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# SELECTION WITH RESTRICTION IN CATTLE

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# SELECTION WITH RESTRICTION IN CATTLE[1]

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#### Summary

Application of new and accurate methods of estimating breeding values and systematic selection based on performance characters have increased the complexity of animal breeding. Negative correlations between the most important characters may be involved, especially in the selection of dairy and beef cattle. This problem demands efficient selection models to maximize genetic improvement of some economic traits and simultaneously to restrict other antagonistic traits to a certain genetic change. We have established a general theory of selection indexes with constraints to impose fixed and proportional restrictions when several selection indexes containing different information sources are applied. A detailed description of the iterative computing procedure used to determine the restricted index weights is presented in this report. For demonstrating the procedure, the effects of fixed and proportional restrictions imposed on genetic change in milk, growth, reproduction and carcass traits of cattle were investigated. Findings show that genetic progress in some economic traits is still possible when genetic change of other economic, but antagonistic traits, is positive or set equal to zero.

(Key Words: Selection Index, Restricted Selection Indexes, Multiple Trait Selection, Cattle Breeding.)

#### Introduction

The goals of genetic improvement in cattle breeding are manifold. Selection may be designed to improve milk, growth, reproductive and carcass characters. One reason for the development of breeds with different performances is selection in a desired direction. Another important reason for breed differences may be negative relationships between the main traits. Highly negative correlations between milk yield and carcass value, as well as between reproductive traits and carcass value, are known.

Negative correlations between carcass value and growth characters or type traits and milk also are likely. The efficiency of cattle used in the dairy and beef industries is influenced by these traits. Restricted selection index methods provide a tool for controlling undesired genetic change in the development of breeds and for directing selection response in different characters according to certain rules.

The general method of index selection for multiple traits in animal breeding was developed by Hazel (1943). The idea of imposing restrictions on the genetic change of some characters was first introduced by Kempthorne and Nordskog (1959). Rao (1962), Tallis (1962), Harville and Reeves (1972), Harville (1974, 1975) and Niebel (1979) have contributed to the development of the restricted selection index theory. The present study is based on a general theory of restricted selection indexes (Niebel and Van Vleck, 1982), wherein fixed and proportional restrictions are introduced when more than one selection index is used in a population. Proportional restriction means that the genetic change in certain characters has to be in a given ratio, while fixed restrictions fix genetic change in designated traits to certain values or certain directions.

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A detailed description of the computation procedure for solving the nonlinear restriction problem is given. Then the restricted index methods are demonstrated on a complex situation in the breeding of dual-purpose cattle. Traits on which fixed and proportional restrictions are imposed are milk, growth, carcass value and calving difficulty.

### Computation Procedure

In this section, only a brief description of the conceptual framework of a general model for restricted selection indexes is presented. The detailed description is given by Niebel and Van Vleck (1982). The main reason is the description of the iterative computing procedure used to obtain the restricted index weights.

*Model*

The basic idea for restricted selection indexes is nearly the same as that for unrestricted indexes, where $\mathbf{I}_i = \mathbf{X}_{ij}\mathbf{b}_i$ is a linear function of the index weights, $\mathbf{b}_i$, and the sources of information, $\mathbf{X}_{ij}$, are measured on the animal or its relatives. The components of $\mathbf{b}_i$ are derived such that the total breeding value, $\mathbf{Y}_{O_i} = \mathbf{Y}_{ij}\mathbf{h}_O$, of the average of selected animals is maximized, where $\mathbf{Y}_{O_i}$ is a linear function of the vector of additive genetic values of the economic traits, $\mathbf{Y}_{ij}$, and the corresponding vector of economic values, $\mathbf{h}_O$. In the following discussion, all traits involved in the total breeding value and in restrictions are defined as economic traits. The weights for the unrestricted selection index are obtained by solving $\mathbf{V}_i\mathbf{b}_i = \mathbf{G}_i\mathbf{h}_O$, where $\mathbf{V}_i$ is the covariance matrix of the standardized information sources $\mathbf{X}_{ij}\sigma_{X_{ij}}^{-1}$ of index i, and $\mathbf{G}_i$ is the covariance matrix between the economic traits and the standardized information sources. The $\mathbf{X}_{ij}$ and $\mathbf{Y}_{ij}$ are assumed to follow a multivariate normal distribution, and $\mathbf{V}_i$ is full rank, symmetric and positive definite. $\mathbf{G}_i\mathbf{h}_O$ equals the vector, $\mathbf{r}_{O_i}$, of the righthand sides.

If restrictions are imposed on the genetic change in certain traits and n different selection indexes are used in the population, the maximization criterion must be redefined for the derivation of n sets of index weights, which will optimize selection response for the whole population:

$$\Delta Y_O/yr = \sum_{i=1}^{n} c_i\sigma_{I_i}/\sum_{i=1}^{n} q_i\ell_i. \quad (1)$$

The use of different selection indexes is common in animal breeding, especially with dairy cattle, where the milk characters are sex-related such that a performance test of sires for milk traits is impossible. The symbols $c_i$ in the formula for the selection response are constants related to index group i. Each $c_i$ is the product of the group selection intensity factor, $\delta_i$, and $q_i$ is the proportion of genes that animals of index group i contribute to the next generation. Those products must be divided by the average generation interval, $\sum_{i=1}^{n} q_i\ell_i$, if selection response is to be expressed in time units.

The restrictions can be summarized in vectors $\overline{\mathbf{g}}_f$ and $\mathbf{p}$, where vector $\overline{\mathbf{g}}_f$ contains fixed restrictions imposed for desired changes in economic traits and $\mathbf{p}$ the proportional restrictions. To form restrictions needed for the computations, transformation matrices $\mathbf{H}, \mathbf{H}_O, \mathbf{S}, \mathbf{S}_O$ have to be defined. The first columns, $\mathbf{h}_j$, of matrix $\mathbf{H}$ are used to form linear combinations of the economic traits for fixed restrictions and the last columns for proportional restrictions. The coefficients due to the traits in the columns of $\mathbf{H}$ are ordered in the same sequence as the covariances of the economic traits in the rows of $\mathbf{G}_i$. Economic traits that are involved in fixed restrictions cannot be involved in proportional restrictions, and vice versa. The number of rows in $\mathbf{H}$ has to correspond to the number of traits contained in $\mathbf{G}_O$. Augmenting $\mathbf{H}$ with vector $\mathbf{h}_O$ results in matrix $\mathbf{H}_O = (\mathbf{h}_O\ \mathbf{H})$. Next, a transformation matrix is needed to form proportional restrictions. Matrix $\mathbf{S}$ contains two submatrices wih nonzero elements on the diagonal. The first submatrix is an identity matrix and is related to fixed restrictions imposed on genetic change in certain economic traits. The second submatrix $\mathbf{E}$ corresponds to proportional restrictions, which are summarized in vector $\mathbf{p}$. Proportional restrictions can be imposed on v groups of traits with $s_j$ economic traits in the $j^{th}$ group such that the genetic change in traits in one group is not influenced by the restrictions on another group. Vector $\mathbf{p}$ is partitioned to subvectors $\mathbf{p}_j$ for $j = 1,\ldots,v$, where the elements of $\mathbf{p}_j$ must be relative values not equal to zero. This submatrix is constructed as:

