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Uncertainty analysis of an irrigation scheduling model for water management in crop production

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ABSTRACT

Irrigation scheduling tools are critical to allow producers to effectively manage water resources for crop production. To be useful, these tools need to be accurate, complete, and relatively reliable. The current work presents an uncertainty analysis and its results for the Mississippi Irrigation Scheduling Tool (MIST) model, showing the margin of error (uncertainty) of the resulting irrigation advice arising solely from the propagation of measurement uncertainty through the MIST calculations. The final relative uncertainty in the water balance value from MIST was shown to be around 9% of that value, which is in the normal range of the margin of error and acceptable for agronomic systems. The results of this research also indicate that accurate measurements of irrigation and rainfall are critical to minimizing errors when using MIST and similar scheduling tools. While developed with data from Mississippi, the results of this uncertainty analysis are relevant to similar tool development efforts across the southern and southeastern United States and other high-rainfall areas, especially for locations lacking high-quality co-located weather stations.

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1. Introduction

Irrigation scheduling is a method of applying water for irrigation of crops based on calculated crop water needs. It improves water management while maximizing crop yields. Modeling and simulation of irrigation requirements to ensure effective water management has been employed in many regions, and a number of irrigation schedulers have been developed (Cancela et al., 2006; Dağdelen et al., 2006; Fortes et al., 2005; Grassini et al., 2011; Popova and Pereira, 2008). The Mississippi Irrigation Scheduling Tool (MIST) was designed for the needs of producers in the Mississippi River Valley Alluvial Flood Plain, a region colloquially known as the Delta (Sassenrath et al., 2013a). Continued and expanding

reliance on ground water for irrigation by crop producers has begun to deplete the alluvial aquifer in the Delta, imperiling future availability of groundwater resources (Powers, 2007). To provide accurate irrigation scheduling for this area, MIST uses daily weather data to calculate the evapotranspiration using standard equations (Allen et al., 2006), and determines daily soil water balance using a checkbook method (Andales et al., 2011).

As with all models, there are differences between in-field reality and model results. Simplifying assumptions useful in models for one region and a specific crop are frequently not appropriate in other regions or for different crops. Therefore, it is necessary to adjust any model to regional climate and crops, and to examine the accuracy of model predictions. Several researchers have evaluated and measured uncertainty in other irrigation scheduling systems (Burt et al., 1997 and Molden et al., 1990), and Chaubey et al. (1999) examined the uncertainty due to regional rainfall. Allen et al. (2011) researched common uncertainty errors arising from measurements of evapotranspiration, and Snyder et al. (2015) proposed improvements on estimates of evapotranspiration to account for microclimates. Pereira et al. (2015) also investigated and updated formulations of crop coefficients and estimates of

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evapotranspiration to improve accuracy. Popova et al. (2006) validated their irrigation modeling system for crops and conditions in Bulgaria. Prats and Picó (2010) performed a similar type of uncertainty analysis of the irrigation scheduling model, using a Monte Carlo method where the uncertainties of the various parameters were considered. Monte Carlo type analysis is useful for analyzing the statistical inference between parameters, but it is computationally expensive due to the convergence test, which requires significant sampling from random distribution and calculation of the equation. Therefore, this method is difficult to use for decision making tools such as an irrigation scheduling tool from a practical standpoint. On the other hand, Taylor series method, the mathematical technique that we use in this manuscript, includes analytical derivations so that the solutions can be obtained through computationally inexpensive calculations.

In this study, we focused on determining the uncertainty of MIST predictions by calculating the propagated uncertainties of input data through the underlying model, one aspect of overall validation of the MIST model. All observational data have measurement and observational uncertainties, and complex sequences of calculations can in some cases result in very large uncertainties in the final number (prediction). Previous research examined potential inaccuracies in the weather database used in the water balance calculations and irrigation decision (Sassenrath et al., 2012), and the spatial variability of rainfall patterns (Sassenrath et al., 2013b). Uncertainty analysis quantifies the degree of error arising from uncertainties in input data (typically measurement uncertainties) during the model calculations. The standards for determination of uncertainty analysis are based in quality assessment methodologies and guidelines developed and revised over time by consortiums of researchers and engineers (e.g., BIPM, 2008; AIAA Standard, 1995). Coleman and Steele (2009) further refined the uncertainty methodology, delineating uncertainties into those that are caused by variability (random) and those that are not (systematic), and their approach is the basis of this analysis.

Herein we examine the uncertainty in all equations and other parameters used by MIST in the calculation of the water balance. We compare the calculated values with trends, and then evaluate the uncertainty associated with all the parameters in the water balance modeling. This gives us an indication of the sources of errors in the measured parameters used in the daily water balance calculations and the contributions of the error sources to the total uncertainty of the daily water balance. This information will be used in subsequent studies to validate the model against soil moisture measurements. The following sections describe the uncertainty analysis methodology (Section 2), the results and discussion (Section 3) deduced from the uncertainty analysis of the MIST web-based application, and conclusions (Section 4) of the current research.

2. Methodology

2.1. Crop growth and data collection

Three crops (corn, *Zea mays*, cotton, *Gossypium hirsutum*, and soybean, *Glycine max*) were grown with common production and irrigation practices, and critical data was recorded and quality assured for use in the uncertainty calculations. Crops were grown at the USDA-ARS Mechanization Farm near Stoneville, MS from 2005 to 2012 using standard agronomic practices for several different planting dates. Plant measurements included emergence date, growth stage, leaf area index and yield. Plant growth was assessed as plant height and plant growth stage based on published stages of development; leaf area index was measured with a LAI Plant Canopy Analyzer (LiCor, Lincoln, NE). Alternatively, canopy

development was measured as percent of incoming sunlight intercepted by the crop canopy using a light bar (LiCor, Lincoln, NE). Yield from small plots was measured at harvest by weight, and on large plots or production farms by using yield monitors on commercial scale harvesting equipment. Soil nutrient and textural composition were analyzed at the Mississippi State University soil testing lab. Soil water content was measured near the rooting zone throughout the growing season using Watermark Soil Moisture Sensors (Irrometer, Inc., Irvine, CA) placed at 15 cm increments to a depth of 1 m. The Watermark sensors measure soil water tension as resistance changes in a solid state electrical resistance sensing device embedded in a granular matrix. Additional measurements were made in production fields in 2010, 2011 and 2012 in collaboration with cooperating producers.

Weather parameters were downloaded from the Mississippi Delta Weather Center network of weather stations as previously described (Sassenrath et al., 2012). Measured weather parameters were tested for accuracy and used to calculate daily reference evapotranspiration rates according to the modified Penman–Monteith method (Allen et al., 2006) in an Excel spreadsheet (Microsoft, Inc.). Crop coefficients were developed from measured crop growth parameters (plant height, leaf area, and percent light interception) as described in Sassenrath et al. (2013a) and Allen et al. (2006). The MIST daily soil water balance was determined for each research and production field using a water balance method (Allen et al., 2006; Andales et al., 2011). All measured, calculated, and constant input parameters for the soil water balance calculations are given in Table 1.

2.2. Uncertainty methodology

Uncertainties in a measured variable can arise from a variety of sources such as an imperfect instrument calibration process, incorrect standards used for calibration, or influence on the measured variable due to variations in ambient temperature, pressure, humidity and vibrations. Uncertainties can also result from unsteadiness in an assumed “steady-state” process being measured, and undesirable interactions between the transducers and environment (Coleman and Steele, 2009). The uncertainties that arise due to variability or randomness of a measured quantity (such as water balance on a given day) are referred to as random standard uncertainty. Uncertainties that do not arise from random variability are called systematic standard uncertainty. The systematic uncertainty can include calibration (bias), data acquisition, data reduction, or conceptual errors.

