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A Modified Theory of Gravitation

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A MODIFIED THEORY OF GRAVITATION[1]

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[1]Costs of publication of this paper were partially defrayed by the author because of its length, its controversial nature and differences of opinion, pro and con, by the reviewers. It is here published on recommendation of the Executive Committee. – Editor

This paper is an approach to the problem of establishing finite limits and values for mass, energy, and force and their relation to our physical world. In view of new discoveries in astronomy and elementary particle studies, there is a need for a theory that includes possible physical limits of matter and their kinderance to cosmological numbers presently undefined. These relationships, in order to be valid or useful, must show common physical properties to both the microcosmic and macrocosmic worlds. This implies a mathematical theory which unitizes our present knowledge with new information to explain the phenomenon of our physical world under condition of two extremes.

The theory presented basically suggests threshold values for discrete particles or masses as they move towards the speed of light in an accelerated frame of reference. The proposals set forth may appear, in context, to be contrary to the concept of infinite mass or the concept of indetermancy of mass. However, the theory proposes an extention of both the Newtonian and Einsteinian principles which gives rise to a new mathematical approach as to the "smallness and bigness" of the universe. In addition, the proposals include a method by which to link Einstein's mass energy equation with Planck's constant and Newton's gravitational constant, in regard to nuclear properties and their broad extention in cosmological principles.

Some of the conclusions resulting from the investigation indicate as follows: (1) quantization of Einstein's mass energy equation; (2) extention of Newtonian mechanics; (3) a finite gravitational mass in which no fundamental particle may exceed this mass as it moves towards the speed of light; (4) energies of quasi-steller bodies are related to discrete and finite masses which are associated with Newton's gravitational constant; (5) application of the Correspondence Principle to Galilean and Newtonian mechanics and their relation to quantum physics; (6) proposed quantitized gravitational equations to explain central nuclear forces and energies; (7) an extension of the Fitzgerald-Lorenz transformation equations; (8) significance and meaning of the elusive value 137 and it's relation to two-pi (2π); (9) application of large cosmological numbers and their relation to the finite world; (10) application of Special Relativity to Newtonian gravity; (11) application of Special Relativity to quantum mechanics; (12) an algebraic solution to general relativity and Poisson's equation of the Newtonian theory.

The writer wishes at the outset of this paper to offer an algebraic presentation of the derived equations and in the concluding portion of the paper give explanations and applications of the equations in support of the foregoing contentions.

A Revised Non-relativistic Approach to Galilean-Newtonian Mechanics in an Accelerated Frame of Reference

The assumptions set forth in this undertaking are by no means conclusive or absolute. These are attempts to bring forth certain enigmas that exist in concurrence with the physical world between uniform and accelerated motion. Specific consideration is given to the Lagrangian field concept in which a mass is constantly accelerated by the intensity of the field towards speed of light in a non-relativistic frame of reference. It is necessary to clarify at this particular time that a non-relativistic algebraic approach is used in order to relate the basis principle of this theory to the relevant assumptions which lead to an expanded relativistic concept at the conclusion of this paper.

If one were to assume the condition of the well known Lagrangian gravitational field concept in which the force of gravity is constant with height, this concept is valid, provided that this force is normal to an infinitely uniform surface. The prime objective of this assumption is to establish a fixed length and time in terms of a constant acceleration in a system of reference. The above contention is by no means restricted and may apply for any constant acceleration. Therefore, for the purpose of demonstration, the magnitude of the earth's gravitational constant at the surface is used in this paper. Hopefully, the proposed example will demonstrate how any gravitational constant facilitates in understanding the parameter of the speed of light in Einstein's mass energy equation. Since the theory of Special Relativity is restricted to uniform motion, it was on this precept that the author became interested in the relation between translational and accelerated motion.

It was noted that a non-relativistic mass starting from a rest position and constantly accelerated by a Lagrangian gravitational field at 980 cm/sec^2 or to the nearest magnitude 10^3 cm/sec^2, until it travels to its terminal velocity at the speed of light will cover a distance of $\cong 4.5 \times 10^{17}$ cm. in $\cong 3 \times 10^7$ secs. Meanwhile, a beam of light traveling for the same interval of time will cover a distance twice as much as the distance covered by a mass with a constant acceleration. Therefore, in order for events (A) and (B) cited below to be in their proper perspective each occurrence must travel their respective distances in the same interval of time, in this case $\veeeq 3 \times 10^7$ secs.

(A) d $\cong 4.5 \times 10^{17}$ cm

(B) 2d $\cong 9 \times 10^{17}$ cm.

From the above implication, a basic postulate may be stated: if a mass starts from a rest position in a Lagrangian field and moves with a constant acceleration to its terminal velocity at the speed of light, the time that it takes to traverse the distance will always be one half the distance it would take light to travel in the same interval of time. Therefore, in this paper, a unit length in terms of a constant acceleration is designated as (d) and the unit length traversed by light in terms of uniform motion is expressed as (2d).

The relatively simple relation is formulated principly from the expression

$$2d = vt \tag{1}$$

which indicates uniform motion and

$$d = \frac{vt}{2} \tag{2}$$

relates to the distance covered by a constantly accelerated particle in a Lagrangran field. These relations are valid with the provision that each event is equal in time and the particle motion terminates at the speed of light. The following equations are an application of the above relation: if

$$2d = vt$$

then the square of the velocity is

$$v^2 = \frac{4d^2}{t^2}. \tag{3}$$

When equation (3) is substituted in the kinetic energy equation

$$E_k = ½mv^2 \tag{4}$$

we obtain

$$Ek = \frac{2md^2}{t^2} \tag{5}$$

In a situation, when the kinetic energy equation (5) is equal in magnitude to the potential energy

$$E_p = ma^d, \tag{6}$$

we may write the expression as

$$\frac{2md^2}{t^2} = mad. \tag{7}$$

Upon solution of this expression, we get the Galilean expression for straight-line motion under constant acceleration,

$$2d = at^2. \tag{8}$$

It is necessary, at this time, to introduce another assumption into the original proposed concept, and that is, $2d = \lambda$, (lambda being either a wavelength or a distance without preference to any distinction). Continuing with this line of reasoning, we know from the expression of linear acceleration that

$$V^2 = 2ad; \quad (9)$$

if we substitute $2d = \lambda$ in equation (9), we get

$$V^2 = a\lambda.$$

Upon substituting the acceleration in equation (9) in the Galilean expression (8) we derive the equation for uniform motion in a straight line.

$$\lambda = vt. \quad (10)$$

Therefore, from expression (9), uniform motion in a circle in which the acceleration toward the center can be written as follows: If

$$\frac{V^2}{\lambda} = a$$

and when λ is equal to $2\pi r$, then

$$\frac{V^2}{r} = 2\pi a. \quad (11)$$

The above relation is contrary to Newtonian equation for uniform motion in a circle, written as

$$\frac{V^2}{r} = a. \quad (12)$$

Again proceeding with the same train of thought, a mass starting from a rest position and constantly accelerated in a circular and/or linear path will have the following respective expressions in which

$$\frac{V^2}{\lambda} = a \quad (13)$$

or

$$\frac{V^2}{2d} = a \quad (14)$$

In a situation in which a particle starts from a rest position and moves in a circular path with a constant acceleration to its terminal velocity at the speed of light, the particle will experience both a centripetal acceleration towards the center (due to a change in direction) and a constant or tangential acceleration moving in a circular path (due to a change in speed). An illustration will best demonstrate the two relations in a single example, if we assume the original premise that $2d = \lambda$ and that $2d = vt$. For illustration purposes, if we envision a non-relativistic mass moving from a rest position to its terminal velocity at the speed of light in a circular path with a constant acceleration of 10^3 cm./sec.^{2}, it will enscribe a circle whose circumferential distance is one light year with a radius of 1.4×10^{17} cm. When the particle reaches its terminal velocity at the speed of light, the mass will have traveled a distance of (2d) or ($2\pi r$) which is a circle. Now if the particle continues in the same circular path with a constant speed, the increase in centripetal acceleration towards the center from its initial constant acceleration of 10^3 cm/sec.^{2} must increase by (2π) to retain the original orbital path. In the second event, if a particle decreases its acceleration toward the center by (2π) upon returning to its initial position, the radius will expand to 9×10^{17} cm. to adjust to the change in angular momentum, and the acceleration towards the center will be 10^3 cm./sec.^{2}. The above relations may be expressed mathematically by the use of the Modified Newtonian equation. Hence we could write,

$$2d = at^2$$
$$2\pi r = 10^3 \times 9 \times 10^{17}$$
$$2\pi r = 9 \times 10^{17}$$
$$r = 1.4 \times 10^{17} \text{ cm.}$$

The acceleration towards the center is

$$\frac{V^2}{r} = 2\pi a$$
$$\frac{9 \times 10^{20}}{1.4 \times 10^{17}} = 6.3 \times 10^3 \text{ cm./sec.}^2$$

therefore, its linear acceleration or constant acceleration in a circular orbit is

$$\frac{V^2}{2\pi r} = a$$
$$\frac{9 \times 10^{20}}{9 \times 10^{17}} = 10^3 \text{ cm./sec.}^2$$

since $2d = 2\pi r$. The same application can be repeated for the second example cited.

$$\frac{V^2}{r} = 2\pi a$$
$$\frac{9 \times 10^{20}}{9 \times 10^{17}} = 10^3 \text{ cm./sec.}^2$$

In the second example, the acceleration towards the center now becomes 10^3 cm./sec.^{2} and the linear acceleration is determined below as,

$$\frac{V^2}{2\pi r} = a$$
$$\frac{9 \times 10^{20}}{2\pi \times 9 \times 10^{17}} = 1.65 \times 10^2 \text{ cm./sec.}^2.$$

A reinterpretation of the original premise may now be stated in terms of energy. If a particle starts from a rest position in a circular Lagrangian gravitational field and undergoes a constant acceleration to its terminal velocity at the speed of light to a distance of (d) or (πr) then the total energy of the system will enscribe a circular path which is equivalent to twice its accelerated distance of (πr). A semicircle (d) or (πr) will represent the kinetic energy and the circumference of the circle (2d) or (2πr) shall indicate the total energy.

