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Abstract

Selection of dairy cows for economic merit requires the use of a model for genetic merit that involves the cross-product of the genetic values for milk and milk fat percentage. A linear and a quadratic index are described as procedures for selection of cows, the linear index being $b_1p_m + b_2p_t$ and the quadratic index being $b_1p_m + b_2p_t + b_3p_m^2 + b_4p_t^2 + b_5p_mp_t$ where p_m and p_t are the phenotypic deviations for milk and milk fat percentage, respectively, and the b's are selection index weights. These indices are compared to one another and to other possible selection procedures in terms of the expected genetic progress in economic merit that would result from their use. Predicted gains are maximum for the quadratic index. Relative to these gains, the gains from the linear index are nearly equivalent, while those from a restricted index and a simplified form of the quadratic index are somewhat less, the extent of the decrease depending on the means for milk and milk fat percentage. Gains in economic merit from selection for milk are only slightly less than those from the quadratic index at all but very high mean levels of milk production. Selection for milk thus appears to be a reasonable selection procedure to improve economic merit, except at high mean levels of milk production. Changes in the parameters of milk production and milk fat percentage and in economic values could affect these comparisons.

Selection index procedures have long been recognized as valuable in the selection of dairy cattle. The objective of selection has not been, and is not presently, as clearly recognized. The relative importance of type and production has often been discussed in general terms, but specific economic values for type characteristics have not yet been obtained. The value of production itself is not clear-cut, since practically all payments for production depend upon the percentage of one of the constituents, usually milk fat percentage (test). Three courses of action seem feasible in selection for economic merit or dollar value: a) to select for milk production alone, assuming that this will increase economic merit considerably, b) to select for milk production holding the percentage of some constituent, say fat, constant as suggested by Butcher et al. (1), or c) to select for production and percentage of some constituent according to their contribution to economic merit. The failure of selection for milk production alone to make maximum gains in economic merit has been suggested in a recent empirical study by Spahr (5), but the relative genetic progress in the economic merit to be expected from these three procedures has not previously been examined.

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The purposes of this paper are a) to describe the use of an explicit procedure for selection based on milk production and test according to their contribution in determining economic merit, and b) to compare expected genetic progress in economic merit by selection by the explicit procedure, by selection for milk holding test constant, and by selection for milk yield or for milk fat yield.

Experimental Procedures

Description of total economic merit. Selection for economic merit depends on the pricing structure for the product sold. In most fluid milk sales, the value of product is given by the expression:

$$D = P[v_1 + v_2(T - T_b)]$$

in which

- D is dollars received,
- P is kilograms of milk produced,
- v_1 is the value per kilogram of milk at the base test,
- v_2 is the test differential, or the change in the value of each kilogram of milk with each change of 0.1% test,
- T is the test of the milk, and
- T_b is the base test.

The value of production used here is \$0.11023/kg of milk at a base test of 3.5%, with a differential of 1.323¢/kg of milk/1% test. As an example, the value for the product of a cow with a genetic capability of 6,350 kg of milk with a 3.6% test would be:

$$D = 6350\ [\$.11023 + \$.01323\ (3.6 - 3.5)], \\ = \$708.40.$$

Gross economic values are used because of lack of knowledge of net economic values, even

though net economic values would be more appropriate. A pricing system based on test differential is not necessarily the best system under present market conditions, nor is it necessarily one that represents the pricing system of dairy cattle production appropriate for the next generation.

The equation representing value of product can also be written as:

$$D = P[v_1 + v_2T - v_2T_b], \\ = P[a_1 + a_2T], \quad [I]$$

in which

a_1 is $v_1 - v_2T_b$, and
a_2 is v_2.

This arrangement results from T_b being a constant for a given pricing system.

It is also worth noting that Equation [I] can be written as:

$$D = Pa_1 + PTa_2,$$

and, since for individual records yield of milk fat is the product of yield of milk by test value of product can also be expressed as:

$$D = a_1P + a_2F, \quad [II]$$

in which

F is yield of milk fat.