The systematic standard uncertainty can be calculated either through Taylor’s Series Method (TSM) or Monte-Carlo Method (MCM). With TSM, the uncertainty U_x can be calculated through a root sum of random uncertainty s_x and systematic uncertainty as specified by Coleman and Steele (2009):

$$U_x = \tau \sqrt{s_x^2 + b_x^2} \quad (1)$$

where τ is the normalized deviation from the mean value for a standard Gaussian distribution.

$$P(\tau) = \frac{1}{\sqrt{2\pi}} \int_{-\tau}^{\tau} e^{-\tau^2/2} d\tau \quad (2)$$

For example, for $P(\tau)=0.95$ or 95% of the confidence, τ is approximately 2 and for $P(\tau) \approx 0.68$ or 68% of the confidence, τ is approximately 1. Here, we use $\tau=2$ for 95% confidence so that the true value of w_t , for any given day in the calculations, is expected to lie within the bounds of 95% of the time. Similar to Eq. (1),

the uncertainty in the result is given by the following equation (Coleman and Steele, 2009):

$$U_r^2 = \left(\frac{\partial r}{\partial X_1} \right)^2 U_{X_1}^2 + \left(\frac{\partial r}{\partial X_2} \right)^2 U_{X_2}^2 + \dots + \left(\frac{\partial r}{\partial X_j} \right)^2 U_{X_j}^2 \quad (3)$$

where r is an experimental result, X_i are measured variables, $U_{X_i}^2$ are the uncertainties in the measured variables X_i and j is the number of model inputs or the measurements of the variables (temperature, humidity, etc.). This equation assumes that the measured values of X_i are independent of one another and the uncertainties in the measured variables are also independent.

By dividing each term in the equation by r^2 , the following equation is obtained from Eq. (3).

$$\frac{U_r^2}{r^2} = \left(\frac{X_1}{r} \frac{\partial r}{\partial X_1} \right)^2 \left(\frac{U_{X_1}}{X_1} \right)^2 + \left(\frac{X_2}{r} \frac{\partial r}{\partial X_2} \right)^2 \left(\frac{U_{X_2}}{X_2} \right)^2 + \dots + \left(\frac{X_j}{r} \frac{\partial r}{\partial X_j} \right)^2 \left(\frac{U_{X_j}}{X_j} \right)^2$$

which can be rearranged to give:

$$\frac{U_r}{r} = \left(\left(\underbrace{\frac{X_1}{r} \frac{\partial r}{\partial X_1}}_{UMF_1} \right)^2 \left(\frac{U_{X_1}}{X_1} \right)^2 + \left(\underbrace{\frac{X_2}{r} \frac{\partial r}{\partial X_2}}_{UMF_2} \right)^2 \left(\frac{U_{X_2}}{X_2} \right)^2 + \dots + \left(\underbrace{\frac{X_j}{r} \frac{\partial r}{\partial X_j}}_{UMF_j} \right)^2 \left(\frac{U_{X_j}}{X_j} \right)^2 \right)^{1/2}, \quad (4)$$

where U_r/r is the relative uncertainty, and the factors U_{X_i}/X_i are the relative uncertainties for each variable (Coleman and Steele, 2009). The factors which multiply the relative uncertainties of

the variables are uncertainty magnification factors (UMF), and are defined as following:

$$UMF_i = \frac{X_i}{r} \frac{\partial r}{\partial X_i}, \quad 1 \leq i \leq J \quad (5)$$

The relative uncertainty is decreased when UMF is less than 1, and the relative uncertainty is increased if the value of the UMF is greater than 1. It should be noted that the UMF are absolute values.

2.2.1. Random uncertainty

The standard deviation of data gives an estimate of the extent of the spread of the random uncertainty bands with a 95% confidence level. The assessment of the random uncertainty requires substantial experimentation on multiple spatial locations, and as such is not part of this analysis.

2.2.2. Systematic uncertainty

For this calculation, we used an estimated value of 3% of error in each measurement as shown in Table 1, which is the standard bias of instrumentation for measuring weather parameters (Sassenrath et al., 2013b). In addition to measurement errors in weather parameters, the systematic uncertainty in the MIST modeling procedure could include errors from measurements of soil water, irrigation application rates or plant growth.

2.3. Water balance calculation

2.3.1. Evapotranspiration

We performed the uncertainty analysis on the water balance equation for each of the measured input parameters for the calculation (Table 1). The MIST equation for the water balance is based on Allen et al. (2006), in which the net water balance of the system is calculated from the previous water balance plus any water added to the system less water removed from the system:

$$w = w(t-1) - \underbrace{ET_o \cdot K_c}_{\text{water loss}} + \underbrace{P_{eff} + I}_{\text{water gain}} \quad (6)$$

Table 1
System/random uncertainty derived from climatological sensor error.

Physical quantity (unit)	Sensor/model number	Data range (min, max, mean)	Accuracy (underline is chosen for the maximum possible error)	Systematic uncertainty (b_r)	Random uncertainty (s_r)
Wind speed (mile/day)	3-Cup anemometer/Wind Sentry Set 03002	(12, 202, 59.9)	±1% typical	0.01	0.03
Maximum temperature (°F)	Thermostat/Vaisala HMP45C	(33, 106, 77.3)	±0.4 °C (−20 °C), ±0.72 °F (−4 °F) ±0.3 °C (0 °C), ±0.54 °F (32 °F) ±0.2 °C (20 °C), ±0.36 °F (68 °F) ±0.3 °C (40 °C), ±0.54 °F (104 °F)	(0.54/ T_r , 0.3/ T_c , 0.3/ T_k) ^a	0.03
Minimum temperature (°F)	Thermostat/Vaisala HMP45C	(23, 83, 55.9)			
Solar radiation (langley/day)	Pyranometer/LI200X	(15, 722, 406.2)	±5% maximum ±3% typical	0.05	0.03
Maximum relative humidity (%)	Relative humidity sensor/Vaisala HMP45C	(71, 100, 93)	At 20 °C: ±2% (0–90% RH); ±3% (90–100% RH) Dependence: ±0.05% RH/°C (40 °C or 0 °C): 20 × 0.05 = ±1% Total: 3 + 1 = ±4%	0.04	0.03
Minimum relative humidity (%)	Relative humidity sensor/Vaisala HMP45C	(14, 97, 46.2)			
Precipitation (in./day)	Tipping bucket rain gauge/TE525 TI, 6" orifice	(0, 2.66, 0.1)	Up to 1 in./h: ±1% 1–2 in./h: ±0, −3% 2–3 in./h: +0, −5%	0.03	0.03

^a Here, we assume that the degree of uncertainty of Fahrenheit and Celsius is proportional as much as 9/5, and Celsius and Kelvin has the same degree of uncertainty due to the same unit scale.

where w is the current day's water balance; $w_{(t-1)}$ is the previous day's water balance; P_{eff} is effective precipitation; and I is irrigation (Allen et al., 2006). Water balance is considered to be zero when the soil profile is full of water; negative values indicate a water deficit from the reference water balance. Because the current day's soil water balance depends on the previous day's water balance, the computations are iterative. The water lost from the system is assumed to occur primarily through evapotranspiration from the crop and soil (a valid assumption during the growing season; Allen et al., 2006), given by $ET_o \cdot K_c$. Here, ET_o is the reference evapotranspiration from a standardized vegetated surface (commonly grass) without shortage of water, and does not depend on crop type, crop development and management practices. K_c is a crop coefficient that varies depending on the crop type and stage of plant growth. In MIST, the reference crop evapotranspiration is calculated using the modified Penman–Monteith (Allen et al., 2006):

$$ET_o = \frac{0.408 \Delta R_n + \gamma(900/(T_{meanC} + 273.16))u_2(e_s - e_a)}{\Delta + \gamma(1 + 0.34u_2)}, \quad (7)$$

where R_n is net radiation at the crop surface [$MJ/m^2/day$], T_{meanC} is the mean of the daily maximum (T_{maxC}) and minimum temperatures (T_{minC}) ($T_{meanC} = (T_{maxC} + T_{minC})/2$) [$^{\circ}C$], u_2 is wind speed at standard 2 m height [m/s], e_s is saturation vapor pressure [kPa], e_a is actual vapor pressure [kPa], Δ is the slope of the vapor pressure curve [$kPa/^{\circ}C$], and γ is the psychrometric constant [$kPa/^{\circ}C$] (0.067 at 38.71 m above sea level for Stoneville, MS). The saturation vapor pressure deficit ($e_s - e_a$) is an accurate indicator of the actual evaporative capacity of the air.

