1973

The Nebraska - South Dakota High School Mathematics Contest

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THE NEBRASKA – SOUTH DAKOTA HIGH SCHOOL MATHEMATICS CONTEST

Henry Miot Cox
The University of Nebraska-Lincoln and
The Mathematical Association of America

The Annual High School Mathematics Examination, now in its twenty-fourth year, is a joint project of The Mathematical Association of America, the Society of Actuaries, Mu Alpha Theta, the National Council of Teachers of Mathematics, and the Casualty Actuarial Society. In Nebraska additional sponsors include the Nebraska Academy of Sciences and the Nebraska Actuaries Club.

The examination is conducted by regional contest chairmen in both the United States and Canada. More than 350,000 students in more than 6,000 high schools participate in the contest. In Nebraska and South Dakota, participation over a six-year period has been as follows:

Contest	Number of Contestants	Number of Schools	Range of Team Scores	[Median of Team Scores in U.S. and Canada]
1967	5,752	202	4.50 to 211.50	[66.0]
1968	6,004	203	8.00 to 299.00	[105.0]
1969	6,769	211	15.00 to 231.75	[97.0]
1970	6,575	203	8.50 to 281.25	[100.0]
1971	6,385	206	11.75 to 232.50	[85.0]
1972	6,158	174	20.50 to 250.75	[90.0]

Questions on the Annual High School Mathematics Examination are prepared for the versatile, imaginative student. Although superiority in the subject is required for a student to do well on the examination, the condition is a necessary but not a sufficient one.

About one-third of the contestants in Nebraska and South Dakota had participated in one or more of the previous examinations, some as many as three times. The following comparisons of mean scores by high school class and by times in contest suggest the effects of experience and maturation:

	Number of times in contest				Combined	Number of
	1	2	3	4	Groups	Contestants
Senior Students	10.37	12.80	17.91	10.80	12.87	1,833
Junior Students	7.43	11.67	9.37		8.67	2,357
Sophomore Students	6.62	6.79			6.64	1,410
Freshmen Students	2.73				2.73	527
(No. of contestants)	(4,020)	(1,516)	(511)	(80)	(6,127)	

Further comparison is afforded by the study of a group of 423 contestants who participated in three successive years and from whose scores the following coefficients of correlation were calculated:

	1970	1971	1972	Mean	Standard Deviation
1970	------	0.60	0.54	6.69	11.21
1971	0.60	------	0.67	13.29	11.55
1972	0.54	0.67	-----	15.67	13.58
Mean	6.69	13.29	15.67	--------	--------
Standard Deviation	11.21	11.55	13.58	-------	--------

Coefficients of correlation computed between the Annual Mathematics High School Examination and the Scholastic Aptitude Test of the College Entrance Examination Board are of the order of 0.40 with Verbal Ability and 0.60 with Mathematical Ability (data available upon request).

Students who score relatively low on mathematics achievement rank low in the contest. However, students who score high on achievement do not *ipso facto* rank high in the contest. It is, of course, intended that the purpose and the structure of the contest examination should be different from that of the achievement examination.

The subject-matter contained in the Annual High School Mathematics Examination includes materials from elementary algebra, plane and simple coordinate geometry, intermediate algebra, and trigonometry. Questions pre-judged to be more difficult are weighed more heavily than the "easier" questions. Questions on the 1972 Examination together with an indication as to difficulty for three groups of students are shown in an appendix. The three groups of students are:

Group I: Top 100 Contestants in Nebraska and South Dakota

Group II: Team Members in Nebraska and South Dakota (3 per school; N = 522)

Group III: All Contestants from Nebraska and South Dakota (N = 6,158)

A broader base for the analysis of the 1972 Examination is provided by a ten per cent sampling of the high scoring contestants throughout the United States and Canada. Table 1 shows for each question the per cent of contestants reading the question, the per cent of contestants attempting the question, the per cent of contestants answering the question correctly, the per cent of contestants answering the question adjusted for guessing($R - \frac{1}{4}W$), and the biserial coeffficient of correlation (contestants with correct responses compared with contestants with incorrect responses). Questions 9, 10, 1, 21, and 14 are found to vary from 78% to 52% in the per cent of adjusted correct responses. The most valid items are found to be 19, 14, and 7 (biserial r's of 0.45, 0.42, and 0.40, respectively).

