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The Purpose of this paper is to apply a branch of mathematics which has recently come to the fore in investigations in the social sciences, namely automata theory. This application will show how automata theory may be used in the formal description of a wide variety of social phenomena.

My basic plan will be to start by describing a broad class of social systems in terms of game-theoretical notions, especially the notions of strategy and strategy mixture. Then I will define automata and show how, under a certain translation of terms, every social system of the class I have taken turns out to be an automaton.

Let us first introduce certain notions of game theory: we take a certain set of external states G_m^t which are potentially relevant to the behavior of any participant in a social system at any time t, where m is the number of such states. We will consider the set of states to contain a record of all past behavior of all participants in the social system, so that any particular participant can consider this in making his future decisions. We let I_n be the set of n participants and S_i the strategy set of the i^{th} participant. The strategy set includes all possible actions that a participant might undertake under the various circumstances he might observe to hold and which are elements of the set G_m^t of external states.

Now as is well known, in many social situations, it is preferable to randomize one's choice of actions so as not to let those with whom one might be in competition be able to predict perfectly one's own actions, since the opponents might then take advantage of their perfect knowledge. If, for example, a batter in a baseball game knew exactly what sequence of curves and fastballs a pitcher were going to throw, he would certainly raise his batting average. Now the problem of randomizing strategies involves the introduction of probability in order to at least describe the proportion of the kinds of actions one chooses, even if their exact sequences be random. Even though an observer could not predict any individual actions the participant might undertake, he would still know their *proportions* and therefore the *probability* that any particular action will be undertaken. The set of probabilities describing the actions of participant i is called a *strategy mixture* and is designated by '$\sigma_i^{k,t}$', which is a function of external state or circumstance k and time t.

At this point we consider an important property of the class of social systems we wish to analyze in this paper. This property is descriptive of those systems whose behavior has a certain minimal stability. This stability is such that all participants in the social system develop certain fixed strategy mixtures which they then retain during the rest of their participation in the social system. We assume that the participants, depending on their personalities, begin their participation with a predisposition towards certain mixtures, which are then changed to new mixtures according to information received in the course of participation and according to other inherent predispositions. Looked at formally, this means we may ascribe to each participant i a transition probability for each strategy mixture $\sigma_i^{k,t}$ that i will accept it, based on his observation of a prior state of the world, viz. a member of G_m^{t-1}. and his former acceptance of another strategy mixture $\sigma_i^{k,\, t-1}$.

It should be mentioned here that the question of stability plays a central role in game-theoretical and decision-theoretical investigations. The approach usually taken in this regard is to apply the notion of dominance to see if a social system defined by the values held by its participants tends to develop stability. The simplest case to investigate is the one where it can be calculated that one strategy dominates all others, and we are also sure that it will eventually be chosen. Other cases are more complicated, e.g. where dominance holds for given strategies only among restricted subsets of the entire set of strategies. I cannot go more deeply into these issues here, but refer those interested in this problem to standard works, such as those of Luce and Raiffa (1) and Fishburn (2). It suffices for the discussion at hand to assume that stability exists for the class of social systems we are treating here and to see what consequences this has in a more formal way.

Let us now recapitulate in a list the concepts we have been discussing:

G_m^t set of m possible external states at time t, including a record of payoffs for all strategies used by participants prior to t

I_n set of n participants in the social system

S_i strategy set of i^{th} participant

$\sigma_i^{k,t}$ m·n strategy mixtures at time t, viz. a probability for each participant i and state k for each strategy $s_{i,h} \in S_i$ such that

$$\sigma_i^{k,t}(s_{i,h}) \geqslant 0 \quad \text{and} \quad \sum_h \sigma_i^{k,t}(s_{i,h}) = 1$$

$p_i^{k,t}$ m·n transition probabilities from $G_m^t \times \left\{\sigma_i^{k,t}\right\}$ to $\left\{\sigma_i^{k,t+1}\right\}$

Using the concepts discussed above, we can now define stability of a social system as occurring whenever all participants in that system, after a preliminary period of adjustment, settle down on a single strategy mixture. Formally, this may be defined as follows: if we let F be the set of stable strategy mixtures, it will have the following property:

$$\sigma_i^{k,t} \in F \quad \text{iff}_{df} \quad \exists t' \forall t''(t'' > t' \rightarrow p_i^{k,t''}(\sigma_i^{k,t}) = 1).$$

Now it will be my purpose to show that stable social systems as characterized above are probabilistic automata (3). To do this, I define probabilistic automata, and then we examine the stable social systems to see if they have the defining properties of probabilistic automata. These are defined as follows. Basically, an automaton is any process which begins in some state, is stimulated by an input and thereupon enters into a new state. The rule which governs its changes of state is given by a transition function M. Furthermore, an automaton is defined by the initial state it is in (because being in one rather than another initial state may prevent it from ever going into some states); and the automaton is defined by a set F of final states, which may be called its "goal states." Formally, we define *probabilistic* automata (where the transitions given by M are not onto definite states, but only onto probabilities of states, which is appropriate where transitions are indeterministic) as follows: $a = \langle \Xi, A, M, a_0\ F \rangle$ is a *probabilistic automaton* iff_{df}