The net radiation R_n , is the difference between incoming radiation from the sun and outgoing radiation emitted by the earth as expressed below (Allen et al., 2006):

$$R_n = R_{ns} - R_{nl} \quad (8)$$

where R_{ns} is the net solar radiation, and R_{nl} is net longwave radiation. The net solar radiation, R_{ns} , is the fraction of the solar radiation that reaches the earth's surface and that is not reflected from the surface (Allen et al., 2006):

$$R_{ns} = (1 - \alpha)R_s \quad (9)$$

where α is the reflection coefficient ($\alpha = 0.23$ for the hypothetical grass reference crop), and R_s is the solar radiation. The net longwave radiation, R_{nl} , is energy loss from the earth, and is calculated as (Allen et al., 2006):

$$R_{nl} = \sigma \left(\frac{T_{maxK}^4 + T_{minK}^4}{2} \right) (0.34 - 0.14\sqrt{e_a}) \left(1.35 \frac{R_s}{R_{so}} - 0.35 \right) \quad (10)$$

where σ is Stefan–Boltzmann constant ($4.903 \times 10^{-9} MJ/K^4 m^2 day$), $T_{maxK/minK}$ is maximum/minimum absolute temperature during the 24-h period, e_a is actual vapor pressure, R_s is solar radiation, and R_{so} is maximum solar radiation in the clear sky at the same location. R_s/R_{so} is relative shortwave radiation, expressing cloudiness of the atmosphere. It should be noted that the ratio cannot exceed one, or $R_s/R_{so} \leq 1.0$. The equation for the maximum solar radiation in the clear sky, R_{so} , at 38.71 m above sea level (an average for the region of study) is calculated by Allen et al. (2006):

$$R_{so} = 0.7492 \frac{24(60)}{\pi} G_{sc} d_r (\omega_s \sin(\varphi) \sin(\delta) + \cos(\varphi) \cos(\delta) \sin(\omega_s)), \quad (11)$$

where G_{sc} is the solar constant ($0.0820 MJ/m^2 min$), d_r is inverse relative distance Earth–Sun $d_r = 1 + 0.033 \cos\left(\frac{2\pi D_c}{365}\right)$, ω_s is sunset hour angle in radian $\omega_s = \arccos(-\tan(\varphi)\tan(\delta))$, φ latitude in radian, δ is solar declination $\delta = 0.409 \sin(2\pi D_c/365 - 1.39)$, and D_c is the number of the day in the year. It should be noted that R_{so} is only based on the location and time, and is independent of climate.

Another factor contributing to the evapotranspiration calculation is the vapor pressure of the atmosphere. The saturated vapor pressure is given by Allen et al. (2006):

$$e^o(T_c) = 0.6108 \cdot e^{(17.27T_c/(T_c+273.3))}, \quad (12)$$

where T_c is temperature in Celsius. Due to the non-linearity of Eq. (11), the mean saturation vapor pressure e_s should be calculated as follows (Allen et al., 2006):

$$e_s = \left(\frac{e^o(T_{maxC}) + e^o(T_{minC})}{2} \right) \quad (13)$$

As we want to know how much water can be vaporized to the air, the actual vapor pressure should be subtracted from the saturation vapor pressure, ($e_s - e_a$). The actual vapor pressure can be derived from relative humidity data so that the mean of the actual vapor pressure, e_a , is (Allen et al., 2006):

$$e_a = \frac{e^o(T_{minC})(RH_{max}/100) + e^o(T_{maxC})(RH_{min}/100)}{2}, \quad (14)$$

where $RH_{max/min}$ is maximum/minimum relative humidity. The slope of the saturation vapor pressure curve, Δ , representing mean temperature and mean saturation vapor pressure is given by (Allen et al., 2006):

$$\Delta = \frac{4098e_s}{(T_{meanC} + 273.3)^2} \quad (15)$$

The last term to consider in the ET_o calculation, wind speed at 2 m, u_2 , is related to the transfer of heat and water vapor from the evaporating surface into the air. To standardize the wind speed measurement to a height of 2 m above the surface we use the equation (Allen et al., 2006):

$$u_2 = \frac{4.87u_z}{\ln(67.8z - 5.42)} \quad (16)$$

where u_z is the measured wind speed at z m above ground surface (m/s), where z is height of the wind speed measurement above ground surface in meter.

2.3.2. Crop coefficients

To determine the crop evapotranspiration rate, a specific crop coefficient is required. The K_c for cotton, soybeans and corn were adjusted from published values based on from in-field measurements of crops during the 2005–2012 growing seasons for each of the planting dates. The crop evapotranspiration under standard conditions, ET_c , is calculated from the reference crop evapotranspiration multiplied by the crop coefficient. Using a crop coefficient (K_c) value, an estimate of the amount of water loss for a certain crop can be computed.

2.3.3. Water inputs – irrigation, rainfall and runoff

Irrigation (I) and effective precipitation (P_{eff}) are water gains in the modeled system, and both can be measured. Sassenrath et al. (2013b) explored the potential variability of rainfall measurements from various sources, noting that daily rainfall measurements can vary widely even across a single crop field.

P_{eff} is the useful amount of daily rainfall reduced by the run-off of water when there is rain in excess of what the soil can retain (Kent, 1973).

$$P_{eff} = \begin{cases} P - Q, & \text{if } P > 0.7 \\ P, & \text{else,} \end{cases} \quad (17)$$

where P is measured precipitation and Q is direct run-off which is defined as follows.

$$Q = \frac{(P - 0.25)^2}{(P - 0.85)} \quad (18)$$

where S is maximum potential retention after run-off begins (Kent, 1973), given by:

$$S = \frac{1000}{CN} - 10, \quad (19)$$

where CN is curve number (limited $0 < CN < 100$). CN depends on the soil type, tillage and condition.

$$CN = \begin{cases} \frac{4.2 \cdot SR}{(10 - 0.058 \cdot SR)}, & \text{if } \sum_{t=t-5}^{t-1} P_t < 1.4 \\ \frac{23 \cdot SR}{(10 - 0.058 \cdot SR)}, & \text{if } \sum_{t=t-5}^{t-1} P_t > 2.0 \\ SR, & \text{else} \end{cases}, \quad (20)$$

Most cropped fields in the Mississippi Delta are planted in straight rows, and the soil has been tilled to a depth greater than 12 in. The soil type can be commonly classified as group “C”, with a moderately high runoff potential due to slow infiltration rate, giving a value of $SR = 85$ Kent, 1973.

Here, $\sum_{t=t-5}^{t-1} P_t$ is the sum of the five previous days' precipitation.

Therefore, CN is adjusted based on the soil condition with respect to the accumulated amount of rain. If the sum of previous rain is less than 1.4 in., the soil is considered “dry” and CN is decreased. If there has been more than 2 in. of rain in the previous 5 days, the soil is considered “wet” and CN is increased. Otherwise “average” conditions are assumed and the default CN is used.

2.4. Uncertainty of the water balance equation

As the water balance equations were cascaded to calculate the daily water balance, the uncertainties were also propagated through the same set of equations, which will include weather measurements such as cumulative wind speed, maximum/minimum air temperature, daily total solar radiation, maximum/minimum relative humidity temperature, and daily total precipitation. These weather measurements potentially include both systematic and random error. Systematic error can be found in the accuracy in each measurement device (Table 1). The total uncertainty of each measurement with 95% confidence level is given as follows.

