A Modified Transhipment Algorithm For Trucking

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ABSTRACT: The transportation of goods and merchandise within any city is an essential part of urban life. These trucks, however, add greatly to the traffic congestion and pollution. Consequently, some approach is desired to reduce the effect. One such approach can be to restrict truck traffic during rush hours and on specific roads. However, a more desirable approach includes minimization of the truck-miles required to transport the merchandise. One such technique is the transhipment algorithm which concerns the flow through an intermediate point. The current algorithm is a minor modification of the distribution problem and it is not very efficient. Thus, the proposed approach is to transform the problem into three, rather than two, dimensions. Current research has been devoted to forward flow only.

INTRODUCTION

Since the advent of the industrial revolution the world has seen a remarkable growth in the size and complexity of their organizations. The small artisans' shops of an earlier era have become the expanding corporations of today. But, as times changed and old complications were done away with, new problems arose. As each company had its own set of goals and responsibilities they tended to cross purposes.

The transportation of goods within one city, or among several cities, is one of the familiar branches of common dispute. Trucks tend to add to traffic congestion and are responsible for a considerable amount of pollution produced by motor vehicles. Therefore, a method of dispersing this problem, or at least easing it in part, is needed. One approach is the limitation of truck-miles and thus limit the time a truck is on the road.

A technique for solving this problem is by using a linear equation, known as the transhipment algorithm, to determine the minimal time a truck needs be on the road. This algorithm, when applied to the trucking situation, has a four-fold purpose. It saves time by reducing the mileage and limiting trucking to the best route. It saves resources, in this case trucks and their maintenance, as one big truck may be used to perform the present task of three smaller trucks. It lessens cost, it uses the least expensive routes for all parties concerned, so there is a minimum cost. Reducing the cost to the shipper then allows the shipper, in theory, to pass it on to consumers. Finally, it cuts back on pollution, using one truck instead of three, you limit the air pollution produced, plus the noise pollution as well. And less time on the road means less time for the production of pollution.

The current algorithm, however, is a mere extension of the distribution problem and not as effective as it might be. It was designed by Alex Orden, and first published in *Management Science* in 1956. It was proven valid, but as time passed, new ideas for solving this problem more quickly were designed. The proposed approach is to transform the problem into three, rather than two, dimensions.

To understand this better, one must realize that the original method was based on a table placed on an xy-graph. The form on which my proposed solution is obtained is on a table on the area and within the volume of a cube. At the present, research has been devoted to the forward flow only.

BACKGROUND: The transhipment problem is a direct extension of the transportation problem. This, in turn, is a special case of linear equation which involves the determining of the optimal shipping pattern. In this case there are three different series and three different types of paths to consider. Yet, to begin, it is better to observe the transportation problem so the introduction of information is step by step.

The transportation problem is similar to the transhipment problem only there do not exist any intermediary points. For example, there exists a source, a factory i (i=1,2,3,...m) that produces a_i (amount in supply) and a destination, a store j (j=1,2,3,...n) with a requirement of b_j units (amount in demand). Supposing that the amount shipped from the factory to the store is directly proportional to the shipping cost. Now, x_{ij} is the amount shipped from factory i to store j; also c_{ij} is the cost per unit shipped along the varying paths. The resulting equation looks like this:

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \, x_{ij}.$$

This is subject to the restrictions that the summation over j of x_{ij} is equal to a_i (the amount shipped is equal to that in supply); the summation over i of x_{ij} is equal to b_j (the amount shipped is equal to that in demand); and that x_{ij} is greater than or equal to zero. This leads to the observations that the amount in supply is equal to the amount in demand which are equal to x_{ij} with the bounds of i and j. However, this is only for convenience sake and the difference may be supplemented by introducing a dummy factory or store with infinite supply and demand respectively.

Therefore, except for the non-negativity restriction the others can be written thus;

$$0 = a_i - \sum_{j=1}^{n} x_{ij} \qquad \text{for all } i = 1,2,3,\ldots m.$$

$$0 = b_j - \sum_{i=1}^{m} x_{ij} \qquad \text{for all } j = 1,2,3,\ldots n.$$

Using Lagrange multipliers we may then write an equation to combine the constraints with the basic objective function z. These multipliers shall be u_i ($i=1,2,3,...m$) and v_j ($j=1,2,3,...n$). Thus one arrives at the final equation;

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{i=1}^{m} u_i(a_i - \sum_{j=1}^{n} x_{ij}) + \sum_{j=1}^{n} v_j(b_j - \sum_{i=1}^{m} x_{ij})$$

$$= \sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij} - u_i - v_j)x_{ij} + \sum_{i=1}^{m} u_i a_i + \sum_{j=1}^{n} v_j b_j.$$

This shows that x_{ij} must be greater than zero or else there is no allocation, then c_{ij} must be equal to u_i plus v_j. In addition, there shall be $(m + n = 1)$ allocations when the program is completed, unless the problem is degenerate, because there are $(m + n)$ unknowns or that is to say u_i and v_j. Any one of these can be assigned arbitrarily, and the rest solved algebraically. This holds true for the transhipment algorithm.

