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Stayability Evaluation as a Categorical Trait and by Considering Other Traits

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ABSTRACT

Correspondence between usual categorical scores where frequencies sum to one and stayability of herd life measured as survival to several age periods is illustrated. The singularity of the variance-covariance matrix of categorically measured traits when each category is a separate trait also is true for stayability, since the measurement for the last period for stayability is always zero, has variance zero, and covariances zero with measurements in preceding periods. Best linear unbiased prediction for stayability of a bull's daughters is achieved by ignoring the last period. A procedure is described for predicting stayability of daughters having variable opportunity for stayability measurements that utilizes production and all available measures of stayability.

INTRODUCTION

Stayability of a bull's daughters is the fraction surviving to a specified age. Everett et al. (4) chose survival to ages 36, 48, 60, 72, and 84 mo as separate stayability traits. Stayability currently is being evaluated at 48 mo (11). The evaluation considers only daughters with opportunity to have survived 48 mo. Stayabilities at various ages make up a major component of profitability (1) when lifetime yields by daughters and their descendants are considered. In these uses, stayability is a categorically scored trait. The purposes of this paper are 1) to show the relationship between mutually exclusive categories of period when survival ceases and stayability, and, more importantly, 2) to propose a procedure for predicting stayability using milk and survival information on all daughters, irrespective of opportunity for survival.

Prediction of frequencies of future progeny for traits scored in mutually exclusive categories such as calving difficulty was proposed with each category a separate trait (2). Descriptions of best linear unbiased prediction (BLUP) (e.g., 6, 7, 8, 10) for single trait and multiple trait evaluation, however, include the assumption that variance-covariance matrices of random elements of the model are nonsingular. Variance-covariance matrices where measurements for each category are 0 to 1 are singular, since the sum of frequencies of the categories is unity. Discussion of the problem (R. Anderson, personal communication) led to the assertion that discarding measurements in one category and using variance-covariance matrices of the remaining categories would lead to BLUP of frequencies of the categories (3). That generalized inverse of the full but singular variance-covariance matrix does satisfy the condition described by Harville (5) for BLUP. Quaas and Van Vleck (13) outlined the procedure to predict frequencies of progeny in categories when the frequencies of categories sum to unity. Stayability as defined here is a categorically scored trait, but the means do not sum to one as they do for mutually exclusive categories.

Correspondence of Stayability Measurements and Scores for Mutually Exclusive Categories of When Survival Ceases

Correspondence is shown by example. Let measurement of 36-mo stayability be 0 if the cow fails to survive past 36 mo, 1 if she survives, and similarly for survival to 48 mo, 60 mo, 72 mo, and 0 for $\geq$ 72 mo. Let measurements of failure to survive be 1 for only one of the periods $\leq$ 36 mo, 37 to 48 mo, 49 to 60 mo, 61 to 72 mo, and $>$ 72 mo and 0 otherwise. Let s be the vector of observations for stayability for the five categories and p be the vector of observations for failure to survive. The correspondence is:

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Condition			Category 1	2	3	4	5	
dies ≤ 36 mo:	s'	= (	0	0	0	0	0	)
	p'	= (	1	0	0	0	0	)
36 mo < dies ≤ 48 mo:	s'	= (	1	0	0	0	0	)
	p'	= (	0	1	0	0	0	)
48 mo < dies ≤ 60 mo:	s'	= (	1	1	0	0	0	)
	p'	= (	0	0	1	0	0	)
60 mo < dies ≤ 72 mo:	s'	= (	1	1	1	0	0	)
	p'	= (	0	0	0	1	0	)
dies > 72 mo:	s'	= (	1	1	1	1	0	)
	p'	= (	0	0	0	0	1	)

The last category of stayability always must be zero to correspond to death in the last or final period and, thus, to match the probability of death in the last period.

