):

$$\underline{C}_R = \frac{1}{TC_R}\left([A]^T\underline{LMP}\right) \qquad Where: TC_R = \sum_{i=1}^{n_l} C_{R_i},\ \forall i \in \Omega \qquad (21)$$

This factor is a surrogate for the level of congestion. In the base case, lines 134 and 154 have the most congestion rent, respectively. Table I shows the first 10 lines with the highest congestion rent factor. The first two lines are congested in the base case. Line 134 has the most LMP difference between the two ends as well. In [11-13], it is shown that the congestion rent factor and LMP difference could be utilized as a sensitivity factor to select the most appropriate lines for compensation. However, the following results illustrate that the congestion rent factor does not always identify the best line to be compensated.

| TABLE I. LINES WITH HIGH CONGESTION RENT FACTOR | |
|---|---|
| Line number | Congestion rent factor(percent) |
| 134 | 32.02555 |
| 154 | 30.29284 |
| 156 | 11.85044 |
| 137 | 7.945686 |
| 138 | 4.324966 |
| 166 | 3.625088 |
| 152 | 2.388287 |
| 140 | 2.010762 |
| 155 | 1.921765 |
| 132 | 1.808252 |

Fig. 1 shows the optimum value of the modified DCOPF model when compensating the ten highest congestion rent factor lines one at a time for different permissible compensation levels.

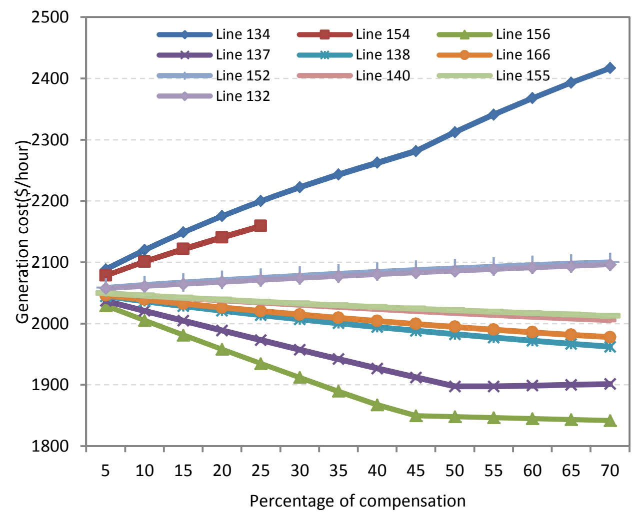


Figure 1. *Generation cost versus compensation level for lines with higher congestion rent.*

For Line 154, compensating more than 25% does not have any impact on the cost. As Fig. 1 shows, installing TCSC on lines with higher congestion rent factors does not always result in reduced cost as is proclaimed in [11]. For lines 134, 154, 152, and 132, increasing the compensation level results in a higher generation cost. Moreover, the merit order of lines is not in accordance with the congestion rent factor. For instance, installing TCSC on Line 156 has the most influence on generation cost. However, line 156 is in third place in Table I. The nonlinear aspect of this problem is obvious in the figure.

Fig. 2 depicts the optimal generation cost for placing one to nine TCSCs for different values of the compensation limit for the IEEE-118 bus test case. This figure shows that: a) the nonlinearity of the optimally compensating lines enhances as the number of lines allowed to be compensated increases, and b) the generation cost is a convex function of the level of compensation provided by TCSCs. Furthermore, Fig. 2 demonstrates that increasing the limit on the number of lines to be compensated does not necessarily lead to finding the optimum generation cost. This implies that a combination of the number of installed TCSCs and the permissible level of their compensation needs to be considered in order to find the minimum generation cost.

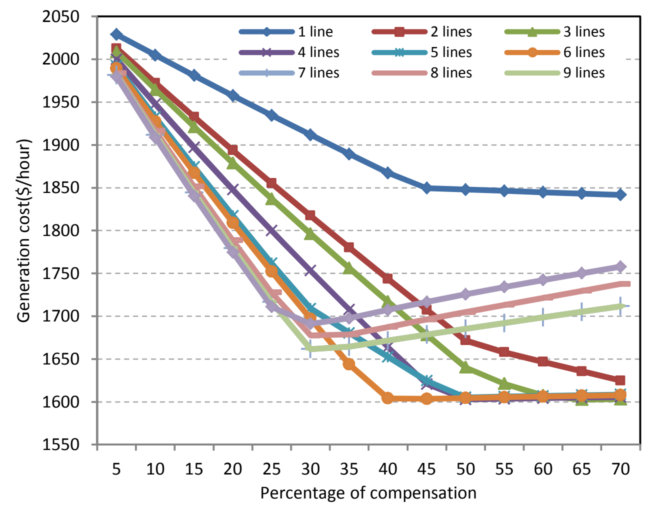


Figure 2. *Generation cost versus compensation level for selected lines.*

Table II shows the cost reduction from the base case when the limit of the number of compensated lines varies between one and ten. The results indicate that placement of three TCSCs with a 65% compensation level for lines $L_3 = \{156,137,140\}$ causes the most generation cost reduction in the system, which is 21.96%.