The total energy of a system is the addition of the kinetic and potential energies. We can now write an equation for the total energy in terms of acceleration. If the Hamiltonian total energy is

$$E_T = E_K + E_p, \tag{15}$$

then substituting in equation (15), the kinetic energy expression (5) and for the potential energy equation (7) we get the relation

$$E_T = \frac{2md^2}{t^2} + mad. \tag{16}$$

From the assumption 2d = λ, we may write the total energy as

$$E_T = \frac{m\lambda d}{t^2} + mad \tag{17}$$

and when

$$t^2 = \frac{2d}{a} \tag{18}$$

the total energy becomes

$$E_T = mad + mad$$

$$E_T = 2mad \tag{19}$$

or

$$E_T = ma\lambda. \tag{20}$$

The singular aspect of the above total energy equation is that when ($a\lambda$) is equal to (c^2), the mass energy equation of Einstein's becomes apparent. In that, the acceleration (a) times the distance (λ) cannot exceed the velocity squared. If the accelerated mass starting from a rest position is of a small magnitude, greater is the distance covered before reaching the speed of light. Conversely, a mass starting from a rest position with a large initial acceleration, shorter is the distance traveled before reaching the speed of light.

The Application of the Correspondence Principle to Modified Newtonian Mechanics

The Correspondence Principle has always been difficult to apply to

Newtonian mechanics. An attempt is made by the use of the Correspondence Principle to demonstrate the facility of the expression

$$\frac{v^2}{\lambda} = a$$

in deriving the basic principles of Newton by the use of algebra. In the modified equations forthcoming, lambda (λ) or (2d) will indicate a circular distance and (r) the radial distance, in which (d) or (πr) indicates the distance of a semicircle. If we equate $2d = vt$ with $\lambda = 2\pi r$, we obtain the expression

$$vt = 2\pi r. \tag{21}$$

By squaring both sides of the equation and then solving for the velocity squared, we can write this expression as

$$v^2 = \frac{4\pi^2 r^2}{t^2} \tag{22}$$

and the Newtonian centripetal acceleration as

$$\frac{v^2}{r} = \frac{4\pi^2 r}{t^2} \;. \tag{23}$$

Using the same equation as above, we can write the linear acceleration as

$$\frac{v^2}{2\pi r} = \frac{2\pi r}{t^2} = a. \tag{24}$$

If we substitute the linear acceleration (24) into the expression for the inertial force

$$F = ma, \tag{25}$$

then the force becomes

$$F = \frac{m 2\pi r}{t^2} \;. \tag{26}$$

We then square the expression $2d = vt$, and solve for the time square to get

$$t^2 = \frac{4d^2}{v^2} \;. \tag{27}$$

Substitute equation (27) for the time squared in equation (26) and we obtain

$$F \frac{4d^2}{v^2} = m\, 2\pi r \tag{28}$$

and the force will be equal to

$$F = \frac{mv^2 2\pi r}{4d^2} \;.$$

Since the assumption is that $2d = \lambda$, then the force becomes

$$F = \frac{mv^2}{2\pi r} \tag{30}$$

or

$$F = \frac{mv^2}{\lambda} \tag{31}$$

or

$$4\pi^2 F = \frac{mv^2}{r} \tag{32}$$

In order to solve for the gravitational force, we need to square equation (28) to get

$$F^2 \frac{16d^4}{v^4} = m^2 4\pi^2 r^2 \quad . \tag{33}$$

in which the force is written as

$$F = \frac{m^2 v^4\, 4\pi^2 r^2}{F 16 d^4} \quad . \tag{34}$$

When $2d = \lambda$, the expression becomes

$$F = \frac{m^2 v^4 4\pi^2 r^2}{F 16 \pi^4 r^4} \quad .$$

or

$$F = \frac{m^2 v^4 \lambda^2}{F \lambda^4} \quad . \tag{35}$$

Newton's gravitational constant is assumed to have the dimensions

$$G = \frac{v^4}{F} \tag{36}$$

when substituted in to equation (35) and like expressions canceled, the modified form for the gravitational force becomes

$$F = \frac{Gm^2}{\lambda^2} \quad . \tag{37}$$

The Newtonian expression where (r) is designated as the radius, may be written as

$$F_N = 4\pi^2 F = \frac{Gm^2}{r^2} \tag{38}$$

and in terms of the potential energy

$$E_N = 2\pi E = \frac{Gm^2}{r} \quad . \tag{38a}$$

Equation (38) and (38a) correspond to Newton's expression for the gravitational force and potential energy. If the assumption $v^2 = a\lambda$ is incorrect and not applicable, then the equation for the inertial force should be written as $F = m2\pi a$, in order to obtain the radius when applied to the above derivation. However, the writer believes that it would be unreasonable to accept the principle of two force equations, one radial, one circumferential as separate entities. Therefore, in the next section of this paper, an attempt will be made to apply modified Newtonian mechanics to quantum physics. It would be noted that in this paper the various accelerations are designated as follows: the linear acceleration, the linear gravitational acceleration and the quntitize linear acceleration are written as (a), the acceleration towards the center as ($2\pi a$) and the gravitational acceleration near the surface of a mass as ($4\pi^2 a$).

Galilean and Modified Newtonian Mechanics as Applied to Quantum Physics

It is the considered opinion of the writer that one of the difficulties in applying classical with quantum physics is due to the dual relationship of the expression, when written as

$$a = \frac{c^2}{\lambda} \qquad (39)$$

where (c) is the speed of light. This portion of the paper will be mainly concerned with above expression as applied to quantum physics. From the following equation

$$\lambda = \frac{c}{f} \qquad (40)$$

when equation (39) is substituted for lambda (λ) in equation (40), we get

$$f = \frac{a}{c} \qquad (41)$$

and in terms of time this becomes

$$t = \frac{c}{a} \; . \qquad (42)$$

If we substitute equation (41) in Planck's relation for the energy of the photon $E = hf$, then

$$E = \frac{ha}{c} \qquad (43)$$

and the linear acceleration for the equation becomes

$$a = \frac{mc^3}{h} \qquad (44)$$

In order to obtain a quantitized expression for the mass energy equation, substitute expression (44) in equation (20), $E_T = ma\lambda$; for the linear acceleration to get

$$E_T = \frac{m^2 c^3 \lambda}{h} \qquad (45)$$

and when

$$\lambda = \frac{c^2}{a} \qquad (46)$$

the energy of the equation becomes

$$E_T = \frac{m^2c^5}{ha} \quad . \qquad (47)$$

We may write the above relation in terms of time by substituting equation (42) in equation (47) and the expression becomes

$$E = \frac{m^2c^4t}{h} \quad . \qquad (48)$$

Since E = F xλ, the equation for the force may be written as

$$F = \frac{m^2c^4t}{h\lambda} \qquad (49)$$

and when $\lambda = ct$, then the force becomes

$$F = \frac{m^2c^3}{h} \quad . \qquad (50)$$

When we substituted the modified Newtonian equation

$$a = \frac{Gm}{\lambda^2} \quad . \qquad (51)$$

for the acceleration in the quantitized energy equation (47), we get

$$E = \frac{mc^5\lambda^2}{hG} \qquad (52)$$

from which the square of lambda (λ^2) is equal to

$$\lambda^2 = \frac{hG}{c^3} \quad . \qquad (53)$$

The above equation has three constants in which lambda square (λ^2) has a specific magnitude. A mass that is associated with such a wave-length may be derived by either the de Broglie expression or by equating equal dimensions for the modified gravitational linear potential energy with the quantum energy equation

$$E = \frac{hc}{\lambda} \quad . \qquad (54)$$

Hence we can write

$$\left[\frac{Gm^2}{\lambda}\right] = \left[\frac{hc}{\lambda}\right] \qquad (55)$$

from which

$$\left[m^2\right] = \left[\frac{hc}{G}\right] \quad . \qquad (56)$$

If we take the expression $c^2 = a\lambda$ and square both sides we get

$$\lambda 2 = \frac{c^4}{a^2} \quad .$$

When the above expression is substituted in equation (52), we obtain

$$E = \frac{mc^9}{hGa^2} \quad . \qquad (57)$$

Substitute the de Broglie expression h = mcλ in equation (52), then

$$E = \frac{c^8}{a^2 \lambda G} \qquad (58)$$

and $\frac{c^2}{a}$ for (λ) to obtain the total energy equation in terms of acceleration

$$E = \frac{c^6}{Ga} \quad . \qquad (59)$$

When solving for the force, since the energy may be written as

$$F\lambda = \frac{c^6}{Ga} \qquad (60)$$

and force as

$$F = \frac{c^6}{Ga\lambda} \qquad (61)$$

by substituting the relation $c^2 = a\lambda$ in equation (61), we obtain

$$F = \frac{c^4}{G} \quad . \qquad (62)$$

The total force in the above equation is given in terms of two constants. It is now possible to derive the equation for the time that is associated with equation (62). Since F = ma, equation (62) becomes

$$a = \frac{c^4}{Gm} \quad , \qquad (63)$$

then substitute the expression for the linear acceleration

$$a = \frac{\lambda}{t^2} \qquad (64)$$

in equation (63) for the linear gravitational acceleration to obtain

$$\lambda = \frac{c^4 t^2}{Gm} \quad . \qquad (65)$$

Upon substituting the de Broglie equation

$$\lambda = \frac{h}{m_o c}$$

for (λ), an equation containing three constants in terms of time becomes

$$t^2 = \frac{hG}{c^5} \quad . \qquad (66)$$

An equation of the acceleration in terms of three constants may be derived from the expression of the total linear force

$$F = \frac{Gm^2}{\lambda^2} \quad .$$

The substitution of the expression

$$\lambda = \frac{C^2}{a}$$

and upon squaring, the above relation for the force becomes

$$F = \frac{Gm^2a^2}{c^4} \tag{67}$$

Equate equation (50) with equation (67) for the 1xforces

$$\frac{Gm^2a^2}{c^4} = \frac{m^2c^3}{h}$$

from which, we obtain the acceleration in terms of three constants

$$a^2 = \frac{c^7}{hG} \quad . \tag{68}$$

When the expression for the linear gravitational acceleration

$$a = \frac{Gm}{\lambda^2}$$

is substituted for the linear acceleration in the Galilean equation

$$2d = at^2 \quad ,$$

the gravitation constant (G) may be written as

$$G = \frac{\lambda^3}{mt^2} \quad . \tag{69}$$

If we equate equation (69) and equation (66) for the time squared

$$\frac{\lambda^3}{Gm} = \frac{hG}{c^5}$$

and solve for $(\lambda)^3$, we obtain

$$\lambda^3 = \frac{hG^2m}{c^5} \tag{70}$$

Since (λ^3) can be used to approximate the volume, and equation in terms of three constants can be derived for the density. Since

$$\lambda^3 = \frac{m}{\rho} = V_o \tag{71}$$

and substituted in equation (70), the expression for the density designated as rho (ρ) becomes

$$\rho = \frac{c^5}{hG^2} \quad . \tag{72}$$

The purpose for deriving the "equation of constants" is to demonstrate in the conclusions the interconnection of the gravitational constant (G) with these constants. In addition, the derived "equation of constants" have in relation to Einstein's Theory of General Relativity.