ACKNOWLEDGEMENTS

Contest Chairmen for Nebraska and South Dakota have been Professor James M. Earl, Chairman until his death on November 28, 1972, of the M.A.A. Committee on High School Contests, and Professor Barbara Buchalter of The University of Nebraska at Omaha. The National Association of Secondary School Principals (U.S.A.) includes the High School Mathematics Examination on its annual Advisory List of Contests and Activities. Questions on the 1972 Examination are reproduced by permission.

TABLE 1.

1972 HIGH SCHOOL MATHEMATICS CONTEST/ EVERY TENTH PAPER NATIONALLY-TEAMS

QUESTION	% READING	% ATTEMPTING	% CORRECT	% ADJUSTED	BISERIAL R
1	100.00	97.07	69.25	62.29	0.233
2	100.00	24.48	14.85	12.45	0.231
3	100.00	73.43	52.93	47.80	0.342
4	100.00	68.83	33.05	24.11	0.383
5	100.00	72.59	36.61	27.62	0.307
6	100.00	78.66	42.05	32.90	0.277
7	100.00	64.23	40.79	34.94	0.397
8	100.00	56.90	19.67	10.36	0.321
9	100.00	84.73	77.82	76.10	0.194
10	100.00	85.36	74.27	71.50	0.227
11	100.00	93.31	48.74	37.60	0.072
12	100.00	58.79	33.26	26.88	0.374
13	100.00	42.26	22.80	17.94	0.103
14	100.00	68.41	55.23	51.94	0.416
15	99.58	49.58	38.49	35.72	0.333
16	99.37	38.91	30.75	28.71	0.326
17	98.95	37.87	13.81	7.79	0.265
18	98.74	46.23	41.84	40.74	0.272
19	98.12	45.61	38.08	36.19	0.446
20	96.65	41.84	38.28	37.40	0.319
21	95.40	75.31	64.44	61.72	0.087
22	90.17	13.60	6.49	4.71	0.287
23	89.12	37.87	7.53	-0.05	0.271
24	86.40	25.31	20.29	19.04	0.281
25	83.89	21.55	15.90	14.49	0.150
26	82.43	35.98	28.03	26.05	0.158
27	79.08	16.53	10.25	8.68	0.391
28	76.99	55.86	44.14	41.21	0.308
29	62.76	13.39	9.41	8.42	0.325
30	60.46	10.25	2.30	0.31	0.173
31	59.21	28.03	12.76	8.94	0.197
32	51.88	15.48	8.58	6.85	0.163
33	47.49	39.96	23.85	19.82	0.290
34	21.55	10.67	5.23	3.87	0.064
35	16.32	16.32	3.77	0.63	0.151