This reflects the fact that due to the part-whole relationship of milk fat and milk production, knowledge of any two of milk production, milk fat production, and test specifies the third characteristic.

Total economic merit, or aggregate genotype, can then be defined for value of product as given in [I] as:

$$M = (\mu_m + g_m)[a_1 + a_2(\mu_t + g_t)] \quad [III]$$

in which

M is total economic merit or dollar value,
μ_m and μ_t are the population means for milk and test, respectively,
g_m and g_t are the genotypic deviations from population means for milk and test, respectively, and,
a_1 and a_2 are as defined above.

For the expression for value of product as given in [II], total merit is

$$M = a_1(\mu_m + g_m) + a_2(\mu_f + g_f) \quad [IV]$$

in which

μ_m and g_m are as defined above,
μ_f is the population mean for milk fat yield, and
g_f is the genotypic deviation from the population mean for milk fat yield.

Indices and comparisons. Several indices are compared for selection of cows with one milk production and one test record each. Four of these indices select directly for economic merit, one selects for milk production while holding test constant, and six select for some single characteristic. Indices for economic merit different from the usual linear selection index are required, since the definition of economic merit [III] contains the cross-product of the genotypic values for two traits.

The four indices that select directly for economic merit are:

(i) The linear index described by Wilton and Van Vleck (10), with the index defined as:

$$I_f = b_f'p_f,$$

in which

I_f is the linear index,
p_f is the vector of phenotypic observations on milk production and test expressed as deviations from means, and
b_f is the vector of selection index weights for p_f, found as

$$P^{-1}G \begin{bmatrix} a_1 + \mu_t a_2 \\ \mu_m a_2 \end{bmatrix}$$

with P being the phenotypic and G the genotypic variance-covariance matrix for milk and test.

(ii) The quadratic index described by Wilton et al. (9), which is an explicit procedure, with the index defined as:

$$I_q = b_q'p_q,$$

in which

I_q is the quadratic index,
p_q is the vector of phenotypic deviations for milk production and test and the squares and cross-products of these phenotypic deviations, and
b_q is the vector of selection index weights.

This quadratic index is equivalent (9) to an index based on substitution of index values for milk and test into the merit equation. Thus, the quadratic index value can be determined as:

$$I_q = I_s = (\mu_m + I_m)[a_1 + a_2(\mu_t + I_t)] \\ - \mu_m(a_1 + a_2\mu_t)$$

in which

I_s is the substitution index value,
μ_m and μ_t are the population means for milk and test, respectively, and
I_m and I_t are the index values for milk and test, respectively, each based on informa-

tion on both milk and test and expressed as deviations from population means.

(iii) A simplified form of the quadratic index, found as:

$$I_{ss} = (\mu_m + I_{m(1)})[a_1 + a_2(\mu_t + I_{t(1)})] - \mu_m(a_1 + a_2\mu_t),$$

in which

I_{ss} is the simplified substitution index,
$I_{m(1)}$ is the index value for milk based on milk production information only, and
$I_{t(1)}$ is the index value for test based on test information only.

(iv) An index based on milk production and milk fat production, appropriate to the model of economic merit given in Equation [IV].

The indices that do not select directly for economic merit are:

(v) An index based on increasing milk production as rapidly as possible while holding test constant, this being a restricted index as developed by Kempthorne and Nordskog (3).

(vi) An index for milk production based on milk production information only.

(vii) An index for milk production based on milk production and test information.

(viii) An index for test based on test information only.

(ix) An index for test based on test and milk production information.

(x) An index for milk fat production based on milk fat production information.

(xi) An index for milk fat production based on milk fat production and milk production information.