Propose Equations for the Nuclear Force and Energy of the Central-Force

The equations developed below is an attempt at deriving a series of relations that may approximate the central force field and its energy. This may be accomplished by the quantization of Newton's gravitational constant (G) associated with a different inverse relationship for the distance. Therefore, the implication implies that gravity may activate the mechanism which operates the nuclear force. Or else our knowledge concerning this physical phenomenon may be understood from the point of view of another frame of reference yet undefined. If equation (44)

$$a = \frac{mc^3}{h}$$

is substituted for the acceleration in equation (59) where

$$E = \frac{c^6}{Ga}$$

the energy of the expression becomes

$$E = \frac{hc^3}{Gm} \quad . \qquad (73)$$

When the de Broglie equation

$$C = \frac{h}{m_o \lambda}$$

is substituted for one of (c's) in (c^3) and in addition insert expression (69)

$$G = \frac{\lambda^3}{mt^2}$$

for (G) in equation (73), we obtain the relation for the energy

$$E = \frac{h^2 c^2 vt^2}{\lambda^4 m^2} \quad , \qquad (74)$$

in terms of force the above relation becomes

$$F = \frac{h^2}{\lambda^3 m} \quad . \qquad (75)$$

Equation (52) is written as

$$E = \frac{mc^5 \lambda^2}{hG} \quad , \qquad (76)$$

if we solve for the force where $E = F\lambda$, the expression becomes

$$F = \frac{mc^5\lambda}{hG} \quad . \tag{77}$$

Since $c^2 = a\lambda$, we obtain the relation for the force as

$$F = \frac{mc^7}{hGa} \quad . \tag{78}$$

By equating equation (75) with equation (78) for the force and then solve for the energy, hence we get

$$E = \frac{h^3Ga}{\lambda^3 mc^5} \quad . \tag{79}$$

If we substitute $mc\lambda$ for one of the (h's) of Plank's constant in equation (79), the expression for the energy becomes

$$E = \frac{h^2Ga}{\lambda^2 c^4} \quad . \tag{80}$$

Since $c^4 = a^2\lambda^2$, the energy of the equation can be written as

$$E = \frac{h^2G}{a\lambda^4} \quad , \tag{81}$$

and the central nuclear force expressed as

$$F = \frac{h^2G}{a\lambda^5} \quad . \tag{82}$$

Equation (81) and (82) will be developed further in the concluding portion of this paper for possible interpretation of the central force field of the nucleus.

The Significance of The Value 137 and Its Relation to two-pi (2π)

The following expressions are used to demonstrate the application of the value 137 and its relation to Coulomb's Law of electric charges and the constant (hc). If we equate equal dimensions for the Modified Newton relation

$$E = \frac{Gm^2}{\lambda} \quad .$$

with the quantum expression

$$E = \frac{hc}{\lambda}$$

then

$$\left[Gm_1{}^2\right] = \left[hc\right] \tag{83}$$

to obtain

$$\left[M_1{}^2\right] = \left[\frac{hc}{G}\right] \cong 29.3 \times 10^{-10} \text{ gm.} \qquad (84)$$

If we repeat the same relations for equal dimension of the Newtonian expression

$$E = \frac{Gm^2}{r^2}$$

with Coulomb's Law, (the constant (k) is assumed in each equation where (e^2) is present)

$$E = \frac{e^2}{r} \quad ,$$

we obtain a similar expression as above

$$\left[Gm_2{}^2\right] = \left[e^2\right] \qquad (85)$$

which

$$m_2{}^2 = \frac{e^2}{G} = 3.45 \times 10^{-12} \text{ gm.} \qquad (86)$$

If we divide $m_1{}^2$ by $m_2{}^2$, as written below

$$\frac{m_1{}^2}{m_x{}^2} = \frac{29.82 \times 10^{-10}}{3.456 \times 10^{-12}} = 860.6 \quad .$$

The division of the value 860.6 divided by 137 gives the following

$$\frac{860.6}{137} = 6.28 = 2\pi$$

therefore

$$\frac{137}{21.8} = 6.28 = 2\pi$$

or approximately we can write

$$137 \cong 2\pi \times 22 \quad . \qquad (87)$$

The quantum expression containing Sommerfeld's fine structure constant stant is

$$\frac{2\pi e^2}{hc} = \frac{1}{137}$$

then

$$\frac{hc}{2\pi e^2} = 137 \quad . \qquad (88)$$

Substitute 137/22 for (2π) in expression $2\pi r = \lambda$, hence we may write

$$137 = \frac{22\lambda}{r} \quad . \qquad (89)$$

Equate equation (88) and (89) for the value 137 to get

$$hcr = 2\pi \times 22e^2\lambda \tag{90}$$

and substitute $mc\lambda$ for (h) to obtain

$$mc^2\lambda = 2\pi \times 2\pi \times 22e^2 \tag{91}$$

Therefore, the energy of the equation becomes

$$E = \frac{2\pi \times 2\pi \times 22e^2}{\lambda} = \frac{137e^2}{r} \tag{92}$$

since $2\pi \times 22 \veeeq 137$ or

$$E = \frac{hc}{\lambda\mathfrak{X}} = \frac{(\hbar)c}{r} \ . \tag{93}$$

The expression for the force therefore, may be written as

$$F = \frac{2\pi \times 2\pi \times 22e^2}{\lambda} \tag{94}$$

$$F = \frac{2\pi \times 2\pi \times 22e^2}{4\pi^2r^2} = \frac{22e^2}{r^2} \tag{95}$$

or

$$F = \frac{hc}{\lambda 2} = \frac{(\hbar)c}{r^2} = \frac{h}{ct^2} = \frac{h}{\lambda t} \tag{96}$$

See errata on page 1

in which $(\hbar)$ is equal to $h/4\pi^2$.

From the above relation for the force in equation (95)

$$F = \frac{2\pi \times 2\pi \times 22e^2}{\lambda^2}$$

or

$$F = \frac{2\pi \times 137e^2}{\lambda^2} \tag{97}$$

when $2\pi \times 22 \cong 137$, by substituting (ma) for the force the equation becomes

$$\lambda^2 = \frac{2\pi \times 137e^2}{ma} \ . \tag{98}$$

Since mc^3 - ha from expression (44), then lambda square (λ^2) is written as

$$\lambda^2 = \frac{2\pi \times 137e^2h}{m^2c^3} \ . \tag{99}$$

If we assume $hc = 2\pi \times 137e^2$, then the equation becomes

$$\lambda^2 = \frac{h^2}{m_o{}^2c^2} \ . \tag{100}$$

The equation for the energy between two static charges is

$$E = \frac{e^2}{r} \tag{101}$$

or written as

$$E = \frac{2\pi e^2}{\lambda} \quad . \tag{102}$$

When solving the above equation for lambda (λ), the expression becomes

$$\lambda = \frac{2\pi e^2}{mc^2} \quad . \tag{103}$$

If we substitute c^2 in equation (103) in equation (100), we obtain the first Bohr radius

$$\lambda = \frac{h^2}{2\pi me^2} \tag{104}$$

or

$$r = \frac{h^2}{4\pi^2 me^2} \quad . \tag{105}$$

When we equate equation (104) with the de Broglie equation for lambda (λ) such as

$$\frac{h^2}{2\pi me^2} = \frac{h}{m_o c} \quad . \tag{106}$$

we obtain Sommerfeld's equation for the fine structure constant

$$\frac{hc}{2\pi c^2} = 137 \quad .$$

When we equate equation (92)

$$E = \frac{2\pi \times 137e^2}{\lambda}$$

or

$$c^2 = \frac{2\pi \times 137e^2}{m\lambda} \tag{107}$$

with equation (100), we get the expression

$$\frac{h^2}{m^2\lambda^2} = \frac{2\pi \times 137e^2}{m\lambda} \quad . \tag{108}$$

By solving for lambda (λ), the relation becomes

$$\lambda = \frac{h^2}{m2\pi \times 137e^2} \quad . \tag{109}$$

The above expression is equivalent in dimension and magnitude when the de Broglie equation is written as

$$\lambda = \frac{h}{m_o c}$$

in which (m_o) is the rest mass of the particle.

The following derivation is an attempt to demonstrate Kepler's Third Law and its relation to the charge square (e^2) and its associated mass. If equation (38a) is equated with equation (92) where

$$\left[\frac{Gm^2}{\lambda}\right] = \left[\frac{2\pi \times 137e^2}{\lambda}\right] \tag{109a}$$

the above relation becomes

$$\left[Gm^2\right] = \left[2\pi \times 137e^2\right] . \tag{109b}$$

Since from equation (69), expressed as

$$G = \frac{\lambda^3}{mt^2} ,$$

then equation (109b) may be written as

$$\frac{\lambda^3 m}{t^2} = 2\pi \times 137e^2 \tag{109c}$$

or

$$\frac{8\pi^3 r^3 m}{t^2} = 2\pi \times 2\pi \times 2\pi \times 3.5e^2 . \tag{109d}$$

The value (3.5) is determined from the relation expressed as ($3.5 \times 2\pi = 7\pi$) or ($\cong 22$) and the value (137) is equal to ($\cong 2\pi \times 22$).

