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Supporting representation-rich problem-solving in high school physics

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High school students' use of representations in physics problem solving

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Abstract

Findings from physics education research strongly point to the critical need for teachers' use of multiple representations in their instructional practices such as pictures, diagrams, written explanations, and mathematical expressions to enhance students' problem-solving ability. In this study, we explored use of problem-solving tasks for generating multiple representations as a scaffolding strategy in a high school modeling physics class. Through problem-solving cognitive interviews with students, we investigated how a group of students responded to the tasks and how their use of such strategies affected their problem-solving performance and use of representations as compared to students who did not receive explicit, scaffolded guidance to generate representations in solving similar problems. Aggregated data on students' problem-solving performance and use of representations were collected from a set of 14 mechanics problems and triangulated with cognitive interviews. A higher percentage of students from the scaffolding group constructed visual representations in their problem- solving solutions, while their use of other representations and problem-solving performance did not differ with that of the

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comparison group. In addition, interviews revealed that students did not think that writing down physics concepts was necessary despite being encouraged to do so as a support strategy.

Keywords: high school physics, modeling physics curriculum, multiple representations, physics problem solving, science education

1 Introduction

High school physics courses that prepare students to meet the performance expectations of the Next Generation Science Standards (NGSS) and the ACT College Career and Readiness Standards have a pedagogical structure that promotes the development of deep conceptual understanding through a broad range of qualitative and quantitative problem-solving skills applied in inquiry learning activities (American Association of Physics Teachers [AAPT], 2015). The NGSS science and engineering practices explicitly name problem-solving, computational thinking, and data analysis skills as critical to learning disciplinary core ideas (NGSS Lead States, 2013). Thus, developing problem-solving skills early in students' formal science education is vital regardless of the career path they choose.

Physics instruction, textbooks, and curriculum materials show that problem-solving is an intrinsic part of any physics course. Problem-solving guides and examples of solved problems are commonly included in physics textbooks; physics teachers typically expect that students will learn from these examples. Unfortunately, problem-solving instruction is known to fail when little attention is given to reasoning modes because focusing primarily on the use of mathematics leads students to prefer formula-centered problem-solving methods (Hestenes, Swackhamer, & Wells, 1992). The high level of abstraction and predominant role of mathematics in physics teaching has been one of the major reasons for problems that many students have with learning physics (Duit, Schecker, Hottecke, & Niedderer, 2014).

To help students solve problems and understand physics concepts, physics education research endorses the use of multiple representations (Rosengrant, Van Heuvelen, & Etkina, 2009). Representations are semiotic resources used to convey the ways of knowing of science such as oral and written language, diagrams, graphs, mathematics, and gestures (Airey & Linder, 2008). More recently, the performance expectations in the NGSS for high school physical science place a strong emphasis on mathematical representations and qualitative meanings of physics principles (NGSS Lead States, 2013). For instance, a key NGSS practice is developing and using models to (a) aid in constructing explanations, (b) understanding abstract relationships, (c) developing questions, (d) generating data to make predictions, and (e) to communicate ideas to others (NGSS Lead States, 2013). Previous studies have shown that modeling develops students' representational competence (Kozma & Russell, 2005). Since scientific practices are interrelated and do not operate in isolation (Bell, Bricker, Tzou, Lee, & Van Horne, 2012), the practice of modeling may lead to mathematical and computational thinking, which is necessary for analyzing and interpreting data (NGSS Lead States, 2013).

2 Literature review

The use of representations can both refer to external and mental representations (Kohl & Finkelstein, 2008) and both types of representations have been the subject of research in science education (Fredlund, Linder, Airey, & Linder, 2014; Kohl & Finkelstein, 2008; Pande & Chandasekharan, 2017; Savinainen, Makynen, Nieminen, & Viiri, 2013). Several studies have shown that the use of representations to make a problem-solving task easier is an expert-like approach (Kohl & Finkelstein, 2008; Nokes, Schunn, & Chi, 2010; Stylianou and Silver, 2004). Experts have been observed to start by visualizing a problem, then performing the conceptual analysis and planning steps before moving to implement a plan, while novices may simply look for plausible formulas without regard for the applicability of concepts (Mason & Singh, 2011). Understanding the effects of using problem-solving heuristics and the differences between expert and novice learners have been extensively studied in science education and cognitive psychology (Leonard, Dufresne, & Mestre, 1996; Lorenzo, 2005; Maloney, 1994; Polya, 2004). Studies in this area have investigated pedagogical strategies and curricular developments to consider ways of helping students develop more robust problem-solving approaches including students' representational skills.

An example of a curricular program that emphasizes the use of multiple representations is physics *modeling instruction,* a research-based curriculum design for high school science and a reform effort supported by the National Science Foundation (NSF). Modeling instruction has had great success at the high school level with its emphasis on active construction of conceptual and mathematical models in an interactive learning community through the use of activities focused on the process of building, validating, and deploying models (Brewe, Kramer, & O'Brien, 2009). In a comparative analysis of *Force Concept Inventory* (FCI) test scores, it was found that high school physics teachers who used the modeling method with fidelity had higher student posttest FCI mean scores and gains than more traditional teaching approaches (Jackson, Dukerich, & Hestenes, 2008). As the weaknesses in traditional physics instruction have been identified through science education research, the development of instructional scaffolding has become a flourishing research theme to seek effective ways to encourage expert-like problem- solving behaviors in students. Since fluid use of multiple representations is an important goal of physics education for successful problem solving (Van Heuvelen & Zou, 2001), it is productive to develop and explore the use of scaffolding strategies to support students. As the development of expertlike traits occurs in stages, the goal of instruction should be to aid students in making effective transitions from one stage to the next (Goldman, 2003).