Let s_i be the observation for survival through period i and p_i be the observation for death in period i. Then by inspection

$$s_i = 1 - \sum_{j=1}^{i} p_j \quad .$$

The mean vector for the p's are the multinomial probabilities

$$\pi' = (\pi_1 \;\; \pi_2 \;\; \pi_3 \;\; \pi_4 \;\; \pi_5)$$

where

$$\sum_{i=1}^{5} \pi_i = 1 \quad ,$$

and the mean vector for the s's is

$$\sigma' = (\sigma_1 \;\; \sigma_2 \;\; \sigma_3 \;\; \sigma_4 \;\; \sigma_5)$$

where

$$\sigma_1 = 1 - \pi_1,$$

$$\sigma_2 = 1 - \pi_1 - \pi_2,$$

$$\sigma_3 = 1 - \pi_1 - \pi_2 - \pi_3,$$

$$\sigma_4 = 1 - \pi_1 - \pi_2 - \pi_3 - \pi_4, \text{ and}$$

$$\sigma_5 = 1 - \pi_1 - \pi_2 - \pi_3 - \pi_4 - \pi_5 = 0.$$

The phenotypic variance-covariance matrix of the p's is well known:

$$P = [P_{ij}] = \begin{bmatrix} \pi_1(1-\pi_1) & -\pi_1\pi_2 & -\pi_1\pi_3 & -\pi_1\pi_4 & -\pi_1\pi_5 \\ -\pi_1\pi_2 & \pi_2(1-\pi_2) & -\pi_2\pi_3 & -\pi_2\pi_4 & -\pi_2\pi_5 \\ -\pi_1\pi_3 & -\pi_2\pi_3 & \pi_3(1-\pi_3) & -\pi_3\pi_4 & -\pi_3\pi_5 \\ -\pi_1\pi_4 & -\pi_2\pi_4 & -\pi_3\pi_4 & \pi_4(1-\pi_4) & -\pi_4\pi_5 \\ -\pi_1\pi_5 & -\pi_2\pi_5 & -\pi_3\pi_5 & -\pi_4\pi_5 & \pi_5(1-\pi_5) \end{bmatrix}$$

Each row and each column sum to zero so that P is singular.

The phenotypic variance-covariance matrix of the s's, $V = [V_{ij}]$, can be derived in terms of the P_{ij}. For example:

$$s_1 = 1 - p_1 \qquad \text{so that}$$

$$\text{Var}(s_1) = V_{11} = \text{Var}(1-p_1) = P_{11};$$

$$s_2 = 1 - p_1 - p_2 \qquad \text{so that}$$

$$\text{Var}(s_2) = V_{22} = V(1-p_1-p_2)$$

$$= V(p_1) + V(p_2) + 2\text{Cov}(p_1, p_2)$$

$$= P_{11} + P_{22} + 2P_{12}; \text{ and}$$

$$\text{Cov}(s_1 s_2) = V_{12} = \text{Cov}(1-p_1,\, 1-p_1-p_2)$$

$$= P_{11} + P_{12}.$$

Similarly,

$$V_{33} = P_{11} + P_{22} + P_{33} + 2(P_{12} + P_{13} + P_{23})$$

$$V_{13} = P_{11} + P_{12} + P_{13}$$

$$V_{23} = P_{11} + P_{22} + 2P_{12} + P_{13} + P_{23}$$

$$V_{44} = P_{11} + P_{22} + P_{33} + P_{44} + 2(P_{12} + P_{13} + P_{14} + P_{23} + P_{24} + P_{34})$$

$$V_{14} = P_{11} + P_{12} + P_{13} + P_{14}$$

$$V_{24} = P_{11} + P_{22} + 2P_{12} + P_{13} + P_{14} + P_{23} + P_{24}$$

$$V_{34} = P_{11} + P_{22} + P_{33} + 2(P_{12} + P_{13} + P_{23}) + P_{14} + P_{24} + P_{34}$$

$$V_{55} = 0 = V_{15} = V_{25} = V_{35} = V_{45}$$

The phenotypic variance-covariance matrices can be partitioned into parts as, for example, the variance-covariance matrices associated with sire, herd-year-season (HYS), and residual effects.

Correspondences between the p's and s's are such that predictions for either system can be converted into the other system.