APPENDIX

1972 Annual High School Mathematics Examination

PART I (3 credits each)	Percent of Correct Responses: Group I (top 100)	Group II (Team)	Group III (all cont.)
1. The lengths in inches of the three sides of each of four triangles I, II, III, and IV are as follows: I 3, 4, and 5 III 7, 24, and 25 II 4, $7\frac{1}{2}$, and $8\frac{1}{2}$ IV $3\frac{1}{2}$, $4\frac{1}{2}$, and $5\frac{1}{2}$ Of these four given triangles, the only right triangles are (A) I and II (B) I and III (C) I and IV (D) I, II, and III (E) I, II, and IV	77%	66%	41%
2. If a dealer could get his goods for 8% less while keeping his selling price fixed, his profit, based on cost, would be increased to (x + 10)% from his present profit of x% which is (A) 12% (B) 15% (C) 30% (D) 50% (E) 75%	18%	7%	5%
3. If $x = \frac{1 - i\sqrt{3}}{2}$ where $i = \sqrt{-1}$, then $\frac{1}{x^2 - x}$ is equal to (A) -2 (B) -1 (C) $1 + i\sqrt{3}$ (D) 1 (E) 2	69%	38%	20%
4. The number of solutions to $\{1,2\} \subseteq X \subseteq \{1,2,3,4,5\}$ where X is a subset of $\{1,2,3,4,5\}$ is (A) 2 (B) 4 (C) 6 (D) 8 (E) None of these	52%	20%	8%
5. From among $2^{1/2}$, $3^{1/3}$, $8^{1/8}$, $9^{1/9}$ those which have the greatest and the next to the greatest values in that order, are (A) $3^{1/3}, 2^{1/2}$ (B) $3^{1/3}, 8^{1/8}$ (C) $3^{1/3}, 9^{1/9}$ (D) $8^{1/8}, 9^{1/9}$ (E) None of these	44%	25%	14%
6. If $3^{2x} + 9 = 10(3^x)$, then the value of $(x^2 + 1)$ is (A) 1 only (B) 5 only (C) 1 or 5 (D) 2 (E) 10	55%	29%	14%
7. If $yz:zx:xy = 1:2:3$, then $\frac{x}{yz} : \frac{y}{zx}$ is equal to (A) 3:2 (B) 1:2 (C) 1:4 (D) 2:1 (E) 4:1	67%	29%	12%
8. If $\lvert x - \log y \rvert = x + \log y$ where x and log y are real, then (A) $x = 0$ (B) $y = 1$ (C) $x = 0$ and $y = 1$ (D) $x(y-1) = 0$ (E) None of these	35%	8%	3%
9. Ann and Sue bought identical boxes of stationery. Ann used hers to write 1-sheet letters and Sue used hers to write 3-sheet letters. Ann used all the envelopes and had 50 sheets of paper left, while Sue used all of the sheets of paper and had 50 envelopes left. The number of sheets of paper in each box was (A) 150 (B) 125 (C) 120 (D) 100 (E) 80	82%	75%	54%
10. For x real, the inequality $1 \le \lvert x - 2 \rvert \le 7$ is equivalent to (A) $x \le 1$ or $x \ge 3$ (B) $1 \le x \le 3$ (C) $-5 \le x \le 9$ (D) $-5 \le x \le 1$ or $3 \le x \le 9$ (E) $-6 \le x \le 1$ or $3 \le x \le 10$	82%	56%	41%

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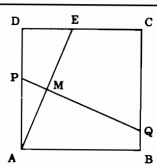
Annual High School Mathematics Examination 1972