In this study, we sought to understand how a guided approach for the use of multiple representations would affect students' problem-solving performance and use of multiple representations at a suburban, Midwestern U.S. public high school. We explored the role of using a scaffolding approach for generating representation-rich solutions to problems in a high school modeling physics course (Table 1) by investigating the following questions:

1. How does the scaffolding strategy of using problem solving tasks affect the students' use of representations and their problem-solving performance?
2. How do students address problem-solving tasks? Which representations do they use and how do they use them? and
3. What are the differences, if any, in misconceptions and problem-solving behaviors that may be related to students' use of representations?

Since physics education literature and the framework supporting the modeling instruction program suggest that expert- like

Table 1 Problem-solving tasks for students

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1. Visual representations

The students were expected to produce a visual representation with the following prompts:

- Draw a diagram(s) that represents your understanding of the problem (chart, graph, sketch, free-body diagram, picture, arrows)
- Label the diagram(s) with symbols of physical quantities given in the problem

2 Reasoning expressed in written language

The students were expected to express their reasoning in written language with the following prompts:

- Identify the key physics concepts that you think are relevant to solving the problem
- Briefly, explain how you will use the key concepts in your procedure for solving the problem and evaluating if your answer is correct

3 Mathematical representations

The students were expected to use mathematical representations with the following prompts:

- Identify the equations that you would need
- Derive the mathematical model that you would need to use in order to find a numerical solution

4 Numerical output

The students were expected to arrive at a numerical solution:

- Identify the numerical values of the physical quantities given in the problem
- Perform the appropriate operations on your derived mathematical model

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problem-solving skills such as the use of multiple representations may be supported by targeted training, we selected specific problem-solving tasks that point to the use of representations and formed a list of these tasks for students' use (**Table 1**). Thus, guided scaffolding involved the use of specific problem-solving tasks, which were assumed to encourage the use of specific representations (e.g., verbal, mathematical, pictorial, and graphical) that may assist in meaning-making. This study aims to contribute to the growing research interest on guiding students to generate their own representations in science to support their learning (Prain & Tytler, 2012).

3 Method

In high school physics, students begin to encounter multistep mathematical problems that require systematic approaches. Most science and mathematics standardized tests include such complex problems to evaluate conceptual knowledge and problem-solving skills. However, evaluation practices that focus solely on test scores may not reveal how the use of representations affects the problem-solving performance of students. Developing and evaluating curricular scaffolding materials for problem solving requires multiple methods to examine overall group performance and specific problem-solving behaviors. We used a multimethod research design to explore the effects of our scaffolding strategy with students.

3.1 Study context

This study was conducted in two algebra-based physics classes during an 18-week semester-long course at a public, U.S. Midwestern, high school with a population of n = 1,785. The school had an even distribution of students by gender (51% female and 49% male students). While several racial groups were represented, 81% of the students were White. A modest proportion of students (18%) were enrolled in a free and reduced lunch program. The school operates on a block schedule, a type of academic scheduling in which a student has fewer classes per day, but each class is scheduled for a longer period of time (i.e., 90 min). Thus, the selected block section physics classes met every day and students finished the course in one semester.

3.2 Student participants

The two honors physics classes in the study were taught by a high school teacher who used a modeling instruction curriculum. One section was arbitrarily assigned as the scaffolding group (SG) and the other section as the comparison group (CG). Preexisting class enrollment was used because random assignment of students to groups was not possible; thus, the study sample was a convenience sample. We gathered demographic data to describe the characteristics of both groups. Two surveys, the *Student Information Survey* (SIS) and the *Force Concept Inventory* (FCI) were administered to the two groups (SG: n = 23, CG: n = 20) at the beginning of the semester to compare the participants in terms of background variables and physics knowledge. The average age of the students (n = 43) in this study was 16.9 (SD = 0.7). The majority were female (60.5%) and were in their senior year (60.5%).

3.3 Physics instruction

The modeling physics course used a broader selection of representations than a comparable traditional course. A detailed description of modeling instruction, its goals and the research that undergirds its methods, can be found in the American Modeling Teachers Association website (https://modelinginstruction.org/). The students were studying Newtonian mechanics at the time that the study occurred. The homework problems used in this study were given to students after the key physics concepts involved in the problems were discussed in class (Appendix A).

3.4 Data sources

We collected data from student artifacts and cognitive interviews to understand how students use multiple representations in solving multistep physics problems. **Figure 1** shows the two data generating phases: (a) collecting problem-solving work of students (*n* = 43) on 14 homework problems for a period of 10 weeks and (b) interviewing selected students (*n* = 12). At least one homework problem was given each week, two problems at most. The teacher discussed and

