January 1982

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L. Dale Van Vleck
University of Nebraska-Lincoln, dvan-vleck1@unl.edu

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L. D. VAN VLECK
Department of Animal Science
Cornell University
Ithaca, NY 14853

ABSTRACT

Theoretical weights for prediction of a son's genetic evaluation from genetic evaluations of his sire and dam are derived. Weights for sire and dam are equal and depend on number of daughters of the son when the bull evaluation is from his daughters alone and cow evaluation is from her records alone. Mixed model procedures which incorporate records of other relatives change theoretical weights. When records of daughters of both the son and his sire are used in evaluations of both, theoretical weights for the sire to predict the son's evaluation are about .50 for many combinations of daughter numbers of the bulls. Including the evaluation of the sire of the cow in her evaluation changes the theoretical weight to predict the son's evaluation from the dam's evaluation only slightly from the situation when only the dam's records are used. In addition, the theoretical weight for the maternal grandsire is nearly zero.

DISCUSSION

The purpose of this note is to show theoretical weights for predicting a son's evaluation from those of his parents. Theoretical weights for the sire's genetic evaluation and dam's genetic evaluation for predicting a progeny are both .5, but when they are used to predict the progeny's evaluation, unless the number of records in the progeny's evaluation is extremely large, the weights are usually less than .5 but may be equal. For example, theoretical weights are equal when records in evaluating the sire are only from his daughters and in evaluating the dam are only from her records. The Northeast Artificial Insemination Sire Comparison (NEAISC), however, uses records of daughters of a sire and daughters of his son to evaluate both the sire and son. The use of the maternal grandsire model (7) complicates the theoretical regression but will not be considered here because sire evaluation in pedigree analyses previously have not considered the maternal grandsire model (1, 2, 4, 5, 6, 8).

Let y_B be the sum of n_B records of daughters of a bull,
y_S be the sum of n_S records of daughters of his sire,
y_G be the sum of n_G records of daughters of his maternal grandsire, and
y_D be the sum of m records of his dam.

Heritability, h^2, and repeatability, r, will be .25 and .50 so that the ratio $(4-h^2)/h^2 = 15$ and the ratio $(1-r)/r = 1$. Phenotypic variance will be denoted as σ^2. All evaluations will be for transmitting ability (one-half additive genetic value) unless otherwise specified. Adjustment for all fixed effects, e.g., herd-year-season effects, will be assumed to be perfect.

Variances and covariances of the sums are:

$$V(y_j) = n_j(n_j + 15)\sigma^2/16 \quad (j = B, S, \text{ or } G)$$
$$V(y_D) = m(m + 1)\sigma^2/2$$
$$\text{Cov}(y_B y_S) = n_B n_S h^2 \sigma^2/8 = n_B n_S \sigma^2/32,$$
$$\text{Cov}(y_B y_D) = n_B m h^2 \sigma^2/4 = n_B m \sigma^2/16,$$
$$\text{Cov}(y_B y_G) = n_B n_G h^2 \sigma^2/16 = n_B n_G \sigma^2/64,$$
$$\text{Cov}(y_D y_G) = m n_G h^2 \sigma^2/4 = m n_G \sigma^2/16.$$

Equal Weighting of Sire's and Dam's Evaluations

Evaluation of the sire from single records of his own daughters is approximately

$$\tilde{s}_S = y_S/(n_S + 15).$$

Received May 8, 1981.

Similarly for the son

$$\tilde{s}_B = y_B/(n_B + 15),$$

and for the dam from her own records (ETA)

$$\tilde{s}_D = y_D/[4(m + 1)].$$

$$\left[V(\tilde{s}_S) = [1/(n_S + 15)]^2 V(y_S)\right] = \left[\frac{n_S}{n_S + 15}\right]\left[\frac{\sigma^2}{16}\right]$$

and

$$\mathrm{Cov}(\tilde{s}_S\tilde{s}_B) =$$

$$[1/(n_S + 15)]\,[1/(n_B + 15)]\,\mathrm{Cov}(y_S y_B)$$

$$= \left[\frac{n_S}{n_S + 15}\right]\left[\frac{n_B}{n_B + 15}\right]\left[\frac{\sigma^2}{32}\right].$$

Thus

$$b_{\tilde{s}_B \cdot \tilde{s}_S} = .5\left[\frac{n_B}{n_B + 15}\right]$$

as is well known. Similarly,

$$V(\tilde{s}_D) = \left[\frac{1}{16}\right]\left[\frac{1}{m + 1}\right]^2\left[V(y_D)\right]$$

$$= \left[\frac{m}{m + 1}\right]\left[\frac{\sigma^2}{32}\right] \text{ and}$$

$$\mathrm{Cov}(\tilde{s}_D\tilde{s}_B) = \left[\frac{m}{m + 1}\right]\left[\frac{n_B}{n_B + 15}\right]\left[\frac{\sigma^2}{64}\right]$$

