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THE IMPROBABILITY OF INDUCTIVE LOGIC

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Various conceptions of the nature of logic are considered, leading to the view that to have a logic is to have a method of calculating the rationality of certain beliefs, or systems of beliefs, on the basis of syntactic properties of sentences. The possibility of developing an inductive logic is then considered in the light of this view. It is concluded that the prospects for developing a complete inductive logic are not good.

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INTRODUCTION

In some arguments, premises and conclusions are so related that if the former are true, so, necessarily, are the latter. Arguments having this property are said to be *valid*; those which lack it are said to be *invalid.* Some invalid arguments are worthless, but others are not. Among the latter are many arguments used by scientists, such as the arguments by which laws are inferred from their instances. These arguments, one wants to say, do not guarantee the truth of their conclusions, but they nevertheless make them more probable. Though not valid, they are *inductively strong.* Thus there arises the idea of an inductive logic, a logic which would provide a method for determining inductive strength, just as deductive logic provides a method for determining validity.

An inductive logic, if one could be developed, would give insight into both the nature and the grounds of scientific inference. The principles of such a logic would be the principles in accordance with which scientific reasoning proceeds, and showing such principles to be *logical* would leave little doubt as to their justifiability. The motives for developing an inductive logic are thus clear. What is less clear is that such a logic is actually possible. In recent years there has been heated controversy on this point, with philosophers such as Carnap and Hempel defending inductive logic, and other philosophers, such as Popper, claiming that there can be no such thing. The purpose of this paper is to consider whether anything worthy of the name "inductive logic" could ever be developed.

THE NATURE OF LOGIC

An inductive logic would presumably do for the notion of inductive strength, or degree of confirmation, what a deductive logic does for the notion of validity. But what *does* a deductive logic do for the notion of validity? What *is* an inductive logic, anyway?

According to one traditional account, logic is the study of *inference, i.e.,* the study of the principles in accordance with which people pass from one belief to another. On this account, inductive logic would be the study of those inferences—*e.g.,* from the belief that a theory's predictions have invariably come out true to the belief that the theory itself is true—which qualify as inductive. Now surely there *are* such inferences; and, surely they could be made an object of a study. Thus on this conception of logic the possibility of there being such a thing as an inductive logic would be beyond question.

Of course there are some deductively invalid inferences which also are not *in*ductive. The conclusion that a man is in for a bad day may be inferred from the premise that his moon is in Aquarius, or from the premise that the local witch doctor has put a curse on him. If logic is the study of the principles in accordance with which people make inferences, then these inferences, too, will fall within the purview of logic. In addition to deductive logic and inductive logic, there will be religious logic, voodoo logic, astrological logic, and so on for any type of inference that may be named. Perhaps it is this line of thought which has led some writers to maintain that there is a special logic of religious or ethical reasoning. Such writers have tended to welcome this conclusion, apparently on the ground that it makes religion and ethics seem every bit as rational as science.

Unfortunately, there is no stopping with ethics and religion. Suppose there is announced the development of a new branch of logic, *gin*ductive logic, which is the study of those inferences that people will accept after consuming a pint of gin. Clearly there *is* a class of such inferences, and clearly they could be made an object of a study; so if logic is the study of inference, gindeductive logic must be considered one of the branches of logic.

The absurdity of this conclusion shows that the conception of logic now before us is too broad. Logic involves the study of inferences, but inferences can be studied without doing logic. Narrower conceptions of the subject must be considered. One such narrower conception, developed by Quine (1966), exploits the notion of a logical word. First, a class of logical words is specified: "not," "or," "and," "all," "some," "=" (in the sense of identity) would be typical candidates for membership. The logical truths are then defined as those truths which invariably turn into other truths when any of their expressions other than a logical word is replaced by a grammatically acceptable substitute. Logic is then defined as the study of logical truths. A pair of sentences will stand in the relation of logical consequence just in case their corresponding conditional is logically true (Quine, 1966:80-81 and 110).