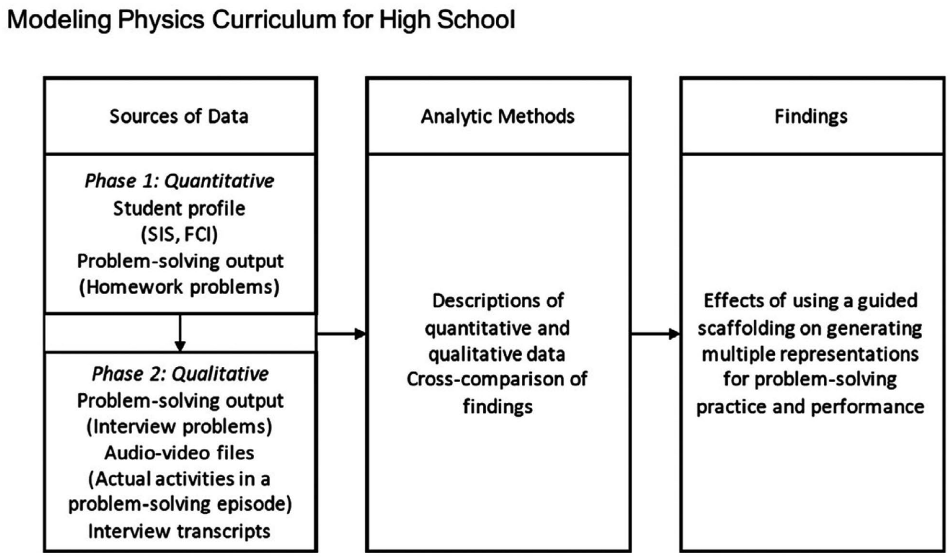

Figure 1 Research design including sources of data, analytic methods, and findings within the context of a modeling physics curriculum

introduced the list of problem-solving tasks to the scaffolding group before the homework problems were given to both groups. A copy of the list of problem-solving tasks was included in all homework assignments given to the scaffolding group. Students were aware that the homework was used for research and not graded by their teacher.

For the interviews, we purposely selected a total of 12 students (6 students from each group) based on their FCI posttest scores relative to their class' average performance. The FCI posttest score was used as an indicator of the students' physics knowledge after instruction. The FCI was a standard component of the high school physics teacher's modeling physics curriculum and it was administered when all topics tested in the FCI were covered. To sample heterogeneity, students with above average, average, and below average FCI posttest scores were recruited to participate in cognitive interviews in which they were instructed to think aloud while solving two physics problems.

The homework problems and interview problems used in this study were situational and well defined (i.e., problems with discrete representations and finite goals). Solving these problems required the student to engage in multiple steps to find an unknown physical quantity. In this article, we would refer to two problems as the "Blowgun" and "Skier" problems to discuss our findings. In the cognitive interviews (i.e., think-aloud interviews), the "Blowgun" problem was given first and the "Skier" problem last. These problems were taken from the physics textbook (Cutnell & Johnson, 2001) used by both groups:

1. The length of the barrel of a primitive blowgun is 1.2 m. Upon leaving the barrel, a dart has a speed of 14 m/s. Assuming that the dart is uniformly accelerated, how long does it take the dart to travel the length of the barrel?
2. A skier is pulled up a slope at a constant velocity by a tow bar. The slope is inclined at 25.0 with respect to the horizontal. The force applied to the skier by the tow bar is parallel to the slope. The skier's mass is 55.0 kg and the coefficient of kinetic friction between the skis and the snow is 0.120. Find the magnitude of the force that the tow bar exerts on the skier.

3.5 *Analytic methods*

The two groups were compared in terms of demographics (i.e., age, sex, year level) and student background variables (i.e., preparedness to take the physics course, most recent math course completed, time since most recent math course was completed, current math course enrollment, expected grade in physics course, expected study time per week and FCI pretest score) to assess if there were meaningful group differences at the beginning of the semester to address potential selection bias (Shadish, Cook, & Campbell, 2002). Comparisons between the two groups were analyzed using (a) independent samples Student's t test for continuous variables, (b) chi-square test for categorical variables, and (c) Fisher's exact test for categorical variables following Cochran's recommendation for 2 × 2 contingency tables with a minimum expected frequency of less than 5 (Kroonenberg & Verbeek, 2018). We hypothesized that there were no statistically significant differences between the two groups at the beginning of the semester. The Bonferroni correction was applied to account for the multiple comparisons used to test the hypothesis and to protect from Type I error. To determine if any of the 10 correlations was statistically significant, the p value must be $p \leq .005$. At the end of the data collection period, we investigated if there were statistically significant differences in FCI posttest scores and aggregate homework scores between the two groups.

To address the research questions, solutions to homework and interview problems were scored using a 3-point rubric (i.e., 0: No work, 1: Incorrect; 2: Inadequate; 3: Complete). A score of zero was given when there was no evidence of problem- solving work; one, when the solution reflected that the student was unable to identify the key concept(s) needed to proceed from one step to the next; (typically showing "plugand- chug" work); two, when the student was able to identify some key concept(s) and was able to come up with a somewhat organized solution (typically shows success in choosing equations and reveals misconceptions that lead to an incorrect answer); and three, when the student was able to identify the key concept(s) needed to proceed from one step to the next (typically shows organized work and an understanding of the problem). The students' total homework scores were used as a measure of performance in problem solving. Similarly, the students' representations were also identified and coded in alignment with the problem-solving tasks (Table 1) given to the scaffolding group (i.e., diagram, written explanations, symbolic math, and numeric math). The homework data were used to compare the problem-solving performance of the two groups, find patterns, and make direct interpretations for comparison with the narrative descriptions of the cognitive interviews.

4 Results

Using data from the *Student Information Survey* (SIS) and FCI pretest scores, the two groups' baselines were compared before the physics teacher introduced the tasks for generating multiple representations in homework problems. Statistical analyses (**Table 2**) indicated that there were no significant differences between the scaffolding and comparison groups in the assessed variables. This showed a high level of homogeneity between the two groups. Although the differences were statistically insignificant when evaluated at the corrected alpha-level, the scaffolding group had slightly more seniors than juniors [SG: 69.6%, CG: 50.0%], thus had more students with a calculus versus a precalculus background [SG: 65.2%, CG: 30.0%]. Most of the students in both groups stated that they felt unprepared or somewhat prepared to deal with the subject [SG: 60.9%; CG: 50.0%]. They were not concurrently enrolled in a math course [SG: 91.3%, CG: 90.0%]. They were mostly expecting a grade of an A after completing the course [SG: 87.0%, CG: 85.0%] and they were anticipating at least 5 hr of study time per week for the course [SG: 87.0%, CG: 70.0%]. The results from the baseline FCI were also relatively equivalent for both groups [SG: 25.48% (10.08), CG: 23.10% (8.12)].