Prediction of Stayability

Stayability for all periods can be predicted jointly from records for all opportunity groups as well as from records on other traits as, for example, milk production by mixed model methods of Henderson (8) and Henderson and Quaas (10). Correct generalized inverses of variance-covariance matrices of survival traits must be used. If stayability traits are used, the last category, e.g., $\geqslant$ 72 mo, usually will be ignored. If traits associated with periods of death are used, then any category can be ignored but usually will be the last.

Let the model be

$$y = X\beta + Zu + e,$$

where the observation vector on n cows, y, is $y' = (y_1' \; y_2' \; \ldots \; y_n')$ with y_i the observations on cow i which would be other traits and the number of stayability observations associated with her opportunity group, X is the incidence matrix for fixed effects associated with the observations, β is the vector of fixed effects, Z is the incidence matrix for random effects, which may have some zero columns, u is the vector of random effects for herd-year-seasons and sires with the traits in order within sires and herd-year-seasons, and e is a vector of random residual effects.

$$E[y] = X\beta \quad \text{and}$$

$$V\begin{bmatrix} u \\ e \end{bmatrix} = V\begin{bmatrix} h \\ s \\ e \end{bmatrix} = \begin{bmatrix} I_h * H_c & O & O \\ O & I_s * S_c & O \\ O & O & R \end{bmatrix}$$

The variance-covariance matrices for the c traits are

H_c for herd-year-season effects,
S_c for sire effects, and
I_h and I_s are identity matrices of the order of number of herd-year-seasons and sires, respectively. If sires are related, I_s is replaced by A, the matrix of numerator relationships among the sires (9),

* denotes the direct product operation which, for example, makes V(h) into a block diagonal matrix with diagonal blocks of H_c,

R is a block diagonal matrix with diagonal blocks associated with an animal and the number of traits measured on the animal. For example, if

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} & r_{15} \\ & r_{22} & r_{23} & r_{24} & r_{25} \\ & & r_{33} & r_{34} & r_{35} \\ \text{Symmetric} & & & r_{44} & r_{45} \\ & & & & r_{55} \end{bmatrix}$$

is the variance-covariance matrix associated with all five traits, then the block diagonal for an animal with only the first record is $r_{11} = R_1$; for an animal with two traits, the block is

$$\begin{bmatrix} r_{11} & r_{12} \\ r_{12} & r_{22} \end{bmatrix} = R_2\text{; etc.}$$

The general mixed model equations (8) are

$$\begin{bmatrix} X'R^{-1}X & X'R^{-1}Z \\ Z'R^{-1}X & Z'R^{-1}Z+G^{-1} \end{bmatrix} \begin{bmatrix} \hat{\beta} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} X'R^{-1}y \\ Z'R^{-1}y \end{bmatrix}$$

where

$$G = \begin{bmatrix} I_h{*}H & O \\ O & I_s{*}S \end{bmatrix}$$

Due to the nature of the equations, the coefficients can be written simply (12, 13). Let

$$R_1 = \begin{bmatrix} r_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$R_2 = \begin{bmatrix} r_{11} & r_{12} & 0 & 0 & 0 \\ r_{12} & r_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{etc.}$$

Let n_{1ij} be the number of measurements in herd-year-season i of daughters of sire j on cows with an observation only on trait 1, n_{2ij} be the number of daughters measured only on traits 1 and 2, etc.

Then the equations are:

$$\left[\begin{array}{ccc|c|ccc} \sum_k n_{k1.}R_k^- & & 0 & \sum_k n_{k1.}R_k^- & (\sum_k n_{k1j}R_k^-\ j=1,\ldots,s) & & \\ & \ddots & \quad + H^{-1} & \vdots & \vdots & & \\ 0 & & \sum_k n_{kh.}R_k^- & \sum_k n_{kh.}R_k^- & (\sum_k n_{khj}R_k^-\ j=1,\ldots,s) & & \\ \hline & & & \sum_k n_{k..}R_k^- & (\sum_k n_{k.j}R_k^-\ j=1,\ldots,s) & & \\ \hline & & & & \sum_k n_{k.1}R_k^- & & 0 \\ & \text{SYMMETRIC} & & & & \ddots & \quad + S^{-1} \\ & & & & 0 & & \sum_k n_{k.s}R_k^- \end{array}\right] \left[\begin{array}{c} \hat{h}_1 \\ \vdots \\ \hat{h}_h \\ \hline \hat{\mu} \\ \hline \hat{s}_1 \\ \vdots \\ \hat{s}_s \end{array}\right] = \left[\begin{array}{c} \sum_k R_k^- y_{k1..} \\ \vdots \\ \sum_k R_k^- y_{kh..} \\ \hline \sum_k R_k^- y_{k...} \\ \hline \sum_k R_k^- y_{k.1.} \\ \vdots \\ \sum_k R_k^- y_{k.s.} \end{array}\right]$$