$$\mathbf{E}_j = \begin{bmatrix} p_{12} & \cdots & p_{1s_1} \\ -p_{11} & & O \\ & \ddots & \\ O & & -p_{11} \end{bmatrix},$$

where $E. \equiv \text{diag}(E_j)$ for $j = 1,\ldots,v$. For example, if the monetary genetic change in milk yield and type score is proportional restricted in the ratio of 10:1, then scalar $p_{11}$ has the value 1 and $p_{12}$ the value 10. Matrix $S_O$ follows by augmenting matrix S with one row and column, such that $S_O = \begin{bmatrix} 1 & O \\ O & S \end{bmatrix}$ and is used later in the computation of restricted economic values. The restriction matrices $R_i$ are defined by

$$R_i \equiv G_iHS \qquad \text{for } i = 1,\ldots,n. \qquad (2)$$

The restriction vector, $a = (\bar{g}_f' \; O)'$, contains subvector $\bar{g}_f$ for the given fixed restrictions and null vector with order $t_p - v$ for the proportional restrictions. In case of single restricted indexes, vector a has to be redefined arbitrarily for the n index groups, such that $\tilde{a}_i = q_i a$, where $\sum_{i=1}^{n} q_i = 1$.

*Single Restricted Index*

With the single restricted index method (SRIM), the weights for a "single" index are derived independently from the weights of other selection indexes used in a population. The objective of SRIM is to maximize the standard deviation of the selection index ($\sigma_{\tilde{I}_i}$) under the condition that the imposed restrictions are fulfilled. The use of SRIM gives optimal results only if the index weights for all animals of a population are based on the same information sources. Otherwise, the theoretical optimum is not achieved, and the use of multiple restricted index methods is advised. The procedure of a general single restricted index is discussed in the following outline. A more detailed description of SRIM is given by Niebel and Van Vleck (1982).

*Preliminary Part.* Solutions to be obtained are $\tilde{\tau}_i$, $\tilde{b}_i$ and $\tilde{\Lambda}_i$, given $c_i$, $V_i$, $R_i$, $\tilde{a}_i$ and $r_{O_i}$. For computational reasons, the nonsingular, symmetric matrix $Q_i$ is defined as $Q_i = [I - V_i^{-1}R_i(R_i'V_i^{-1}R_i)^- R_i']V_i^{-1}$, where the part in the square brackets is idempotent and $V_i^{-1}$ is the inverse of the positive definite matrix of covariances among the information sources.

Rank conditions are that rank $(c_iR_i'V_i^{-1})$ must equal the rank of $(c_iR_i'V_i^{-1}\tilde{a}_i)$ and rank of $(c_iR_i)$ must be less than rank of $(V_i)$.

*Direct Solution.* The direct solution is $\tilde{\tau}_i = c_i^{-1}V_i^{-1}R_i(R_i'V_i^{-1}R_i)^-\tilde{a}_i + [1-\tilde{a}_i'(c_i^2R_i'V_i^{-1}R_i)^-\tilde{a}_i]^{1/2}(r_{O_i}'Q_ir_{O_i})^{-1/2}Q_ir_{O_i}$.

If no fixed restrictions are imposed or the fixed restrictions are set equal to zero, the direct solution with SRIM simplifies to $\tilde{\tau}_i = (r_{O_i}'Q_ir_{O_i})^{-1/2}Q_ir_{O_i}$.

*Restricted Index Weights.* The weights for the single restricted selection index can be obtained with the equation $\tilde{b}_i = (r_{O_i}'\tilde{\tau}_i)\tilde{\tau}_i$. The term $r_{O_i}'\tilde{\tau}_i$ is proportional to selection response and, in the positive case, identical with the standard deviation of the selection index, $\sigma_{\tilde{I}_i}$. However, $r_{O_i}'\tilde{\tau}_i$ can be negative if severe restrictions are imposed. The Lagrange multipliers $\tilde{\Lambda}_i$ can be obtained from $\tilde{\Lambda}_i = (R_i'V_i^{-1}R_i)^-\{R_i'V_i^{-1}r_{O_i} - (r_{O_i}'Q_ir_{O_i})^{1/2}[c_i^2 - \tilde{a}_i'(R_i'V_i^{-1}R_i)^-\tilde{a}_i]^{-1/2}\tilde{a}_i\}$. If the imposed restrictions cannot be fulfilled, the following equalities will not be true: $\tilde{\tau}_i'V_i\tilde{\tau}_i = 1$ and $c_iR_i'\tilde{\tau}_i = \tilde{a}_i$.

*Multiple Restricted Index*

The objective of the multiple restricted index method (MRIM) is to maximize the selection response of the whole population when restrictions are imposed on the genetic change of some economic traits and one or several different selection indexes are used. In the calculations given below, the index weights for the multiple restricted indexes are derived by mean of the Newton-Raphson algorithm (Stoer and Bulirsch, 1980). The derivation and definitive equations of MRIM are given by Niebel and Van Vleck (1982). Therefore, only the procedure for achieving solutions from the definitive equations is described.

*Preliminary Part.* Solutions to be obtained are $\phi_i$, $\lambda_i$, $\tau_i$ and $b_i$ for $i = 1,\ldots,n$ and $\Lambda$, given $c_i$, $V_i$, $G_i$, $R_i$, $r_{O_i}$, for $i = 1,\ldots,n$ and a. Via the Cholesky decomposition, the symmetric, nonsingular, positive definite and standardized covariance matrixes $V_i$ for $i = 1,\ldots,n$ can be decomposed, such that $V_i = L_iL_i'$, where $L_i$ is a lower triangular matrix. The inverse of $L_i$ is needed, or more efficiently, $L_i$ can be used in combination with back-substitution for the following iterative steps. Then the condition that rank $(D_1',\ldots,D_n')$ is equal to rank $(D_1',\ldots,D_n', a)$ must be checked. For simplification, define

$$\left.\begin{aligned} D_i &\equiv c_iL_i^{-1}R_i \\ d_i &\equiv c_iL_i^{-1}r_{O_i} \end{aligned}\right\} \quad \text{for } i = 1,\ldots,n.$$

Starting points for iteration can be

$$\left.\begin{aligned} \lambda_i^{(O)} &= -\|d_i\|_2 \\ \tilde{\phi}_i^{(O)} &= \frac{d_i}{\|d_i\|_2} \end{aligned}\right\} \text{ for } i = 1,\ldots,n.$$

*Iterative Part.* The iterative computing algorithm for Newton-Raphson procedure is:

for round k = 1,2,...:

$$B_O^{(k)} = O$$

$$v_O^{(k)} = a.$$

for i = 1,...,n:

$$B_i^{(k)} = B_{i-1}^{(k)} - [D_i'D_i - D_i'\tilde{\phi}_i^{(k-1)}\tilde{\phi}_i^{(k-1)\prime}D_i]/\lambda_i^{(k-1)} \text{ and}$$

$$v_i^{(k)} = v_{i-1}^{(k)} - [(\lambda_i^{(k-1)} + \tilde{\phi}_i^{(k-1)\prime}d_i)\, D_i'\tilde{\phi}_i^{(k-1)} - D_i'd_i]/\lambda_i^{(k-1)}$$