An equation may be written in terms of Kepler's Third Law in which the period square (t^2) of a wave is directly proportional to the radius cube (r^3) of a circle or a cycle. It is as follows:

$$\frac{r^3}{t^2} = \frac{3.5e^2}{m} \tag{109e}$$

or

$$t^2 = \left(\frac{m}{3.5e^2}\right) r^3 . \tag{109f}$$

CONCLUSION

The material presented in the conclusion consists of extension of the equations formulated in the early part of the paper and in addition to demonstrate the similarities between the Modified Newtonian Theory and General Relativity. In the preceding portion of the paper, equation (53)

$$\lambda^2 = \frac{hG}{C^3}$$

contains three constants and has value of 4.1×10^{-33} cm. for its wavelength. A wavelength of this order, considering the particle at rest, has a corresponding mass whose magnitude is 5.4×10^{-5} gm. The mass may be derived from either the de Broglie equation or from the expression (56) written as

$$\left[m^2\right] = \left[\frac{hc}{G}\right] .$$

Similar results for the mass and wavelength were reported as of this writing by Columbia University Professor, Lloyd Motz, ("Science News," May 1972, Vol. 101, No. 20). The time and acceleration related to the above gravitational mass is also in terms of three constants, from equations (66) and (68) and written respectively as

$$t^2 = \frac{hG}{c^5}$$

and

$$a^2 = \frac{c^7}{hG} .$$

Their respective values are 1.4 x 10^{-43} sec. and 2.2 x 10^{53} cm./sec^2. If the above constants are placed in equation (69)

$$G = \frac{\lambda^3}{mt^2} ,$$

we obtain the value 6.67 x 10^{-8} for the magnitude of the constant (G). When the acceleration and time, derived from "equation of constants," are placed into the Galilean equation

$$2d = at^2$$

in which $\lambda = 2d$, lambda becomes 4.1 x 10^{-33} cm. A wavelength similar in magnitude derived from expression

$$\lambda^2 = \frac{hG}{C^3} .$$

However, if the mass of 5.4 x 10^{-5} gm. is used in the revised Newtonian expression (37), some interesting aspects of this constant mass becomes apparent for the force equation,

$$F = \frac{Gm^2}{\lambda^2} = \frac{6.67 \times 10^{-8} (5.4 \times 10^{-5})^2}{(4.1 \times 10^{-33})^2}$$

$$F = 1.2 \times 10^{49} \text{ dynes.}$$

The same value for the force also is obtained from the two constant equation (62) written as

$$F = \frac{c^4}{G} = 1.2 \times 10^{49} \text{ dynes.}$$

Another interesting posture of equation (62) is observed when quation (69) is substituted for the constant (G) and written as

$$F\lambda^2 = \frac{\lambda^3 m}{t^2}$$

therefore, the force becomes

$$F = \frac{\lambda^3 m}{t^2} \quad \cdot \quad \left[\frac{1}{\lambda^2}\right] \qquad (110)$$

in which

$$\frac{\lambda^3 m}{t^2} = hc. \qquad (111)$$

Since $F\lambda^2 = hc$, the force may be expressed similar to equation (96) as

$$F = \frac{hc}{\lambda^2} \quad .$$

The left-hand expression of the equation (111) states that the wavelength cube (λ^3) multiplied by its associated mass is inversely proportional to the time squared (t^2). The time is determined by the interval of time it takes the speed of light to traverse the distance of its wavelength. Time squared (t^2) may also be determined from the Galilean expression $2d = at^2$ in which the acceleration is calculated from the expression

$$a = \frac{c^2}{\lambda} \quad .$$

If these conditions are satisfied, then the left-hand expression (111) will always be a constant equal to (hc). Therefore from equation (96)

$$F = \frac{hc}{\lambda^2}$$

when (λ) is equal to 4.1×10^{-33} cm.

$$F = 1.2 \times 10^{49} \text{ dynes.}$$

Again to demonstrate the consistancy of the large dimensional force, the quantitized relation (50)

$$F = \frac{m^2 c^3}{h} \quad ,$$

if the mass 5.4×10^{-5} gm. is inserted in equation, the force becomes

$$F = 1.2 \times 10^{49} \text{ dynes.}$$

From relationship (83) which is expressed as

$$\left[Gm^2\right] = \left[hc\right] \quad ,$$

we arrive at a dimensionless relation

$$K^1 = \frac{hc}{Gm^2} \quad . \qquad (112)$$

When the rest mass of the proton is placed into the equation, it results in a

dimensionless value of 1.1×10^{39}. The square root of this value is equal to $\cong 3.2 \times 10^{19}$. In
Special Relativity, the implication is that the proton mass may increase $\cong 3.2 \times 10^{19}$ times its original rest mass. In the event the mass of the meson is used in equation (112), then the dimensionless value becomes $\cong 4.8 \times 10^{40}$, and the square root of this dimension is $\cong 2.2 \times 10^{20}$. Similarly, a dimensionless value may be calculated for the electron which gives a value of $\cong 3.7 \times 10^{45}$ and the square root of this dimension is $\cong 1.9 \times 10^{22}$. This implies there may be a finite particle associated with Einstein's mass tranformation equation. The transformation may be written as follow for a single mass:

$$K^1 = m_1 = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (113)$$

and for the mass squared ($m_o{}^2$) (114)

$$K^2 = m_1{}^2 = \frac{m^2{}_o}{\left[\sqrt{1 - \frac{v^2}{c^2}}\right]^2}$$

It is interesting to note that if the mass of the proton is placed in equation (114) and travels to one-half the speed of light, the mass will increase $\cong 15.0$ per cent and the mass squared $\cong 33.0$ per cent greater than its rest mass. The above contention also implies that no mass can be accelerated with a force greater than 1.2×10^{49} dynes and/or no two fundamental gravitational masses may experience a force greater than 1.2×10^{49} dynes. The critical mass associated with this gravitational phenomenon is $\cong 5.4 \times 10^{-5}$ gm. This relationship may be demonstrated from equation (73)

$$E = \frac{hc^3}{Gm}$$

since $E = F\lambda$, the force is

$$F = \frac{hc^3}{Gm\lambda} \qquad (115)$$

Equation (115) indicates that the force in constant, regardless of the size of the mass as long as it is accompanied by its associated wavelength. It will be shown further in the paper that this relationship does not hold for the energy in equation (73). Another means of demonstrating the consistancy of a maximum force associated with a finite particle is the relation of cosmological numbers with the force in equation (50). They are as follow:

Proton mass: $\cong 1.67 \times 10^{-24}$ gm.

$$F = \frac{m^2c^3}{h} \quad 1.1 \times 10^{10} \text{ dynes} \times 1.1 \times 10^{39} \cong 1.2 \times 10^{49} \text{ dynes}$$

Meson mass: $\cong 2.5 \times 10^{-25}$ gm.

$$F = \frac{m^2c^3}{h} = 2.5 \times 10^8 \text{ dynes} \times 4.8 \times 10^{40} \cong 1.2 \times 10^{49} \text{ dynes}$$

Electron mass: $\cong 9.1 \times 10^{-28}$ gm.

$$F = \frac{m^2c^3}{h} = 3.8 \times 10^3 \text{ dynes} \times 3.7 \times 10^{45} \cong 1.2 \times 10^{49} \text{ dynes}$$

If we examine the equations for the energies (59) and (73), one may conclude that larger energies are associated with small masses and conversely it indicates that small energies are derived from large masses equation (73). When considering acceleration in relation to equation (59), we find equally this contrary situation: large initial accelerations result in small energies and small initial accelerations give rise to large energies.

Consider expression (59)

$$E = \frac{c^6}{Ga}$$

and equation (73)

$$E = \frac{hc^3}{Gm} \quad ,$$

we can see from the nature of these expressions the peculiarties as stated above. However, when a comparison is made between equation (59) and expression (47)

$$E = \frac{m^2c^5}{ha} \quad ,$$

we may note that equation (47) the mass and acceleration are not constant. Now, if an acceleration value similar in magnitude is inserted in both equations (59) and (47) with the proviso that the mass square (m^2) is equal to (5.4×10^{-5} gm.)^2, in equation (47), the energies for both equations are similar. This suggests to the writer that a small mass will increase in dimension as it accelerates toward the speed of light until it reaches a finite value. Larger energies occur when the initial accelerations are small and vice versa.

In the preceding portion of this paper, cosmological numbers were discussed in relation to force; now a similar analogy can be demonstrated for the energies. The following dimensionless cosmological values multiplied by the rest mass energy of the particle give the total energy of system.

Proton: rest mass energy, 1.5×10^{-3} ergs

$$(1 \times 10^{39})^2 \cong 1 \times 10^{78} \times 1.5 \times 10^{-3} \cong 1 \times 10^{75} \text{ ergs}$$

Meson: rest mass energy, 2.2×10^{-4} ergs

$$(4.8 \times 10^{40})^2 = 2.3 \times 10^{81} \times 2.2 \times 10^{-4} \cong 5.0 \times 10^{77} \text{ ergs}$$

Electron: rest mass energy, 81×10^{-8} ergs

$$(3.7 \times 10^{45})^2 = 1.4 \times 10^{91} \times 81 \times 10^{-8} \cong 1.1 \times 10^{85} \text{ ergs}$$

The sets of values presented above can be mathematically calculated from equation (59) and (47), provided we use the minimum gravitational acceleration associated with a particle and the mass in equation (47) has a finite value of $(5.4 \times 10^{-5}\ \mathrm{gm})^2$. The minimum linear gravitational acceleration for both equations is determined from expression (51)

$$a = \frac{Gm^2}{\lambda^2}$$

and the maximum linear acceleration associated with a particle or "rest mass acceleration" designated as (a_o) is derived from equation (39)

$$a_o = \frac{c^2}{2\pi r}$$

or

$$a_o = \frac{c^2}{\lambda} \quad .$$

In equation (47), the rest mass energy of a particle may be determined from the "rest mass linear acceleration" and its associated particle, to give results similar to $E = m_o c^2$.

The data presented in the early portions of this paper demonstrated the mechanics of the derived equations. The following equations to be set forth are an attempt to show the relativistic relations and the high energies that are associated with these equations, when the source is in motion. In addition, gravitational and quantum expressions which have correspondingly relativistic equivalents are presented to display their similarities. Therefore, if equations (47) and (50) represent comparative non-relativistic relations for the energy and force, in which the acceleration is variable in equation (47), we can rewrite the expression as follow for the energy for a particle with a specific initial acceleration:

$$E = \frac{C^6}{G} \left[\frac{1}{a}\right] \quad \text{limits } a \rightarrow \infty \qquad (116)$$

$$E = \frac{C^6}{G} \left[\frac{1}{a}\right] \quad \text{limits } a \rightarrow 0 \qquad (117)$$

$$F = \frac{C^4}{G} \qquad (118)$$

The above relations stipulate that if the initial acceleration is large as it approaches infinity, the energy of the system is small. Conversely, if the initial acceleration is a small dimension as it approaches zero, the energies are large. The force is finite and no mass may acquire an acceleration or experience a total force greater than stated in equation (118), since the speed of light is a constant and a limiting factor for the total system.