As previously denoted the transhipment is similar to the transportation problem, however, it adds warehouses in between to provide a wider variety of routes and costs. Whereas, in one case it is cheaper to ship from factory to a store, in another it might be cheaper to ship it with a regular run to a warehouse and then distribute it to the store-from there. Luckily, the transhipment problem can be broken down to factor nearly the same way as the transportation problem.

There are actually three parts to the problem as shown in Fig. 1. This is designated by the direction of the flow between series. There are three: A series are the manufacturers (denoted $A_1, A_2, A_3, ...$) and are the source of the supply a_i; B series are the warehouses (denoted $B_1, B_2, B_3, ...$) and may be any point in the system as long as there is a path to the point; and C series are the retailors (denoted $C_1, C_2, C_3, ...$) these hold the final demand. As well as these there are three different paths: forward flow, the flow directed towards the demand which includes direct and excludes any that meet the following paths' requirements; backward flow, flow away from the demand with similar costs as the forward but, not necessarily the same; and flow within a series; example, from A_1 to A_2. Present research by the author has developed a solution to the forward flow only since it can be equated without the use of any other part.

The new algorithm was based on the theory that placing the operations on a cube, instead of a plane, would minimize elements and improve accessibility. In Fig. 2, it is explained how each series is taken into account; A encompasses the cube horizontally, while B and C oppose each other rising vertically on adjacent sides, descending on the opposite, and crossing both the top and bottom planes. Thus the area is utilized, while within the volume of the cube the shift between each plane takes place. The operation of the forward flow takes place on the three visible planes. Unfolding the planes for

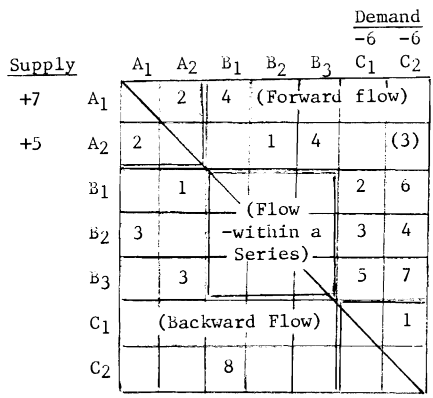

Fig. 1

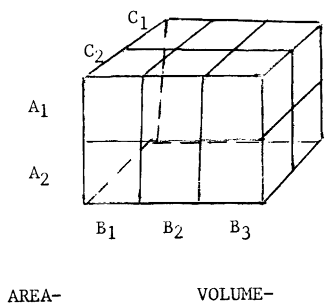

Fig. 2

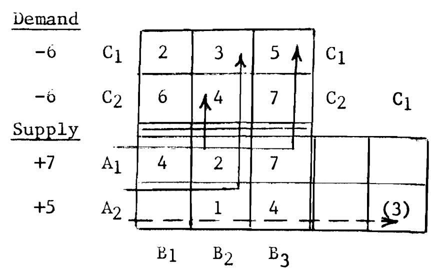

Fig. 3

better observation the diagram is similar to Fig. 3. It is noticeable that all traffic is at right angles to reach the final destination when passing through an intermediary point, the direct path is not. However, for convenience sake, the direct path can be altered to conform with the right angle pattern by adding a dummy intermediary point with a like cost into its cell but, a zero shipping cost out. Still, the problem may be solved in either form but the author recommends the latter form as easier.

There yet remains the determining of the necessary and sufficient optimal conditions before the equation can be worked. The following conditions were designed by Marvin M. Johnson, Ph.D., University of Nebraska, and are used with his permission. The effective objective function and the constraints for this problem are as follows:

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} e_{ij}x_{ij} + \sum_{j=1}^{n} \sum_{k=1}^{p} e_{jk}x_{jk}.$$

which is subject to the following points; Input to the warehouses is equal to the amount in supply. Input to the stores is equal to the amount in demand. Input to the warehouses is equal to their outputs. Supply is equal to demand and x_{ij} and x_{jk} are non-negative. Here e_{ij} and e_{jk} are used to denote cost per unit since one of the series is already C.