Solve $B_n^{(k)}\Lambda^{(k)} = v_n^{(k)}$, where $B_n^{(k)}$ is symmetric and positive definite

$$\Delta_O^{(k)} = O.$$

for i = 1,...,n:

$$\lambda_i^{(k)} = \tilde{\phi}_i^{(k-1)\prime}d_i - \tilde{\phi}_i^{(k-1)\prime}D_i\Lambda^{(k)},$$

$$\phi_i^{(k)} = [(\lambda_i^{(k-1)} - \lambda_i^{(k)})\tilde{\phi}_i^{(k-1)} - D_i\Lambda^{(k)} - d_i]/\lambda_i^{(k-1)},$$

$$\tilde{\phi}_i^{(k)} = \phi_i^{(k)}/\|\phi_i^{(k)}\|_2 \text{ and}$$

$$\Delta_i^{(k)} = \Delta_{i-1}^{(k)} + \|\tilde{\phi}_i^{(k)} - \tilde{\phi}_i^{(k-1)}\|_2 + \|\lambda_i^{(k)} - \lambda_i^{(k-1)}\|_2,$$

where $\Delta_n^{(k)}$ is a measure of convergence. If $\Delta_n^{(k)}$ is small enough $\tilde{\phi}_i^{(k)} \approx \phi_i^{(k)} = \phi_i$, $\lambda_i^{(k)} = \lambda_i$ for i = 1,...,n and $\Lambda^{(k)} = \Lambda$ are the solutions. Otherwise, proceed with the next iterative step.

*Restricted Index Weights.* Vector $\tau_i$ is a linear combination of the solution vector $\phi_i$ and is $\tau_i = L_i^{-1}\phi_i$, for i = 1,...,n. The vector of weights, $b_i$, for selection index i is given by $b_i = c_i^{-1}\phi_i'd_i\tau_i$[4]. The term $c_i^{-1}\phi_i'd_i$ is proportional to the selection response of index group i and, if positive, is identical with the standard deviation of the selection index $\sigma_{I_i}$. If restrictions imposed on the index weights are too severe and cannot be fulfilled by the selection differentials available in the n index groups, the following equations cannot be true: $\phi_i'\phi_i = 1$ or equivalently $\tau_i'V_i\tau_i = 1$, for i = 1,...,n and

$$a = \sum_{i=1}^{n} c_i R_i'\tau_i.$$

If the equations are not true, the imposed restrictions have to be made less severe. Under the MRIM method, during each iterative round only two matrix × vector multiplications and the solution of one equation system with order of the number of the restriction equations are needed in addition to the check for the imposed restrictions. The number of iterative rounds required depends on the desired accuracy of the index weights. In the application of restricted selection indexes calculated with MRIM, only a few iterative rounds usually are needed. If an accuracy of six decimal places for the index weights is required, about four to 10 iterative rounds are adequate, depending, however, on the severity of the imposed restrictions.

### *Restricted Economic Values*

Reasons for using restricted selection indexes are poor knowledge of the economic values of some traits, uncertainty about the economic future and the detrimental change of economic traits. In the last three decades the number of such traits has grown as a result of improved performance of the animals and higher standards set by the cattle industry. In the application of restricted selection indexes, knowledge of loss in breeding progress for economically well-defined traits by the imposed restrictions is important. Of real interest are the economic values needed for the total breeding value in the derivation of the index weights for an unrestricted selection index such that the imposed restrictions are also fulfilled. With those values, which will be called "restricted" economic values, the results of severe restrictions can be shown. If, from an economic standpoint, some

[4] Examples for computing the index weights can be obtained from the authors.

of the economic values are not justified, the imposed restrictions should be redefined and new index weights for the changed conditions calculated. For the computation of the restricted economic values $h_O^*$, transformation matrices, $H_O$ and $S_O$, the solution vector of the Lagrange multipliers, $\underset{\sim}{\Lambda}$, and the vector of the economic values, $h_O$, are needed. The vector of restricted economic values, derived with MRIM ($h^*$), is $h^* = H_O S_O \begin{pmatrix} 1 \\ \underset{\sim}{\Lambda} \end{pmatrix}$, and with SRIM ($h_i^*$) for index i, correspondingly, $h_i^* = H_O S_O \begin{pmatrix} 1 \\ \underset{\sim}{\Lambda}_i \end{pmatrix}$.

TABLE 1. ECONOMIC TRAITS

| Performance group | Economic trait |
|---|---|
| Milk | Milk yield<br>Type score |
| Growth | Daily gain<br>Growth capacity |
| Carcass | Muscle content |
| Reproduction (fertility) | Calving difficulty |

## Results

### *Economic and Genetic Parameters*

The purpose of the following section is to show the effects of using restricted selection indexes in breeding dairy, beef or dual-purpose cattle. The economic and genetic parameters used in the present study may not be suitable in the application of the procedure to actual production situations. They are described only for application and demonstration of the restricted selection technique. The economic system is related to American dairy, beef and(or) dual-purpose cattle. By definition, the economic traits are involved in total breeding value and can be part of the restrictions. The economic traits used in the calculations are summarized in table 1.

Multiple-trait selection requires determination of the discounted economic values such that the genetic contribution of all traits to the index value can be summarized in a single term. The discounted economic value of an economic trait is dependent on the time lag between investment and first genetic return. The sooner that the genetic progress contributes to more efficient cattle production, the greater the discounted economic value will be. This also means that the longer the time between the birth of genetically improved animals and the time of the first genetic return, the lower will be the value of one unit of a specified economic trait.

The computation procedure for the economic traits is based on Adelhelm et al. (1972) and Henze et al. (1980). One procedure for evaluating the discounted economic values is the discounted gene flow method suggested by Hill (1974) and McClintock and Cunningham (1974). This method provides accurate discounted economic values. For simplification an alternative formula for discounting the genetic return is given at the bottom of table 4. In agreement with Brascamp (1978), this method provides nearly the same results as the discounted gene flow method.

In the computation of the discounted economic values, the following assumptions are made: (1) an investment period of 17.25 yr; (2) a lag of 7.25 yr between the time of investment and birth of genetically improved animals for all traits, where the transmission of the selection response on the dam of sire and sire of sire paths requires an increased interval of two generations; (3) a period of 10 yr after the birth of genetically improved animals; (4) separate computations for each trait for discounting selection response and (5) a discounting factor of 6% for risk, where, in accordance with Smith (1978), consideration of the inflation rate is not necessary.

In accordance with Adelhelm et al. (1972), one unit of economic value is related to one cow for 1 yr, such that all genetic return realized in the progenies due to this cow year is included. In table 2, heritabilities, genetic standard deviations and correlation coefficients of selection and economic traits are summarized. Test procedures for sire and cows are defined in table 3. The relative economic values of the economic traits pertain to the economic system in USA and are listed in table 4[5]. A brief description of the selection and the economic traits is provided below.

*Milk Yield.* Unit of milk yield is 1 kg fat-corrected milk based on a level of 3.5% fat.

[5] A short description of the procedure for computing the economic values can be obtained from the authors.