- Wind speed at height z (u_z)

$$\frac{U_{u_z}}{u_z} = (b_{u_z}^2 + s_{u_z}^2)^{1/2} = 2[0.01^2 + 0.03^2]^{1/2} = 0.0006 \quad (21)$$

- Maximum/minimum temperature in Fahrenheit ($T_{\max F/\min F}$)

$$\begin{aligned} \frac{U_{T_{\max F/\min F}}}{T_{\max F/\min F}} &= (b_{T_{\max F/\min F}}^2 + s_{T_{\max F/\min F}}^2)^{1/2} \\ &= 2 \left[\left(\frac{0.54}{T_{\max F/\min F}} \right)^2 + 0.03^2 \right]^{1/2} \end{aligned} \quad (22)$$

- Solar radiation (R_s)

$$\frac{U_{R_s}}{R_s} = (b_{R_s}^2 + s_{R_s}^2)^{1/2} = 2(0.05^2 + 0.03^2)^{1/2} = 0.1166 \quad (23)$$

- Maximum/minimum relative humidity temperature ($RH_{\max/\min}$)

$$\begin{aligned} \frac{U_{RH_{\max/\min}}}{RH_{\max/\min}} &= (b_{RH_{\max/\min}}^2 + s_{RH_{\max/\min}}^2)^{1/2} \\ &= 2(0.04^2 + 0.03^2)^{1/2} = 0.1 \end{aligned} \quad (24)$$

- Precipitation (P)

$$\frac{U_P}{P} = (b_P^2 + s_P^2)^{1/2} = 2(0.03^2 + 0.03^2)^{1/2} = 0.0848 \quad (25)$$

Because some terms such as temperature share the same form of the uncertainty equation, those equations are compactly represented without loss in understandability, i.e., $T_{\max F/\min F}$. Also, most of the measurements are taken in English units at the weather station, and converted to SI units such as centigrade during the calculation process. Therefore, we need to define the uncertainties of the temperatures converted to SI units.

- Maximum/minimum/mean temperature in centigrade ($T_{\max C/\min C/\text{mean C}}$)

$$\frac{U_{T_{\max C/\min C/\text{mean C}}}}{T_{\max C/\min C/\text{mean C}}} = 2 \left[\left(\frac{0.3}{T_{\max C/\min C/\text{mean C}}} \right)^2 + 0.03^2 \right]^{1/2} \quad (26)$$

- Maximum temperature in Kelvin ($T_{\max K/\min K}$)

$$\frac{U_{T_{\max K/\min K}}}{T_{\max K/\min K}} = 2 \left[\left(\frac{0.3}{T_{\max K/\min K}} \right)^2 + 0.03^2 \right]^{1/2} \quad (27)$$

These uncertainties propagate within the water balance equation in MIST, giving a relative uncertainty of the final water balance Eq. (6) as follows.

$$\begin{aligned} \frac{U_w}{w} &= \left[\underbrace{\left(\frac{w_{(t-1)}}{w} \frac{\partial w}{\partial w_{(t-1)}} \right)}_{UMF_1}^2 \left(\frac{U_{w_{(t-1)}}}{w_{(t-1)}} \right)^2 \right. \\ &\quad + \underbrace{\left(\frac{ET_o}{w} \frac{\partial w}{\partial ET_o} \right)}_{UMF_2}^2 \underbrace{\left(\frac{U_{ET_o}}{ET_o} \right)}_{(29)}^2 + \underbrace{\left(\frac{K_c}{w} \frac{\partial w}{\partial K_c} \right)}_{UMF_3}^2 \left(\frac{U_{K_c}}{K_c} \right)^2 \\ &\quad \left. + \underbrace{\left(\frac{P_{eff}}{w} \frac{\partial w}{\partial P_{eff}} \right)}_{UMF_4}^2 \underbrace{\left(\frac{U_{P_{eff}}}{P_{eff}} \right)}_{(39)}^2 + \underbrace{\left(\frac{I}{w} \frac{\partial w}{\partial I} \right)}_{UMF_5}^2 \left(\frac{U_I}{I} \right)^2 \right]^{1/2}, \end{aligned} \quad (28)$$

where $U_{w_{(t-1)}/w_{(t-1)}}$, U_{ET_o}/ET_o , U_{K_c}/K_c , $U_{P_{eff}}/P_{eff}$, and U_I/I are the relative uncertainty of the previous day's water balance, the reference evapotranspiration, the crop coefficient, the effective precipitation, and the irrigation, respectively. The terms paired with the relative uncertainty terms are UMF terms, and there are 5 UMF terms in the water balance equation (Table 2). UMF_i enclosed in a bracket refers to the UMF index number in Table 2. By plugging the reduced form of the UMFs in the table and the relative uncertainty values in the equation, we can obtain the relative uncertainty of the MIST water balance. However, the relative uncertainty in the equation does not have a simple solution because some of the uncertainties, such as U_{ET_o}/ET_o and $U_{P_{eff}}/P_{eff}$, have sub-equations that are cascaded with other sub-equations. To correctly determine the uncertainty, we need to include all of the equations until it reaches the relative uncertainties of the weather data. In order to refer to the relevant sub-equation, index (i) is used under the bracket. Therefore, the final uncertainty of

Table 2

Uncertainty magnification factor (UMF) for water balance equation.