The parameters concerning milk and milk fat in this study (Table 1) are based on variances reported by Harville (2) and Van Vleck (8) and on average heritabilities and correlations reported in the literature. The variances are extrapolations to 1967 values by use of the regression of variance on time (8). The parameters concerning test are determined from knowledge of the relationships among milk production, milk fat production, and test as based on expectations. The means for milk and milk fat production are 6,404 and 232 kg, respectively, as reported for Holsteins in the May, 1967, A.I. Sire Summary prepared by Cornell University. The corresponding mean test is determined by expectations to be 3.635%. The parameters of milk and test are considered to be constant throughout, and the parameters concerning milk fat production are determined at each combination from the relationships among the three traits. The parameters concerning milk fat production are thus slightly different at each combination of means (Table 2).

TABLE 1. Within-herd parameters of milk, milk fat, and milk fat percentage (test).[a]

	Milk	Milk fat	Test
	Variances and covariances		
Milk	1,525,420 (381,355)	49,283.9	−96.116
Milk fat	10,268.9	1,966 (491.5)	2.488
Test	−55.721	1.811	.0934 (.0599)
	Heritabilities and correlations		
Milk	1.00 (.25)	.90	−.25
Milk fat	.75	1.00 (.25)	.18
Test	−.37	.33	1.00 (.64)

[a] Phenotypic parameters on the diagonal and above, genotypic parameters in parentheses and below the diagonal, with milk and milk fat measured in kilograms and test as a per cent.

The indices are compared first by the indexing of 25 hypothetical individuals. The 25 individuals arise from taking all possible combinations of −2, −1, 0, +1, and +2 standard devations from the Holstein mean for both milk and test. This procedure is then repeated at nine combinations of mean levels of milk and test.

The relative usefulness of each index for increasing economic merit is represented by its relative selection efficiency (4). Genetic progress in economic merit to be expected from the use of the indices is given by the usual formula:

$$\Delta M = \frac{\sigma_{MI}}{\sigma_I} \cdot \frac{z}{p}$$

in which

ΔM is genetic progress in total merit,
σ_{MI} is the covariance between total merit and index,
σ_I is the standard deviation of the index,
z is the height of the ordinate of the normal distribution at the point of truncation, and
p is the proportion of individuals selected.

This formula is valid for cases in which the index and merit have a bivariate normal distribution. Merit is not normally distributed when it contains the product of two variables that are normally distributed, but a normal

TABLE 2. Parameters[a] concerning milk fat production at various combinations of mean levels of milk and milk fat percentage (test).

Mean levels[b] Milk	Test	h	r_{pmf}	r_{gmf}
	1	.23	.94	.86
	2	.23	.95	.88
1	3	.23	.96	.90
	4	.23	.97	.92
	5	.23	.97	.93
	1	.25	.90	.76
	2	.24	.92	.80
2	3	.24	.93	.83
	4	.23	.94	.86
	5	.23	.95	.88
	1	.27	.86	.66
	2	.26	.88	.71
3	3	.25	.90	.75
	4	.24	.91	.78
	5	.24	.93	.81
	1	.29	.81	.56
	2	.28	.84	.61
4	3	.27	.86	.66
	4	.26	.88	.71
	5	.25	.90	.74
	1	.32	.75	.46
	2	.30	.79	.52
5	3	.29	.82	.58
	4	.28	.84	.63
	5	.27	.86	.67

[a] Heritability (h), phenotypic correlation with milk (r_{pmf}), and genetic correlation with milk (r_{gmf}).

[b] Mean levels of milk and test, where the values of 1, 2, 3, 4, and 5 represent 3,930, 5,170, 6,400, 7,640, and 8,870 kg for milk and 3.01, 3.32, 3.64, 3.95, and 4.26% for test, respectively.

distribution may be a reasonable approximation. For a constant selection intensity relative genetic progress depends on σ_{MI}/σ_I and not on z/p, so lack of normality is not so important for predicting relative genetic progress as for predicting absolute genetic progress.