Thus,

$$b_{\tilde{s}_B \cdot \tilde{s}_D} = .5\left[\frac{n_B}{n_B + 15}\right]$$

as for $b_{\tilde{s}_B \cdot \tilde{s}_S}$ and as is also well known. These equal weights for sire and dam approach .5 as the number of daughters of the son becomes larger. When other information is used to evaluate the sire, son, and dam, the weights may not be equal nor a simple function of n_B.

Unequal Weights for Sire and Dam When Relationship Between Sire's and Son's Daughters are Considered

Regression of Son's Sire Comparison (SC) on Sire's SC. The example will include only one son per sire, both of which are assumed unrelated to all other bulls. The σ^2 will be set equal to 1 since in the regression coefficients it will cancel out. The variance ratio is 15.

The numerator relationship matrix is

$$A = \begin{bmatrix} 1 & .5 \\ .5 & 1 \end{bmatrix} \text{ and } 15A^{-1} = \begin{bmatrix} 20 & -10 \\ -10 & 20 \end{bmatrix}.$$

Then the mixed model equations for the sire comparisons, $\hat{s}_S$ and $\hat{s}_B$, are:

$$\begin{bmatrix} n_S + 20 & -10 \\ -10 & n_B + 20 \end{bmatrix}\begin{bmatrix} \hat{s}_S \\ \hat{s}_B \end{bmatrix} = \begin{bmatrix} y_S \\ y_B \end{bmatrix}.$$

Thus

$$\begin{bmatrix} \hat{s}_S \\ \hat{s}_B \end{bmatrix} = \frac{1}{d}\begin{bmatrix} n_B + 20 & 10 \\ 10 & n_S + 20 \end{bmatrix}\begin{bmatrix} y_S \\ y_B \end{bmatrix},$$

where $d = (n_S + 20)(n_B + 20) - 100$ for convenience. Then

$$\hat{s}_S = [(n_B + 20)y_S + 10y_B]/d,$$

$$\hat{s}_B = [10y_S + (n_S + 20)y_B]/d,$$

$$V(\hat{s}_S) = (1/d)^2\,[(n_B + 20)^2 n_S(n_S + 15) + 100n_B(n_B + 15) + 10(n_B + 20)n_S n_B]/16$$

and

$$\mathrm{Cov}(\hat{s}_S\hat{s}_B) = (1/d)^2\,\{[20]\,[(n_B + 20)n_S(n_S + 15) + (n_S + 20)n_B(n_B + 15)] + [n_B n_S]\,[(n_S + 20)(n_B + 20) + 100]\}/32.$$

Thus the regression of $\hat{s}_B$ on $\hat{s}_S$ is

$$b_{\hat{s}_B \cdot \hat{s}_S} = \frac{[10]\,[(n_B + 20)n_S(n_S + 15) + (n_S + 20)n_B(n_B + 15) + [2n_S n_B]\,[(n_S + 20)(n_B + 20) + 100]}{(n_B + 20)^2 n_S(n_S + 15) + 100n_B(n_B + 15) + 10(n_B + 20)n_S n_B}$$

For many combinations of n_B and n_S, the weight is about .5 but can be larger, as shown in Table 1.

TABLE 1. Coefficients for regression of son's sire comparison on his sire's sire comparison.

No. of daughters		
Bull	Sire	$b_{\hat{s}_B \cdot \hat{s}_S}$
40	40	.615
40	1000	.505
60	1000	.505
1000	1000	.507

TABLE 2. Coefficients for regression of son's sire comparison, $\hat{s}_B$, on his dam's estimated transmitting ability, $\hat{s}_D$.

No. of daughters of bull	No. of daughters of his sire	$b_{\hat{s}_B \cdot \hat{s}_D}$
40	40	.343
40	1000	.334
60	1000	.375
100	1000	.417
1000	1000	.490

Regression of Son's SC on Dam's Estimated Transmitting Ability (ETA) From Own Records. If the dam's ETA is based on only m of her records, the ETA is

$$\hat{s}_D = b_D y_D$$

as before, where

$$b_D = 1/[4(m + 1)].$$

Note

$$V(\hat{s}_D) = b_D m/8,$$

$$Cov(y_D, y_S) = 0 \text{ and}$$

$$Cov(y_D, y_B) = n_B m/16.$$

Then

$$Cov(\hat{s}_D, \hat{s}_B)$$

$$= Cov\left[b_D y_D, \frac{10y_S + (n_S + 20)y_B}{d}\right]$$

$$= (n_S + 20)n_B m/16] [b_D/d]$$

and

$$b_{\hat{s}_B \cdot \hat{s}_D} = .5[n_B(n_S + 20)/d].$$

The theoretical weight depends primarily on the number of records by daughters of the bull as shown in Table 2.