Target uncertainty	UMF index	Reduced form of UMF term	Corresponding uncertainty
$\frac{U_w}{w}$	1	$\frac{w_{(t-1)}}{w} \frac{\partial w}{\partial w_{(t-1)}} = \frac{w_{(t-1)}}{w}$	$\frac{U_{w_{(t-1)}}}{w_{(t-1)}}$
	2	$\frac{ET_o}{w} \frac{\partial w}{\partial ET_o} = \frac{ET_{oKc}}{w}$	$\frac{U_{ET_o}}{ET_o}$
	3	$\frac{K_c}{w} \frac{\partial w}{\partial K_c} = \frac{K_c ET_o}{w}$	$\frac{U_{K_c}}{K_c}$
	4	$\frac{P_{eff}}{w} \frac{\partial w}{\partial P_{eff}} = \frac{P_{eff}}{w}$	$\frac{U_{P_{eff}}}{P_{eff}}$
	5	$\frac{I}{w} \frac{\partial w}{\partial I} = \frac{I}{w}$	$\frac{U_I}{I}$
$\frac{U_{ET_o}}{ET_o}$	6	$\frac{\Delta}{ET_o} \frac{\partial ET_o}{\partial \Delta} = \Delta \left(\frac{0.408 R_n}{0.408 \Delta R_n + \gamma(900/(T_{meanC} + 273.15))u_2(e_s - e_a)} - \frac{1}{\Delta + \gamma(1 + 0.34u_2)} \right)$	$\frac{U_{\Delta}}{\Delta}$
	7	$\frac{R_n}{ET_o} \frac{\partial ET_o}{\partial R_n} = \frac{0.408 \Delta R_n}{0.408 \Delta R_n + \gamma \frac{900}{T_{meanC} + 273.15} u_2(e_s - e_a)}$	$\frac{U_{R_n}}{R_n}$
	8	$\frac{T_{meanC}}{ET_o} \frac{\partial ET_o}{\partial T_{meanC}} = \frac{-T_{meanC} \gamma u_2(e_s - e_a)(900/(T_{meanC} + 273.15)^2)}{0.408 \Delta R_n + \gamma(900/(T_{meanC} + 273.15))u_2(e_s - e_a)}$	$\frac{U_{T_{meanC}}}{T_{meanC}}$
	9	$\frac{u_2}{ET_o} \frac{\partial ET_o}{\partial u_2} = u_2 \left(\frac{\gamma(900/(T_{meanC} + 273.15))(e_s - e_a)}{0.408 \Delta R_n + \gamma(900/(T_{meanC} + 273.15))u_2(e_s - e_a)} - \frac{0.34\gamma}{\Delta + \gamma(1 + 0.34u_2)} \right)$	$\frac{U_{u_2}}{u_2}$
	10	$\frac{e_s}{ET_o} \frac{\partial ET_o}{\partial e_s} = \frac{(u_2 900 \gamma e_s / (T_{meanC} + 273.15))}{0.408 \Delta R_n + \gamma(900/(T_{meanC} + 273.15))u_2(e_s - e_a)}$	$\frac{U_{e_s}}{e_s}$
	11	$\frac{e_a}{ET_o} \frac{\partial ET_o}{\partial e_a} = \frac{-(u_2 900 \gamma e_a / (T_{meanC} + 273.15))}{0.408 \Delta R_n + \gamma(900/(T_{meanC} + 273.15))u_2(e_s - e_a)}$	$\frac{U_{e_a}}{e_a}$
	12	$\frac{T_{meanC}}{\Delta} \frac{\partial \Delta}{\partial T_{meanC}} = -\frac{T_{meanC}}{T_{meanC} + 237.3}$	$\frac{U_{T_{meanC}}}{T_{meanC}}$
	13	$\frac{e_s}{\Delta} \frac{\partial \Delta}{\partial e_s} = 1$	$\frac{U_{e_s}}{e_s}$
	14	$\frac{e^o(T_{maxC})}{e_s} \frac{\partial e_s}{\partial e^o(T_{maxC})} = \frac{e^o(T_{maxC})}{e^o(T_{maxC}) + e^o(T_{minC})}$	$\frac{U_{e^o(T_{maxC})}}{e^o(T_{maxC})}$
	15	$\frac{e^o(T_{minC})}{e_s} \frac{\partial e_s}{\partial e^o(T_{minC})} = \frac{e^o(T_{minC})}{e^o(T_{maxC}) + e^o(T_{minC})}$	$\frac{U_{e^o(T_{minC})}}{e^o(T_{minC})}$
	16	$\frac{T_{maxC}}{e^o(T_{maxC})} \frac{\partial e^o(T_{maxC})}{\partial T_{maxC}} = \frac{4098.171 T_{maxC}}{(T_{maxC} + 273.3)^2}$	$\frac{U_{T_{maxC}}}{T_{maxC}}$
$\frac{U_{e^o(T_{minC})}}{e^o(T_{minC})}$	17	$\frac{T_{minC}}{e^o(T_{minC})} \frac{\partial e^o(T_{minC})}{\partial T_{minC}} = \frac{4098.171 T_{minC}}{(T_{minC} + 273.3)^2}$	$\frac{U_{T_{minC}}}{T_{minC}}$
$\frac{U_{R_n}}{R_n}$	18	$\frac{R_{ns}}{R_n} \frac{\partial R_n}{\partial R_{ns}} = \frac{R_{ns}}{R_{ns} - R_{nl}}$	$\frac{U_{R_{ns}}}{R_{ns}}$
	19	$\frac{R_{nl}}{R_n} \frac{\partial R_n}{\partial R_{nl}} = -\frac{R_{nl}}{R_{ns} - R_{nl}}$	$\frac{U_{R_{nl}}}{R_{nl}}$
$\frac{U_{R_{ns}}}{R_{ns}}$	20	$\frac{R_s}{R_{ns}} \frac{\partial R_{ns}}{\partial R_s} = 1$	$\frac{U_{R_s}}{R_s}$
$\frac{U_{R_{nl}}}{R_{nl}}$	21	$\frac{T_{maxK}}{R_{nl}} \frac{\partial R_{nl}}{\partial T_{maxK}} = \frac{4T_{maxK}^4}{T_{maxK}^4 + T_{minK}^4}$	$\frac{U_{T_{maxK}}}{T_{maxK}}$
	22	$\frac{T_{minK}}{R_{nl}} \frac{\partial R_{nl}}{\partial T_{minK}} = \frac{4T_{minK}^4}{T_{maxK}^4 + T_{minK}^4}$	$\frac{U_{T_{minK}}}{T_{minK}}$
	23	$\frac{e_a}{R_{nl}} \frac{\partial R_{nl}}{\partial e_a} = \frac{-0.07\sqrt{e_a}}{0.34 - 0.14\sqrt{e_a}}$	$\frac{U_{e_a}}{e_a}$
	24	$\frac{R_s}{R_{nl}} \frac{\partial R_{nl}}{\partial R_s} = \frac{1.35R_s}{1.35R_s - 0.35R_{s0}}$	$\frac{U_{R_s}}{R_s}$
$\frac{U_{e_a}}{e_a}$	25	$\frac{RH_{max}}{e_a} \frac{\partial e_a}{\partial RH_{max}}, \frac{e^o(T_{minC})}{e_a} \frac{\partial e_a}{\partial e^o(T_{minC})} = \frac{e^o(T_{minC})RH_{max}}{e^o(T_{minC})RH_{max} + e^o(T_{maxC})RH_{min}}$	$\frac{U_{RH_{max}}}{RH_{max}}, \frac{U_{e^o(T_{minC})}}{e^o(T_{minC})}$
	26	$\frac{RH_{min}}{e_a} \frac{\partial e_a}{\partial RH_{min}}, \frac{e^o(T_{maxC})}{e_a} \frac{\partial e_a}{\partial e^o(T_{maxC})} = \frac{e^o(T_{maxC})RH_{min}}{e^o(T_{minC})RH_{max} + e^o(T_{maxC})RH_{min}}$	$\frac{U_{RH_{min}}}{RH_{min}}, \frac{U_{e^o(T_{maxC})}}{e^o(T_{maxC})}$
$\frac{U_{u_2}}{u_2}$	27	$\frac{u_z}{u_2} \frac{\partial u_2}{\partial u_z} = 1$	$\frac{U_{u_z}}{u_z}$
$\frac{U_{P_{eff}}}{P_{eff}}$	28	$\frac{P}{P_{eff}} \frac{\partial P_{eff}}{\partial P} = \frac{P}{P - Q}$	$\frac{U_P}{P}$
	29	$\frac{Q}{P_{eff}} \frac{\partial P_{eff}}{\partial Q} = \frac{-Q}{P - Q}$	$\frac{U_Q}{Q}$
$\frac{U_Q}{Q}$	30	$\frac{P}{Q} \frac{\partial Q}{\partial P} = \frac{P(P + 1.8S)}{(P - 0.2S)(P + 0.8S)}$	$\frac{U_P}{P}$

Fig. 1. Diagram of uncertainty analysis for water balance equation.

the water balance equation is obtained through a recursive process. The entire process for the water balance equation is depicted in Fig. 1. The remaining relative uncertainty terms of the equation, $U_{w(t-1)}/w_{(t-1)}$, U_{K_c}/K_c , and U_I/I , do not have sub-equations. $U_{w(t-1)}/w_{(t-1)}$ is simply from the previous day's uncertainty of the water balance and its initial uncertainty is set to 10%, $U_{w(0)}/w_{(0)} = 0.1$. U_{K_c}/K_c is developed from a different model and is not based on the climate measurement in this work, so $U_{K_c}/K_c = 0$. The uncertainty in the amount of water applied in an irrigation application has been studied is not considered in this work, so the uncertainty of irrigation is set to 10%, $U_I/I = 0.1$.

As defined in Eq. (6), w results in a water balance of zero when there is rain or irrigation, while the excessive water is removed from the soil through surface run-off. Zero water balance is convenient to measure the reference water balance, but it may introduce a numerical error when $w \approx 0$ appears in the denominator in each term of Eq. (28). Especially during the rainy season, the whole term causes a divide-by-zero problem so that the total uncertainty cannot have a value or excessively high value. This leads to a difficult and erroneous calculation in the uncertainty analysis. To avoid the divide-by-zero problem, we assign $UMF_{1-5} = 0$, and hence $U_w/w = 0$ if $w = 0$ in the numerical implementation of the uncertainty analysis.

The sub-equation for U_{ET_o}/ET_o of the reference crop evapotranspiration Eq. (7) is calculated similarly to the U_w/w calculation.

$$\begin{aligned} \frac{U_{ET_o}}{ET_o} &= \left[\left(\frac{\Delta}{ET_o} \frac{\partial ET_o}{\partial \Delta} \right)^2 \left(\frac{U_{\Delta}}{\Delta} \right)^2 + \left(\frac{R_n}{ET_o} \frac{\partial ET_o}{\partial R_n} \right)^2 \left(\frac{U_{R_n}}{R_n} \right)^2 \right. \\ &\quad \left. + \left(\frac{\gamma}{ET_o} \frac{\partial ET_o}{\partial \gamma} \right)^2 \left(\frac{U_{\gamma}}{\gamma} \right)^2 + \left(\frac{T_{meanC}}{ET_o} \frac{\partial ET_o}{\partial T_{meanC}} \right)^2 \left(\frac{U_{T_{meanC}}}{T_{meanC}} \right)^2 \right]^{1/2} \end{aligned}$$

$$\begin{aligned} &+ \left(\frac{u_2}{ET_o} \frac{\partial ET_o}{\partial u_2} \right)^2 \left(\frac{U_{u_2}}{u_2} \right)^2 + \left(\frac{e_s}{ET_o} \frac{\partial ET_o}{\partial e_s} \right)^2 \left(\frac{U_{e_s}}{e_s} \right)^2 \\ &+ \left(\frac{e_a}{ET_o} \frac{\partial ET_o}{\partial e_a} \right)^2 \left(\frac{U_{e_a}}{e_a} \right)^2 \right]^{1/2} \end{aligned} \quad (29)$$