4.1 Analysis of FCI posttest and problem-solving homework scores

Four students from the scaffolding group (17.4%) were removed from the study because they withdrew from the course or transferred to another section. Due to incomplete homework and missing FCI posttest for these students, their data were not included in the analysis. Two students from the comparison group (10.0%) were unable to take the FCI posttest.

Table 2. Demographic profiles of the two groups

Characteristics	*Scaffolding group (n = 23)*	*Comparison group (n = 20)*	*p value*
Age in years, mean (*SD*)	17.0 (0.6)	16.7 (0.8)	.069*
Sex, *n* (%)			
Male	9 (39.1)	8 (40.0)	.954**
Female	14 (60.9)	12 (60.0)	
Year level, *n* (%)			
Junior	7 (30.4)	10 (50.0)	.191**
Senior	16 (69.6)	10 (50.0)	
Preparedness to take the physics course, *n* (%)			
Unprepared/Somewhat prepared	14 (60.9)	10 (50.0)	.474**
Prepared/Very well prepared	9 (39.1)	10 (50.0)	
Last math course completed, *n* (%)			
Precalculus/Algebra/Trigonometry	8 (34.8)	14 (70.0)	.021**
Calculus	15 (65.2)	6 (30.0)	
Semester when last math course was completed, *n* (%)			
Last semester	18 (78.3)	15 (75.0)	>.999***
Two semesters ago or more	5 (21.7)	5 (25.0)	
Current math course enrollment, *n* (%)			
Not enrolled in a math course	21 (91.3)	18 (90.0)	>.999***
Enrolled in a math course	2 (8.7)	2 (10.0)	
Expected grade in the physics course, *n* (%)			
A (90–100)	20 (87.0)	17 (85.0)	>0.999***
B (80–84.9) or B+ (85–89.9)	3 (13.0)	3 (15.0)	
Expected study time per week, *n* (%)			
Less than 5 hr	3 (13.0)	6 (30.0)	.263***
5 or more hours	20 (87.0)	14 (70.0)	
FCI pretest: percent correct, mean (*SD*)	25.5 (10.1)	23.1 (8.1)	.396*

Abbreviations: *n*, number; *SD*, standard deviation.
*Student's *t*-test; **Pearson Chi-square; ***Fisher's exact test.

The FCI posttest average scores of both classes were higher than the entry threshold for understanding Newtonian mechanics (Hestenes & Halloun, 1995), which is 60% [SG: M = 77.00% (SD = 12.02), CG: M = 73.44% (SD = 12.16)]. Hake (1998) documented FCI data for over 6,000 high school and college students and showed that in reform-based courses using nontraditional teaching methods, high school students averaged about 65% on the FCI posttest, while average FCI posttest score for students in a modeling physics course was 74%. The two classes involved in this study had average scores comparable to the national average for modeling physics courses.

We ran an independent-samples t test to determine if there were differences in FCI posttest scores and problem-solving performance

Table 3. FCI posttest and problem-solving performance of the two groups

Characteristics	*Scaffolding group (n = 19)*	*Comparison group (n = 18)*	*p-value**
FCI posttest: percent correct, mean (*SD*)	77.00 (12.02)	73.44 (12.16)	.377
Aggregate homework score, mean (*SD*)	18.00 (10.23)	18.40 (11.34)	.909

*Student's *t* test.

based upon cumulative homework scores between the two groups. The group differences were not statistically significant (**Table 3**). After completing units on Newtonian mechanics, both groups' performance on the FCI posttest was relatively equivalent. Similarly, after 10 weeks of problem-solving homework, the aggregated homework data showed that both groups had a similar performance on the 14 problems. The mean scores of the scaffolding and comparison groups were 18.0 (*SD* = 10.2) and 18.4 (*SD* = 11.2), respectively.

While there was no significant difference between the aggregated problem-solving score of both groups, the mean score per problem of the scaffolding group was higher in 9 of 14 (64%) problems (Figure 2). To explore possible explanations for differences in performance, we compared the percentage of students drawing a visual representation per problem (**Figure 2**). The data showed that in 9 of 14 (64%) problems, the percentage of students who used visual representations was at least 20% higher in the scaffolding group. We coded each problem based on the representations used by the students (e.g., Lake Problem, Appendix B) and we found that the scaffolding appeared to have resulted in an increased use of visual representations, but there was no apparent difference in the students' use of verbal and mathematical representations. Specifically, students' least used representation in problem solving from both groups was the verbal type, which involves expressing reasoning in written language. Students did not identify the key concepts they used nor explained their mathematical solutions to the given problems.