Regression of Son's SC on Dam's ETA (With Maternal Grandsire SC Included). The Northeast cow ETA procedure uses the NEAISC of her sire as part of the information to evaluate the cow (3). The purpose of the following exercise was to determine if that information changes the theoretical regression of a son's NEAISC on his dam's ETA, which includes the maternal grandsire (MGS) NEAISC. The result of this simplified approximation indicates that the theoretical regression is approximately the same as if the NEAISC of the cow's sire were not included.

The approximation in the ETA program involves including the genetic value of the sire of the cow in the solution vector with the diagonal coefficient and right-hand side corresponding to what the selection index equations would have been to obtain twice the NEAISC (records are assumed perfectly adjusted for their herd-year seasons). The ETA program actually predicts additive genetic value or 2ETA which is then divided by 2 to obtain ETA.

If the cow had no other paternal sibs in the herd and if her dam has no records, the pair of equations involving the cow and her sire are (ignoring herd-year-season effects) (3):

$$[4t/3 + n(1 - r)/(4 - h^2)]\hat{g}_G - [t/3]\hat{g}_D = 2\hat{s}_G t[4 + (n - 1)h^2]/(4 - h^2)$$

$$[-t/3]\hat{g}_G + [4t/3 + mk/(m + k)]\hat{g}_D = ky_D/(m + k),$$

where $n = n_G$ is the number of other daughters of G to give equivalent accuracy for $\hat{s}_G$ as obtained from the NEAISC, m is the number of records of the cow, and the coefficients 4/3 and $-1/3$ are from the inverse of the relationship matrix. When $t = (1-r)/h^2 = 2.4$ for $r = .4$ and $h^2 = .25$, $k = (1-r)/(r-h^2) = 4$ as used in the

TABLE 3. Coefficients for regression of son's sire comparison on dam's estimated transmitting ability (incorporating her sire's proof) and on dam's ETA and her sire's proof.

Dam no. of records	No. of daughters			Weights		
	Bull	Sire	MGS[a]	Bull on dam	Bull on Dam	and MGS[a]
1	40	40	40	.343	.322	.022
1	40	1000	1000	.334	.323	.009
1	60	1000	1000	.375	.375	.000
1	1000	1000	1000	.490	.490	.000
3	40	40	40	.319	.306	.018
3	40	1000	1000	.314	.292	.025
3	60	1000	1000	.353	.334	.021
3	1000	1000	1000	.461	.436	.027

[a]Maternal grandsire.

ETA program. An approximation for y_G is $(n + 15)\hat{s}_G$. The equations can be rewritten as

$$(3.2 + .16n)\hat{g}_G - 1.6\hat{g}_D = .32y_G$$

$$-1.6\hat{g}_G + [3.2 + 4m/(m + 4)]\,\hat{g}_D = 4y_D/(m + 4).$$

From these equations $V(\hat{s}_D) = V(\hat{g}_D)/4$, $Cov(\hat{s}_D,\tilde{s}_B)$, $Cov(\hat{s}_D,\hat{s}_G)$, $V(\hat{s}_G)$, and $Cov(\hat{s}_G,\tilde{s}_B)$ can be approximated to approximate the theoretical regression of $\tilde{s}_B$ on $\hat{s}_D$ and the multiple regression of $\tilde{s}_B$ on $\hat{s}_D$ and $\hat{s}_G$ for various numbers of records on the dam and of daughters of the bull, his sire, and maternal grandsire. A representative set of theoretical weights are in Table 3.

Including the maternal grandsire evaluation in the dam's ETA changes the theoretical regression of son on dam little as seen from comparison of Tables 2 and 3. Table 3 also shows when the MGS is included in the dam's ETA that the weight for the MGS to predict her son is nearly zero. The surprising result is that the weights for the dam ETA become slightly smaller for more records on the dam, essentially because with more records on the dam the variance of the dam's ETA increases more than the covariance between the dam's ETA and her son's evaluation when her sire's evaluation is included in her evaluation.

ACKNOWLEDGMENTS

This research was supported in part by a grant from Eastern Artificial Insemination Cooperative, Inc., Ithaca, NY.

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