$\Xi \quad = \quad \xi_0,\ldots,\xi_p$ is the set of (elementary) inputs

$A \quad = \quad a_0,\ldots, a_q$ is the set of (internal) states

M is a transition function onto probabilities, from $A \times \Xi$ onto $[0,1]^{q+1}$

(this is a q+1 matrix), such that for any pair $(a,\xi) \in A \times \Xi$;
$M(a,\xi) = (p_0(a,\xi),\ldots, p_q(a,\xi))$, and $p_h(a,\xi) \geqslant 0$ and $\sum_h p_h(a,\xi) = 1$

a_0 is the initial (internal) state of a

F is the set of final states

Let us see how the stable social systems may be understood as automata. Perhaps the basic step is to determine what in the social system is to be taken as its internal state. As a preliminary step, we might consider the case of a single participant. Clearly, the state of the individual participant which interests us is whichever strategy mixture he has chosen for his actions; and the external stimulus which guides his actions is the state $S^k \in G_m$ which holds at that time. We may assume that G_m is the same for everyone, and that in particular, the past actions of other participants is public knowledge for all participants. Everyone acts on the same knowledge about external events, and hence the external state S^k can be taken as the input for the social system as a whole. But what about the internal state of the social system? This is determined by the internal states of all the participants, and hence may be taken as the n-tuple of all n participants' strategy mixtures at any time, viz. $\langle\sigma_1^{k,t},\ldots, \sigma_n^{k,t}\rangle$. The set of all these n-tuples corresponds to the set A of internal states of the automation. The set G_m corresponds to the set Ξ of external inputs. The individual probability functions $p_i^{k,t}$ do not correspond directly to the transition function M because the latter yields probabilities of internal states of the automaton, whereas the former is a probability function merely of an individual participant's internal state. What we need is a probability of the n-tuple of strategy mixtures which is the internal state of the social system: this is what we are interpreting as the automaton, and not the individual participant. For this purpose, a new probability function is introduced, $P^{k,t}$, which is determined by the $p_i^{k,t}$'s, and is a function of $\langle\sigma_1^{k,t},\ldots, \sigma_n^{k,t}\rangle$. The set of all these new P functions corresponds to the domain of the automaton's transition function M, so that we have the following situation: for every n-tuple of strategy mixtures which hold for the social system at any time t and for every state of the world, that is, for every $\langle\sigma_1^{k,t},\ldots, \sigma_n^{k,t}\rangle$ and S^k, there is a uniquely determined $P^{k,t}(\sigma_1^{t+1},\ldots, \sigma_n^{t+1})$. This is in accord with the characterization of stable social systems as given above.

Two concepts remain for the interpretation of the social system as automata: the initial and final states. The final states of the automaton clearly correspond to the set F of stable strategy mixtures. The initial states simply correspond to the n-tuple of strategy mixtures to which the participants are predisposed at the very beginning of their participation, however these are determined: whether by chance or by inherited instinct or by learning from previous participations in other social systems.

The upshot of our investigation may be set down in the following list, which gives the automaton-interpretation of our social systems which we had been looking for:

Probabilistic Automaton	**Stable Social System**
$A = [a_0,\ldots, a_q]$	$[\langle\sigma_1^{1,0},\ldots, \sigma_n^{1,0}\rangle ,\ldots, \langle\sigma_1^{m,tq},\ldots, \sigma_n^{m,tq}\rangle]$
$\Xi = [\xi_0,\ldots, \xi_p]$	$G_m = [S_1,\ldots, S_m]$
M	$[P^{1,0},\ldots, P^{m,tq}]$
F	F
a_0	$\langle\sigma_1^0,\ldots, \sigma_n^0\rangle$

My conclusion is that the interpretation of stable social systems as automata succeeds, because it has been shown that they satisfy all the formal defining conditions of automata. It may be mentioned that there are other ways to interpret social systems as automata, such as a way developed by Werner Leinfellner. This approach views social systems more from the point of view of value theory for an individual, and the system is analyzed as a participant playing against the rest of the social system; his strategies are the internal states, the strategy n – 1-tuples of the rest of the system are the external states, and the transition functional domain consists of evaluations the individual makes knowing his previous strategy mixture and the strategy mixtures of the other participants.

The principle outcome of this paper is to show how a new and fruitful mathematical technique may be applied to the analysis of social systems. It is not to present any particular social facts, but rather a way to come by them in the future.

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P. C. Fishburn: *Decision and Value Theory,* Wiley, 1964; cf. Section 3.7 "Dominance among Strategies" and Chapters 5 and 6.

Cf. the definition given in M. O. Rabin: "Probabilistic Automata" in E. F. Moore: *Sequential Machines*, Addison-Wesley, 1964.