TABLE 2. HERITABILITIES, GENETIC STANDARD DEVIATIONS, PHENOTYPIC CORRELATIONS (ABOVE DIAGONAL) AND GENETIC CORRELATIONS (BELOW DIAGONAL)

| Selection or economic trait | $h^2$ | Gentic standard deviation | Correlation | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | MY | TS | DG | GC | MC | CA2 | CO2 | CA3 | CO3 |
| | – % – | | | | | | | | | | |
| Milk yield (MY), kg | 25 | 566 | 1. | –.10 | .10 | .40 | –.30[a] | .20 | –.10 | .20 | –.10 |
| Type score (TS), points | 25 | 2 | –.10 | 1. | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Daily gain (DG), g | 25 | 59 | .10 | 0 | 1. | .40 | 0 | .20 | –.20 | .20 | –.20 |
| Growth capacity (GC), kg | 30 | 36.2 | .40 | 0 | .40 | 1. | –.10 | .30 | –.10 | .30 | –.10 |
| Muscle content (MC), % | 30 | 1.25 | –.60[a] | 0 | 0 | –.20 | 1. | .30[b] | .30[b] | .30[b] | .30[b] |
| Calving difficulty due to calf, category 2 (CA2), % | 6 | 10 | .20 | 0 | .20 | .30 | .60[b] | 1. | –.20[c] | .20 | –.20[c] |
| Calving difficulty due to cow, category 2 (CO2), % | 3 | 5 | –.10 | 0 | –.10 | –.10 | .60[b] | –.20[c] | 1. | –.20[c] | .20[c] |
| Calving difficulty due to calf, category 3 (CA3), % | 15 | 16 | .20 | 0 | .20 | .30 | .60[b] | .20 | –.20[c] | 1. | –.20[c] |
| Calving difficulty due to cow, category 3 (CO3), % | 8 | 8 | –.10 | 0 | –.10 | –.10 | .60[b] | –.20[c] | .20 | –.20[c] | 1. |

[a]Variable, see table 8.

[b]Variable, see table 8.

[c]Variable, see table 7.

TABLE 3. TEST PROCEDURE FOR SELECTION OF SIRES AND COWS

| Information source or selection trait[a] | Sires | Cows |
|---|---|---|
| (1) Milk yield (MY) | Progeny test, first lactation, 50 daughters | Performance test, first lactation |
| (2) Type score (TS) | Progeny test, 50 daughters | Performance test |
| (3) Daily gain (DG) | Progeny test, average daily live weight gain on field test, 50 daughters | Performance test average daily live weight gain to an age of 18 mo |
| (4) Growth capacity (GC) | Progeny test, predicted optimal final weight of fattening animals, 50 daughters | Performance test, predicted optimal final weight of fattening animals, 50 daughters |
| (5) Muscle content (MC) | Performance test, visual muscle score | Performance test, visual muscle score |
| (6) Calving difficulty due to calf, category 2 (CA2) | Progeny test, 100 calvings | Progeny test, first calving |
| (7) Calving difficulty due to cow, category 2 (CO2) | Progeny test, 50 daughters first calving | Performance test, first calving |
| (8) Calving difficulty due to calf, category 3 (CA3) | Progeny test, 100 calvings | Progeny test, first calving |
| (9) Calving difficulty due to cow, category 3 (CO3) | Progeny test, 50 daughters, first calving | Performance test, first calving |

[a]Standard test procedure contains all information sources listed except for muscle content of sires and cows.

TABLE 4. RELATIVE ECONOMIC VALUES

| Economic trait | Return from selection response per cow and yr, $ (a) | Time lag, yr from: Investment to birth of genetically improved animals (b) | Time lag, yr from: Birth of genetically improved animals to beginning of return (c) | Discounting coefficients[a] (d) | Economic values, $ 10*(a)*(d) |
|---|---|---|---|---|---|
| Milk yield (MY), kg | .176 | 7.25 | 5 | .278 | .49 |
| Type score (TS), points | 10 | 7.25 | 5 | .278 | 28 |
| Daily gain (DG), g | .57 | 7.25 | 1.1 | .557 | 3.15 |
| Growth capacity (GC), kg | .048 | 7.25 | 4.6 | .304 | .146 |
| Muscle content (MC), % | 10 | 7.25 | 4.7 | .296 | 30 |
| Calving difficulty due to calf, category 2 (CA2), % | −.17 | 7.25 | 3 | .414 | −.7 |
| Calving difficulty due to cow, category 2 (CO2), % | −.17 | 7.25 | 3 | .414 | −.7 |
| Calving difficulty due to calf, category 3 (CA3), % | −.70 | 7.25 | 3 | .414 | −2.8 |
| Calving difficulty due to cow, category 3 (CO3), % | −.70 | 7.25 | 3 | .414 | −2.8 |

[a] $(d) = \{1 - [(q^{(c)}j-1)q^{10-(c)}j]/(q^{10}-1)\}\, q^{(b)}j$, where risk factor is 6%, q = 1 + .06 = 1.06 and investment period in years = 7.25 + 10 = 17.25.

This expression is highly correlated with income from milk sales. For simplification, marginal net return for milk yield is calculated as price of fat-corrected milk minus costs for grain, cooling and transportation charge. Age correction factors were used to compute the amount from milk sales dependent on time of return and quantity of milk produced in the different lactations, as described by Van Vleck and Everett (1976). The phenotypic and genetic parameters of milk yield correspond to those for Holstein-Frisian cattle.

*Type Score.* The economic values for type are derived from the value per genetic standard deviation for milk yield. It is assumed that the value per genetic standard deviation of type score amounts to 20% of that for milk yield.

*Daily Gain.* The improvement of daily gain results in a shorter freshening and fattening period and reduces labor, housing, maintenance feeding and capital costs and holds for all categories of cattle included in one cow year. The economic value of daily gain depends on the replacement rate of heifers and the number of beef units produced per cow year and is calculated on the basis of break even costs in accordance with Knoblauch et al. (1979) and Fox and Black (1980). The phenotypic and genetic parameters for the model calculations are based mainly on a review by Petty and Cartwright (1966) and reported by Harville and Henderson (1966), Brown et al. (1972) and Calo et al. (1973).

*Growth Capacity.* Adelhelm et al. (1972) defined growth capacity as final fattening weight achieved with a fixed body fat content in a standard environment. An increase in growth capacity results in increased weight at slaughter and heavier cows. The economic values for growth capacity of both these factors are opposite and, for purposes of simplification, will be ignored. The economic value for growth capacity also is a function of the calf price, price for beef and beef produced per cow year. Low calf price and small numbers of fattened steers result in small economic values for growth capacity.

*Muscle Content.* Carcass value is influenced by several factors. However, in defining growth capacity as slaughter weight at a fixed proportion of fat, it is necessary to distinguish only between slaughter animals with the same fat content. An appropriate criterion for carcass value seems to be high muscle content of shrunk live weight. Genetic progress in muscle content means simultaneous decreases in the proportions of bones, skin and tendons, i.e., an improvement in the meat to bone ratio. In the evaluation of the net return per unit of muscle content, it is assumed that bones, skin and tendons are worthless. Other characters helping to determine carcass value are shape and location of the muscles. In computing the economic value for muscle content, it will be assumed that increases in muscle results in higher-priced cuts of beef. The computation of the economic values was based on the research of Henze et al. (1980). The genetic and phenotypic parameters for the calculations were drawn from experiments with serially slaughtered purebred cattle (Reichen, 1972; Andersen, 1978; Berg et al., 1978; Nour et al., 1980). It must be considered that genetic parameters obtained from purebred populations can be quite different from parameters obtained from crossbreeding experiments (Koch et al., 1976; Koch and Dikeman, 1977). The unit of economic value for muscle content is 1% of shrunk live weight. In estimating the breeding values of muscle content performance test for muscle score is assumed for sires.

