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Two Views Of Quantif1cation

Charles W. Sayward
University of Nebraska-Lincoln, csayward1@unl.edu

TWO VIEWS OF QUANTIFICATION

CHARLES W. SAYWARD

Department of Philosophy
University of Nebraska-Lincoln 68588

In this paper I discuss the distinction between the referential and substitutional interpretations of one of the basic concepts of modern logic–the concept expressed by "for all x." I try to bring out what is at stake in choosing between the two. Finally, I argue for the referential interpretation on the grounds that the substitutional interpretation allows defective formulation of sound principles.

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One remarkable thing about modern logic is that the entire apparatus rests on just three basic concepts: the statement connective "not both . . . and ____" "for all x" ("(x)" for short) and "=". There is controversy regarding the second of these concepts. One view has it that

(1) A universally quantified statement *(x) A*, is true just in case every object satisfies A.

This is the referential view. Another view has it that

(2) A universally quantified statement, *(x) A*, is true just in case every substitutive instance of A is true.

This is the substitutional view.

You may think that the distinction between (1) and (2) is trivial. You would be wrong. Consider the innocuous claim

(3) $(Ex)\, x > 0$

"Something is greater than O." The quantifier *"(Ex)"* is defined as "not *(x)* not." This definition gives us these results. If you take (1) as your understanding of *"(x),"* then (3) is true just in case there exists at least one object greater than O. This means we have to admit the numbers greater than O into our ontology as actually existing! The mind boggles. If you take the substitutional view, all you have to say is that (3) is true just in case some sentence of the form "$x > 0$" is true. And you can say this without thereby admitting that numbers actually exist.

At this point, one probably favors (2). But the problem is that there are good reasons for not adopting (2). A major contribution to the study of formal languages has been made by Alfred Tarski. Tarski proposed that any definition of the set of true sentences in a language, L, must be such that, for any sentences x and S, if S gives the truth conditions of x in L, then

$$x \text{ is true in } L \equiv S$$

should be derivable from the definition. This has been referred to in the literature as convention T. The use of convention T has led to some really remarkable work in the study of formal systems. Recently, it has been shown that, in the case of languages at least as rich as elementary number theory, convention T cannot be satisfied using (2). It can be satisfied only if (1) is adopted.

The defender of (2) can say: So much the worse for convention T. We simply have the challenge now of finding something in its place.

My purpose in this paper is to raise doubts as to whether it is worthwhile even to try to meet this challenge. I shall argue that the use of (2) allows defective formulation or sound principles. It should be rejected on these grounds alone.

I take as a starting point an argument recently proposed by Snyder (1971). Snyder argues that the absurdity, "All meaningless sentences are both true and false," could be deduced from four very plausible principles, which he uses the following formulas to express.

(4) $\ulcorner p \urcorner$ is true $\equiv p$

(5) $\ulcorner p \urcorner$ is false $\equiv -p$

(6) $\ulcorner p \urcorner$ is true $\supset \ulcorner p \urcorner$ is meaningful

(7) $\ulcorner p \urcorner$ is false $\supset \ulcorner p \urcorner$ is meaningful.

The formulas tautologically imply

(8) $-\ulcorner p \urcorner$ is meaningful $\supset \cdot \ulcorner p \urcorner$ is true & $\ulcorner p \urcorner$ is false.

Letting the letter *"p"* range over sentences (8) is supposed to symbolize "All meaningless sentences are both true and false."

The basic problem with all this is traceable to the fact that '$\ulcorner p \urcorner$' means "the result of putting p for the letter '*p*' in '*p*.'" That is,

(9) $\ulcorner p \urcorner = p.$

Rewriting (4) and (5) in accordance with this fact yields

p is true $\equiv p$

p is false $\equiv -p$

which makes one doubt the interest of Snyder's deduction. Seen in this light, (4) and (5) have as instances notoriously ill-formed sentences (e.g., "Snow is white is true $\equiv$ snow is white"). Of what interest, then, is Snyder's point?

The interest comes out when we seek other verbalizations of the principles that (4)–(8) are supposed to express. Suppose we adopt the substitutional view of quantification. Then the expressions we seek are these:

(10) (p)("p" is a sentence $\supset$ · "p" is true $\equiv p)$

(11) (p)("p" is a sentence $\supset$ · "p" is false $\equiv -p)$

(12) (p) ("p" is a sentence & "p" is true · $\supset$ "p" is meaningful)

(13) (p) ("p" is a sentence & "p" is false · $\supset$ "p" is meaningful).

These sentences entail

(14) (p) ("p" is a sentence & $-$ "p" is meaningful · $\supset$ "p" is true & "p" is false)

which is our new expression of "All meaningless sentences are both true and false."

Just how interesting or paradoxical it is that (14) follows from (10)–(13) depends upon how plausible one finds (10)–(13) to be. The latter set of sentences is certainly defective in some respect since it entails a sentence that is not true. Two options are open at this point. First, one might agree that (10)–(13) correctly express the relevant principles concerning meaningfulness, truth, and falsity; the defectiveness of (10)–(13) would then be traceable to these principles. Second, one might disagree that (10)–(13) correctly express the relevant principles; the defectiveness of (10–(13) would then be traceable to the fact that we have used the substitution interpretation of the quantifiers in trying to express these principles.

I think the second explanation is the better one. To support this contention, I shall argue that plausible expressions of the four principles can be obtained using the referential interpretation of the quantifiers, and that, so expressed, the principles do not have the absurd result that all meaningless sentences are both true and false.

The expression of the first principle is this:

(x) (x is a sentence $\supset$ the result of putting x for the letter "x" in " 'x' is true $\equiv x$" is a true sentence).

Using quasi-quotation, this can be expressed less cumbersomely:

(15) (x) (x is a sentence $\supset \ulcorner$ "x" is true $\equiv x \urcorner$ is a true sentence).

The rest of the principles are expressed as follows:

(16) (x) (x is a sentence $\supset \ulcorner$"x" is false $\equiv -x \urcorner$ is a true sentence).

(17) (x) (x is a sentence & x is true · $\supset x$ is meaningful).

(18) (x) (x is a sentence & x is false · $\supset x$ is meaningful).

It is a simple matter to prove that

(19) (x) (x is a sentence & $-$ x is meaningful · $\supset$. x is true & x is false).

does not follow from (15)–(18).

REFERENCE

Snyder, A. 1971. "On Semantic Interpretations of Truth Functions." Phil. Stu. 22:34-36.