4.2 Problem-solving cognitive interviews

In this section, we provide examples of the think-aloud cognitive interviews conducted with selected representative students from the

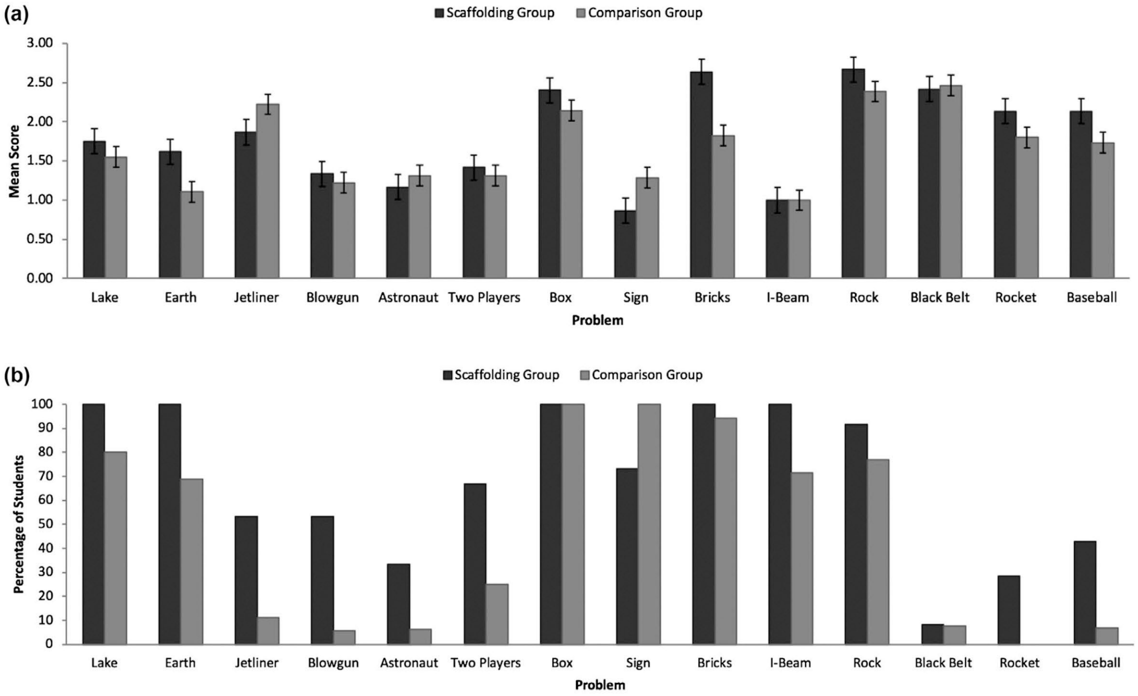

Figure 2. Students' performance (top) and use of visual representations (bottom) in the 14 homework problems. The graphs show the mean score of the groups per problem and the percentage of students who used visual representations per problem

two groups. The audio–video files of the interviews were transcribed to examine students' use of representations and other problem-solving patterns. We only present selected narrative descriptions of how students from both groups solved the "Blowgun" problem due to limited space. From the scaffolding group, Noah and Cat were selected as examples. As scaffolding group members, they were regular users of visual representations alongside their mathematical solutions. However, they differed in class performance and problem-solving performance, Noah had the highest FCI pre- and posttest scores in his group while Cat's scores were below the class average. Joshua and Mary were selected from the comparison group. Joshua was an atypical case because he regularly used visual representations even without prompting from the scaffolding guide. However, Mary was a typical comparison group member who occasionally used visual representations in problem solving. Like Noah, Joshua had the highest FCI pre- and posttest scores in his group.

4.2.1 Scaffolding group member: Noah

Noah is an example of a regular user of visual and mathematical representations in solving problems. He had the highest pre- and post-FCI scores (53% and 93%) in his class and he was relatively successful in problem solving. In Noah's interview, he reported that he uses visual representations to aid his understanding of the problem. We also observed that despite being relatively successful in problem solving compared to the rest of his class, Noah also exhibited some novice-like problem-solving behaviors such as formula-seeking and exploration of which equation would work for a particular problem.

Noah started by drawing an accurate picture based on the values given in the problem. In his representation, the dart is leaving the barrel at 14 m/s. He continued reading the problem aloud and then quickly decided what to do next. He said, "I'm probably going to use the quadratic." He correctly plugged in the values and recognized that he had two unknown variables in the equation: "...we don't know the acceleration and time. We have to figure that out." He set aside his equation with two unknown variables, looked at his equation sheet and mumbled, "What am I going to use?" He said he was looking for time and explained that it had been a while since he solved a similar problem. He then thought of using what he called as *v-vax*, quickly worked through the equation, and found a value for acceleration, which he plugged into the equation he previously derived from the *quadratic*. He successfully found the value for time but did not check his work. He said that he usually does not check his work, but he would think about the final answer for a second and see if it makes sense. He said, "A fifth of a second for a dart to leave a blowgun is pretty reasonable."

Noah explained that he started with the *quadratic* because that was his favorite equation, stating that it works for many different kinds of problems. He further explained that when he was stuck while working on the problem, he had to look for a different equation to suit his needs better with only one unknown variable. When asked about the purpose of his illustration, he said it is a lot easier for him to look at a picture instead of only reading a problem; drawing helps improve his understanding and avoid reading a problem repeatedly without really understanding it.

4.2.2 Scaffolding group member: Cat

Cat is an example of a regular user of visual and mathematical representations but was unsuccessful in solving both interview problems correctly. Her pre- and post-FCI scores (17% and 63%) were below the class average. When interviewed, she explained that she sometimes generates visual representations that may or may not provide useful information in problem solving.

After reading the problem, Cat said she would first draw a picture because visuals help her. She drew a picture of a gun and wrote down the length of the gun's barrel and the velocity of the dart. She did not identify whether the velocity was the dart's initial velocity or final velocity. She read the problem again and then looked at her equation sheet. She used an equation meant for motion at constant velocity. She copied the equation and then inserted the values and divided 1.2 m by 14 m/s to get a value for time and a final answer of 0.0857 s. Cat said that to check if her answer is correct, she could work back into the equation or use a different equation. She also said that she does not normally check her answers and commented that maybe she should check them.