Part II (4 credits each)	Percent of Correct Responses		
	Group I (Top 100)	Group II (Team)	Group III (all cont.)
11. The value(s) of y for which the following pair of equations $x^2 + y^2 - 16 = 0$ and $x^2 - 3y + 12 = 0$ may have a real common solution, are (A) 4 only (B) −7, 4 (C) 0, 4 (D) no y (E) all y	57%	42%	24%
12. The number of cubic feet in the volume of a cube is the same as the number of square inches in its surface area. The length of the edge expressed as a number of feet is (A) 6 (B) 864 (C) 1728 (D) 6 × 1728 (E) 2304	53%	20%	6%
13. Inside square ABCD (See figure) with sides of length 12 inches, segment AE is drawn where E is the point on DC which is 5 inches from D. The perpendicular bisector of AE is drawn and intersects AE, AD, and BC at points M, P, and Q respectively. The ratio of segment PM to MQ is (A) 5:12 (B) 5:13 (C) 5:19 (D) 1:4 (E) 5:21 	33%	20%	9%
14. A triangle has angles of 30° and 45°. If the side opposite the 45° angle has length 8, then the side opposite the 30° angle has length (A) 4 (B) $4\sqrt{2}$ (C) $4\sqrt{3}$ (D) $4\sqrt{6}$ (E) 6	68%	39%	17%
15. A contractor estimated that one of his two bricklayers would take 9 hours to build a certain wall and the other 10 hours. However, he knew from experience that when they worked together, their combined output fell by 10 bricks per hour. Being in a hurry, he put both men on the job and found that it took exactly 5 hours to build the wall. The number of bricks in the wall was (A) 500 (B) 550 (C) 900 (D) 950 (E) 960	60%	31%	13%
16. There are two positive numbers that may be inserted between 3 and 9 such that the first three are in geometric progression while the last three are in arithmetic progression. The sum of those two positive numbers is (A) $13\frac{1}{2}$ (B) $11\frac{1}{4}$ (C) $10\frac{1}{2}$ (D) 10 (E) $9\frac{1}{2}$	50%	17%	7%
17. A piece of string is cut in two at a point selected at random. The probability that the longer piece is at least x times as large as the shorter piece is (A) $\frac{1}{2}$ (B) $\frac{2}{x}$ (C) $\frac{1}{x+1}$ (D) $\frac{1}{x}$ (E) $\frac{2}{x+1}$	18%	8%	4%
18. Let ABCD be a trapezoid with the measure of base AB twice that of base DC, and let E be the point of intersection of the diagonals. If the measure of diagonal AC is 11, then that of segment EC is equal to (A) $3\frac{2}{3}$ (B) $3\frac{3}{4}$ (C) 4 (D) $3\frac{1}{2}$ (E) 3	55%	36%	18%
19. The sum of the first n terms of the sequence $1, (1+2), (1+2+2^2), \ldots (1+2+2^2+\ldots+2^{n-1})$ in terms of n is (A) 2^n (B) $2^n - n$ (C) $2^{n+1} - n$ (D) $2^{n+1} - n - 2$ (E) $n \cdot 2^n$	60%	23%	9%

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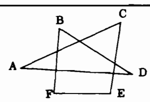
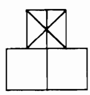
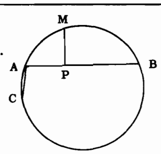
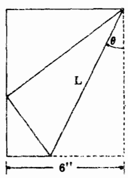
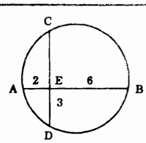
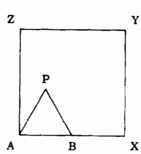
1972 Annual High School Mathematics Examination	Percent of Correct Responses		
	Group I (Top 100)	Group II (Team)	Group III (all cont.)
20. If $\tan x = \frac{2ab}{a^2 - b^2}$ where $a > b > 0$ and $0° < x < 90°$, then $\sin x$ is equal to (A) $\frac{a}{b}$ (B) $\frac{b}{a}$ (C) $\frac{\sqrt{a^2 - b^2}}{2a}$ (D) $\frac{\sqrt{a^2 - b^2}}{2ab}$ (E) $\frac{2ab}{a^2 + b^2}$	39%	22%	10%
PART III (5 credits each)			
21. If the sum of the measures in degrees of angles A, B, C, D, E, and F in the figure to the right is 90n, then n is equal to (A) 2 (B) 3 (C) 4 (D) 5 (E) 6 	55%	55%	30%
22. If $a \pm bi$ $(b \neq 0)$ are imaginary roots of the equation $x^3 + qx + r = 0$ where a, b, q, and r are real numbers, then q in terms of a and b is (A) $a^2 + b^2$ (B) $2a^2 - b^2$ (C) $b^2 - a^2$ (D) $b^2 - 2a^2$ (E) $b^2 - 3a^2$	8%	2%	1%
23. The radius of the smallest circle containing the symmetric figure composed of the 3 unit squares shown at the right is  (A) $\sqrt{2}$ (B) $\sqrt{1.25}$ (C) 1.25 (D) $\frac{5\sqrt{17}}{16}$ (E) None of these	8%	4%	1%
24. A man walked a certain distance at a constant rate. If he had gone $\frac{1}{2}$ mile per hour faster, he would have walked the distance in four-fifths of the time; if he had gone $\frac{1}{2}$ mile per hour slower, he would have been $2\frac{1}{2}$ hours longer on the road. The distance in miles he walked was (A) $13\frac{1}{2}$ (B) 15 (C) $17\frac{1}{2}$ (D) 20 (E) 25	20%	9%	4%
25. Inscribed in a circle is a quadrilateral having sides of lengths 25, 39, 52, and 60 taken consecutively. The diameter of this circle has length (A) 62 (B) 63 (C) 65 (D) 66 (E) 69	19%	13%	6%
26. In the circle to the right, M is the mid-point of arc CAB and segment MP is perpendicular to chord AB at P. If the measure of chord AC is x and that of segment AP is (x + 1), then segment PB has measure equal to (A) 3x + 2 (B) 3x + 1 (C) 2x + 3 (D) 2x + 2 (E) 2x + 1 	15%	25%	18%
27. If the area of ΔABC is 64 square inches and the geometric mean (mean proportional) between sides AB and AC is 12 inches, then sin A is equal to (A) $\frac{\sqrt{3}}{2}$ (B) $\frac{3}{5}$ (C) $\frac{4}{5}$ (D) $\frac{8}{9}$ (E) $\frac{15}{17}$	14%	5%	2%

