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J. W. Wilton
Cornell University

L. Dale Van Vleck
University of Nebraska-Lincoln, dvan-vleck1@unl.edu

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J. W. WILTON [1] and L. D. VAN VLECK
Department of Animal Science, Cornell University, Ithaca, New York

Abstract

Procedures for selecting sires are compared in terms of making maximum genetic progress in economic merit. A quadratic index, a simplified form of the quadratic index and a linear index to evaluate economic merit, and a restricted index to improve milk as much as possible while holding milk fat percentage constant, all result in nearly equal expected genetic progress in economic merit. Selection for milk production results in less expected genetic progress in economic merit, the extent of the decrease depending on the economic value of milk production at a given base test and on the test differential.

Evaluation of sires for economic merit depends on the economic values and on the means of milk production and test. The simplified form of the quadratic index appears to be a useful index, since sire economic merit index values can be calculated for any mean levels of milk production and test and for any economic values, once index values for milk and test have been found.

The economic value of dairy cattle production depends on the amount of milk and the percentage of milk fat under most present pricing systems. Selection for economic merit among cows with one record each has been discussed in an earlier paper (5).

The purpose of this paper is to examine sire selection for economic merit by comparing the expected genetic progress in economic merit resulting from the use of different indices, and by examining changes in sire index values at several combinations of levels of milk production and milk fat percentage (test).

Experimental Procedures

1. *Description of total economic merit.*

The value of milk production can be expressed as:

$$D = P[v_1 + v_2(T - T_b)],$$

in which

D is dollars received,
P is kilograms of milk produced,
v_1 is the value per kilogram of milk at a base test,
v_2 is the test differential, or the change in the value of each kilogram of milk with each change of 1.0% in test,
T is the test of the milk, and
T_b is the base test.

The genetic value for total economic merit can then be expressed as:

$$M = (\mu_m + g_m)[a_1 + a_2(\mu_t + g_t)],$$

in which

M is total economic merit,
μ_m and μ_t are the population means for milk and test, respectively,
g_m and g_t are the genotypic deviations from population means for milk and test, respectively,
a_1 is $v_1 - v_2T_b$, the value per kilogram of milk at 0.0% test, and
a_2 is v_2.

Values of \$.1102/kilogram of milk at a base test of 3.5% for a_1 and \$.0132/kilogram/1.0% test for a_2 were used as basic representative economic values with other values used to illustrate the effect of changes in economic values. Means of 6,400 kg for milk and 3.635% for test were taken as the current Holstein means.

2. *Comparisons of expected genetic progress.*

The expected genetic progress in economic merit is given for six possible selection procedures:

a) Selection directly for economic merit by a substitution index:

$$I_s = (\mu_m + I_m)[a_1 + a_2(\mu_t + I_t)] - \mu_m(a_1 + a_2\mu_t), \qquad \text{(I)}$$

in which

I_s is the substitution index, which is equivalent to a quadratic index (4),
μ_m and μ_t are the population means for milk and test, respectively,
I_m and I_t are the selection index values for milk and test, respectively—each based on information on both milk and test, and
a_1 and a_2 are as described above;

b) selection directly for economic merit by a simplified form of the substitution index:

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[1] Present address: Department of Animal Science, University of Guelph, Guelph, Ontario, Canada.

$$I_{ss} = (\mu_m + I_{m(1)})[a_1 + a_2(\mu_t + I_{t(1)})] - \mu_m(a_1 + a_2\mu_t), \quad \text{(II)}$$

in which

I_{ss} is the simplified index,
$I_{m(1)}$ is the selection index value for milk based on milk information only, and
$I_{t(1)}$ is the selection index value for test based on test information only;

c) selection directly for economic merit by a linear index based on milk production and test, as developed by Wilton et al. (4),
d) selection for maximum improvement in milk production while holding test constant as developed by Kempthorne and Nordskog (2), and described by Butcher et al. (1);
e) selection for milk production; and
f) selection for milk fat production.