Next, using the Lagrangian function to eliminate the basic variables, the technique assures global rather than local optimality. The multipliers are similar to those used in the transportation problem but there is an extra one as we have introduced a new series. These are u_i $(i=1,2,3,...m)$, r_j $(j=1,2,3,...n)$, and v_k $(k=1,2,3,...p)$. The resulting function is:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} e_{ij}x_{ij} + \sum_{j=1}^{n} \sum_{k=1}^{p} e_{jk}x_{jk} - \sum_{i=1}^{m} u_i(\sum_{j=1}^{n} x_{ij} - a_i) - \sum_{k=1}^{p} v_k(\sum_{j=1}^{n} x_{jk} - b_k)$$

$$- \sum_{j=1}^{n} r_j(\sum_{i=1}^{m} x_{ij} - \sum_{k=1}^{p} x_{jk}) - s(\sum_{i=1}^{m} a_i - \sum_{k=1}^{p} b_k)$$

$$- \sum_{i=1}^{m} \sum_{j=1}^{n} w_{ij}(x_{ij} - z_{ij}^2) - \sum_{j=1}^{n} \sum_{k=1}^{p} t_{jk}(x_{jk} - y_{jk}^2)$$

So that the Lagrange function is optimal at the same point as the minimum of the object function, either the multipliers must be equal to zero or the terms following must. The simplified function looks like this:

$$= \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}(e_{ij} - u_i - r_j - v_k) + \sum_{j=1}^{n} \sum_{k=1}^{p} x_{jk}(e_{jk} - u_i + r_j - v_k)$$

$$+ \sum_{i=1}^{m} u_i(a_i - \sum_{j=1}^{n} x_{ij}) + \sum_{k=1}^{p} v_k(b_k - \sum_{j=1}^{n} x_{jk}) + \sum_{j=1}^{n} r_j(\sum_{k=1}^{p} x_{jk} - \sum_{i=1}^{m} x_{ij})$$

$$+ s(\sum_{k=1}^{p} b_k - \sum_{i=1}^{m} a_i) + \sum_{i=1}^{m} \sum_{j=1}^{n} w_{ij}(z_{ij}^2 - x_{ij}) + \sum_{j=1}^{n} \sum_{k=1}^{p} t_{jk}(y_{jk}^2 - x_{jk}).$$

Now the conditions for optimality must be restated from this.

$w_{ij} = e_{ij} - u_i - r_j$ for Stage 1, and $t_{jk} = e_{jk} - v_k + r_j$ for Stage 2.

Stage 1 indicates the flow from factory to warehouse, and Stage 2 indicates the flow from warehouse to store. The amount in supply must be equal to the number shipped in Stage 1, and the amount in demand must be equal to the number shipped in Stage 2. This means supply and demand are equal, and ends with the restriction that the number shipped cannot be negative.

With the use of partial derivitives we also obtain this:
$w_{ij}z_{ij} = O$ and $w_{ij} \geqslant O$ for Stage 1; $x_{ij} = z_{ij}^2$ non-negative;

$t_{jk}y_{jk} = O$ and $t_{jk} \geqslant O$ for Stage 2; and, $x_{jk} = y_{jk}^2$ non-negative.

Then either w_{ij} (t_{jk} for Stage 2) or z_{ij} (y_{jk} for Stage 2) or both equal zero when the solution is optimal. However, assuming that z_{ij} (y_{jk}) is zero then $x_{ij} = z_{ij}^2 = 0$ ($x_{jk} = y_{jk}^2 = O$). But, if the amount shipped is equal to zero then there is no problem. Because of this w_{ij} (t_{jk}) cannot be greater than zero as this forces the amount to be shipped to retain the value of zero. Therefore, w_{ij} (t_{jk}) must equal zero, and then $e_{ij} = u_i + r_j$ ($e_{jk} = v_k - r_j$). And these are all the conditions which must be met for optimality.