The expected genetic progress in total merit resulting from the use of an index for selection for milk is equivalent to:

$$\Delta M = (\mu_m + \Delta g_m)[a_1 + a_2(\mu_t + \Delta g_t)] - \mu_m(a_1 + a_2\mu_t),$$

in which

Δg_m is the genetic gain in milk, and
Δg_t is the correlated response in test.

A similar equivalence holds for selection for test. The correlated response in a characteristic is given by the known formula:

$$\Delta g_i = b_{g_i \cdot I}\, \Delta I,$$

in which

Δg_i is the gain in the i^{th} characteristic,
$b_{g_i \cdot I}$ is the regression of the i^{th} characteristic on the index, and
ΔI is the change in the index value $(\frac{z}{p}\sigma_I)$.

This equation for correlated response holds for the response in characteristics involved in total merit when selection is directly for total merit itself, as well as when selection is for some single trait.

Results and Discussion

The linear (i), quadratic (ii), and milk and milk fat (iv) indices yield approximately the same index values for the 25 possible individuals (Table 3). The quadratic index appears to place more emphasis on test than does the linear index when milk levels are high, since index values found by the quadratic index show smaller differences from one level of milk production to the next and greater differences between cows at a common level for milk when milk deviations are above zero. The opposite trend when milk deviations are below zero indicates that the quadratic index emphasizes test slightly less than does the linear index at these lower levels of milk.

The milk and milk fat index (iv) is expected to give similar index values to those found by the quadratic index because of the relationships among milk, milk fat, and test. The values are not exactly identical because the mean for milk fat is not exactly the product of the mean of milk by the mean of test and, hence, at a deviation of zero for both milk and test the deviation for milk fat is not zero. Further failure of the index for milk and milk fat to be identical to the quadratic index could result from lack of validity in the assumptions of normality underlying the equations giving the relationships among milk, milk fat, and test.

The index values for the various possible individuals as calculated by the quadratic index change with changes in the means for milk and test (Table 4). The ranking of the individuals also changes considerably with changes in the means. As a result of these changes in rank, animals with high test would be selected against when the mean for milk is low, so that greater progress could be made in milk production. At higher means for milk, rankings are based more on test. This agrees with the findings of Spahr (5) that changes in milk fat percentages become more important when the starting milk fat percentage is low and when the production level is high.

TABLE 3. Index values for several individuals as calculated by linear (L), quadratic (Q), simplified (S), milk and milk fat (MF), and restricted (R) indices at the Holstein means for milk and milk fat percentage (test).

Milk (kg)	Deviations of individuals Test (%)	Milk fat[a] (kg)	Indices L	Q	S	MF	R
	−.62	−112	−72.5	−70.7	−99.7	−68.6	−78.8
	−.31	−101	−68.1	−66.8	−84.4	−66.0	−71.4
−2,470	0	−89	−63.7	−63.6	−69.2	−63.4	−63.9
	.31	−77	−59.3	−61.0	−53.9	−60.8	−56.4
	.62	−64	−55.0	−59.0	−38.6	−58.2	−48.9
	−.62	−76	−40.6	−40.3	−66.8	−38.5	−46.9
	−.31	−60	−36.2	−35.8	−50.8	−35.0	−39.4
−1,235	0	−44	−31.9	−31.8	−34.6	−31.6	−31.9
	.31	−28	−27.5	−28.5	−18.5	−28.2	−24.5
	.62	−12	−23.1	−25.8	−2.4	−24.7	−17.0
	−.62	−39	−8.8	−10.0	−33.8	−8.3	−15.0
	−.31	−19	−4.4	−4.7	−16.9	−4.1	−7.5
0	0	1	0.0	0.0	0.0	0.2	0.0
	.31	21	4.4	4.1	16.9	4.5	7.5
	.62	41	8.8	7.6	33.8	8.7	15.0
	−.62	−2	23.1	20.4	2.4	21.8	17.0
	−.31	22	27.5	26.5	18.5	26.9	24.5
1,235	0	46	31.9	31.9	34.6	32.0	31.9
	.31	70	36.2	36.7	50.8	37.1	39.4
	.62	93	40.6	40.9	66.8	42.2	46.9
	−.62	36	55.0	50.9	38.6	52.0	48.9
	−.31	63	59.3	57.6	53.9	57.9	56.4
2,470	0	91	63.7	63.8	69.2	63.8	63.9
	.31	118	68.1	69.4	84.4	69.7	71.4
	.62	146	72.5	74.3	99.7	75.6	78.8