where U_{γ}/γ is the relative uncertainty of the psychrometric constant, and $U_{\gamma}/\gamma = 0$ because γ is a constant that includes only geological information (elevation above sea level) that causes almost no uncertainty. Also, we ignore the UMF term paired with U_{γ}/γ . The relative uncertainty of the mean temperature in Celsius, $U_{T_{meanC}}/T_{meanC}$, is already calculated in Eq. (26). Other terms are specified next. First, U_{Δ}/Δ of the slope of the mean saturation vapor pressure curve, Eq. (15), is given as:

$$\begin{aligned} \frac{U_{\Delta}}{\Delta} &= \left[\left(\frac{T_{meanC}}{\Delta} \frac{\partial \Delta}{\partial T_{meanC}} \right)^2 \left(\frac{U_{T_{meanC}}}{T_{meanC}} \right)^2 \right. \\ &\quad \left. + \left(\frac{e_s}{\Delta} \frac{\partial \Delta}{\partial e_s} \right)^2 \left(\frac{U_{e_s}}{e_s} \right)^2 \right]^{1/2} \end{aligned} \quad (30)$$

where $U_{T_{meanC}}/T_{meanC}$ is in Eq. (26), and $\frac{U_{T_{meanC}}}{T_{meanC}}$ is

$$\frac{U_{e_s}}{e_s} = \left[\left(\frac{e^o(T_{maxC})}{e_s} \frac{\partial e_s}{\partial e^o(T_{maxC})} \right)^2 \left(\frac{U_{e^o(T_{maxC})}}{e^o(T_{maxC})} \right)^2 \right]^{1/2}$$

$$+ \left[\left(\underbrace{\frac{e^o(T_{\min C})}{e_s} \frac{\partial e_s}{\partial e^o(T_{\min C})}}_{UMF_{15}} \right)^2 \left(\underbrace{\frac{U_{e^o(T_{\min C})}}{e^o(T_{\min C})}}_{(33)} \right)^2 \right]^{1/2} \quad (31)$$

with

$$\frac{U_{e^o(T_{\max C})}}{e^o(T_{\max C})} = \left[\left(\underbrace{\frac{T_{\max C}}{e^o(T_{\max C})} \frac{\partial e^o(T_{\max C})}{\partial T_{\max C}}}_{UMF_{16}} \right)^2 \left(\underbrace{\frac{U_{T_{\max C}}}{T_{\max C}}}_{(26)} \right)^2 \right]^{1/2} \quad (32)$$

and

$$\frac{U_{e^o(T_{\min C})}}{e^o(T_{\min C})} = \left[\left(\underbrace{\frac{T_{\min C}}{e^o(T_{\min C})} \frac{\partial e^o(T_{\min C})}{\partial T_{\min C}}}_{UMF_{17}} \right)^2 \left(\underbrace{\frac{U_{T_{\min C}}}{T_{\min C}}}_{(26)} \right)^2 \right]^{1/2} \quad (33)$$

Second, U_{R_n}/R_n of net radiation, Eq. (8), is calculated as follows:

$$\frac{U_{R_n}}{R_n} = \left[\left(\underbrace{\frac{R_{ns}}{R_n} \frac{\partial R_n}{\partial R_{ns}}}_{UMF_{18}} \right)^2 \left(\underbrace{\frac{U_{R_{ns}}}{R_{ns}}}_{(35)} \right)^2 + \left(\underbrace{\frac{R_{nl}}{R_n} \frac{\partial R_n}{\partial R_{nl}}}_{UMF_{19}} \right)^2 \left(\underbrace{\frac{U_{R_{nl}}}{R_{nl}}}_{(36)} \right)^2 \right]^{1/2} \quad (34)$$

where

$$\frac{U_{R_{ns}}}{R_{ns}} = \left[\left(\underbrace{\frac{R_s}{R_{ns}} \frac{\partial R_{ns}}{\partial R_s}}_{UMF_{20}} \right)^2 \left(\underbrace{\frac{U_{R_s}}{R_s}}_{(23)} \right)^2 \right]^{1/2} \quad (35)$$

with U_{R_s}/R_s in Eq. (23) and $U_{R_{nl}}/R_{nl}$ of the net long wave radiation Eq. (10) is represented as follows:

$$\begin{aligned} \frac{U_{R_{nl}}}{R_{nl}} = & \left[\left(\underbrace{\frac{T_{\max K}}{R_{nl}} \frac{\partial R_{nl}}{\partial T_{\max K}}}_{UMF_{21}} \right)^2 \left(\underbrace{\frac{U_{T_{\max K}}}{T_{\max K}}}_{(27)} \right)^2 \right. \\ & + \left(\underbrace{\frac{T_{\min K}}{R_{nl}} \frac{\partial R_{nl}}{\partial T_{\min K}}}_{UMF_{22}} \right)^2 \left(\underbrace{\frac{U_{T_{\min K}}}{T_{\min K}}}_{(27)} \right)^2 \\ & + \left(\underbrace{\frac{e_a}{R_{nl}} \frac{\partial R_{nl}}{\partial e_a}}_{UMF_{23}} \right)^2 \left(\underbrace{\frac{U_{e_a}}{e_a}}_{(37)} \right)^2 + \left(\underbrace{\frac{R_s}{R_{nl}} \frac{\partial R_{nl}}{\partial R_s}}_{UMF_{24}} \right)^2 \left(\underbrace{\frac{U_{R_s}}{R_s}}_{(23)} \right)^2 \\ & \left. + \left(\frac{R_{so}}{R_{nl}} \frac{\partial R_{nl}}{\partial R_{so}} \right)^2 \left(\frac{U_{R_{so}}}{R_{so}} \right)^2 \right]^{1/2} \quad (36) \end{aligned}$$

where $U_{T_{\max K}}/T_{\max K}$, $U_{T_{\min K}}/T_{\min K}$, and U_{R_s}/R_s are already calculated in Eqs. (27) and (23), respectively. We assume that $U_{R_{so}}/R_{so} = 0$ because the corresponding equation for the maximum solar radiation under clear-sky conditions, Eq. (11), is based on constants

and not measurements, and hence there is almost zero uncertainty. U_{e_a}/e_a of the mean of the actual vapor pressure Eq. (14) is

$$\begin{aligned} \frac{U_{e_a}}{e_a} = & \left[\left(\underbrace{\frac{RH_{\max}}{e_a} \frac{\partial e_a}{\partial RH_{\max}}}_{UMF_{25}} \right)^2 \left(\underbrace{\frac{U_{RH_{\max}}}{RH_{\max}}}_{(24)} \right)^2 \right. \\ & + \left(\underbrace{\frac{RH_{\min}}{e_a} \frac{\partial e_a}{\partial RH_{\min}}}_{UMF_{26}} \right)^2 \left(\underbrace{\frac{U_{RH_{\min}}}{RH_{\min}}}_{(24)} \right)^2 \\ & + \left(\underbrace{\frac{e^o(T_{\max C})}{e_a} \frac{\partial e_a}{\partial e^o(T_{\max C})}}_{UMF_{26}} \right)^2 \left(\underbrace{\frac{U_{e^o(T_{\max C})}}{e^o(T_{\max C})}}_{(32)} \right)^2 \\ & \left. + \left(\underbrace{\frac{e^o(T_{\min C})}{e_a} \frac{\partial e_a}{\partial e^o(T_{\min C})}}_{UMF_{25}} \right)^2 \left(\underbrace{\frac{U_{e^o(T_{\min C})}}{e^o(T_{\min C})}}_{(33)} \right)^2 \right]^{1/2} \quad (37) \end{aligned}$$

where $U_{RH_{\max}}/RH_{\max}$, $U_{RH_{\min}}/RH_{\min}$, $U_{(e^o(T_{\max C})/e^o(T_{\max C}))}$, and $U_{e^o(T_{\min C})/e^o(T_{\min C})}$ are given in Eqs. (24), (32) and (33), respectively. It should be noted that the reduced form of $(RH_{\max}/e_a)(\partial e_a/\partial RH_{\max})$ and $\{e^o(T_{\min C})/e_a\} \{\partial e_a/\partial e^o(T_{\min C})\}$ are the same. Also, $(RH_{\min}/e_a)(\partial e_a/\partial RH_{\min})$ and $\{e^o(T_{\max C})/e_a\} \{\partial e_a/\partial e^o(T_{\max C})\}$ are the same.