The next group of equations are expressions described earlier in the foregoing. An attempt, now, is made to show their relation to the comparative or standard relations set above in comparison to their relativistic equivalents, when the source is in motion. The transformation equations for the total system are as follows:

$$E_T = \frac{Gm^2}{2\pi r} \tag{119}$$

$$E_K = \frac{Gm^2}{4\pi r}$$

$$\frac{1}{2} mv^2 = \frac{Gm^2}{4\pi r}$$

$$V^2 = \frac{Gm}{2\pi r}$$

$$\beta_1{}^2 = \frac{v^2}{c^2} = \frac{Gm}{2\pi rc^2} = \frac{Gm}{\lambda c^2} . \tag{120}$$

Mass

$$m_1 = \frac{m_o}{\sqrt{1 - \beta^2{}_1}} \tag{121}$$

$$m_1{}^2 = \frac{m_o{}^2}{\left[\sqrt{1 - \beta^2{}_1}\right]^2} \tag{122}$$

Length

$$\lambda 1 = \lambda o \sqrt{1 - \beta_1{}^2} \tag{123}$$

$$\lambda 1^2 = \lambda o^2 \left[\sqrt{1 - \beta_1{}^2}\right]^2 \tag{124}$$

Time

$$t_1 = \frac{t_o}{\sqrt{1 - \beta_1{}^2}} \tag{125}$$

$$t_1{}^2 = \frac{t^2{}_o}{\left[\sqrt{1 - \beta_1{}^2}\right]^2} \tag{126}$$

I. Relativistic Mass Energy Equations

(a) Non-Relativistic

$$E = m_o c^2$$

(b) Relativistic

$$E = \frac{m_o c^2}{\sqrt{1 - \beta_1{}^2}} \tag{127}$$

$$E = F\lambda \qquad\qquad E = F\lambda$$

$$E = \frac{m_o c^2}{\lambda} \cdot \frac{c^2}{a} \qquad E = \frac{m_o c^4}{\sqrt{1-\beta_1{}^2}\ \lambda\sqrt{1-\beta_1{}^2}} \cdot \frac{1}{a} \quad (128)$$

II. Modified Relativistic Newtonian Equations

(a) Non-Relativistic (b) Relativistic

$$E = \frac{Gm_o{}^2}{\lambda} \qquad E = \frac{Gm_o{}^2}{\left[\sqrt{1-\beta_1{}^2}\right]^2 \lambda\sqrt{1-\beta_1{}^2}} \quad (129)$$

$$E = F\lambda \qquad\qquad E = F\lambda$$

$$E = \frac{Gm_o{}^2.}{\lambda^2} \cdot \frac{c^2}{a} \qquad E = \frac{Gm_o{}^2c^2}{\sqrt{1-\beta_1{}^2}\ \lambda^2\left[\sqrt{1-\beta_1{}^2}\right]^2} \cdot \frac{1}{a} \quad (130)$$

III. Relativistic Quantum Equations

(a) Non-Relativistic (b) Relativistic

$$E = \frac{hc}{\lambda} \qquad E = \frac{hc}{\lambda\sqrt{1-\beta_1{}^2}} \quad (131)$$

$$E = F\lambda \qquad\qquad E = F\lambda$$

$$E = \frac{hc}{\lambda^2} \cdot \frac{c^2}{a} \qquad E = \frac{hc^2}{\lambda^2\left[\sqrt{1-\beta_1{}^2}\right]^2} \cdot \frac{1}{a} \quad (132)$$

IV. Relativistic Quantitized Mass Energy Equation:

(a) Non-Relativistic (b) Relativistic

$$E = \frac{m^2{}_o c^3}{h} \cdot \frac{c^2}{a} \qquad E = \frac{m_o{}^2 c^5}{\left[\sqrt{1-\beta_1{}^2}\right]^2 h} \cdot \frac{1}{a} \quad (133)$$

$$F = \frac{m_o{}^2 c^3}{h} \qquad F = \frac{m^2{}_o c^3}{\left[\sqrt{1-\beta_1{}^2}\right]^2 h}$$

In the event, there is a many-particle system, a general equation perhaps may be written for the energy, in regard to equation (133)

$$E^n = \frac{m^{(n+1)}c^{(2n+3)}}{\sqrt{1-\beta^2}\ (n+1)\,h} \cdot \frac{1}{a} \quad . \quad (134)$$

The foregoing transformations are indicative of the total energy of the system. Since Newtonian concepts owe their relationship to the radius of a circle ($2\pi r$), then the Lorenze-Fitzgerald and revised transformations may be written accordingly. In the Modified Newtonian Theory, the Newtonian expression (38a) is written as

$$E_N = 2\pi E = \frac{Gm^2}{r} \quad .$$

and the total linear expression (37) is designated as

$$E = \frac{Gm^2}{\lambda} \quad .$$

When the potential energy equation (38a) is equal to the kinetic energy, the relation becomes

$$E_K = \frac{Gm^2}{r} \tag{135}$$

therefore

$$\frac{1}{2} mv^2 = \frac{Gm^2}{r} \tag{136}$$

$$\beta_2{}^2 = \frac{v^2}{c^2} = \frac{2Gm}{rc^2} \quad . \tag{137}$$

Mass

$$m_1 = \frac{m_o}{\sqrt{1 - \beta_2{}^2}} \tag{138}$$

$$m_1{}^2 = \frac{m^2{}_0}{\left[\sqrt{1 - \beta_2{}^2}\right]^2} \tag{139}$$

Length

$$r_1 = r_o \sqrt{1 - \beta_2{}^2} \tag{140}$$

$$r_1{}^2 = r_o{}^2 \left[\sqrt{1 - \beta_2{}^2}\right]^2 \tag{141}$$

Time

$$t_1 = \frac{t_o}{\sqrt{1 - \beta_2{}^2}} \tag{142}$$

$$t_1{}^2 = \frac{t^2{}_o}{\left[\sqrt{1 - \beta_2{}^2}\right]^2} \tag{143}$$

It is believed that the material just presented may assist in understanding quasi-steller bodies in which small masses are associated with large energies. The writer is unqualified to express the philosophical and physical meanings of some of the relativistic equations, other than to mention that it appears that as the force dimenishes, the acceleration lessens with time. Uniform motion approaches – but never reaches – an absolute frame of reference regarding the motion of matter. Therefore, Special Relativity is restricted to a frame of reference which has an apparent uniform motion. Otherwise, according to equation (47), if the acceleration were to cease, the energy would become infinite. If this be the case, then small accelerations characteristic of large energies may be due to looking back in time. The above contentions concerning accelerations appear to have application to the Machian Principle. A zero force due to zero acceleration is applicable to inertial frames of references and non-zero forces due to acceleration are applicable to all frames of reference. This implies that Newtonian concepts may be applied to the latter condition, (Casper and Noer, 1972).

At the beginning of this paper a premise was introduced which stated that the total energy of the system will enscribe a circle. The premise was that a semicircle would be designated as (d) or (πr) and it represents the kinetic energy of a Lagraingian gravitational field and that the whole circle (2d) or 2πr represents the total energy. For example in equation (51), if the wavelength of the meson is $2\pi \times 1.4 \times 10^{-13}$ cm. and its associated mass of $\cong 2.5 \times 10^{-25}$ gm. were inserted in the equation, the linear gravitational acceleration would have a value of $\cong 2 \times 10^{-8}$ cm./sec.^{2}. When placed in equation (59), the total energy would have a value of $\cong 5.4 \times 10^{77}$ ergs. The rest mass associated with this large energy ($\cong 5.4 \times 10^{77}$ ergs) is $\cong 6 \times 10^{56}$ gms. and the distance or the circumference of a circle designated as (λ or 2πr) is related to the total energy of the system and has a value of $\cong 4.7 \times 10^{28}$ cms. From the relation

$$E = m_o c^2 - \frac{Gm^2}{\lambda} \qquad (144)$$

if the appropriate dimensions are inserted in the above relation, the total energy of the system should be zero. For purpose of demonstration then,

$$E = 6 \times 10^{56} \times 9 \times 10^{20} - \frac{6.67 \times 10^{-8}(6 \times 10^{56})^2}{4.7 \times 10^{28}} \text{ ergs}$$

$$E = 5.4 \times 10^{77} - 5.1 \times 10^{77} \text{ ergs}$$

$$E = \cong 0 \text{ erg.}$$

Similar results for the total energy may be obtained from equation (133)

$$E = \frac{m^2 c^5}{\left[\sqrt{1 - \frac{v^2}{c^2}}\right]^2 ha}$$

if we assume that the meson mass squared $(m^2) = 2.5 \times 10^{-25}$ gm. does not exceed 4.8×10^{40} times its original mass or $(5.4 \times 10^{-5}$ gm.$)^2$ as it moves towards the speed of light and the acceleration (a) is 2 x 10^{-8} cm./sec.^{2}. The meson mass has been used as an example since it represents the middle mass between the electron and the proton.

Since the kinetic energy in a circular Lagrangian gravitational field is the distance of a semicircle or (πr), and since for the purposes of this paper, the distance (πr) is the length in which the particle traveled to its terminal velocity at the speed of light. Therefore, when the kinetic energy is equal to the gravitational potential energy we may write the expression as

$$\tfrac{1}{2}mv^2 = \frac{Gm^2}{\pi r}$$

If the velocity square (v^2) approaches the velocity square (c^2) or $(v^2 \rightarrow c^2)$, then the equation for the radius (r) becomes

$$r = \frac{2Gm}{\pi c^2} \quad . \qquad (145)$$

The above relation is the radius of the universe predicted by Einstein from the Theory of General Relativity using Gaussian-Reinmmian Geometry.