[a] Calculated as [(Mean for milk + deviation for milk) × (Mean for test + deviation for test)] − Mean for milk fat.

The relative emphasis being placed on milk and test in the quadratic index is difficult to determine. It is apparently close to the relative emphasis in the linear index and this value can be determined (Table 5) as $b_m\sigma_{p_m}/b_t\sigma_{p_t}$ where b_m and b_t are the weights for milk and test, respectively, in the linear index and σ_{p_m} and σ_{p_t} are the phenotypic standard deviations for milk and test, respectively. The negative values of relative importance have little meaning, simply indicating again that test should be selected against at a low mean for milk. The changes in the importance of milk relative to test further indicate that low test and high milk production are conditions that result in decreased emphasis on milk relative to test. Milk should apparently receive over seven times as much emphasis as test in selection of cows at the present Holstein means for milk and test. Even at a higher level of milk and a lower level of test, the emphasis on milk should be about four times greater than that on test.

The relative weights of milk and test in Table 5 can be compared to a relative emphasis of milk to test of 4.4 for the restricted selection index. There exist several combinations of means at which the restricted selection index gives the same relative weight to milk and test as does the linear index.

Selection efficiency, relative to the quadratic index, is near unity for the linear (i) and the milk and milk fat (iv) indices at the Holstein mean (Table 6) and at all other combinations of means (Table 7). The quadratic index apparently offers little advantage over a linear index in making genetic progress in total merit, based on these two particular traits and their pricing structure. However, it does offer possible advantages computationally by allowing substitution of index values into the merit equation. Use of a simplified index results in about 93–95% as much expected genetic progress as the quadratic index. Thus, although computationally advantageous this procedure does not appear very acceptable. The restricted selection index is only slightly less efficient than the quadratic index at the present Holstein means for milk and test, indicating that

TABLE 4. Index values for several individuals as calculated by the quadratic index at nine combinations of means for milk and milk fat percentage (test).