4.2.3 Comparison group member: Joshua

Joshua was a regular user of visual and mathematical representations and was relatively successful in problem solving. He had the highest pre- and post-FCI scores (43% and 93%) in his class. Joshua used visual representations to aid his understanding of the problem. Despite being relatively successful in problem solving, Joshua also tended to engage in formula-seeking until he gained a more complete understanding of how to proceed.

During the cognitive interview, Joshua read the problem and drew a picture. He labeled his picture with the information given. He said that the dart leaving the blowgun was traveling at 14 m/s and then identified that he needed to find how long it would take the dart to travel the length of the barrel. He said that he needed an equation that includes distance and velocity and decided that he would start with the *quadratic*. He wrote the equation, plugged in the known values, and noticed that acceleration was missing. "This isn't right," he said, "I'm trying to figure out if this is the right equation because my brain is telling me right now that this is for falling bodies.” He reasoned that since the problem described the dart’s motion as uniformly accelerated, there was an acceleration from zero to 14 m/s.

Joshua reviewed his equation sheet, wrote down the given values again, and identified the missing values. Joshua realized that he needed to find the acceleration of the dart before he could find the unknown time of travel. He said he would try to find the acceleration by using *v-vax* first, and then he would come back to the *quadratic*. After finding the acceleration, Joshua said, “That just seems that it has gone way too fast, but it makes sense. I’m going to run through it again, sort of, in my head.” He checked his calculation and then he plugged in the value of the acceleration to the *quadratic*. He used dimensional analysis to check the units and recorded his final answer.

Joshua said that he started with the *quadratic* because he was thinking of falling bodies but then he realized that was not the case for the given problem. When he was stuck, he said he had to go back to his equation sheet and see if there was something else that might work better. He also explained that he drew a picture because it helps him if he can visualize something. He said that he could check if his answer is right by plugging the value back to the equation.

4.2.4 Comparison group member: Mary

Mary is an occasional user of visual representations and was unsuccessful in solving both problems given during the cognitive interview. Her pre- and post-FCI scores (23% and 63%) were below the class average. In her interview, she explained that she does not draw diagrams for problems that seem simple, like the “Blowgun” problem. She drew a free body diagram for the “Skier” problem, but she mainly used numerical manipulations of given values.

Mary read the problem and then looked at her equation sheet. She said that she was trying to find time, so she will use a velocity equation. She wrote down the given values and then tried to do numerical manipulation but stopped and said she was trying to remember the lessons from the last semester. She said that she did not like the first equation she tried to use. She entered numbers in her calculator and then wrote “.0857 s.” She said that she took 1.2 m and divided it by the velocity to get the time. In the interview, Mary explained that she would usually try to figure out what variables are given and then find an equation in which she could plug in values to get what she is looking for. If she is stuck, she explained, “Usually, I try to do different things that I’m not sure that’ll work just to see if I can get a logical answer.” She also commented that if she were taking a test, she would review her solution and try to plug her answer back into an equation to ensure that it works, but she does not usually check her homework.

4.3 Triangulation of results

Results from the homework data supported our cognitive interview findings. There were homework problems in which students typically used visual representations even without having an explicit prompt for using multiple representations in their written solution (e.g., Joshua). In solving force problems, students customarily used a visual representation in the form of a free-body diagram. When students were asked why they drew a diagram for the “Skier” and not the “Blowgun” problem, they often stated that they were taught to use a force diagram for “those types of problems” and that a diagram helps.

The limited use, or lack of use, of physics concepts was observed in the interview data. Students were not influenced by the given problem-solving tasks to describe or explain the concepts they used in problem solving (e.g., Noah and Cat). Although mathematical reasoning was apparent in some of the students’ solutions, the interview data suggest that students seemed to believe they were demonstrating expertise by quickly finding equations and lacing them together more than being able to apply physics concepts. The problems were treated as basic math problems in which the students attempted to find an unknown value based on a set of given parameters. For instance, students who were unable to solve the “Blowgun” problem demonstrated a common error of plugging in the given value, 14 m/s as the dart’s initial velocity, based on verbal cues in the problem without careful analysis (e.g., Cat and Mary). In the case of SG students, those who drew a diagram to depict the motion of the dart typically solved the problem successfully. On the other hand, students who identified the problem as “fairly easy” claimed that they did not need to draw a diagram. In solving the “Blowgun” problem, four out of six

CG students (67%) began solving the problem by searching for useful equations, one (17%) drew a picture and one (17%) wrote down given information. Alternatively, three out of six SG students (50%) began solving the problem by drawing a picture and then labeled it with known values, two (33%) of them searched for equations, and one (17%) wrote down given information. Both the homework data and interview data showed that the problem-solving tasks may have influenced SG students to start solving the problem by drawing a picture or writing down given values since the majority of CG students immediately searched for an equation from their equation sheet during the interviews.

The data from the "Skier" problem showed that while students use free-body diagrams they may lack the ability to interpret and use their diagrams to construct mathematical expressions. Although all SG students drew a free-body diagram, misconceptions such as the idea that the normal force acting on an object is always equal and opposite to the object's weight led them to draw incorrect representations. Students also customarily drew the friction force vector in the negative x-axis without considering an object's direction of motion. They cited physics concepts with limited understanding of what they were saying. Students who were relatively successful in solving problems on the applications of Newton's laws of motion appeared to have the habit of analyzing their diagrams and they constructed mathematical equations consistent with their free-body diagrams. On the other hand, students who were least successful also drew free-body diagrams but focused on manipulation of equations without evaluating if their diagram is an accurate representation of the described mechanical system.