If we desire the radius in terms of density as in Einsteinian General Relativity substitute the density equation $(m = V_o p)$ in equation (145), resulting in

$$r = \frac{2GM_o\rho}{\pi c^2} \quad . \qquad (146)$$

When adjusting the volume (V_o) in terms of Guassian Volume which is

$$V_o = 2\pi^2 r^3$$

and then substituted in equation (146), we get

$$r^2 = \frac{c^2}{4\pi G\rho}$$

A Proposed Quantitized Gravitational Nuclear Energy and Force Equation

Several equations which may explain some of the properties of the central force (CF) and energy of the nucleus. Equation (81) for the energies is expressed as

$$E_{cf} = \frac{h^2 G}{a\lambda^4} \qquad (147)$$

which also may be written as

$$E_{cf} = \frac{h^2 G}{a\pi 16\pi^4 r^4} \qquad (148)$$

(see errata on page 1)

The equation (82) for the forces is expressed as

$$F_{cf} = \frac{h^2G}{a\lambda^5} \tag{149}$$

and also written as

$$F_{cf} = \frac{h^2G}{a\pi 32\pi^5 r^5} . \tag{150}$$

The symbol for radius is (r) in which the nuclear force range is 1.4 x 10^{-13} cm. and the linear gravitational acceleration is derived from the relation

$$a\pi = \frac{Gm}{\lambda 2} \tag{151}$$

Since the rest mass wavelength of the proton is the radius of the meson, the wavelength of meson is therefore 8.8 x 10^{-13} cm., according to equations (92) and (93). The resulting linear gravitational acceleration has a magnitude of ≅ 2.0 x 10^{-8} cm./sec.^{2}. The value obtained for the nuclear energy when (r) is 1.4 x 10^{-13} cm. is 2.2 c 10^{-4} erg or 140MeV and the magnitude of the force is 2.5 x 10^8 dynes. If we vary the distance (r) to 4 x 10^{-13} cm. or 4 fermi, then the binding energy at this distance is 3.5 x 10^{-6} erg or ≅ 2.2 MeV and the force becomes 1.4 x 10^6 dynes. When the electrical force between two protons is calculated at a distance of 4 x 10^{-13} cm. or 4 fermi, the electrical force has approximately the same intensity of 1.6 x 10^6 dynes as the nuclear force. Also, the range of the nuclear force greater than 4 x 10^{-13} cm., the electrostatic force greatly exceeds the nuclear force. When the radius is reduced to 1.2 x 10^{-13} or 1.2 fermi, the nuclear force is 36 times as strong as the electrical force. The data presented adheres closely to the values accumulated from empirical nuclear studies, in regard to proton-proton, two body problems, (Adler, 1963).

From the proposed nuclear equation, it appears that the nuclear force may be gravitational in aspect; however, it is seemingly known that nuclides have intrinsic angular momentum or "spin," among other properties of motion. The gravitational interpretation is probably more apparent than real. If Einstein's Principle of Equivalents is applied in the following equation (43)

$$E = \frac{ha_o}{c}$$

in which the force is

$$F = \frac{ha_o}{2\pi rc} \tag{152}$$

we obtain similar values for the force and energy as in equation (150) but their accelerations are different from those in the gravitational interpretation. Also the short range nuclear force is not apparent. The "rest mass acceleration" or linear acceleration for the above equations can be derived from expression (39)

$$a_o = \frac{c^2}{\lambda}$$

in which lambda (λ) is 8.8×10^{-13} cm. and the acceleration is 1×10^{33} cm./sec.^{2} When the gravitational linear acceleration $\cong 2 \times 10^{-8}$ cm./sec.^{2} is placed in equation (152) in linear of the "rest mass" or linear acceleration $\cong 1 \times 10^{33}$ cm./sec.^{2} the results are comparable to those obtained from equation (37), $F = Gm^2/\lambda^2$. In equation (37) the meson mass squared $(m)^2$ is ($\cong 2.5 \times 10^{-25}$ gm.)^2 and the distance squared $(\lambda)^2$ or $(2\pi r)^2$ is ($\cong 8.8 \times 10^{-13}$ cm.)^2. It is also worth noting that, if the "rest mass" or linear acceleration $\cong 1 \times 10^{33}$ cm./sec.^{2} were placed in equation (150) and the distance $(r)^5$ is ($\cong 1.4 \times 10^{-13}$ cm.)^5, the central nuclear force would have a magnitude of $\cong 5 \times 10^{-33}$ dyne, similar to equation (37). The foregoing discussion may be interpreted mathematically as follow:

$$F = \frac{ha/2\pi rc}{ha_o/2\pi rc} = \frac{h^2G/a32\pi^5r^5}{h^2G/a_o32\pi^5r^5} =$$

$$\frac{h^2G/a32\pi^5r^5}{Gm^2/4\pi^2r^2} = \frac{2.5 \times 10^8 \text{ dynes}}{5 \times 10^{-33} \text{ dyne}} \cong 5 \times 10^{40}$$

The dimensionless cosmological number of $\cong 5 \times 10^{40}$ is a value which was derived earlier in the paper.

It appears that due to the difference in our frame of references nuclear forces and energies can be interpreted as a quantitized gravitational phenomenon. Under these circumstances then, in a gravitational system the radius of the proton wavelength at a rest position, in a static system, would be $\cong 2.2 \times 10^{-14}$ cm. and its wavelength would have a value of $\cong 1.4 \times 10^{-13}$ cm. Two protons waves in juxtaposition will have a radial distance from its center of masses of $\cong 4.4 \times 10^{-14}$ cm. Any distance less than $\cong 4.4 \times 10^{-14}$ cm. will experience a repulsive force. Several other possible nuclear force equations which may have some merit are as follow:

If the equation for the constant (G) written as

$$G = \frac{\lambda^3}{mt^3}$$

were substituted in equation (150), the quantum expression for the force becomes

$$F = \frac{h^2}{m_p\lambda^3} \tag{153}$$

$$F = \frac{h^2}{m_p\, 8\pi^3 r^3} \qquad (154)$$

In the above equation when the radius (r) is 4 x 10^{-13} or 4 fermi, where (m_p) is the mass of the proton the range of the nuclear force is similar to equation (150). However, as the radius decreased the results vary slightly than those derived from the quantitized gravitational force equation.

A modified gravitational equation for the nuclear central force may be derived by substituting ($m^2c^2\lambda^2$) for (h^2) in equation (150) and the resulting equation for the force becomes

$$F_{cf} = \frac{Gm^2c^2}{a\lambda^3} \qquad (155)$$

or

$$F_{cf} = \frac{Gm_p{}^2c^2}{a\, 8\pi^3 r^3} \quad . \qquad (156)$$

The linear gravitational acceleration (a) for the above equation is derived from expression (51), which is written as

$$a = \frac{Gm_p}{\lambda^2}$$

in which the mass (m) is the proton and lambda (λ) is the rest mass wavelength of the proton. Since it is assumed in this paper that the rest mass wavelength (λ) for the proton or the radius of a meson wavelength is $\cong$ 1.4 x 10^{-13} cm. as a result, the magnitude for the proton linear acceleration becomes 5.5 x 10^{-6} cm./sec.^{2}. In the above force equation (156) the nuclear central force has a range slightly less than 6 x 10^{-13} cm. or $\cong$ 6 fermi.

Comparison of the Modified Newtonian Theory of Gravitation and General Relativity

Earlier in this paper several "equations of constants" were derived for the mass, wavelength, and force to show their relationship to the gravitational constant. The discrete values mentioned above are also intimately linked to kappa(k), a constant in General Relativity. If we take the expression

$$G = \frac{\lambda c^2}{m}$$

and rewrite the equation as

$$\frac{G}{c^2} = \frac{\lambda}{m} \quad ,$$

we have a basic equation which is related to kappa (k) of General Relativity.

In addition, we may write another equation for part of the above relation derived from equation (62)

$$F = \frac{C^4}{G} \quad .$$

By rewriting equation (62) as

$$\frac{G}{c^2} = \frac{C^2}{F} \qquad (157)$$

we can equate four similar expressions from above to a constant (k)

$$k = \frac{G}{c^2} = \frac{\lambda}{m} = \frac{c^2}{F} = \frac{Gt^2}{\lambda^2} \quad . \qquad (158)$$

If we multiply each expression by 8π, the above relations may be written as follow:

$$k = \frac{8\pi G}{c^2} = \frac{8\pi\lambda}{m} = \frac{8\pi c^2}{f} = \frac{8\pi Gt^2}{\lambda^2} \quad . \qquad (159)$$

When inserting the proper constants derived from "equations of constants" for each expression, the results will be the same as kappa (k_1), a constant of General Relativity,

$$k_1 = \frac{8\pi \times 6.67 \times 10^{-8}}{9 \times 10^{20}} = 1.86 \times 10^{-27} \qquad (159a)$$

$$k_2 = \frac{8\pi\lambda}{m} = \frac{8\pi \times 4.1 \times 10^{-33}}{5.4 \times 10^{-5}} = 1.86 \times 10^{-27} \qquad (159b)$$

$$k_3 = \frac{8\pi C^2}{F} = \frac{8\pi \times 9 \times 10^{20}}{1.2 \times 10^{49}} = 1.86 \times 10^{-27} \qquad (159c)$$

$$k_4 = \frac{8\pi\, Gt^2}{\lambda^2} = \frac{8\pi \times 6.67 \times 10^{-8} \times [1.4 \times 10^{-43}]^2}{[4.1 \times 10^{-33}]^2} = 1.79 \times 10^{-27} \qquad (159d)$$

The forthcoming relation is an attempt to adjust the Quantum Theory of Electromagnetism to the Modified Newtonian Theory. At this point the writer wishes to hazard a conjecture as to the relationship of the value 137 to General Relativity. From expression (94) written as

$$F = \frac{2\pi \times 137\, e^2}{\lambda^2}$$

and expression (53), where

$$\lambda^2 = \frac{hG}{c^3} \quad ,$$

we may write

$$\frac{hG}{c^3} = \frac{2\pi \times 137e^2}{F} \tag{160}$$

upon substitution of lambda square (λ^2) for equation (53) in equation (94). By a series of substitutions and multiplications we can derive the following equations:

$$\frac{hGF}{c^3} = 2\pi \times 137e^2 \tag{161}$$

since F = ma, then

$$\frac{hGma}{c^3} = 2\pi \times 137e^2 \quad . \tag{162}$$

When the acceleration (a) is equal to c^2/λ, the above expression becomes

$$\frac{hcGm}{\lambda c^2} = 2\pi \times 137e^2 \tag{163}$$

since $hc = 2\pi \times 137e^2$. We may write the above equation as

$$2\pi \times 137e^2 \frac{GM}{\lambda c^2} = 2\pi \times 137e^2$$

or as

$$2\pi \times 2\pi \times 7\pi e^2 \frac{Gm}{c^2} = 2\pi \times 2\pi \times 7\pi e^2 \tag{164}$$

owing to the relation that $137 \cong 2\pi \times 22$, and 22 is equal to $\cong 7\pi$. The value of 7π is the average of 6π and 8π. These are two values or multiples of these values which constantly occur in General Relativity. Therefore, we may write a relation which is the average value for the constant, $\cong 2\pi \times 137e^2$.