Deviations of individuals		Means for milk (kg)								
		3,930			6,400			8,870		
		Means for test (%)								
Milk (kg)	Test (%)	3.01	3.64	4.26	3.01	3.64	4.26	3.01	3.64	4.26
	−.62	−54.4	−57.2	−60.0	−67.9	−70.7	−73.4	−81.4	−84.1	−86.9
	−.31	−56.3	−59.9	−63.6	−63.2	−66.8	−70.5	−70.1	−73.7	−77.4
−2,470	0	−58.7	−63.3	−67.9	−59.0	−63.6	−68.3	−59.4	−64.0	−68.6
	.31	−61.7	−67.3	−72.8	−55.5	−61.0	−66.6	−49.3	−54.8	−60.4
	.62	−65.3	−71.8	−78.3	−52.6	−59.0	−65.5	−39.8	−46.2	−52.7
	−.62	−26.6	−27.0	−27.5	−39.9	−40.3	−40.8	−53.2	−53.6	−54.1
	−.31	−27.7	−29.1	−30.4	−34.4	−35.8	−37.2	−41.1	−42.5	−43.9
−1,235	0	−29.4	−31.7	−34.0	−29.5	−31.8	−34.2	−29.7	−32.0	−34.3
	.31	−31.7	−34.9	−38.1	−25.3	−28.5	−31.7	−18.9	−22.1	−25.3
	.62	−34.6	−38.7	−42.9	−21.6	−25.8	−29.9	−8.6	−12.8	−17.0
	−.62	1.3	3.2	5.0	−11.8	−10.0	−8.1	−24.9	−23.1	−21.2
	−.31	1.0	1.9	2.8	−5.6	−4.7	−3.8	−12.2	−11.2	−10.3
0	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	.31	−1.6	−2.5	−3.4	5.0	4.1	3.2	11.6	10.6	9.7
	.62	−3.7	−5.6	−7.4	9.4	7.6	5.7	22.5	20.7	18.8
	−.62	29.2	33.4	37.6	16.3	20.4	24.6	3.3	7.5	11.7
	−.31	29.6	32.9	36.1	23.2	26.5	29.7	16.8	20.1	23.3
1,235	0	29.4	31.7	34.0	29.6	31.9	34.2	29.7	32.0	34.4
	.31	28.6	30.0	31.3	35.3	36.7	38.1	42.0	43.4	44.8
	.62	27.2	27.6	28.1	40.5	40.9	41.4	53.8	54.2	54.7
	−.62	57.2	63.7	70.2	44.4	50.9	57.4	31.6	38.1	44.6
	−.31	58.3	63.9	69.4	52.1	57.6	63.2	45.9	51.4	57.0
2,470	0	58.8	63.5	68.1	59.2	63.3	68.4	59.5	64.1	68.8
	.31	58.8	62.5	66.1	65.7	69.4	73.0	72.6	76.3	79.9
	.62	58.1	60.8	63.6	71.5	74.3	77.1	85.0	87.8	90.5

TABLE 5. Weights for milk and milk fat percentage (test) and the emphasis on milk relative to test in the linear index at several combinations of levels of milk and test.[a]

Milk means (kg)	Test means (%)				
	3.01	3.32	3.64	3.95	4.26
	.02379[b]	.02474	.02566	.02659	.02754
3,930	−4.038[c]	−5.530	−7.022	−8.514	−10.006
	−23.8[d]	−18.1	−14.8	−12.6	−11.1
	.02385	.02480	.02573	.02665	.02760
5,170	6.513	5.021	3.529	2.037	.545
	14.8	20.0	29.5	52.9	204.6
	.02392	.02487	.02579	.02672	.02767
6,400	17.064	15.572	14.080	12.588	11.096
	5.7	6.5	7.4	8.6	10.1
	.02399	.02493	.02586	.02681	.02773
7,640	27.614	26.122	24.630	23.138	21.646
	3.5	3.9	4.2	4.7	5.2
	.02405	.02500	.02593	.02687	.02780
8,870	38.165	36.673	35.181	33.689	32.197
	2.5	2.7	3.0	3.2	3.5

[a] Weights for milk and test are the same for the quadratic index as for the linear index, while the weights for milk squared, test squared, and the cross-product of milk and test are .0000000126, —3.09807, and .00192, respectively, at all combinations of means.

[b] Weights for milk (b_m).

[c] Weights for test (b_t).

[d] Relative emphasis ($b_m\sigma_{p_m}/b_t\sigma_{p_t}$).

TABLE 6. Expected genetic progress in total merit and changes in milk, milk fat percentage (test), and milk fat from various selection procedures at Holstein mean production and test.