5 Discussion

Our rationale for exploring how the use of a scaffolding for generating representations in problem solving was to understand how students use multiple representations as they engage in solving multi-step physics problems. Using our findings from the homework data and think-aloud interviews, we return to address the questions that guided our inquiry.

5.1 Research question 1

Our first task was to examine how the scaffolding strategy described in Table 1 affected students' use of representations and problem-solving performance. In terms of students' use of representations, we found that SG students incorporated visual representations in the homework problems more than CG students. However, we found that the scaffolding did not increase students' use of verbal representations in the form of written descriptions and explanations in their problem-solving work. The use of mathematical representations was central to the problem-solving work of students from both groups. Overall, while the use of the scaffolding may be associated with an increased use of visual representations, SG and CG students showed similar patterns of use of verbal and mathematical representations.

In terms of problem-solving performance, there was no statistically significant difference in the aggregated homework scores between the two groups. While there was an observed increase in the use of visual representations in the SG group, it was not associated with an increase in their problem-solving performance. Other representations such as verbal representations to express reasoning remained underdeveloped for both groups. In the student interviews, we found that most students did not think about the concepts, which could explain their inability to express why they were doing what they were doing. These results suggest that developing students' representational skills in problem solving remains to be a challenging task that requires more research-based instructional interventions. Other studies have found that in order to develop students' representational skills as they apply to problem solving, an instructional environment that encourages the use of multiple representations across all aspects of the course should be promoted (Kohl & Finkelstein, 2006). In our study, it appears that although the modeling instruction design supported the use of multiple representations during laboratory activities and completion of worksheets, most students were not using multiple representations throughout the course and may require consistent instigation. This directly connects to the NGSS scientific practice of modeling. Even in a high-achieving group of students, the students still sidestepped using multiple representations that might have assisted their learning of core physics concepts.

5.2 Research question 2

Our second task was to examine how students address problem-solving tasks while solving problems and to determine which representations they used and how they used them. From the interview data, we found that three out six SG students (50%) began solving the interview problem by drawing a picture, while four of six CG students (67%) immediately looked for equations from their equation sheet. We also found that students who did not draw a diagram considered the problem to be easy and assumed that visual aids were not needed since they could proceed with picking the right equations to generate a solution. Although students claimed that the problem was simple, four out of six students (67%) in both groups did not solve the problem correctly. In both groups, students who did not draw a visual representation of the problem failed to solve it. In a previous study, Lin and Singh (2011) found that a common difficulty among introductory physics students during problem solving was that they did not draw a free-body diagram, resulting in many analytical mistakes.

Students in both groups used a combination of visual and mathematical representations in their problem solving, but the use of trial-and-error in applying equations seemed to be most students' default mode. Although SG students incorporated visual representations in their problem-solving work, their next step was to refer to their equation sheet and find an equation that might work. The equation sheet appeared to be a valuable scaffolding for the students and if used properly, it could be a useful aid in problem solving. Overall, students from both groups demonstrated novice-like problem-solving approaches such as formula-seeking, which showed that they had a weak understanding of the underlying conceptual basis of the equations (Mason & Singh, 2011). The interview data showed that even when they found the right equation, they may inaccurately interpret the physical meaning of the given quantities. Our findings support problem-solving approaches that put an emphasis on writing qualitative descriptions for solving problems (Leonard et al., 1996) such as the two-column solution promoted by Docktor, Strand, Mestre, and Ross (2015) in their conceptual approach to problem solving.

5.3 Research question 3

Our final task was to examine misconceptions and problem- solving behaviors that may be related to students' use of representations. Since the homework data provided limited information about the actual problem-solving process, it was necessary to observe the students as they explained how they approached solving well-defined, multistep problems. The cognitive interviews showed that students were less likely to succeed in solving a problem if the visual representation they constructed was based upon solely the problem's surface features and if they generated a representation only as a part of a routine procedure. Finding an answer, any answer even if it was the wrong answer, appeared to be the major concern of most of the students from both groups rather than attempting to fully understand the underlying physics concepts. Thus, apart from the increased use of visual representations among SG students, the problem-solving approach favored by students typically involved the use of their equation sheet to find an equation to use. Similarly, misconceptions were revealed in free-body diagrams illustrated by the students in both groups. Previous studies have shown that misconceptions can originate from various sources (Kikas, 2004). For instance, students may tend to remember patterns from the example problems modeled in class. Thus, teachers should be thorough when drawing free-body diagrams. Other studies have recommended that sufficient instructional time should be devoted to teaching students how to draw correct free-body diagrams as students are more likely to succeed in problem solving if they are able to represent the problem correctly with one (Rosengrant et al., 2009).

5.4 Implications for instruction

Our findings indicate that the scaffolding used in this study only had the desired effect in the students' use of visual representations. Although more SG students used visual representations, their performance as a group did not appear to differ from the comparison group since the visual aids they created varied in quality. In both groups, relatively successful students drew diagrams that they later used to choose equations and operations.

As beginning learners of formal physics, high school students are inexperienced problem solvers who need guidance on how to use various representations as a problem-solving strategy. We anticipate that students would have a positive attitude toward completing problem-solving tasks if the use of visual representations and verbal explanations are explicitly modeled with example problems. However, if the modeled procedure mainly focuses on merely identifying what the given values are, and which quantities are missing, the problem- solving process can become overly formula centered, which can result in students failing to acknowledge the significance of understanding the underlying physics concepts. Kohl and Finkelstein (2006) previously found that a pervasive use of different representations and multiple representations by teachers in a learning environment appears to broaden students' representational skills. Modeling the use of multiple representations should not be limited to problems involving the use of free-body diagrams. Explicit instruction on analyzing situations in terms of concepts and using multiple representations may better support expert-like problem-solving behaviors and possibly lead to greater success. Follow-up work on how students may be supported in better understanding why the use of multiple representations is useful in problem solving would be a productive research endeavor, especially in light of continued reform efforts embodied in the NGSS.