This conception rejects the credentials of voodoo logic and astrological logic, while accepting those of the classical logic of quantification with identity. Moreover it can accommodate such developments as modal logic and deontic logic by adding expressions such as "it is necessary that" and "it is obligatory that" to the list of logical words. (Quine did not deny that modal logic is *logic*; only that it admits of an acceptable interpretation.)

Can this conception be so extended as to count inductive logic as a logic? At first sight, the answer may seem to be "yes." "Probably" or "it is probable that" could be added to the list of logical words. Truths containing only this expression and other logical words essentially would then count as logical. By analogy with the relation of logical consequence, the relation of confirmation between sentences P and Q would be defined as logical truth of the conditional $\ulcorner \text{Probably } (P \supset Q) \urcorner$.

There is, however, a problem here. $\ulcorner P \supset Q \urcorner$ is equivalent to $\ulcorner {\sim}P \vee Q \urcorner$. Now clearly the probability of a disjunction is always at least as high as the probability of its least probable disjunct. Thus the probability of $\ulcorner {\sim}P \vee Q \urcorner$, and hence of $\ulcorner P \supset Q \urcorner$ will be high whenever the probability of P is low, regardless of the probability of Q; similarly, the probability of $\ulcorner {\sim}P \vee Q \urcorner$ and $\ulcorner P \supset Q \urcorner$ will be high whenever the probability of Q is high. $\ulcorner \text{Probably } (P \supset Q) \urcorner$ will therefore be true whenever the probability of P is low or the probability of Q is high, the probability of the other component notwithstanding.

Now consider what happens when P is a statement to which our inductive logic assigns low probability. This means that high probability must be assigned to $\ulcorner P \supset Q \urcorner$. Thus $\ulcorner \text{Probably } (P \supset Q) \urcorner$ will count as a logical truth when "probably" is reckoned among the logical words. Hence, if confirmation of Q by P is equated with logical truth of $\ulcorner \text{Probably } (P \supset Q) \urcorner$, it will follow that P confirms Q. Similarly, every statement to which our inductive logic assigns low probability will, on the conception of logic now before us, confirm any statement whatsoever. By similar reasoning it could be shown that every statement to which our inductive logic assigns high probability will, on that same conception of logic, be confirmed by any statement whatsoever.

The problem is that inductive logic, as standardly developed, disallows these results. On the standard conception, a statement P confirms a statement Q just in case the probability of Q on the assumption that P is true is higher than it would be without that assumption (Haack, 1978:17; Skyrms, 1975:9-11). Mere high probability of Q or low probability of P is never sufficient, of itself, to insure that P confirms Q.

In contrast to standard deductive logic, which equates consequence with logical truth of the corresponding conditional, standard inductive logic does not equate confirmation of Q by P with logical truth of $\ulcorner \text{Probably } (P \supset Q) \urcorner$. Thus standard inductive logic does not, after all, count as logic under Quine's conception.

Should this result be thought of as an objection to Quine or as an objection to inductive logic? Consider an example. Suppose it is proposed, as a principle of inductive logic, that if a statement e is of the form $\ulcorner \Phi\, a \;\&\; \Psi\, \beta \urcorner$ and a statement h is of the form $\ulcorner (\gamma)\ (\Phi\, \gamma \supset \Psi\, \gamma) \urcorner$, then e confirms h. It might be doubted, of course, that this principle is a *correct* one, but what does not seem to be in doubt is that the principle *is* a principle of inductive *logic*. On Quine's conception of logic, establishing that the principle is a logical principle involves finding some logical truth to which it corresponds. Since there does not seem to *be* such a logical truth, and since, in any event, it is implausible to make the logical character of the principle depend upon finding such a truth, Quine's conception of logic as the study of so-called logical truths must be rejected as too narrow, in that it excludes from the realm of logic principles which clearly belong there.