1972 Annual High School Mathematics Examination

	Percent of Correct Responses		
	Group I (top 100)	Group II (team)	Group III (all Cont.)
28. A circular disc with diameter D is placed on an 8 × 8 checkerboard with width D so that the centers coincide. The number of checkerboard squares which are completely covered by the disc is (A) 48 (B) 44 (C) 40 (D) 36 (E) 32	56%	39%	18%
29. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ for $-1 < x < 1$, then $f\left(\frac{3x+x^3}{1+3x^2}\right)$ in terms of f(x) is (A) $-f(x)$ (B) $2f(x)$ (C) $3f(x)$ (D) $[f(x)]^2$ (E) $[f(x)]^3 - f(x)$	12%	5%	2%
30. A rectangular piece of paper 6 inches wide is folded as in the diagram so that one corner touches the opposite side. The length in inches of the crease L in terms of angle θ is (A) $3 \sec^2 \theta \csc \theta$ (B) $6 \sin \theta \sec \theta$ (C) $3 \sec \theta \csc \theta$ (D) $6 \sec \theta \csc^2 \theta$ (E) None of these  	1%	1%	0%
PART IV (6 credits each)			
31. When the number 2^{1000} is divided by 13, the remainder in the division is (A) 1 (B) 2 (C) 3 (D) 7 (E) 11	14%	9%	4%
32. Chords AB and CD in the circle to the right intersect at E and are perpendicular to each other. If segments AE, EB, and ED have measures 2, 6, and 3 respectively, then the length of the diameter of the circle is (A) $4\sqrt{5}$ (B) $\sqrt{65}$ (C) $2\sqrt{17}$ (D) $3\sqrt{7}$ (E) $6\sqrt{2}$  	11%	8%	3%
33. The minimum value of the quotient of a (base ten) number of three different nonzero digits divided by the sum of its digits is (A) 9.7 (B) 10.1 (C) 10.5 (D) 10.9 (E) 20.5	31%	19%	6%
34. Three times Dick's age plus Tom's age equals twice Harry's age. Double the cube of Harry's age is equal to three times the cube of Dick's age added to the cube of Tom's age. Their respective ages are relatively prime to each other. The sum of the squares of their ages is (A) 42 (B) 46 (C) 122 (D) 290 (E) 326	4%	4%	1%
35. Equilateral triangle ABP (See figure) with side AB of length 2 inches is placed inside square AXYZ with side of length 4 inches so that B is on side AX. The triangle is rotated clockwise about B, then P, and so on along the sides of the square until P returns to its original position. The length of the path in inches traversed by vertex P is equal to (A) $20\pi/3$ (B) $32\pi/3$ (C) 12π (D) $40\pi/3$ (E) 15π  	6%	3%	2%

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