where y_{kij} is the vector of totals of daughters of sire j which have opportunity for measurements on the first k traits in HYS i with other elements equal to zero. When $k = 1$, only the first element of the vector is nonzero; when $k = 2$, the nonzero elements will be totals for traits one and two on those cows having opportunity for just those two traits; and R_k^- is the generalized inverse of R_k obtained from the direct inverse of the nonzero block augmented by zeros in the remaining rows and columns.

Example

In the example, herd-year-seasons will be ignored, but in practice each HYS set of equations probably would be absorbed after they are accumulated. Trait 1 will be milk yield, and traits 2 through 4 will be measures of stayability to 36, 48, 60, and 72 mo. The example will consider only the two cases where 1) all animals have an opportunity to stay in the herd more than 72 mo, and 2) all animals have an opportunity to survive more than 48 mo. Suppose

$$R_5 = \begin{bmatrix} 742.19 & 3.60175 & 2.90100 & 2.36750 & 1.67025 \\ & .15500 & .13550 & .11600 & .09650 \\ & & .20350 & .17425 & .14500 \\ \text{SYMMETRIC} & & & .23250 & .19350 \\ & & & & .24200 \end{bmatrix} \text{, and}$$

$$S_5 = \begin{bmatrix} 39.06 & .26525 & .25200 & .21650 & .16775 \\ & .00500 & .00450 & .00400 & .00350 \\ & & .00650 & .00575 & .00500 \\ & & & .00750 & .00650 \\ \text{SYMMETRIC} & & & & .00800 \end{bmatrix}$$

Sire 1 has 10 daughters with total milk production of 1500 in the first lactation (milk to nearest 100 units); 8 survived to 36 mo, 7 to 48 mo, 4 to 60 mo, 2 to 72 mo, and all are dead sometime after 72 mo.

Sire 2 has 20 daughters with total milk production of 2800; 18 survived to 36 mo, 16 to 48 mo, 10 to 60 mo, 6 to 72 mo, and all are dead sometime after 72 mo.

The mixed model equations become:

$$
\left[\begin{array}{ccccccccccccccc}
.0457 & -1.1781 & .0786 & -.0769 & .1688 & .0152 & -.3927 & .0262 & -.0256 & .0563 & .0305 & -.7854 & .0524 & .0513 & .1125 \\
 & 493.49 & -310.51 & 1.9833 & -4.1916 & -.3927 & 164.50 & -103.50 & .6611 & -1.3972 & -.7854 & 329.00 & -207.01 & 1.3222 & -2.7944 \\
 & & 617.10 & -308.62 & .2895 & .0262 & -103.50 & 205.70 & -102.87 & .0965 & .0524 & -207.01 & 411.40 & -205.74 & .1930 \\
 & & & 617.10 & -308.77 & -.0256 & .6611 & -102.87 & 205.70 & -102.92 & -.0513 & 1.3222 & -205.74 & 411.40 & -205.84 \\
 & & & & 371.19 & .0563 & -1.3972 & .0965 & -102.92 & 123.73 & .1125 & -2.7944 & .1930 & -205.84 & 247.46 \\
 & & & & & .0557 & -2.3487 & -.3014 & -.2207 & .4263 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & 789.70 & -451.31 & 10.086 & -24.187 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & 935.62 & -464.93 & -2.8972 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & & 933.91 & -468.34 & 0 & 0 & 0 & 0 & 0 \\
 & \text{SYMMETRIC} & & & & & & & & 549.72 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & & & & .0795 & -2.7414 & -2.752 & -.2464 & .4826 \\
 & & & & & & & & & & & 954.20 & -554.82 & 10.747 & -25.584 \\
 & & & & & & & & & & & & 1141.3 & -567.80 & -2.8007 \\
 & & & & & & & & & & & & & 1139.61 & -571.26 \\
 & & & & & & & & & & & & & & 673.44
\end{array}\right]
\left[\begin{array}{c}
\hat{\mu}_1 \\ \hat{\mu}_2 \\ \hat{\mu}_3 \\ \hat{\mu}_4 \\ \hat{\mu}_5 \\ \hat{s}_{11} \\ \hat{s}_{21} \\ \hat{s}_{31} \\ \hat{s}_{41} \\ \hat{s}_{51} \\ \hat{s}_{12} \\ \hat{s}_{22} \\ \hat{s}_{32} \\ \hat{s}_{42} \\ \hat{s}_{52}
\end{array}\right]
=
\left[\begin{array}{r}
5.5982 \\ 20.5884 \\ 71.3244 \\ -40.2745 \\ -24.3228 \\ 1.9900 \\ .2277 \\ 23.9879 \\ -13.6335 \\ -9.0328 \\ 3.6082 \\ 20.3609 \\ 47.3365 \\ -26.6410 \\ -15.2899
\end{array}\right]
$$

The solutions are:

Trait	$\hat{\mu}$	$\hat{s}_1$	$\hat{s}_2$
1	143.93	1.79	−1.79
2	.865	−.006	.006
3	.765	−.006	.006
4	.464	−.008	.008
5	.263	−.010	.010

The $\mu \times \mu$ block is $30R_5^{-1}$, the $\mu \times$ sire 1 block $10R_5^{-1}$, the $\mu \times$ sire 2 block is $20R_5^{-1}$, the sire 1 diagonal block is $10R_5^{-1} + S_5^{-1}$, and the sire 2 block is $20R_5^{-1} + S_5^{-1}$. The right-hand sides (RHS's) for the equations are:

$$
R_5^{-1}\begin{bmatrix} 1500 + 2800 \\ 8 + 18 \\ 7 + 16 \\ 4 + 10 \\ 2 + 6 \end{bmatrix}
$$

The RHS's for the s_1 and s_2 are equations are:

$$
R_5^{-1}\begin{bmatrix} 1500 \\ 8 \\ 7 \\ 4 \\ 2 \end{bmatrix} \quad \text{and} \quad R_5^{-1}\begin{bmatrix} 2800 \\ 18 \\ 16 \\ 10 \\ 6 \end{bmatrix}
$$

If the daughters of sire 1 and sire 2 had opportunity to survive for 48 mo or more, then the total vectors for sire 1 and sire 2 would be:

$$
\begin{bmatrix} 1500 \\ 8 \\ 7 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 2800 \\ 18 \\ 16 \end{bmatrix}
$$

Now R_3 should be used, but S remains the same as before

$$
R_3 = \begin{bmatrix} 742.19 & 3.60175 & 2.90100 \\ & .15500 & .13550 \\ \text{SYMMETRIC} & & .20350 \end{bmatrix}
$$

The mixed model equations now become:

$$
\begin{bmatrix}
.0456 & -1.1759 & .1328 & .0152 & -.3920 & .0443 & 0 & 0 & .0304 & -.7839 & .0885 & 0 & 0 \\
 & 493.44 & -311.79 & -.3920 & 164.48 & -103.93 & 0 & 0 & -.7839 & 328.96 & -207.86 & 0 & 0 \\
 & & 353.13 & .0443 & -103.93 & 117.71 & 0 & 0 & .0885 & -207.86 & 235.42 & 0 & 0 \\
 & & & .0557 & -2.3480 & -.2833 & -.1951 & .3700 & 0 & 0 & 0 & 0 & 0 \\
 & & & & 789.69 & -451.74 & 9.4248 & -22.790 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & 847.63 & -362.06 & -2.9937 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & 728.21 & -365.42 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & 425.99 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & & .0709 & -2.7399 & -.2390 & -.1951 & .3700 \\
 & & & & & & & & & 954.17 & -555.67 & 9.4248 & -22.790 \\
 & & & & & & & & & & 965.34 & -362.06 & -2.9937 \\
 & & \text{SYMMETRIC} & & & & & & & & & 728.21 & -365.42 \\
 & & & & & & & & & & & & 425.99
\end{bmatrix}
\begin{bmatrix}
\hat{\mu}_1 \\ \hat{\mu}_2 \\ \hat{\mu}_3 \\ \hat{s}_{11} \\ \hat{s}_{21} \\ \hat{s}_{31} \\ \hat{s}_{41} \\ \hat{s}_{51} \\ \hat{s}_{12} \\ \hat{s}_{22} \\ \hat{s}_{32} \\ \hat{s}_{42} \\ \hat{s}_{52}
\end{bmatrix}
=
\begin{bmatrix}
5.6199 \\ 20.0614 \\ 19.5494 \\ 1.9978 \\ .0375 \\ 5.8929 \\ 0 \\ 0 \\ 3.6221 \\ 20.0240 \\ 13.6565 \\ 0 \\ 0
\end{bmatrix}
$$

The solutions are:

Trait	$\hat{\mu}$	$\hat{s}_1$	$\hat{s}_2$
1	143.93	1.79	−1.79
2	.865	−.006	.006
3	.765	−.006	.006
4	....	−.006	.006
5	....	−.007	.007

To predict stayability for 60 and 72 mo, outside estimates of mean frequencies are needed although differences among sires can be predicted from the solutions.

If combinations of opportunity groups were used, R_3 would be augmented by two rows and two columns of zeros, R_2 by 3 rows and columns, etc.

Prediction of Multinomial Frequencies

The procedure for prediction of categorical frequencies from records classified according to time of leaving the herd is almost exactly the same as for stayability. The equations have the same form if the generalized inverses are chosen to eliminate the last category, i.e., death later than 72 mo.

The totals on the RHS's for $\hat{\mu}$, $\hat{s}_1$, and $\hat{s}_2$ are:

$$\begin{bmatrix} 4300 \\ 4 \\ 3 \\ 9 \\ 6 \end{bmatrix}, \begin{bmatrix} 1500 \\ 2 \\ 1 \\ 3 \\ 2 \end{bmatrix} \text{ and } \begin{bmatrix} 2800 \\ 2 \\ 2 \\ 6 \\ 4 \end{bmatrix} \text{ or } \begin{bmatrix} 4300 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 1500 \\ 2 \\ 1 \end{bmatrix} \text{ and } \begin{bmatrix} 2800 \\ 2 \\ 2 \end{bmatrix}$$

if opportunity is to more than 48 mo.

$$R_5 = \begin{bmatrix} 742.18 & -3.6017 & .7007 & .5335 & .6972 \\ & .1550 & -.0195 & -.0195 & -.0195 \\ & & .0875 & -.0098 & -.0098 \\ \text{SYMMETRIC} & & & .0875 & -.0098 \\ & & & & .0875 \end{bmatrix},$$

$$S_5 = \begin{bmatrix} 39.062 & -.2652 & .0132 & .0355 & .04875 \\ & .0050 & -.0005 & -.0005 & -.0005 \\ & & .0025 & -.00025 & -.00025 \\ \text{SYMMETRIC} & & & .0025 & -.00025 \\ & & & & .0025 \end{bmatrix}$$

The solutions for all opportunities and for opportunity to go to 48 mo are:

	Opportunity past 72 mo	Opportunity past 48 mo
$\hat{\mu}_1$	143.93	143.93
$\hat{\mu}_2$	.1354	.1354
$\hat{\mu}_3$	.100	.1000
$\hat{\mu}_4$	.3005	. . . .
$\hat{\mu}_5$	.2007	. . . .
$\hat{s}_{11}$	1.79	1.79
$\hat{s}_{21}$	.0064	.0062
$\hat{s}_{31}$	−.0001	−.0002
$\hat{s}_{41}$	.0015	−.0000
$\hat{s}_{51}$	.0021	.0011
$\hat{s}_{12}$	−1.79	−1.79
$\hat{s}_{22}$	−.0064	−.0062
$\hat{s}_{32}$	.0001	.0002
$\hat{s}_{42}$	−.0015	.0000
$\hat{s}_{52}$	−.0021	−.0011

These solutions can be converted to those obtained from the stayability measurements by the conversion equations given earlier for the means.