The expected genetic progress in economic merit is based on the sire as the object of evaluation and the comparisons of expected genetic progress are based on 50 daughters per sire with one record each. This number corresponds roughly to a first proof in a young sire proving program. The phenotypic parameters used here are:

$$\sigma^2_{\bar{p}m} = \frac{1}{n}\left[\sigma^2_{pm} + (n-1)(.25)(\sigma^2_{gm})\right],$$

$$\sigma_{\bar{p}m\bar{p}t} = \frac{1}{n}\left[\sigma_{pmpt} + (n-1)(.25)(\sigma_{gmgt})\right], \text{ and}$$

$$\sigma^2_{\bar{p}t} = \frac{1}{n}\left[\sigma^2_{pt} + (n-1)(.25)(\sigma^2_{gt})\right],$$

in which

$\sigma^2_{\bar{p}m}$, $\hat{\sigma}_{\bar{p}m\bar{p}t}$, and $\sigma^2_{\bar{p}t}$ are the phenotypic variances and covariance of the average milk production and test of daughters, having values of 123,938 kg^2, −15.574 kg%, and .0166%^{2}, respectively.
σ^2_{pm}, σ_{pmpt}, and σ^2_{pt} are the phenotypic variances and covariance of single records for milk production and test, having values of 1,525,420 kg^2, −96.116 kg%, and .0934%^{2}, respectively,
σ^2_{gm}, σ_{gmgt}, and σ^2_{gt} are the genotypic variances and covariances of single records for milk production and test, having values of 381,355 kg^2, −55.721 kg%, and .0599%^{2}, respectively,
n is 50, and
.25 represents the additive genetic relationship of paternal half sisters.

Expected genetic progress per generation for each procedure comes from the usual formula:

$$\Delta M = \sigma_{MI}/\sigma_I \cdot z/p$$

in which

ΔM is expected genetic progress in economic merit,
σ_{MI} is the covariance between the genetic value for economic merit and the index used for selection,
σ_I is the standard deviation of the index used for selection,
z is the height of the ordinate of the normal distribution at the point of truncation, and
p is the proportion of individuals selected.

Results and Discussion

1. *Expected genetic progress.*

Expected genetic progress in economic merit is almost equal for the quadratic, simplified, linear, and restricted indices at present Holstein means for milk at \$.1102/kilogram with a \$.0132/kilogram/1.0% differential (Table 1). The nearly equal progress expected from the use of a simplified index for sire evaluation contrasts with a relative selection efficiency (RSE) of 94% for the simplified index in selection of cows with one record each (5). This difference is due to the greater accuracy in evaluating each characteristic in a sire proof, so that the other characteristic provides correspondingly less information about the characteristic being indexed. These results indicate that sire selection could accurately be done by the simplified index, as well as easily done because of flexibility with respect to changes in economic values and means.

The relative selection efficiency of selection for milk only is not as high as any of the indices using information on both milk production and test. It is also lower than the 98% relative selection efficiency for cow selection (5). This is due to a greater accuracy of milk evaluation with a corresponding greater correlated reduction in test. This selection for milk fat is lower than that for milk, as in cow selection (5) at present means.

The expected change in test resulting from the use of a quadratic index is negative (Table 1), as is that from the linear and simplified indices. Thus, as for cow selection (5), an optimum selection procedure at present Holstein means and representative economic values would lead to a reduction in test.

The effects of changes in economic values are given in Table 2 for the quadratic, simplified, and milk indices. The relative selection efficiency of the simplified index is nearly unity at other combinations of economic values, as well as at the representative values just dis-

TABLE 1. Expected genetic progress per generation in economic merit and changes in milk, milk fat percentage, and milk fat from various procedures of sire selection at Holstein mean milk production and test.[a]

Index procedure	Expected genetic progress in economic merit	Progress relative to the quadratic index	Change in milk	Change in test	Change in milk fat
	($)[b]		(kg)	(%)	(kg)
Linear	55.99	.999	513.9	−.018	
Quadratic	56.02	1.000	513.6	−.018	
Simplified	55.91	.998	502.0	−.004	
Restricted	55.82	.996	498.8	.000	
Milk	53.40	.953	542.1	−.079	14.6
Milk fat	51.69	.923	406.6	.072	19.5

[a] Milk valued at $.1102/kilogram with a $.0132/kilogram/1.0% differential and means of 6,400 kg for milk and 3.635% for test.
[b] Selection intensity is one phenotypic standard deviation (selection of one sire out of 20 would give a selection intensity of 2.1 standard deviations).

cussed. The accuracy of the simplified index thus appears to be unaffected by changes in economic values. The relative selection efficiency of selection for milk increases with increased values of milk at a common test differential, but decreases with increased test differentials at a common value of milk. Future economic values can considerably affect this criterion of selection for milk. If the value for milk increased markedly while the test differential remained constant, it would be possible for the relative selection efficiency of selection for milk to be nearly unity. If the economic values used were not correct, the gain in economic merit from selection for milk could be even greater than the gain in economic merit from selection by the quadratic index based on these incorrect values.