*Calving Difficulty.* During the last 20 yr, calving difficulty has become a costly problem in the dairy and beef industries. The development of sire and cow evaluation procedures for categorical traits by Quaas and Van Vleck (1980) allows the utilization of covariances between categories, such that the different categories of calving difficulty can be economically well-defined traits. In this study, calving difficulty due to cow and calving difficulty due to calf is distinguished. Calving difficulty due to cow measures the indirect or maternally caused genetic effect and calving difficulty due to calf the direct genetic effect caused from the calf. Four traits of calving difficulty are considered: due to calf, category 2; due to cow, category 2; due to calf, category 3, and due to cow, category 3. Calving difficulty of category 1 is easy calving, has no economic value, is autocorrelated with categories 2 and 3 and, therefore, is not used. The computations of the economic values of calving difficulty are based on the values suggested by Cady (1980), while the phenotypic and genetic parameters are drawn from Philipsson (1976), Pollak and Freeman (1976) and Cady (1980). The unit of economic value for calving difficulty is the monetary loss associated with a 1% increase in frequency of the problem, where the values are negative.

The formula for genetic gain per year is given in equation (1) and must be specified for the model calculations:

$$\Delta Y_O/yr = c_B\sigma_{I_B} + c_C\sigma_{I_C},$$

where

$$c_B = q_B\delta_B/\sum_{i=1}^{n} q_i\ell_i = (.5)\,(1.4)/(6) = .117 \text{ and}$$

$$c_C = q_C\delta_C/\sum_{i=1}^{n} q_i\ell_i = (.5)\,(.98)/(6) = .082.$$

$c_B$ and $c_C$ are constants needed to establish the equations for the restricted selection indexes.

*Efficiency of Restriction Methods*

The SRIM and MRIM were described previously. The outcome of selection practiced when the genetic change of calving difficulty is restricted to a certain amount is shown in table 5.

The economic values of calving difficulty due to calf are set equal to zero. With genetic change in calving difficulty, more problems with dystocia would be expected if unrestricted selection indexes were applied. By the imposition of restrictions on the genetic change in calving difficulty, deterioration can be averted, but the expense of restriction is a decline in selection response for important economic traits. However, if MRIM is used instead of SRIM, the loss in selection response can be reduced. The superiority of MRIM increases with more severe restrictions. In the calculations described below only MRIM is used.

*Fixed Restrictions on Calving Difficulty Depend on Test Procedure*

In table 6 the effects of the test procedures in combination with severity of imposed restrictions on the genetic change in calving difficulty are summarized. The economic results are calculated from the discounted economic values. The economic values of calving difficulty are set equal to zero. The expected selection responses can be improved when the growth traits also are included in the sire and cow evaluation. This effect is increased with restrictions on genetic change in calving difficulty and when muscle content is included in the test and selection procedure. It is worth noting (last column of table 6) that the genetic progress for carcass cannot be made with any of the fixed restrictions.

The restricted economic values represent the values needed for total breeding value to fulfill the imposed restrictions when MRIM is used. The result of more severe restrictions imposed on the genetic change in calving difficulty is higher restricted economic values for all traits of calving difficulty.

For a purebred population, but not for between breed matings (Monteiro, 1969), it can be supposed that relatively large cows have fewer calving difficulties than small cows and vice versa. The reciprocal pattern is observed with large bulls, whose calves cause more dystocia than those of smaller bulls (Cady, 1980). This means that large-framed cows have less difficulty when they give birth. From this relationship, negative correlations between

TABLE 5. EFFICIENCY OF DIFFERENT RESTRICTION METHODS FOR DIFFERENT FIXED RESTRICTIONS ON CALVING DIFFICULTY

| | Selection response per cow and yr[a] | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | CA2 | CA3 | CA2 | | CA3 | | | |
| Method | Population (fixed restricted), % | | Sire, % | Cow, % | Sire, % | Cow, % | $ | % |
| Unrestricted index | No | No | .35 | .12 | .63 | .21 | 49.3 | 100 |
| "Single" restricted index | 0 | 0 | 0 | 0 | 0 | 0 | 40.6 | 82 |
| | −.25 | −.25 | −.125 | −.125 | −.125 | −.125 | 28.3 | 58 |
| | −.25 | −.50 | −.125 | −.125 | −.25 | −.25 | 25.4 | 52 |
| Multiple restricted index | 0 | 0 | −.08 | .08 | −.13 | .13 | 42.1 | 86 |
| | −.25 | −.25 | −.31 | .06 | −.34 | .09 | 35.3 | 72 |
| | −.25 | −.50 | −.30 | .05 | −.56 | .06 | 32.6 | 66 |

[a]For explanation of abbreviations, see table 3.

TABLE 6. SELECTION RESPONSES AND RESTRICTED ECONOMIC VALUES FOR DIFFERENT TEST PROCEDURES AND RESTRICTIONS

| Test procedure[a] | Restriction (MRIM)[b] fixed | Selection response per cow and yr | | | | | Restricted economic values | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $ | % | Milk, % | Growth, % | Carcass, % | CA2 $/% | CO2 $/% | CA3 $/% | CO3 $/% |
| Milk (1, 2)<br>Reproduction (6, 7) | No | 41.0 | 84 | 94 | 11 | −5 | 0 | 0 | 0 | 0 |
| | CA2 = CA3 = 0% | 34.4 | 70 | 109 | 4 | −13 | −13 | 0 | −7 | 0 |
| | CA2 + CO2 = 0%<br>CA3 + CO3 = 0% | 35.1 | 72 | 108 | 4 | −12 | −14 | −14 | −8 | −8 |
| | CA2 + CO2 = −.25%<br>CA3 + CO3 = −.50% | 24.8 | 51 | 124 | 4 | −20 | −25 | −25 | −14 | −14 |
| Milk (1, 2)<br>Reproduction (6, 7, 8, 9) | No | 41.2 | 84 | 94 | 10 | −4 | 0 | 0 | 0 | 0 |
| | CA2 = CA3 = 0% | 34.7 | 71 | 108 | 3 | −11 | −13 | 0 | −7 | 0 |
| | CA2 + CO2 = 0%<br>CA3 + CO3 = 0% | 35.4 | 72 | 108 | 5 | −13 | −13 | −13 | −7 | −7 |
| | CA2 + CO2 = −.25%<br>CA3 + CO3 = −.50% | 26.2 | 54 | 122 | −1 | −21 | −22 | −22 | −12 | −12 |
| Milk (1, 2)<br>Reproduction (6, 7, 8, 9)<br>Growth (3, 4) | No | 48.9 | 100 | 66 | 37 | −3 | 0 | 0 | 0 | 0 |
| | CA2 = CA3 = 0% | 42.2 | 86 | 72 | 36 | −8 | −14 | 0 | −7 | 0 |
| | CA2 + CO2 = 0%<br>CA3 + CO3 = 0% | 42.9 | 88 | 72 | 37 | −9 | −15 | −15 | −8 | −8 |
| | CA2 + CO2 = −.25%<br>CA3 + CO3 = −.50% | 32.7 | 67 | 78 | 37 | −15 | −25 | −25 | −13 | −13 |
| Milk (1, 2)<br>Reproduction (6, 7, 8, 9)<br>Growth (3, 4)<br>Carcass (5) | No | 51.3 | 105 | 65 | 33 | 2 | 0 | 0 | 0 | 0 |
| | CA2 = CA3 = 0% | 47.0 | 96 | 71 | 35 | −6 | −7 | 0 | −5 | 0 |
| | CA2 + CO2 = 0%<br>CA3 + CO3 = 0% | 48.3 | 99 | 68 | 34 | −2 | −6 | −6 | −5 | −5 |
| | CA2 + CO2 = −.25%<br>CA3 + CO3 = −.50% | 43.1 | 88 | 70 | 34 | −4 | −11 | −11 | −8 | −8 |

[a]For explanation of numbers in parentheses and abbreviations, see table 3.