DISCUSSION

There are two major unanswered questions. The procedure requires estimates of variances and covariances which are unbiased by selection. Stayability, almost by definition, involves selection. For example, only cows with relatively high milk yield in first lactation may be allowed to have further lactations (stayability). If milk yield in the first lactation can be considered a nonselected trait, then use of proper variances and covariances will result in prediction of stayability unbiased by selection (8). The procedure of Rothschild et al. (15) might result in unbiased estimates of the variances and covariances. The computing difficulties may be much more severe with more than two traits (14).

The second question deals with inclusion of fixed effects other than a mean vector in the model. Theoretically, if frequencies change with fixed effects, the variance-covariance structure also changes. Whether relatively small changes have much influence is unknown. Biases in prediction of sire effects may occur when herd-year-season effects are correlated with sire effects. Biases can be removed by computing as if herd-year-season effects are fixed for continuous traits (8). Can this be done also for categorical traits or must computation always be with herd-year-season effects random?

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REFERENCES

1 Bakker, J. J., R. W. Everett, and L. D. Van Vleck. 1980. A method of calculating a profitability index for sires. J. Dairy Sci. (Accepted).

2 Cady, R. A., R. D. Anderson, and L. D. Van Vleck. 1978. Evaluation of sires for calving difficulty. Page 96 *in* Genetics research. 1977–78 Report to Eastern AI Coop., Inc.

3 Cady, R. A., and L. D. Van Vleck. 1979. Evaluation of bulls for calving difficulty. Page 74 *in* Genetics research. 1978–79 Report to Eastern AI Coop., Inc.

4 Everett, R. W., J. F. Keown, and E. E. Clapp. 1976. Production and stayability trends in dairy cattle. J. Dairy Sci. 59:1532.

5 Harville, D. 1976. Extension of the Gauss-Markov theorem to include the estimation of random effects. Ann. Stat. 4:384.

6 Henderson, C. R. 1973. Sire evaluation and genetic trend. Page 10 *in* Proc. Anim. Breeding Genet. Symp. in Honor of Dr. Jay L. Lush. Am. Soc. Anim. Sci. and Am. Dairy Sci. Assoc., Champaign, IL.

7 Henderson, C. R. 1974. General flexibility of linear model techniques for sire evaluation. J. Dairy Sci. 57:963.

8 Henderson, C. R. 1975. Best linear unbiased estimation and prediction under a selection model. Biometrics 31:423.

9 Henderson, C. R. 1976. A simple method for computing the inverse of a numerator relationship matrix used in prediction of breeding values. Biometrics 32:69.

10 Henderson, C. R., and R. L. Quaas. 1976. Multiple trait evaluation using relatives' records. J. Anim. Sci. 43:1188.

11 Keown, J. 1979. Cow stayability. . .How do you measure it? Eastern Artif. Breeders Coop. 36:2.

12 Lee, A. J. 1979. Mixed model, multiple trait evaluation of related sires when all traits are recorded. J. Anim. Sci. 48:1079.

13 Quaas, R. L., and L. D. Van Vleck. 1980. Categorical trait sire evaluation by best linear unbiased prediction of future progeny category frequencies. Biometrics 36:117.

14 Rothschild, M. F., and C. R. Henderson. 1979. Maximum likelihood estimates of parameters of first and second lactation milk records. J. Dairy Sci. 62:990.

15 Rothschild, M. F., C. R. Henderson, and R. L. Quaas. 1979. Effects of selection on variances and covariances of simulated first and second lactations. J. Dairy Sci. 62:996.