$$2\pi \times 2\pi \times (1 \pm 7\pi)e^2 \frac{Gm}{2\pi rc^2} = 2\pi \times 2\pi \times (1 \pm 7\pi)e^2. \tag{165}$$

Divide each side of expression (165) by (e^2) to obtain a relation in which the left-hand expression is equal to the right-hand expression, since in the above identity equation

$$\frac{Gm}{2\pi rc^2}$$

is a dimensionless expression equal to one when the values from the "equations of constants" are inserted for $2\pi r \cong 4.1 \times 10^{-33}$ cm. and $m \cong 5.4 \times 10^{-5}$ gm.,

$$2\pi \times 2\pi \times (1 \pm 7\pi) \frac{Gm}{2\pi rc^2} = 2\pi \times 2\pi \times (1 \pm 7\pi). \tag{168}$$

If we divide expression (168) into two equations as a matter of convenience we obtain

(1) $2\pi \times 2\pi \times 6\pi \frac{Gm}{2\pi rc^2} = 2\pi \times 2\pi \times 6\pi$ (169)

(2) $2\pi \times 2\pi \times 8\pi \frac{Gm}{2\pi rc^2} = 2\pi \times 2\pi \times 8\pi.$ (170)

The *a priori* conclusions to be presented probably appeals more to logic than experience. (It is implied for equation (169) and (170) that the division of the left-hand expression by the right-hand expression using various combinations of pi (π), two-pi (2π) and whole numbers one may develop approximately 32-34 identity parameters). It appears that the value $2\pi \times (1 \pm 137)$ has an important consideration in the Quantum Electromagnetic Theory as well as the Modified Newtonian Theory. However, the writer wishes to list six (6) of the possible 32-34 parameters which meet the requirements of General Relativity as examples. The resulting identities are as follows:

(1) $\frac{6\pi^2 Gm}{2\pi rc^2} = \frac{3\pi Gm}{r\,c^2}$ = Precession of the geodesic

(2) $\frac{12\pi^2 Gm}{2\pi rc^2} = \frac{6\pi Gm}{rc^2}$ = advance of the perihelion (unadjusted to the ellipse)

(3) $\frac{8\pi Gm}{2\pi rc^2} = \frac{4Gm}{rc^2}$ = the bending of light in a gravitational field

(4) $\frac{4Gm}{2\pi rc^2} = \frac{2Gm}{\pi rc^2}$ = Einstein's radius of the universe

(5) $\frac{4\pi Gm}{2\pi rc^2} = \frac{2Gm}{rc^2}$ = Schwartzchild radius

(6) $\frac{2\pi Gm}{2\pi rc^2} = \frac{Gm}{rc^2}$ = Gravitational red-shift

Some of the above relations may be calculated mathematically from the relativistic transformation expressions (138) through (143) and then expanding the function ($\sqrt{1 - \beta_2{}^2}$) by the use of the Binomial Theorum. The rule is to take the Newtonian solution and apply the general relativistic effects, (Ney, 1962).

In addition to the identities just presented, there are at least several variations of equation (53)

$$\lambda^2 = \frac{hG}{c^3}$$

which contain in part some of the basic relations of General Relativity. In the first situation, in order to convert the Modified Newtonian Theory in terms of relativity, the calculations are as follow: Equation (53) is an expression related to the total energy of the system, therefore one-half the total of the above expression becomes

$$r^2 = \frac{4hG}{4\pi^2 C^3} \qquad (171)$$

or

$$r = \sqrt{\frac{hG}{\pi^2 c^3}} \quad . \qquad (172)$$

One-half of the total system of equation (53) is comparable to $(2)^2$ or 4 divided into the radius square (r^2). When the total energy is written as

$$E = F\lambda$$

or

$$E = f2\pi r \qquad (173)$$

and we substitute the force in equation (50), in equation (173), the total energy now becomes

$$E = \frac{m^2 c^3}{h} \cdot 2\pi r. \qquad (174)$$

When substituting the relation (172) for the radius (r) in equation (174), the expression for the energy is written as

$$E = \frac{m^2 c^3}{h} \; 2\pi \sqrt{\frac{hG}{\pi^2 c^3}} . \qquad (175)$$

Square both sides of equation (75)

$$E^2 = \frac{m^4 c^6}{h^2} \quad 4\pi^2 \left[\sqrt{\frac{hG}{\pi^2 c^3}} \right]^2 \qquad (176)$$

and then simplify, the energy equation becomes

$$E = \frac{4Gm^2}{2\pi r} \qquad (177)$$

or

$$E = \frac{2GMm}{\pi r} \qquad (178)$$

or

$$mc^2 = \frac{2GMm}{\pi r} \quad . \qquad (179)$$

If we cancel the mass (m) in equation (179), we derive Einstein's relation for the radius of the universe

$$r = \frac{2GM}{\pi c^2} \quad .$$

The expression for the force in terms of General Relativity is

$$F = \frac{4Gm^2}{\lambda^2} \qquad (180)$$

or

$$F = \frac{GMm}{\pi^2 r^2} \quad . \tag{181}$$

Since the force is equal to the expression mass x acceleration (ma), the derived relation becomes

$$a = \frac{Gm}{\pi^2 r^2} \quad . \tag{182}$$

When equation (182) is substituted in equation (47) for the acceleration (a), we get

$$E = \frac{m^2 c^5 \pi^2 r^2}{hGm} \quad . \tag{183}$$

Since Plank's constant (h) is equal to (mcλ), equation (183) is written as

$$E = \frac{c^4 \pi r}{2G} \quad . \tag{184}$$

If the expression $c^2 = 2\pi ar$ is substituted in equation (184) for (πr), the energy of the equation becomes

$$E = \frac{c^6}{4Ga} \quad . \tag{185}$$

The force derived from the above equation is

$$F = \frac{c^6}{4Ga\lambda} \quad , \tag{186}$$

since the energy is equal to (F x λ) in the Modified Theory. If the expression for the acceleration (a) written as

$$a = \frac{c^2}{2\pi r} \quad ,$$

the force in equation (186) becomes

$$F = \frac{c^4}{4G} \quad . \tag{187}$$

When the expression for the energy in equation (185)

$$E = \frac{c^6}{4Ga}$$

is equated with the energy equation (20)

$$E = ma\lambda,$$

we get

$$ma\lambda = \frac{c^6}{4Ga} \quad . \tag{188}$$

By substituting the acceleration in equation (182)

$$a = \frac{Gm}{\pi^2 r^2}$$

in the left-hand relation of equation (188), the derived relation is

$$\frac{Gm^2 2\pi r}{\pi^2 r^2} = \frac{c^6}{4Ga} \quad . \tag{189}$$

If we rewrite equation (189) as

$$\frac{8\pi G}{c^2} = \frac{c^4 \pi^2 r}{Gm^2 a} \quad , \tag{190}$$

the resulting left-hand relation is Einstein's constant kappa (k_1) of General Relativity, as expressed in equation (159a).

From equation (69)

$$G = \frac{\lambda^3}{mt^2} \quad ,$$

if lambda (λ^3) is equated to the Gaussian volume ($\lambda^3 = V_o = 2\pi^2 r^3$), and the mass (m) is equal to the Einsteinian expression

$$m = \frac{\pi r c^2}{2G} \quad ,$$

and then substituted in equation (69), then in terms of General Relativity the radius of the universe also may be written as

$$4\pi r^2 = c^2 t^2 \tag{191}$$

or

$$r = \frac{ct}{2\sqrt{\pi}} \tag{192}$$

An important relation relevant to both General Relativity and Newton's Theory of Gravitation is the Poisson equation. One may derive this expression by means of algebra provided that Gaussian concepts are applied to the Modified Newtonian expression (69)

$$\lambda^3 = Gmt^2 \quad .$$

In Gaussian terms, the volume of a sphere is written as ($V_o = 2\pi^2 r^3$) and the surface area as $s = \underline{16} r^2$. If we equate the radius square (r^2) in the expression for the volume, the derived results may be written as

$$\frac{\pi s}{16} = \frac{V_o}{2\pi^2 r} \quad . \tag{193}$$

Since the volume is equal to ($V_o = \lambda^3 = Gmt^2$) and upon substitution in equation (193) the relation becomes

$$\frac{\pi s}{16} = \frac{Gmt^2}{2\pi^2 r} \quad . \tag{194}$$

When the mass (m) is equal to (ρ V_o), the resulting equation is written as

$$\frac{\pi s}{16} = \frac{G\rho V_o t^2}{2\pi^2 r}. \quad (195)$$

If we substitute ($\lambda^3 = 8\pi^3 r^3$) for the volume (V_o), we get

$$\frac{\pi s}{16} = 4\pi G\rho t^2 r^2. \quad (196)$$

Since the surface area (s) is equal to $\frac{16}{\pi}$ r^2, equation (197) becomes

$$\frac{16\pi}{16\pi t^2} = 4\pi G\rho \quad (197)$$

or

$$\frac{16\pi c^2}{16\pi r^2} = 4\pi G\rho. \quad (198)$$

In terms of vector analysis, the above expression becomes

$$\triangle\phi = 4\pi G\rho \quad (199)$$

in which the left-hand relation is the gradient of the potential phi (ϕ), or the potential gravitational field.