[b]MRIM = multiple restricted index method.

calving difficulty due to calf and calving difficulty due to cow can be expected. However, large-framed female calves that die at birth cannot give birth, and thus the observed correlations will not coincide with the real situation. Another aspect in this connection would be selection on calving difficulty due to calf without involving traits of calving difficulty due to cow. Higher negative correlations between calving difficulty due to calf and calving difficulty due to cow would lead to preference for small-framed animals, again a reason for dystocia. In table 7, the correlations of calving difficulty due to calf with calving difficulty due

TABLE 7. SELECTION RESPONSE FOR DIFFERENT CORRELATION COEFFICIENTS BETWEEN CALVING DIFFICULTY DUE TO CALF AND COW WHEN GENETIC CHANGE OF CALVING DIFFICULTY IS RESTRICTED TO ZERO[a]

| $r_{CA,CO}$ | Selection response per cow and yr | | | | | | Loss in selection response by restriction (MRIM)[b], % |
|---|---|---|---|---|---|---|---|
| | $ | MY, kg | CA2, % | CO2, % | CA3, % | CO3, % | |
| 0 | 44.2 | 63 | .11 | −.11 | .21 | −.21 | 9 |
| −.20 | 42.9 | 62 | .06 | −.06 | .12 | −.12 | 12 |
| −.40 | 40.9 | 60 | −.03 | .03 | −.06 | .06 | 17 |
| −.60 | 41.0 | 58 | −.08 | .08 | −.25 | .25 | 19 |

[a]For explanation of abbreviations, see table 3.
[b]MRIM = multiple restricted index method.

TABLE 8. SELECTION RESPONSE FOR DIFFERENT CORRELATIONS WHEN GENETIC CHANGE IN CALVING DIFFICULTY AND MUSCLE CONTENT IS RESTRICTED TO ZERO

| Genetic (phenotypic) correlations | | Selection response per cow and yr[a] | | | | | | Loss in selection response by restriction (MRIM)[b], % |
|---|---|---|---|---|---|---|---|---|
| | | $ | MY, kg | CA2, % | CO2, % | CA3, % | CO3, % | |
| $r_{MC,MY}$ | −.20 (−.10) | 36.1 | 45 | −.05 | .05 | −.16 | .16 | 29 |
| | −.40 (−.20) | 29.6 | 27 | −.08 | .08 | −.23 | .23 | 42 |
| | −.60 (−.30) | 25.4 | 10 | −.08 | .08 | −.24 | .24 | 50 |
| | −.80 (−.40) | 23.4 | 0 | −.07 | .07 | −.20 | .20 | 54 |
| $r_{MC,C}$ | .20 ( .10) | 25.0 | 2 | .02 | −.02 | .02 | −.02 | 51 |
| | .40 ( .20) | 24.3 | 2 | .05 | −.05 | −.15 | .15 | 52 |
| | .60 ( .30) | 25.4 | 10 | −.08 | .08 | −.24 | .24 | 50 |
| | .80 ( .40) | 26.8 | 18 | −.10 | .10 | −.29 | .29 | 37 |

[a]For explanation of abbreviations, see table 3.
[b]MRIM = multiple restricted index method.

TABLE 9. SELECTION RESPONSE FOR DIFFERENT FIXED AND PROPORTIONAL RESTRICTIONS AND DIFFERENT TEST PROCEDURES

| Restriction (MRIM)[a] | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Proportional (monetary)[b] | | Test[c] | | Selection response per cow and year | | | |
| Fixed[b] | MY:TS | DG:MC | procedure | $ | Relative, % | Milk, % | Growth, % | Carcass, % |
| No | No | No | a | 48.9 | 100 | 67 | 37 | −3 |
| | | | b | 49.2 | 101 | 67 | 36 | −3 |
| | 5:1 | 5:1 | a | 35.4 | 72 | 62 | 32 | 5 |
| | | | b | 36.5 | 75 | 62 | 32 | 6 |
| | 10:1 | 5:1 | a | 37.8 | 77 | 70 | 25 | 5 |
| | | | b | 39.2 | 80 | 69 | 26 | 5 |
| CA2 + CO2 = 0<br>CA3 + CO3 = 0 | 5:1 | 5:1 | a[d] | 17.3 | 36 | 134 | −28 | −5 |
| | | | b | 28.5 | 58 | 79 | 18 | 4 |
| | 10:1 | 5:1 | a | 18.0 | 37 | 143 | −36 | −7 |
| | | | b | 30.1 | 62 | 87 | 11 | 2 |

[a]MRIM = multiple restricted index method.

[b]Explanation of abbreviations, see table 3.

[c]a = standard procedure; b = standard procedure plus performance test on muscle score for sires, see table 3.

[d]Computations with MRIM are given in the appendix.

to cow are varied in a reasonable range, while the genetic change in calving difficulty is restricted to zero. The relative loss in selection response increases proportionally with higher correlation coefficients.

*Fixed Restrictions for Calving Difficulty and Carcass Value*

Selection for milk traits without any attention to beef characters may lead to a decline in carcass value (e.g., Jersey, Guernsey, Ayrshire, Dexters, Ankole). On the other hand, selection for valuable carcasses leads to considerable problems of dystocia (e.g., Blue White Belgium, Charolais, Limousin, Fleckvieh, Gelbvieh, Piamontesa). Little information is available about the correlations among carcass value, milk yield and calving difficulty. These correlations are central to the problem of balancing change in milk yield, carcass value and dystocia in cattle breeding. The expected responses with different correlations, when the genetic change in calving difficulty and muscle content is restricted to zero are summarized in table 8.

The results demonstrate: (1) Losses from combined restrictions on calving difficulty and carcass value are large when no information on carcass value is included in the test and restriction procedure. (2) As shown in further calculations, losses due to a combined restriction on carcass value and calving difficulty dependent on the correlations could be decreased from 10 to about 30% when records for muscle score for sire and cow evaluation are available. A similar result was described by Cunningham and Gjedrem (1970) when direct information on the restricted traits was not available or could not be used. (3) Loss in selection response declines with higher correlations between muscle content and calving difficulty and increases with higher negative correlations between muscle content and milk yield.

*Fixed and Proportional Restrictions*

The examples for restricted selection indexes have all dealt with fixed restrictions where the genetic change of certain traits is equated to a fixed amount. In complex situations, fixed restrictions alone may not be sufficient, as for instance, in the case of combined selection for milk yield, calving ease, no deterioration of the carcass value, improved daily gain and type score. However, knowledge of economic importance of type score is lacking. Moreover, negative relationships between type characters and milk yield were found by Everett et al. (1976) and Van Vleck et al. (1980). In the following calculations, the coefficient of correlation between type and milk is assumed to be −.1. In table 9, the economic values of genetic changes in milk and type score and, independently, the economic values of changes in daily gain and muscle content are proportionally restricted to given ratios, while the fixed restrictions imposed on genetic change in calving difficulty are zero. The responses obtained when direct information sources on muscle score for sire evaluations were available were compared with those obtained when such sources were not available. By imposing only proportional restrictions on milk yield and type as well as on daily gain and muscle content, one moves all traits in the positive direction. However, a deterioration in calving difficulty is expected. When proportional and fixed restrictions are imposed, genetic improvement in the economic traits and monetary loss by restriction depend on availability of information on muscle content for sire evaluation. All traits are moved in the positive direction when selection indexes include performance tests on sires for muscle conent. Otherwise, a negative change in carcass value and low genetic return are to be expected.