An idea of the emphasis that should be placed on milk relative to test in selection of sires can be obtained by considering the selection index weights for the linear index (Table 3). The relative emphasis on milk to test of 5.5 in sire selection at present Holstein means, with milk valued at $.1102/kilogram with a $.0132/kilogram/1.0% differential, is slightly less than the 7.4 ratio in selection of cows (5) for similar conditions. The relative emphasis is, however, greater than might be inferred from the study by Spahr (3), who found that test was 47% as important as milk in determining income for daughters of Holstein sires used in artificial insemination. The changes in the degree of emphasis on milk relative to test follow the same pattern as the relative selection efficiency for selection for milk. As the value of milk increases at a common test differential, the emphasis on milk should be increased and as the differential increases at a common value for milk, the emphasis on milk should be decreased.

TABLE 2. Expected genetic progress per generation in economic merit ($)[a] from sire proofs on 50 daughters at Holstein mean milk production and milk fat percentage at nine combinations of economic values.

Economic values		Index		
Milk[b]	Differential	Quadratic	Simplified	Milk
($/kg)	($/kg/1.0%)			
.1102	.0132	56.02	55.91 (.998)[c]	53.40 (.953)
	.0154	56.02	55.92 (.998)	52.34 (.934)
	.0176	56.21	56.11 (.998)	51.29 (.913)
.1213	.0132	61.70	61.59 (.998)	59.37 (.962)
	.0154	61.59	61.48 (.998)	58.31 (.947)
	.0176	61.66	61.55 (.998)	57.26 (.929)
.1323	.0132	67.44	67.31 (.998)	65.34 (.969)
	.0154	67.24	67.11 (.998)	64.28 (.956)
	.0176	67.20	67.07 (.998)	63.23 (.941)

[a] Selection intensity = 1.
[b] Price/kilogram of milk at 3.5% test.
[c] Selection efficiency relative to the quadratic index (R.S.E.).

TABLE 3. Selection index weights for the linear index and their relative size.

Economic values		Weights		$\frac{b_m\sigma_{p_m}}{b_t\sigma_{p_t}}$
Milk[a]	Differential	Milk (b_m)	Test (b_t)	
($/kg)	($/kg/ 1.0%)			
.1102	.0132	.1689	123.42	5.52
	.0154	.1693	148.89	4.59
	.0176	.1696	174.37	3.93
.1213	.0132	.1852	120.48	6.21
	.0154	.1857	145.95	5.14
	.0176	.1861	171.43	4.39
.1323	.0132	.2018	117.54	6.94
	.0154	.2022	143.01	5.71
	.0176	.2029	168.48	4.86

[a] Price/kilogram of milk at 3.5% test.

2. *Changes in index values.*

Economic merit index values for sires depend on the means for milk and test and on economic values (Table 4). The sire that is high for milk but low for test has a higher index value than the sire that is lower for milk but higher for test, in a situation in which the mean for milk is fairly low, the mean for test is fairly high, and the differential is $.0132/kilogram/1.0%. This result is not unexpected, but it is worth noting that the economic merit index values can be determined for any situation and the best sires can be chosen accordingly.

The relative importance of changes in means and economic values is indicated by the correlations between sire economic merit index values under various situations (Table 5). These correlations are based on the index values for 65 Holstein sires in the September 1967 Artificial Insemination Sire Summary prepared at Cornell University. The correlations between economic merit index values at different combinations of means are high for both differentials. The correlations between index values at different differentials are also high for all combinations of means. Changes in means appear to be potentially as important as changes in differentials.

The importance of changes in means and differentials in determining economic merit index values can be examined by comparing the expected progress in economic merit, if some other mean or differential than the one in the model of merit is used in the index. The expected progress from using a $.0176/kilogram/1.0% differential in the simplified index when the model of merit includes a $.0132/kilogram/1.0% differential is 97% of the expected progress from the use of a $.0132/kilogram/1.0% differential in the simplified index (for milk valued at $.1102/kilogram at present Holstein means). The loss in economic merit from the use of this incorrect differential would be about $.35 per cow per year and would be cumulative over years. Changes in means that resulted in lower correlations between sire index values would result in correspondingly greater economic loss.