Index	ΔM	$\Delta M/\Delta M_q$	Δ Milk	Δ Test	Δ Milk fat
	($)		(kg)	(%)	(kg)
Linear	31.04	1.000	291.6	-.019	......
Quadratic	31.06	1.000	291.5	-.019	......
Simplified	24.97	.939	222.7	.049	......
Milk and milk fat	31.01	.999	291.8	-.019	9.3
Restricted	30.90	.995	275.8	.000	......
Milk (1)[a]	30.60	.985	308.8	-.045	8.3
Milk (2)	27.24	.877	326.9	-.105	5.2
Test (1)	-4.28	-.134	-182.3	.196	......
Test (2)	-3.48	-.112	-175.4	.196	......
Milk fat (1)	29.47	.949	231.6	.041	11.1
Milk fat (2)	24.72	.796	144.1	.100	11.7

[a] Numbers in parentheses indicate whether the index for the trait under selection was based on information on one or two characteristics.

selection keeping test constant is almost the correct procedure at present means. At other mean levels of milk and test, the relative selection efficiency of the restricted index is practically unity; at still other levels it is considerably less than unity.

Selection for milk production based on milk information only gives the greatest expected genetic progress in total economic merit of all procedures for selection for a single trait at the present Holstein means and at all other combinations of means except those at the highest level of milk production considered. At this high level of milk, selection for milk fat production based on milk fat production information only appears preferable. The rela-

TABLE 7. Relative selection efficiency from various selection procedures at several combinations of mean levels of milk and milk fat percentage (test).[a]

Mean levels		Selection procedure[b]									
Milk	Test	L	S	MF	R	M(1)	M(2)	T(1)	T(2)	F(1)	F(2)
	1	.999	.935	.998	.963	.992	.941	-.309	-.284	.927	.852
	2	1.000	.935	.998	.959	.992	.946	-.320	-.296	.935	.886
1	3	1.000	.935	.998	.956	.992	.949	-.331	-.306	.940	.911
	4	1.000	.935	.998	.953	.991	.953	-.340	-.316	.944	.930
	5	1.000	.935	.998	.950	.991	.956	-.349	-.325	.947	.944
	1	.999	.936	.999	.986	.991	.905	-.206	-.181	.929	.770
	2	.999	.936	.999	.983	.992	.912	-.222	-.197	.937	.814
2	3	1.000	.935	.999	.980	.993	.918	-.237	-.212	.944	.850
	4	1.000	.935	.999	.977	.994	.924	-.251	-.226	.948	.878
	5	1.000	.935	.999	.974	.994	.929	-.264	-.239	.952	.901
	1	.999	.940	.998	.998	.977	.856	-.097	-.071	.933	.714
	2	.999	.939	.998	.996	.981	.867	-.118	-.092	.942	.758
3	3	1.000	.939	.999	.995	.985	.877	-.138	-.112	.949	.796
	4	1.000	.938	.999	.993	.987	.886	-.156	-.130	.954	.828
	5	1.000	.937	.999	.991	.989	.894	-.173	-.147	.958	.856
	1	.999	.946	.998	.998	.952	.796	.015	.041	.939	.691
	2	.999	.945	.998	.999	.959	.812	-.011	.015	.948	.727
4	3	1.000	.944	.998	1.000	.965	.826	-.035	-.009	.955	.761
	4	1.000	.942	.998	.999	.971	.838	-.057	-.031	.960	.792
	5	1.000	.941	.999	.999	.975	.850	-.078	-.052	.964	.820
	1	.999	.954	.998	.986	.915	.727	.125	.151	.947	.692
	2	.999	.952	.998	.990	.926	.747	.096	.121	.955	.720
5	3	1.000	.950	.998	.994	.936	.765	.068	.094	.962	.747
	4	1.000	.948	.998	.996	.945	.782	.043	.069	.967	.773
	5	1.000	.947	.998	.998	.952	.797	.019	.045	.970	.798

[a] Relative to selection by the quadratic index.
[b] The selection procedures are linear (L), simplified (S), milk and milk fat (MF), and restricted index (R), and selection for milk, test, or fat based on information on one or two characteristics denoted by M(1), M(2), T(1), T(2), F(1), and F(2), respectively.

tive selection efficiency of selection for test is negative in most cases, so that total emphasis on test would result in a loss in returns.