EXAMPLE:

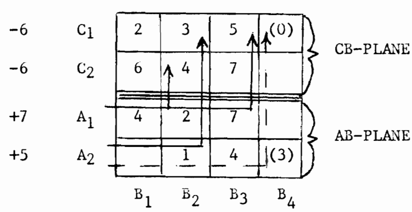

Fig. 4

		B1	B2	B3
Demand				
-6	C1	2	3	5
-6	C2	6	4	7
Supply				
+7	A1	4	2	7
+5	A2		1	4

Fig. 5

		B1	B2	B3	
-6	C1	2	3	5	
-6	C2	6	4	7	
+7	A1	2	0	5	-2
+5	A2		0	3	-1

Fig. 6

		B1	B2	B3	
-6	C1	0	1	3	-2
-6	C2	2	0	3	-4
+7	A1	2	0	5	-2
+5	A2		0	3	-1

Fig. 7

METHOD: Now that all the rules have been established, the final step is the method used to work the equation. In Fig. 4, there is an example of a common problem which shall be utilized to explain the method. In the example there isn't any direct flow, but it has been explained how it would be dealt with. The first instruction is to draw a chart and fill in the pertinent information available as shown in Fig. 5.

Subtract the smallest number in each A row in the AB-plane from every number in that row until there is a zero in at least one cell of that row. This would appear as in Fig. 6. Next, subtract the smallest number in each C row in the CB-plane from every number in that row until there is at least one zero in one cell of each row, as in Fig. 7.

To allocate the supply to the routes, start in the AB-plane with the upper left cell and move outward until a zero is located. If there are two zeroes in a column, pick the one that does not require the other to choose a path that is not optimal. Remember that if something is added to AB-plane, it must also be added to CB-plane.

To make sure that the required conditions are met, it is noted that the amount in supply (12) is equal to that in demand (12). Since the number in the supply series varies from seven to five it is obvious that to meet the demand of six each, the seven must be split into one and six. An arbitrary assignment of six from A_1 to C_1 has been made and the others have one remaining choice, to satisfy C_2. In Fig. 8, arrows indicate the allotments to each cell. Checking to see if this meets the requirements of number of occupied cells is done this way. Since $(m + n - 1)$ is the number used for the transportation problem, then the transhipment problem should be similar but with one element more. Let $(m + n - 1) = r$, then $(r + p - 2) = (m + n + p - 3)$. Placing our information in the formula the result is four; thus, there should be, and are, four occupied cells in the graph.

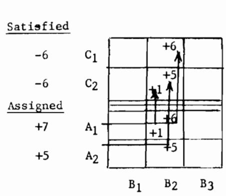

Fig. 8

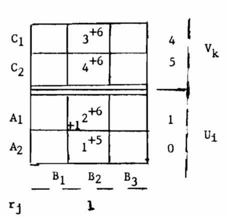

Fig. 9

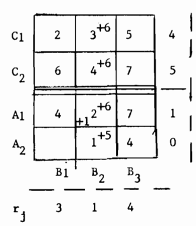

Fig. 10

To prove that the answer is optimal, there are two methods. The first is trial and error, checking each set of paths separately. This is a long and tiresome method. It is better to follow the rules set previously. These are $e_{ij} = u_i + r_j$ (or $e_{jk} = v_k - r_j$). To attain these multipliers is easy enough. Take the lowest number in any column that contains an allocation and subtract it from any number in that column in the AB-plane and add it to any number in the same column in the CB-plane. This then lends itself to determining v_k and u_i algebraically. Subtract the remainder of that number till a zero is placed in each assigned cell. Next, add the unassigned and determine the rest of r_j by subtracting the smallest number until there is a zero in each column. Now you are ready to check for optimality. Plug each of the values into the appropriate one of the two equations, $e_{ij} = u_i + r_j$ ($e_{jk} = v_k - r_j$). If all the results are not greater than or equal to zero then the problem has not reached optimality. You must reassign by changing the allocations to occupy the cell which is less than zero.

The work and final results are shown in Fig. 9 and 10. As our problem proves optimal by means of the two equations, it only remains to find the total optimal cost. However, in deriving the answer we discovered as well that this was a special case as there are two patterns of optimality. These are:

UNITS		**COST**		**TOTAL**
6	x	5	=	30
1	x	6	=	6
5	x	5	=	25
	Total			61

UNITS		**COST**		**TOTAL**
6	x	6	=	36
1	x	5	=	5
5	x	4	=	20
	Total			61

Can optimality be arrived at in both cases? The answer is obviously yes, since the results are equal. This then is the solution and pattern to reach it.

CONCLUSION

In conclusion the method used to solve this problem is a simplified version of the original. Having solved the equation for forward flow the doors to backward flow and series flow are opened and future research may be devoted to establishing a simpler method of factoring the problem. However, the final result should be a simplified and easiêr method to work the problem opening up its use for calculations of trucking in the near future.

ACKNOWLEDGEMENTS

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myself, and through whose help I was able to understand the necessary and sufficient conditions for the optimization.

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