If the class of "logical truths" is too narrow to encompass the principles involved in the study of logic, it is natural to suppose that logic could be characterized as the study of some broader class of truths, say the class of analytic truths. This class would surely number among its members any principle of confirmation worthy of being included in an inductive logic. Unfortunately, however, it also encompasses principles which do not seem to belong to logic of any kind. "All horses are animals" is presumably analytic, yet it does not, and should not, figure as a principle of any logic. The class of analytic truths is thus seen to be too broad to be the subject-matter of logic.

This conclusion applies equally to the classes of *a priori* truths and of necessary truths. "All horses are animals" is *a priori* and necessary as well as analytic, and thus serves to show that logic cannot be identified as the study of the truths in any one of these three traditional categories.

Another familiar proposal concerning the nature of logic holds that logic is essentially metalinguistic: To be a logical principle is to be a principle *about* expressions. As it stands this is too broad: "'Caesar crossed the Rubicon' is true" would count as a principle of logic. Still, this account seems to be on the right track. Not all metalinguistic principles are logical, but all logical principles *are* metalinguistic. Logical principles can be distinguished from other metalinguistic principles by the following characteristics: First, logical principles provide the basis for making certain calculations (*e.g.,* the calculations involved in constructing a truth table); in this respect they obviously differ from many other metalinguistic principles. Second, logical principles effect, or at least aim to effect, a reduction of non-syntactic properties of sentences to syntactic properties of sentences. The calculations licensed by the principles are based on syntactic properties of sentences, but the purpose of the calculations is to show that sentences have certain non-syntactic properties (*e.g.,* truth) or stand in certain non-syntactic relations (*e.g.,* implication). With what kinds of non-syntactic properties is logic concerned? The answer seems to be that logic is concerned with those properties and relations that pertain to the *rationality* of our beliefs. Notions such as validity and consistency, for example, are tied to our conception of rationality in that a person who believed a statement known to be inconsistent, or who accepted the premises of an argument known to be valid while rejecting its conclusion, would be regarded as irrational.

These points together lead to the following characterization: A logic is a system for determining the rationality of certain beliefs, or combinations of beliefs, by means of calculations based on syntactic properties of sentences. This account fits syllogistic and quantificational logic, as well as such further developments as modal logic and deontic logic. It also excludes such deviations as astrological logic, because failure to accept the principles of astrology does not signify irrationality.

With regard to inductive logic the account gives reasonable results without begging any question. It properly accords the probability calculus the status of a partial inductive logic, in that failure to assign probabilities in accordance with the calculus is a sufficient reason for ascribing irrationality. On the other hand, the question whether there could be an inductive logic going beyond the probability calculus is left unresolved. The notions of rationality which inductive logic strives to explicate—probability, confirmation, evidence—do indeed form part of our conception of rationality. A person whose beliefs violate the laws of probability, or are not well confirmed, or are not in accord with the evidence can justly be called irrational. If, therefore, there *were* a system of principles on the basis of which questions of probability, confirmation, and evidence could be settled by calculation, such a system *would* be a logic. On the other hand, no such system of principles presently exists. According to the conception of logic now before us, therefore, no complete system of inductive logic presently exists.

All of this—that the probability calculus is a partial inductive logic, that a general method for calculating probabilities and confirmation relations on the basis of syntactic features would be an inductive logic, that there is not now a complete inductive logic—accords with pre-analytic intuition. Our account of logic thus seems to give reasonable results for inductive logic, as well as for other types of logic. This account may, therefore, be taken as providing at least some clarification of the concept of logic, and the prospects for inductive logic may be considered within the framework it provides.

THE PROSPECTS FOR INDUCTIVE LOGIC

If logic is what we have said it is, what is to be made of the question "Is inductive logic possible?" If logic in general is concerned with the rationality of beliefs, inductive logic is concerned with the rationality of beliefs arrived at inductively, with the rationality of what might be called inductively determined beliefs. In accordance with what has now become fairly common usage, "induction" can be understood to cover not only inductive generalization, but any non-deductive inference in which the premises are observation-sentences. An inductive logic would then be concerned with the rationality of such inferences. It would provide a way of calculating, on the basis of syntactical features of sentences, the extent to which a hypothesis is confirmed (made probable by) certain observations. The question whether such a logic is possible is the question whether a method of calculating degrees of confirmation on the basis of syntactical features can be found.