The last term that we need to consider in U_{ET_o}/ET_o is the uncertainty of the wind speed at 2 m height.

$$\frac{U_{u_2}}{u_2} = \left[\left(\underbrace{\frac{u_z}{u_2} \frac{\partial u_2}{\partial u_z}}_{UMF_{27}} \right)^2 \left(\underbrace{\frac{U_{u_z}}{u_z}}_{(21)} \right)^2 + \left(\frac{z}{u_2} \frac{\partial u_2}{\partial z} \right)^2 \left(\frac{U_z}{z} \right)^2 \right]^{1/2} \quad (38)$$

where $U_z/z = 0$ as height z is constant over time and does not cause uncertainties.

We have found all relative uncertainty terms in U_{ET_o}/ET_o . Next, we need to find the terms for the effective precipitation Eq. (17) in U_w/w .

$$\frac{U_{P_{eff}}}{P_{eff}} = \left[\left(\underbrace{\frac{P}{P_{eff}} \frac{\partial P_{eff}}{\partial P}}_{UMF_{28}} \right)^2 \left(\underbrace{\frac{U_P}{P}}_{(25)} \right)^2 + \left(\underbrace{\frac{Q}{P_{eff}} \frac{\partial P_{eff}}{\partial Q}}_{UMF_{29}} \right)^2 \left(\underbrace{\frac{U_Q}{Q}}_{(40)} \right)^2 \right]^{1/2} \quad (39)$$

where U_P/P is the relative uncertainty of precipitation given in Eq. (25), and U_Q/Q is the relative uncertainty of run-off and is given by

$$\frac{U_Q}{Q} = \left[\left(\underbrace{\frac{P}{Q} \frac{\partial Q}{\partial P}}_{UMF_{30}} \right)^2 \left(\underbrace{\frac{U_P}{P}}_{(25)} \right)^2 + \left(\frac{S}{Q} \frac{\partial Q}{\partial S} \right)^2 \left(\frac{U_S}{S} \right)^2 \right]^{1/2} \quad (40)$$

where U_S/S is the uncertainty of maximum potential retention Eq. (19).

$$\frac{U_S}{S} = \left[\left(\frac{CN}{S} \cdot \frac{\partial S}{\partial CN} \right)^2 \left(\frac{U_{CN}}{CN} \right)^2 \right]^{1/2} \quad (41)$$

where U_{CN}/CN is the uncertainty of the curve number given in Eq. (20), and $U_{CN}/CN=0$ because the curve number equation is based on constants. The above equations were then programmed to calculate the uncertainty involved in the MIST estimation of the water balance.

3. Results and discussion

An example of the soil water balance calculation from weather data using an early-planted corn crop in 2012 is shown in Fig. 2. The solid blue line represents the water balance with the scale on the right Y-axis, and the red dashed line represents the effective precipitation where the run-off is subtracted with the scale on the left Y-axis. The solid black bar indicates the one irrigation of 1.6 in. made during the 2012 season. For production agriculture in high rainfall areas such as the Mississippi Delta, heavy spring rains typically eliminate the requirement for pre-planting and early season irrigation applications. Research is ongoing to determine optimal termination dates of irrigation, which will be crop specific. Therefore, we will limit our interest to the time period between planting and harvesting throughout the analysis. For production regions requiring pre-planting irrigation scheduling, modification of the early-season water balance would be required.

The inaccuracies of sensors used for this work are listed in Table 1. As discussed in Section 2.2, UMF values help to define the variables that influence the uncertainty equation and estimate the amount of the influence of each variable. A UMF less than 1 decreases the uncertainty of a variable, while a UMF more than 1 increases the uncertainty of a variable. Fig. 3 presents the UMFs and the associated uncertainties of all of the MIST variables. Fig. 3(a) shows the 30 calculated UMF values for soybean data for a 20 day period from development plotted in a 3-dimensional (3D) plot. Fig. 3(b) shows uncertainties paired with 30 UMFs (as shown in the last column of Table 2). With the exception of the white colored bars in Fig. 3(a), most of the UMF values are less than 2. The exceptions, highlighted in white, include UMF 24 (solar radiation with respect to net longwave solar radiation $(R_s/R_{nl})(\partial R_{nl}/\partial R_s)$), UMF 29 (relative

Fig. 3. 3-Dimensional plot of early-planted corn of (a) UMF and (b) paired uncertainty according to 30 terms in Table 2.

uncertainty of direct run-off $(Q/P_{eff})(\partial P_{eff}/\partial Q)$ and UMF 30 (precipitation with respect to runoff, $(P/Q)(\partial Q/\partial P)$). Even though the values of UMF 24 and 30 are relatively high, the associated uncertainties are rather low as can be seen in the white colored bars in Fig. 3(b), resulting in an overall low uncertainty. On the other hand, UMF 29 is very low even though the paired uncertainty is high; hence it prevents propagating a large error to the next stage of the uncertainty calculation. As we can see in Fig. 3(b), most of the uncertainty values are less than 10% and the uncertainty values are not greatly influenced by the large UMFs (such as UMFs 24, 29, and 30). All uncertainties from the measurement devices and assumptions are propagated through the uncertainty equations up to the final uncertainty equation for the water balance, which is given in Eq. (28). Therefore, UMFs and the paired uncertainties in the water balance uncertainty equation are important, particularly those of $w_{(t-1)}$ (UMF 1), ET_o (UMF 2), P_{eff} (UMF 4) and I (UMF 5).

Fig. 4 shows the temporal changes in the UMFs of the water balance uncertainty equation over the course of the growing season. There are several key points to note here. First, UMFs for $w_{(t-1)}$ remain close to one. This is appropriate, as the day-to-day variations of the water balance and its uncertainty are usually negligible. Here the UMF for water balance is $w_{(t-1)}/w$ and so this ratio does not change much if the water balance from one day to another does not change much. Secondly, the UMF for $w_{(t-1)}$ is affected by UMFs of P_{eff} and I , as seen on days 158 and 163 in subfigure (d) for I . This clearly shows that a change in P_{eff} or I will have a notable change in the UMF and the uncertainty of the calculated water balance. Thus, one can infer

Fig. 2. Plot of water balance calculation results. (For interpretation of the references to color in text near the figure citation, the reader is referred to the web version of this article.)

Fig. 4. $UMF_{1,2,4,5}$ of (a) corn (early), (b) corn (late), (c) cotton, and (d) soybean.

that the uncertainties associated with measuring and calculating P_{eff} and I are high. This leads to large errors in predicting the water balance on days when a rainfall occurs or when the field is irrigated. Lastly, the UMF for ET_0 is minimal (≈ 0) on most days, which implies that the rate of change in the water balance with respect to ET_0 is very small, and thus, the impact of the uncertainty of ET_0 to the uncertainty of water balance is minimal. This will be explored more completely in future research.

When the paired uncertainties (relative) of the four UMFs discussed in Fig. 4 are considered (Fig. 5), they are generally low for

most of the crop season. As anticipated, $U_{w(t-1)/w(t-1)}$ is immediately influenced by UMFs and the uncertainties of the associated variables from the previous day. For instance, in Fig. 5(a) UMFs of P_{eff} day 163 increases the water balance uncertainty of day 165 from 10% to 40%, and in Fig. 5(d) for the days 154, 158, and 163, the UMFs of P_{eff} and I greatly affect the uncertainty of the water balance.

The uncertainties associated with P_{eff} and I are a major concern for the uncertainty quantification of an irrigation scheduling process. Here, the high degree of spatial variability of rainfall means that the rainfall reported by the weather station or by

Fig. 5. Paired uncertainty of $UMF_{1,2,4,5}$ of (a) corn (early), (b) corn (late), (c) cotton, and (d) soybean.