| TABLE II. INFLUENCE OF LINE COMPENSATION IN GENERATION COST | | |
|---|---|---|
| Line numbers | Optimal setting (%) | Cost reduction (%) |
| $L_1$ =156 | 70 | 10.32 |
| $L_2$ =156,137 | 70 | 20.88 |
| $L_3$ =156,137,140 | 65 | 21.96 |
| $L_4$ =156,137,140,138 | 50 | 21.95 |
| $L_5$ =156,137,140,138,166 | 50 | 21.83 |
| $L_6$ =156,137,140,138,166,149 | 55 | 21.91 |
| $L_7$ =156,137,140,138,166,149,145 | 30 | 17.66 |
| $L_8$ =156,137,140,138,166,149,145,167 | 30 | 18.31 |
| $L_9$ =156,137,140,138,166,149,145,167,139 | 30 | 19.08 |
| $L_{10}$ =156,137,140,138,166,149,145,167,139,144 | 30 | 17.64 |

Compensating lines $L_3 = \{156,137,140\}$ by 65% changes the generation of units at buses 25, 61, 87, and 111 compared to the base case. Generators at Busses 25, 61, and 111 increase their output. On the other hand, a generator at Bus 87 decreases its output. A generator at Bus 69 has the lowest marginal cost, and it is fully committed in both base cases and when compensating three lines. The most expensive generator placed at Bus 92 is not dispatched in both base cases and when three lines are compensated. The second most expensive generator is the generator at Bus 87 in which its output decreases when compensating lines $L_3 = \{156,137,140\}$. Moreover, flows of 178 or 96% of transmission lines change by installing TCSC on $L_3 = \{156,137,140\}$. Line 150, between Buses 87 and 86, takes the most change from 70.73MW to almost zero. This line is the only line that connects the most expensive committed generator at Bus 87 to the rest of the network. So as expected, installing TCSCs in the system in order to reduce cost shifts the generation from expensive units toward cheaper ones.

As is indicated in Fig. 3, both increasing the number of TCSCs and changing their settings have nonlinear effects on the generation cost. In this figure, the X axis represents the number of TCSCs or number of compensated lines for $\eta = 1$ to $\eta = 10$; the Y axis represents the compensation level from 5% to the maximum of 70%; and, finally, the Z axis represents the percentage cost reduction compared to the cost of generation for the base case.

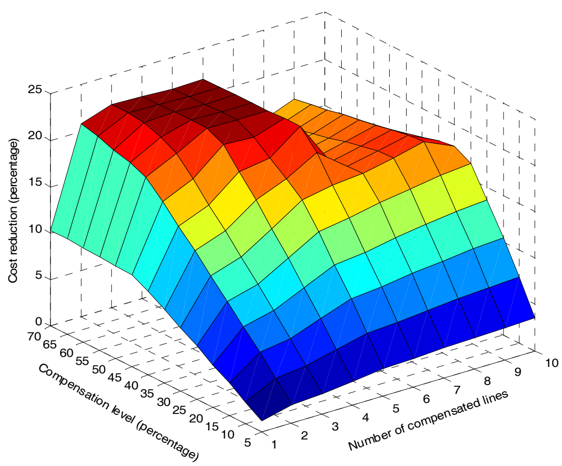


Figure 3. Generation cost versus number of compensated lines and level of compensation

## V. Conclusion

In this paper, we investigated an approach to allocating TCSC based on MILP and Taylor series expansion. To demonstrate the efficacy of the procedure, we apply this model to the IEEE 118-bus test case system. Comparing the results of this study to former works show that compensating those lines with higher congestion rent does not always lead to the best results. However, because of the nonlinear nature of this allocation problem, it is essential to approximate nonlinear constraint by first order Taylor series. To hedge this drawback, we solve the problem iteratively and search through the solution space. Results of this study verify that by independently increasing the number of TCSCs or increasing the compensation level, the optimal solution may not be obtained and not result in a better answer.

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