According to Guassian concepts concerning some of the properties of three dimensional space, if space has a constant positive curvature such a space will have similar characteristics of a sphere. This quasi-like sphere, in addition to having a finite constant curvature, when constructed, will have a finite volume designated as, ($V_o = 2\pi^2 r^3$). Therefore, in accordance to Gaussian reasoning a sphere with a small radius will assume volume which is, ($V_o = \frac{4\pi r^3}{3}$). Upon expansion of such a sphere to its maximum limit the radius will have a value of (πr), in which the radius (r) will be the radius of curvature of space, (Schatzman, 1968). Assuming the foregoing conclusions is the correct interpretation in regard to General Relativity, then under these assumptions a new series of equations may be written to contain the entire Gaussian space in which the volume is equal to ($V_o = 2\pi^2 r^3$). When the cosmological conditions are such that the universe fills the entire space of a Gaussian volume, it may be assumed from the Schwartzchild radius, $Gm/rc^2 = ½$, the inability of light and matter to escape from its peripheral boundaries. In this situation, light should travel around in geodesics inside this critical radius, (Ney, 1962). The following equations are an attempt to demonstrate this relation. From equation (53)

$$\lambda^2 = \frac{hG}{c^3},$$

if $c^2 = a\lambda$ is substituted in part for (c^3), then the above equation becomes

$$\lambda^3 = \frac{hG}{ca}. \quad (200)$$

When the volume, ($\lambda^3 = V_o = 2\pi^2 r^3$) is substituted in equation (200) for lambda cube (λ^3), the expression may be written as

$$2\pi^2 r^3 = \frac{hG}{ca} \,. \qquad (201)$$

or

$$r^2 = \frac{hG}{\pi c^3} \qquad (202)$$

or

$$r = \sqrt{\frac{hG}{\pi c^3}} \,. \qquad (203)$$

If equation (203) is substituted in equation (174) for the radius (r)

$$E = \frac{m^2 c^3}{h} \quad 2\pi r$$

and repeat the algebraic mechanics as performed from equation (174) through (187), the following expressions are derived:

$$\text{(a) } E = \frac{2GMm}{r} \qquad \text{(b) } c^2 = \frac{4GM}{r} \qquad \text{(c) } E = \frac{C^6}{8\pi Ga} \qquad (204)$$

$$\text{(a) } F = \frac{GMm}{\pi r^2} \qquad \text{(b) } a = \frac{GM}{\pi r^2} \qquad \text{(c) } F = \frac{C^4}{4\pi G} \qquad (205)$$

Equation (204b) is an important relation in General Relativity and is referred as the Schwartzchild radius or singularity.

In the event equation (202), the radius square (r^2) is divided by $(\sqrt{2})^2$ or (2) and the algebraic mechanics performed, as in the foregoing, the equation resulting are as follow:

$$\text{(a) } E = \frac{4GMm}{r} \qquad \text{(b) } c^2 = \frac{4GM}{r} \qquad \text{(c) } E = \frac{C^6}{8\pi Ga} \qquad (206)$$

$$\text{(a) } F = \frac{2GMm}{\pi r^2} \qquad \text{(b) } a = \frac{2GM}{\pi r^2} \qquad \text{(c) } F = \frac{C^6}{8\pi G} \qquad (207)$$

When equation (206b) is written as

$$\alpha = \frac{4GM}{rc^2} \,, \qquad (208)$$

then alpha (α) becomes the total angle of deflection of light in a gravitational field.

A similar procedure may be used for a sphere whose volume is ($\lambda^3 = V_o = \frac{4}{3}\pi r^3$), in which equation (53)

$$\lambda^2 = \frac{hG}{c^3}$$

is written as equation (200)

$$\lambda^3 = \frac{hG}{ca} \quad .$$

If the volume of a sphere ($V_o = \frac{4}{3}\pi r^3$) is substituted for lambda cube (λ^3), then equation (200) becomes

$$r^2 = \frac{3\,hG}{2c^3} \qquad (209)$$

or

$$r = \sqrt{\frac{3hG}{2c^3}} \quad . \qquad (210)$$

When the radius (r) in equation (210) is substituted in equation (174)

$$E = \frac{m^2c^3}{h}\; 2\pi r$$

and the algebraic procedure is repeated, as above, the equations resulting are as follow:

$$\text{(a) } E = \frac{3\pi GMm}{r} \qquad \text{(b) } c^2 = \frac{3\pi GM}{r} \qquad \text{(c) } E = \frac{C^6}{6\pi^2 Ga} \qquad (211)$$

$$\text{(a) } F = \frac{3GMm}{2r^2} \qquad \text{(b) } a = \frac{3GM}{2r^2} \qquad \text{(c) } F = \frac{C^4}{6\pi^2 G} \qquad (212)$$

If we rewrite equation (211b) as

$$\theta = \frac{3\pi GM}{rc^2} \quad , \qquad (213)$$

then theta (θ) is the angle per revolution in radians for the precession of the geodesic effect in General Relativity.

If equation (209) the radius square (r^2) is divided by $(\sqrt{2})^2$ or (2), a new series of equations similarly developed in the earlier procedures may be derived. They are as follow:

$$\text{(a) } E = \frac{6\pi GMm}{r} \qquad \text{(b) } c^2 = \frac{6\pi GM}{r} \qquad \text{(c) } E = \frac{C^6}{12\pi^2 Ga} \qquad (214)$$

$$\text{(a) } F = \frac{3GMm}{r^2} \qquad \text{(b) } a = \frac{3GM}{r^2} \qquad \text{(c) } F = \frac{C^4}{12\pi^2 G} \qquad (215)$$

When equation (214b) is written as

$$\sigma = \frac{6\pi GM}{rc^2} \quad , \qquad (216)$$

sigma (σ) gives the radians per revolution for the precession of the perihelion. The above equation is not adjusted for the eccentricity of a circular path.

It is believed, that the algebraic material presented in this portion of the paper may be one method, among others, in which to arrive at the space-time metrics of a nonuniform gravitational field of General Relativity.

In conclusion in regard to the Modified Newtonian Theory, the writer wishes to present several density equations which are related to force and acceleration. There is no attempt made to adjust these equations to any sphericity or special space curvatures unless designated. The following expressions are used to demonstrate the relation of force to approximate the maximum density of mass. If the force is expressed as

$$F = ma$$

and the mass (m) is equal to the Volume (V_o) multiplied by the density designated as rho (ρ), we may write the relation as

$$F = V_o \rho a \quad . \qquad (217)$$

Since lambda cube (λ^3) may be expressed as the volume and is equal to Gmt^2, we obtain expression

$$F = [G\rho t^2] \; ma \quad . \qquad (218)$$

If we substitute expression (72)

$$p = \frac{c^5}{hG^2}$$

for the density in equation (218), the equation becomes

$$F = \left[\frac{c^5 t^2}{hG}\right] \; . \; ma \quad . \qquad (219)$$

The bracketted expression (219), upon substitution of values in which the time squared (t^2) was derived from a three constant equation (66), gives a dimensionless value of one. Therefore, we may set the bracketted expression equal to a constant in which

$$K = \frac{c^5 t^2}{hG} \qquad (220)$$

and the force becomes

$$F = Kma \quad .$$

From expression (218), if we solve for the density (ρ), we get

$$\rho = \frac{F}{Gmt^2 a} \quad . \qquad (221)$$

Since (at^2) is equal to (2d) or (λ) and the force is equal to (ma), the expression for the density in terms of acceleration is

$$\rho = \frac{a^2}{Gc^2} \qquad (222)$$

or

$$a = G\rho\lambda \qquad (223)$$

when $c^2 = a\lambda$. If we substitute equation (223) for the acceleration in equation (47) where

$$E = \frac{m^2c^5}{ha}$$

we get

$$m^2c^5 = hE\,G\rho\lambda. \qquad (224)$$

Dividing both sides of the equation by mc^2, hence

$$mc^3 = hG\lambda\rho \qquad (225)$$

since $\lambda = c^2/a$ and $mc^3 = ha$, the force may be written as

$$F = \frac{hG}{c} \cdot \rho\,. \qquad (226)$$

We may obtain now the expression of the density in terms of three constants as in equation (72) but derived in another manner by equating the force of equation (62) with expression (226). When

$$\frac{c^4}{G} = \frac{hG}{c} \cdot \rho \qquad (227)$$

and then solve for the density (ρ), we have a three constant equation to give the maximum density

$$\rho\text{Max.} = \frac{c^5}{hG^2}$$

The maximum density for a particle whose mass is $\cong 5.4 \times 10^{-5}$ gm. and its wavelength or circumference is $\cong 4.1 \times 10^{-33}$ cm., will have a mass-to-volume ratio of $\cong 8 \times 10^{92}$ gm./cm.^{3}. The magnitude of this value may be calculáted from equation (72), as expressed above.

If we assume that the maximum density for mass is indicative of orderliness, then the maximum disorder of the distribution of mass may be expressed in terms of entropy of the system, delta ($\triangle$) s. Since the equation for the entropy is expressed as

$$\triangle s = \frac{q}{T_{ko}} \qquad (228)$$

and the relation for the Ideal Gas Law is expressed as

$$\rho v_o = nRT_k o. \qquad (229)$$

We may substitute the absolute temperature ($T_k o$) in equation (228) to get

$$\triangle s\rho v_o = nRq\,.$$

Since volume (V_o or λ^3) is equal to Gmt^2), then the entropy of the total system is

$$\triangle s = \frac{q}{T_{ko}} \cdot \frac{\lambda^3}{Gmt^2} \qquad (230)$$

or in terms of density and time written as

$$\triangle s = \frac{q}{T_{ko}} \cdot \frac{1}{G\rho t^2}\ , \qquad (231)$$

since (m) is equal to (V_o/p). The dimensionless value related to the entropy equation, when adjusted to the Guassian volume may be expressed as uniform gravitational field.

$$\triangle s = \frac{q}{T_{ko}} \cdot \frac{4\pi r^2}{c^2 t^2}\ .$$

The derivation is accomplished by substituting equation (145) in equation (230) for the mass and lambda cube (λ^3) for the volume, ($V_o = 2\pi^2 r^3$).

The writer would like to terminate this paper with the comment that it appears as though General Relativity and Newton's Theory of Gravitation are a derivative or a proportional fraction of the Modified Gravitational Theory. It is believed that General Relativity is one of several theories which may be derived algebraically from the Modified Theory of Gravitation to give a more simplified understanding of the metrical nature of space due to curvature or properties related to a nonuniformed gravitational field.

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