Fig. 6. Water balance and its uncertainty of (a) corn (early), (b) corn (late), (c) cotton, and (d) soybean for one complete growing season.

NEXRAD measurements may not reflect actual rainfall on a farmer's fields (Sassenrath et al., 2013b). This is especially true for spatially diverse rains where one part of the farmer's field may receive heavy rain, while another part may receive no rain whatsoever. The local weather station or the NEXRAD data would report, however, that the farmer's entire field received the same amount of rain. Additionally, farmers may not quantitatively measure the amount of irrigation water applied and hence the amount of the water irrigated is left to subjective speculations. The use of local flow meters to accurately measure the amount of water applied will address this potential error. Indeed, efforts are underway to improve the metering of agricultural wells (Brandon, 2014). Further, errors can arise in the irrigation application system, especially for old systems that have not been properly

maintained or measured (I. McCann, personal communication). Errors in amount of irrigation water applied through sprinkler systems can be off by more than 40% in some cases, due to missing or broken sprinkler heads, and clogged or misaligned nozzles. And finally, environmental conditions (e.g., gusty winds) can lead to errors in irrigation water application from sprinkler systems (O'Shaughnessy et al., 2013). These issues lead to substantial levels of uncertainties in measured amounts of rainfall and irrigation. As such, the current uncertainty analysis clearly points out the need for a better methodology to measure local rainfall and amount of irrigation water applied. The need for improved irrigation application systems to enhance performance of water management has been long recognized as a world-wide problem (Augier et al., 1995).

Fig. 7. Water balance over the course of one growing season with standard deviation error (a) corn (early), (b) corn (late), (c) cotton, and (d) soybean.

Finally, Fig. 6 shows the calculated water balance and the final uncertainty of the relative water balance. The final water balance results showed values usually within acceptable variability ranges of around 15%. The notable exception to this is on the days where the water balance is close to zero. This occurs because of the high confidence level (95%) and the numerical error in dividing-by-zero on the days on which irrigation or precipitation occurred. Those uncertainties are as high as 58%. However, the plotted uncertainty is the relative uncertainty, and the relative uncertainty depends on the actual amount of the water balance to assess the total uncertainty. This becomes clearer if we look at the standard deviation (or total uncertainty) instead of the relative uncertainty and how the uncertainty changes according to the water balance each day. Fig. 7 shows the water balance with the standard deviation of error derived from the present uncertainty analysis. As seen, the error is bounded in a reasonable range as small as 0 in. and as large as about ± 0.5 in. as water balance in the soil declines below 10 in. This gives a more immediately useful range of uncertainty or error when predicting the water balance. These results constitute an assessment of the predictive accuracy of the MIST tool when used for similar crops, soils, and climate.

4. Conclusions

In summary, the cascading equations within MIST, as specified by FAO56 (Allen et al., 2006), were used to assess the total uncertainty (equivalent to one standard deviation) of the water balance calculated by MIST, due to uncertainties in the input observational data. Because the UMF values were primarily based on manufacturers' reports of instrument uncertainty, neglecting possible contributions from other sources of error in observations, this is a "best case" analysis. The following are the primary findings of this research:

- MIST was run and the uncertainty analyses were conducted for three different crops (corn, cotton and soybean) with four different planting seasons and planting dates. The model predictions of daily water balance as implemented in MIST were found to be within acceptable error bounds (± 0.5 in.).
- The analysis illustrated how the UMFs and the associated uncertainties all tied into the calculation of the final water balance uncertainty (Fig. 3). Any sudden and large increase in certain UMFs and their associated uncertainties resulted in sharply higher uncertainty in the water balance calculations.
- Rainfall and irrigation were by far the most significant variables of the MIST irrigation model (Figs. 4 and 5) in impact on the model uncertainty.
- The uncertainty in the predicted water balance due solely to measurement uncertainties was determined to be within acceptable ranges (Fig. 6). Given more accurate local data for rainfall and irrigation, a smaller uncertainty of the MIST-predicted water balance can be assured.

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Appendix A. Appendix

List of Equations

- Eq. (1) Total experimental uncertainty
 Eq. (2) Two-tailed Gaussian probability
 Eq. (3) General uncertainty
 Eq. (4) Relative uncertainty
 Eq. (5) Uncertainty magnification factor (UMF_i)
 Eq. (6) Water balance equation (w)
 Eq. (7) Reference crop evapotranspiration (ET_o)
 Eq. (8) Net radiation (R_n)
 Eq. (9) Net solar radiation (R_{ns})
 Eq. (10) Net longwave radiation (R_{nl})
 Eq. (11) Maximum solar radiation in the clear sky (R_{so})
 Eq. (12) Saturation vapor pressure ($e^o(T_c)$)
 Eq. (13) Mean saturation vapor pressure (e_s)
 Eq. (14) Mean actual vapor pressure (e_a)
 Eq. (15) Slope of mean saturation vapor pressure curve (Δ)
 Eq. (16) Wind speed at 2 m (u_2)
 Eq. (17) Effective precipitation (P_{eff})
 Eq. (18) Direct run-off, Q
 Eq. (19) Maximum potential retention after run-off begins, S
 Eq. (20) Curve number, CN
 Eq. (21) Relative uncertainty of wind speed at height z , $\frac{U_{u_z}}{u_z}$
 Eq. (22) Relative uncertainty of maximum/minimum temperature in Fahrenheit, $\frac{U_{T_{max F}}}{T_{max F}}, \frac{U_{T_{min F}}}{T_{min F}}$
 Eq. (23) Relative uncertainty of solar radiation, $\frac{U_{R_s}}{R_s}$
 Eq. (24) Relative uncertainty of maximum/minimum relative humidity temperature, $\frac{U_{RH_{max}}}{RH_{max}}, \frac{U_{RH_{min}}}{RH_{min}}$
 Eq. (25) Relative uncertainty of precipitation, $\frac{U_p}{p}$
 Eq. (26) Relative uncertainty of maximum/minimum/mean temperature in Celsius, $\frac{U_{T_{max C}}}{T_{max C}}, \frac{U_{T_{min C}}}{T_{min C}}, \frac{U_{T_{mean C}}}{T_{mean C}}$
 Eq. (27) Relative uncertainty of maximum/minimum temperature in Kelvin, $\frac{U_{T_{max K}}}{T_{max K}}, \frac{U_{T_{min K}}}{T_{min K}}$
 Eq. (28) Relative uncertainty of water balance, $\frac{U_w}{w}$
 Eq. (29) Relative uncertainty of reference crop evapotranspiration, $\frac{U_{ET_o}}{ET_o}$
 Eq. (30) Relative uncertainty of slope of mean saturation vapor pressure curve, $\frac{U_{\Delta}}{\Delta}$
 Eq. (31) Relative uncertainty of mean saturation vapor pressure, $\frac{U_{e_s}}{e_s}$
 Eq. (32) Relative uncertainty of saturation vapor pressure of maximum temperature in Celsius, $\frac{U_{e^o(T_{max C})}}{e^o(T_{max C})}$
 Eq. (33) Relative uncertainty of saturation vapor pressure of maximum temperature in Celsius, $\frac{U_{e^o(T_{min C})}}{e^o(T_{min C})}$
 Eq. (34) Relative uncertainty of net radiation, $\frac{U_{R_n}}{R_n}$
 Eq. (35) Relative uncertainty of net solar radiation, $\frac{U_{R_{ns}}}{R_{ns}}$
 Eq. (36) Relative uncertainty of net longwave radiation, $\frac{U_{R_{nl}}}{R_{nl}}$
 Eq. (37) Relative uncertainty of mean actual vapor pressure, $\frac{U_{e_a}}{e_a}$
 Eq. (38) Relative uncertainty of wind speed at 2 m, $\frac{U_{u_2}}{u_2}$
 Eq. (39) Relative uncertainty of effective precipitation, $\frac{U_{P_{eff}}}{P_{eff}}$
 Eq. (40) Relative uncertainty of direct run-off, $\frac{U_Q}{Q}$
 Eq. (41) Relative uncertainty of maximum potential retention, $\frac{U_S}{S}$

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