Changes in economic values and means can be taken into consideration by using the simplified index with the appropriate means and economic values. The appropriate means could be those for some population in some geographical area or for some single herd considered to be the population of interest. The best mating for an individual cow could also be determined, using estimates of the genetic values for milk and test for that cow as the means in the sire index.

The index value for the future offspring of some mating can be found as:

$$I_o = I_s + I_d/2,$$

in which I_o, I_s, and I_d are the index values for economic merit of the offspring, sire, and dam, respectively. In terms of economic values, means, and index values for milk and test I_o can be written as:

TABLE 4. Economic merit index values as calculated by the simplified index (II) for two sires at two combinations[a] of means of milk production and test for two test differentials with milk at a base test of 3.5% worth $0.1102/kilogram.

Sire	Milk index	Test index	Differential $.0132/kg/1.0%		Differential $.0176/kg/1.0%	
	(kg)	(%)	Combination 1	Combination 2	Combination 1	Combination 2
1	580	−0.15	51.20	46.00	50.90	40.00
2	270	+0.25	50	55.30	58.80	63.90

[a] Combination 1 is a mean of 5,900 kg for milk production and a mean of 3.90% for test. Combination 2 is a mean of 7,700 kg for milk production and a mean of 3.30% for test.

TABLE 5. Correlations between sire economic merit index values at several combinations of mean levels of milk and milk fat percentage.[a]

Mean levels		5,440 kg			6,350 kg			8,160 kg		
Milk	Test	3.30%	3.60%	3.90%	3.30%	3.60%	3.90%	3.30%	3.60%	3.90%
(kg)	(%)									
5,440	3.30	.994	1.000	1.000	.999	.999	1.000	.988	.991	.993
	3.60	1.000	.995	1.000	.998	.999	.999	.987	.989	.991
	3.90	.999	1.000	.996	.997	.998	.999	.985	.988	.990
6,350	3.30	.998	.996	.994	.992	1.000	1.000	.995	.996	.997
	3.60	.999	.998	.996	1.000	.993	1.000	.993	.995	.996
	3.90	1.000	.999	.998	.999	1.000	.994	.992	.994	.995
8,160	3.30	.978	.974	.970	.990	.987	.983	.987	1.000	.999
	3.60	.984	.980	.976	.994	.991	.988	1.000	.989	1.000
	3.90	.988	.985	.982	.997	.994	.992	.998	1.000	.990

[a] Correlations of index values at different combinations of mean levels of milk production and test for a differential of \$.0132/kilogram/1.0% above the diagonal and for a differential of \$.0176/kilogram/1.0% below the diagonal and at the two differentials for each combination of levels on the diagonal.

$$I_o = (a_1 + a_2\mu_t)I_{ms} + a_2\mu_m I_{ts} + a_2 I_{ms} I_{ts} + (a_1 + a_2\mu_t)I_{md} + a_2\mu_m I_{td} + a_2 I_{md} I_{td},$$

in which

I_{ms}, I_{ts}, I_{md}, and I_{td} are index values for milk and test for the sire and milk and test for the dam, respectively (based on information on either one or both of the traits),

a_1 and a_2 are the economic values for milk and test as described previously, and μ_m and μ_t are the appropriate mean levels for milk and test, determined as in the preceding discussion.

This equation can be rearranged as:

$$I_o = (a_1 + a_2\mu_t)\left[\frac{I_{ms} + I_{md}}{2}\right] + a_2\mu_m \left[\frac{I_{ts} + I_{td}}{2}\right] + a_2 \left[\frac{I_{ms}I_{ts} + I_{md}I_{td}}{2}\right].$$

A product of index values for milk and test is thus involved in indexing for economic merit, although this term does not contribute greatly to I_o.

Conclusions

Relative expected genetic progress in economic merit is nearly equal for a quadratic, a simplified, a linear, and a restricted index for evaluation of dairy sires. Selection of sires for milk production only would not be as efficient as selection by the simplified index in increasing economic merit for any of the economic values considered. Relative efficiency increases, however, as the value of milk increases at an unchanging test differential.

Changes in sire economic merit index values can occur with changes in means of milk and test and in economic values, even though correlations between index values are high. The simplified index (II) appears to be a useful index to evaluate sires in any situation, because index values for milk and test can be used with any means for production and test and any economic values to determine sire index values for economic merit.

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