The present findings in general confirm those of Tabler and Touchberry (6, 7) that selection for milk is preferable to selection for milk fat. However, they failed to consider selection for total merit directly, which as shown here is the most desirable procedure in terms of increasing genetic merit for economic value. The present findings conflict with those of Spahr (5), who found that test was 47% as important as milk in determining income for daughters of AI Holstein sires and that milk fat accounted for 91% and milk for 78% of the variance in income. The present study, however, is based on cow selection, whereas Spahr's study was based on sires used in artificial insemination.

The changes in milk and test expected to occur under each type of selection (Table 8) are also quite informative. Even though test receives a positive weight in the linear index in all cases except at the lowest level of milk considered, the change in test is negative until the mean for milk is quite high. This results from the negative correlation between milk and test. The changes in both milk and test resulting from use of either the linear or quadratic index are intermediate to other procedures, as would be expected. Some progress in economic merit with the quadratic index comes from consideration of the squares and cross-product of milk and test. In contrast to the linear and quadratic indices, the simplified index leads to an increase in test at all combinations of means, even though this may not be desirable.

Results of this study are specific for the economic values used, for the phenotypic and genetic parameters used, and for selection for cows with one record each. Changes in economic values might result in significant changes in the emphasis to be placed on milk relative to test and in the point at which test should be decreased rather than increased. Changes in phenotypic and genetic correlations between milk and test and in the ratios of milk standard deviations to test standard deviations might also affect the results, but such changes should not be large. Selection of sires will be considered in a subsequent paper.

TABLE 8. Expected changes in milk and milk fat percentage (test) from selection by linear (L), quadratic (Q), and simplified (S) indices at several combinations of mean levels of milk and test.[a]

Mean levels		Change in milk (kg)			Change in test (%)		
Milk	Test	L	Q	S	L	Q	S
1	1	313	313	259	—.053	—.053	.018
	2	314	314	261	—.055	—.055	.015
	3	315	315	263	—.057	—.057	.013
	4	316	316	265	—.059	—.059	.011
	5	317	317	267	—.061	—.061	.009
2	1	301	301	237	—.032	—.032	.038
	2	303	303	240	—.036	—.036	.035
	3	305	305	244	—.039	—.039	.032
	4	307	307	247	—.042	—.042	.029
	5	308	308	249	—.044	—.044	.026
3	1	285	285	214	—.011	—.011	.056
	2	288	288	218	—.015	—.015	.053
	3	292	292	223	—.019	—.019	.049
	4	294	294	227	—.023	—.023	.046
	5	297	297	230	—.026	—.026	.043
4	1	265	265	190	.011	.011	.074
	2	270	270	196	.006	.006	.070
	3	275	275	200	.001	.001	.066
	4	279	279	205	—.003	—.003	.062
	5	282	282	210	—.008	—.008	.059
5	1	243	243	166	.033	.033	.089
	2	249	249	172	.027	.027	.085
	3	255	255	178	.021	.021	.081
	4	260	260	184	.016	.016	.078
	5	265	265	189	.012	.012	.074

[a] Changes in milk and test from other selection procedures are the same for all combinations of means as for the Holstein means as in Table 6.

Conclusions

A quadratic index appears to be a useful index for selecting cows for economic merit. The expected genetic gain in economic merit is maximum for the quadratic index and the computation of the index is reasonably simple and flexible with changing economic values and mean levels of milk and test. Selection by a linear index or a restricted index offers no advantages computationally, whereas selection by a simplified form of the quadratic index, although computationally easy, would result in considerably less genetic progress in economic merit.

Selection for milk production could be used effectively as an alternative to the quadratic index for selection of cows in situations other than those in which the mean for milk production is quite high. Selection for milk production is computationally easier than selection by the quadratic index, results in nearly equal predicted genetic gains in economic merit, and does not require knowledge of future economic values. Selection for milk fat production would in most cases be less efficient than selection for milk production, whereas selection for test would result in a decrease in economic merit.

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