6 Conclusions

Conceptual knowledge is vital in K-12 problem solving, but high school students are still inexperienced in applying physics concepts to problem solving. There is a need for developing instructional scaffolding to help students engage in problem solving as a cognitive activity by analyzing problems in terms of underlying physics concepts and using multiple representations. Modeling, constructing explanations, and communicating reasoning are only some of the NGSS scientific practices that are addressed when a curriculum-wide support for the use of multiple representations are designed and implemented. Further study of instructional practices that emphasizes verbal reasoning in problem solving is warranted in order to identify which specific elements of instruction should be modified to inhibit backward problem-solving tendencies such as formula- seeking and equation manipulation at the expense of deep conceptual learning.

7 Limitations of the study

The implications for instruction we have discussed in this article are not definitive and could be explored further through research with a larger sample size and stricter control of group compositions. Since a sample of convenience was used in this study, it would be better to think in terms of translating the results to a comparable situation rather than generalizing the findings to the same context and content (Lincoln & Guba, 1985). Such a context would be a high school honors physics course with a modeling physics curriculum design. Another issue to consider in interpreting the results of this study is the sample distribution by gender. There were slightly more female than male students in both groups and research has shown that boys tend to have stronger visual spatial skills due to gender bias in providing supportive activities for the development of those skills (Eliot, 2009). This study was also constrained by the use of homework scores as a measure of problem-solving performance. Although there were no outliers and the distribution of homework scores were found to be normal, the study was not protected from potential diffusion over groups and contamination effects since students were completing their homework outside a laboratory setting. The findings also suggest that the scaffolding should be modified to increase the intervention differential between the groups. Specifically, the scaffolding group should be encouraged to use the multiple representation tasks throughout the course, especially the use of verbal reasoning to ensure reliable intervention fidelity.

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Appendix A

Table A1 Key physics concepts in the 14 homework problems

Topic/Problem	*Key physics concepts*
Kinematics	
1 Lake	Distance, displacement
2 Earth	Average speed, average velocity
3 Jetliner	Average acceleration
4 Blowgun	Uniformly accelerated motion
5 Astronaut	Uniformly accelerated motion
6 Two Players	Uniformly accelerated motion
Dynamics	
7 Box	Application of Newton's first law of motion, Superposition of forces, Static equilibrium
8 Sign	Application of Newton's first law of motion, Superposition of forces, Static equilibrium
9 Bricks	Application of Newton's first law of motion, Superposition of forces, Dynamic equilibrium
10 I-beam	Application of Newton's first law of motion, Superposition of forces, Dynamic equilibrium
11 Rock	Application of Newton's second law of motion, Superposition of forces
12 Black Belt	Application of Newton's second law of motion, Average acceleration
13 Rocket	Application of Newton's second and third laws of motion, Apparent weight
14 Baseball	Application of Newton's second law of motion, Frictional force

Appendix B

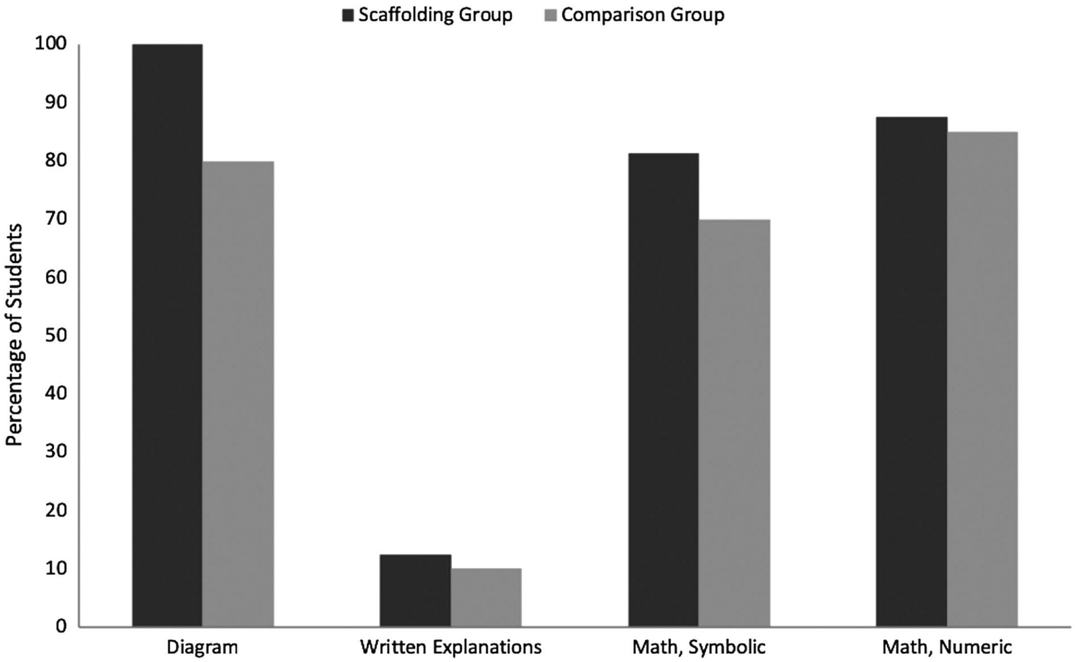

Figure B1 Students' use of representations in the "Lake" problem