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## APPENDIX A

Example for table 9.

*Preliminary Part*

Codes:

Index: 1 = sire; 2 = cow.

Traits in selection indices V, $L^{-1}$-matrices and $r_O$-vectors for sires and cows:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| MY | TS | DG | GC | MC | CA2 | CA3 | CO2 | CO3. |

Traits in columns of $G_i$ and in rows of $S_O$,S:

| 1 | | | |
|---|---|---|---|
| GC | | | |
| unrestricted | | | |
| 2 | 3 | 4 | 5 |
| CA2 | CO2 | CA3 | CO3 |
| fixed restricted | | | |
| 6 | 7 | 8 | 9 |
| MY | TS | DG | MC |
| proportional restricted | | | |

$c' = (.117 \quad .082)$.

$$V_1 = \begin{bmatrix} .08125 & & & & & & & \\ -.00812 & .08125 & & & & \text{Symmetric} & & \\ .00812 & 0 & .08125 & & & & & \\ .03484 & 0 & .0348 & .09350 & & & & \\ -.00412 & 0 & -.00412 & -.00432 & .02735 & & & \\ -.00546 & 0 & -.00546 & -.00547 & .00640 & .03960 & & \\ .00806 & 0 & .00806 & .01296 & -.00410 & -.00543 & .02485 & \\ .01159 & 0 & .01159 & .01875 & -.00532 & -.00742 & .00670 & .04712 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} 1 & & & & & & & \\ -.1 & 1 & & & & \text{Symmetric} & & \\ .1 & 0 & 1 & & & & & \\ .4 & 0 & .4 & 1 & & & & \\ -.1 & 0 & -.1 & -.1 & 1 & & & \\ -.1 & 0 & -.1 & -.1 & .2 & 1 & & \\ .01225 & 0 & .01225 & .02012 & -.00424 & -.00693 & 1 & \\ .01936 & 0 & .01936 & .03182 & -.0067 & -.01095 & .00949 & 1 \end{bmatrix}$$

$$L_1^{-1} = \begin{bmatrix} 3.50823 & & & & & & & 0 \\ .35259 & 3.52590 & & & & & & \\ -3.56170 & -.03562 & 3.52609 & & & & & \\ -1.53029 & -.15303 & -1.51328 & 3.88616 & & & & \\ .24408 & .02441 & .24137 & .10084 & 6.09146 & & & \\ .22702 & .02270 & .22450 & .09943 & -1.12286 & 5.15602 & & \\ -.28718 & .02872 & -.28399 & -.65116 & .67168 & .64190 & 6.74348 & \\ -.27834 & -.02783 & -.27525 & -.63825 & .54874 & .58386 & -.59826 & 4.93175 \end{bmatrix}$$

$$L_2^{-1} = \begin{bmatrix} 1 & & & & & & & 0 \\ .10050 & 1.00504 & & & & & & \\ -.10152 & -.01015 & 1.00509 & & & & & \\ -.43665 & -.04366 & -.43180 & 1.18866 & & & & \\ .07862 & .00786 & .07774 & .03844 & 1.00979 & & & \\ .06525 & .00652 & .06452 & .03190 & -.18931 & 1.02738 & & \\ -.00506 & -.00051 & -.00500 & -.01560 & .00084 & .00420 & 1.00024 & \\ -.00795 & -.00080 & -.00786 & -.02454 & .00132 & .00660 & -.00875 & 1.00063 \end{bmatrix}$$

$$
G_1 = \begin{bmatrix}
3.62 & .5 & -.125 & .8 & -.2 & 141.5 & -.05 & 1.475 & -.1875 \\
0 & 0 & 0 & 0 & 0 & -14.15 & .5 & 0 & 0 \\
3.62 & .5 & -.125 & .8 & -.2 & 14.15 & 0 & 14.75 & 0 \\
9.9138 & .8216 & -.1369 & 1.3145 & -.2191 & 62.0022 & 0 & 6.4631 & -.0685 \\
-.3135 & -.1732 & .4330 & -.2771 & .1386 & -4.9017 & 0 & -.5110 & .0650 \\
-.5119 & -.2828 & .1414 & -.4525 & 1.1314 & -8.0044 & 0 & -.8344 & .1061 \\
1.3301 & 1.2247 & -.1225 & .3919 & -.1960 & 13.8641 & 0 & 1.4452 & .0919 \\
2.1030 & .3873 & -.1936 & 3.0984 & -.3098 & 21.9211 & 0 & 2.2851 & .1452
\end{bmatrix}
$$

$$
G_2 = \begin{bmatrix}
7.24 & 1 & -.25 & 1.6 & -.4 & 283. & -.1 & 2.95 & -.375 \\
0 & 0 & 0 & 0 & 0 & -28.3 & 1 & 0 & 0 \\
7.24 & 1 & -.25 & 1.6 & -.4 & 28.3 & 0 & 29.5 & 0 \\
19.8276 & 1.6432 & -.2739 & 2.6291 & -.4382 & 124.004 & 0 & 12.9263 & -.1369 \\
-.6270 & -.3464 & .8660 & -.5543 & .2771 & -9.8034 & 0 & -1.0219 & .1299 \\
-1.0239 & -.5657 & .2828 & -.9051 & 2.2627 & -16.0089 & 0 & -1.6688 & .2121 \\
1.3301 & 1.2247 & -.1225 & .3919 & -.1960 & 13.8641 & 0 & 1.4452 & .0919 \\
2.1030 & .3873 & -.1936 & 3.0984 & -.3098 & 21.9211 & 0 & 2.2851 & .1452
\end{bmatrix}
$$

$$
\begin{aligned}
r'_{01} &= (\,67.4848 \quad 7.0665 \quad 53.925 \quad 50.1334 \quad -2.1086 \quad -3.4433 \quad 14.2957 \quad 22.6034) \\
r'_{02} &= (134.970 \quad 14.1330 \quad 107.849 \quad 100.267 \quad -4.2171 \quad -6.8865 \quad 14.2957 \quad 22.6034)
\end{aligned}
$$

$$
H_O = \begin{bmatrix}
.15 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
.49 & 0 & 0 & 1 & 0 & 0 & 0 \\
28 & 0 & 0 & 0 & 1 & 0 & 0 \\
3.15 & 0 & 0 & 0 & 0 & 1 & 0 \\
30 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

$$
S_O = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & -285.710 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & -47.6190
\end{bmatrix}
$$

$h_O$ and $H$ can be obtained from $H_O = (h_O \; H)$ and $S$ from $S_O = \begin{pmatrix} 1 & O \\ O & S \end{pmatrix}$

$$
c_1 R_1 = c_1 G_1 HS = \begin{bmatrix}
.043875 & .070200 & 18.2269 & 12.1722 \\
0 & 0 & -18.3696 & 0 \\
.043875 & .070200 & 1.65555 & 17.2575 \\
.080104 & .128167 & 7.25426 & 11.3763 \\
.030397 & -.016212 & -.573499 & -4.21656 \\
-.016546 & .079422 & -.936520 & -6.88562 \\
.128966 & .022927 & 1.62210 & -3.42680 \\
.022657 & .326260 & 2.56477 & -5.41824
\end{bmatrix}
$$

$$
c_2 R_2 = c_2 G_2 HS = \begin{bmatrix}
.061500 & .098400 & 25.5488 & 17.0618 \\
0 & 0 & -25.7488 & 0 \\
.06500 & .098400 & 2.32060 & 24.1900 \\
.112283 & .179653 & 10.1684 & 15.9463 \\
.042608 & -.022724 & -.803879 & -5.9104 \\
-.023193 & .111327 & -1.31273 & -9.6516 \\
.090386 & .016069 & 1.13686 & -2.4017 \\
.015879 & .228661 & 1.79753 & -3.7974
\end{bmatrix}
$$

$$
D_1 = c_1 L_1^{-1} R_1 = \begin{bmatrix}
.153924 & .246278 & 63.9442 & 42.7028 \\
.015470 & .024752 & -58.3428 & 4.29179 \\
.139080 & .222528 & -.000002 & 56.5161 \\
.177762 & .284420 & .604613 & -.532015 \\
.214542 & -.051751 & 1.63815 & -17.4014 \\
-.091671 & .472146 & .628968 & -22.9992 \\
.802253 & .071148 & .051536 & -46.1650 \\
-.033813 & 1.51213 & 1.16905 & -46.4045
\end{bmatrix}
$$

$$
D_2 = c_2 L_2^{-1} R_2 = \begin{bmatrix}
.061500 & .098400 & 25.5488 & 17.0618 \\
.006181 & .009890 & -23.3108 & 1.71478 \\
.055569 & .088911 & .000001 & 22.5809 \\
.080057 & .128091 & 1.05310 & 1.05949 \\
.056957 & -.000656 & 1.565616 & -2.13339 \\
-.020332 & .137177 & .676597 & -5.61427 \\
.087975 & .012728 & .844519 & -2.90373 \\
.011272 & .223402 & 1.328433 & -4.56759
\end{bmatrix}
$$

$$
d_1' = c_1 (L_1^{-1} r_{O_1})' = (27.7000 \quad 5.69911 \quad 19.4050 \quad 1.03792 \quad 2.55894 \quad 2.01066 \quad 2.95237 \quad 3.97016)
$$

$$
d_2' = c_2 (L_2^{-1} r_{O_2})' = (11.0675 \quad 2.27707 \quad 7.75324 \quad 1.07111 \quad 1.53355 \quad 1.04788 \quad .94081 \quad 1.47990)
$$

*Iterative Part*

$$
B_2^{(1)} = \begin{bmatrix}
-.0210 & & \text{Symmetric} & \\
.0019 & -.0748 & & \\
.0689 & .1737 & -231.1 & \\
1.120 & 2.734 & -.2753 & -244.1
\end{bmatrix}
$$

$$
B_2^{(2)} = \begin{bmatrix}
.0041 & & \text{Symmetric} & \\
.0006 & .0146 & & \\
.0664 & .1154 & 51.52 & \\
-.0861 & -.2732 & 16.33 & 60.49
\end{bmatrix}
$$

$$
B_2^{(4)} = \begin{bmatrix}
-.0663 & & \text{Symmetric} & \\
-.0102 & -.2358 & & \\
-1.089 & -1.895 & -845.3 & \\
1.348 & 4.348 & -268.2 & -984.5
\end{bmatrix}
$$

$$
B_2^{(5)} = \begin{bmatrix}
-.0663 & & \text{Symmetric} & \\
-.0102 & -.2358 & & \\
-1.089 & -1.895 & -845.18 & \\
1.348 & 4.348 & -268.2 & -984.5
\end{bmatrix}
$$

| Round k | $V^{(k)'}$ 1 | 2 | 3 | 4 | $\Lambda^{(k)'}$ 1 | 2 | 3 | 4 | $\Delta_1^{(k)}$ |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | – | – | – | – | – |
| 1 | .3809 | .7077 | 58.568 | 79.397 | −152.14513 | −99.14510 | −.37059 | −2.32337 | 336.55088 |
| 2 | −.0659 | −.1212 | −10.000 | −13.433 | −19.51321 | −12.59724 | −.04758 | −.29387 | .04608 |
| 4 | 1.0672 | 1.9802 | 164.328 | 223.070 | −19.37212 | −12.63354 | −.04718 | −.29602 | .00184 |
| 5 | 1.0671 | 1.9800 | 164.310 | 223.045 | −19.37212 | −12.63354 | −.04718 | −.29602 | .00000 |

$\underset{\sim}{\Lambda} = \underset{\sim}{\Lambda}^{(5)}$

| Round k | Index i | $\bar{\phi}_i^{(k)'}$ 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | $\lambda_i^{(k)}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | .79551 | .16367 | .55729 | .02981 | .07349 | .05774 | .08479 | .11402 | −34.82056 |
| 1 | 1 | .46479 | .51983 | −.22940 | .46603 | .32679 | .36456 | .01240 | −.06397 | 202.80958 |
| 2 | 1 | .46416 | .51967 | −.23028 | −.46605 | .32713 | .36469 | .01335 | −.06383 | −12.63792 |
| 4 | 1 | .47047 | .51946 | −.22386 | −.46421 | .32665 | .36380 | .01389 | .06300 | −12.64508 |
| 5 | 1 | .47047 | .51946 | −.22386 | −.46421 | .32665 | .36380 | .01389 | −.06300 | −12.64508 |
| 0 | 2 | .79163 | .16287 | .55457 | .07661 | .10969 | .07495 | .06729 | .10585 | −13.98068 |
| 1 | 2 | .51296 | .54977 | −.22187 | −.51007 | .20959 | .27926 | −.02034 | −.05385 | 82.67938 |
| 2 | 2 | .51357 | .54963 | −.22154 | −.50969 | .20980 | .27930 | −.02003 | −.05363 | −4.77366 |
| 4 | 2 | .49855 | .55046 | −.23722 | −.51624 | .20893 | .27980 | −.02192 | −.05692 | −4.76778 |
| 5 | 2 | .49855 | .55046 | −.23722 | −.51624 | .20893 | .27980 | −.02192 | −.05692 | −4.76778 |

$\underset{\sim}{\phi}_i = \bar{\underset{\sim}{\phi}}_i^{(5)} = \underset{\sim}{\phi}_i^{(5)}$

for i = 1,2

$\lambda_i = \lambda_i^{(5)}$

*Restricted Index Weights*

| Index i | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.799658 | 1.928164 | .087055 | −1.703722 | 1.556055 | 1.847885 | .131372 | −.310700 | |
| 2 | .838621 | .581709 | .019343 | −.594941 | .157913 | .286996 | −.021430 | −.056957 | |
| | | | | | $b_i$ | | | | $\sigma_{I_i}$ |
| 1 | 299.709 | 206.414 | 9.319 | −182.387 | 166.579 | 197.820 | 14.064 | −33.261 | 107.052 |
| 2 | 49.988 | 34.674 | 1.153 | −35.463 | 9.413 | 17.107 | −1.277 | −3.395 | 59.607 |