A partial answer to this question is provided by the existence of the probability calculus. This calculus provides a method for determining the probabilities of truth-functional compounds, given a knowledge of the probabilities of their components, together with a knowledge of their syntactical (specifically truth-functional) structure. It thus constitutes a partial inductive logic. Such a logic is thus not only possible but actual.

It is clear, however, that much more could be hoped for, namely, a logic which allows determination of the probabilities of statements on the basis of more fine-grained syntactical analysis, *e.g.,* analysis of quantificational structure; or, a logic which elucidates the relation between theory and observation, *e.g.,* between scientific laws and their instances. Whether a logic fulfilling these aspirations can be constructed will now be considered.

One argument against the possibility of an expanded inductive logic could appeal to the allegedly holistic character of confirmation. According to the holistic doctrine, championed in recent years by Quine, it is large blocks of theory, rather than isolated sentences, which are confirmed by experience (Quine, 1953:41). If this view is correct, then, it might be argued, no inductive logic could ever be developed, for, if hypotheses do not even stand in relations of confirmation to observation reports, then, obviously, there could be no such thing as determining whether such relations hold on the basis of calculations.

The holistic view could, however, be treated as showing not that inductive logic must be abandoned, but that it has been misconceived. If inductive logic was supposed to codify a relation of confirmation between isolated statements and their evidence, then of course, holism would foreclose the possibility of inductive logic. But why must this conception of inductive logic be accepted? Presumably inductive logic is directed at the confirmation-relation. It need not be said, however, that observation-reports and isolated hypotheses are what stand in this relation. Leaving the door open for holism, we can be noncommital as to the nature of the statements that get confirmed. If the holist is correct, it is theories that get confirmed, but a theory can be regarded simply as a conjunction of statements. The question whether there could be an inductive logic would then turn on the possibility of codifying the confirmation of theories by observation reports. An affirmative answer to this question is obviously consistent with holism.

In its insistence that whole theories rather than isolated hypotheses are confirmed, holism affirms the complexity of one side of the confirmation-relation. Barker (1965:226), on the other hand, affirmed the complexity of the other side of the relation, the evidential side. His view, in effect, was that all inductive arguments are enthememattic. In advancing an inductive argument, a speaker claims that his conclusion is probable in the light of everything that we know. Thus,

> In an inductive argument the explicitly stated premises are only a tiny part, although usually the most noteworthy part, of the indefinitely vast amount of information about the world upon which the conclusion depends. Each bit of this known but unstated information has a bearing upon whether the argument is inductively valid. But where reasoning involves relevant premises so rich that we cannot be sure even of stating them completely, we cannot expect to be able to impute to the premises and conclusion any specific logical form in virtue of which the argument would be valid or invalid.

When Barker spoke of logical form, he had in mind what we have called syntax; and when he spoke of an argument as being inductively valid, he meant that its premises confirm its conclusion. He thus maintained, in effect, that there cannot be syntactical criteria for confirmation.

Judging from the passage just quoted, it would appear that Barker's argument for this view turns on his claim that the premises of an inductive argument are so rich that "we cannot be sure even of stating them completely." In short, inability to state the premises of inductive arguments entails the impossibility of inductive logic.

By way of assessing this entailment, imagine that there were a way of calculating, on the basis of syntactic properties of these sentences, whether e confirms h for every pair of sentences e and h. Surely under these circumstances an inductive logic would be available. Yet at the same time, a way of determining what statements should be included among the suppressed premises of a given inductive argument might very well be lacking. How is it that we could have an inductive logic and yet still not be able to assess certain inductive arguments? An answer emerges if the analogous situation is considered with regard to deductive logic, where there actually is a calculus of the kind described. In this case there is clearly a logic, yet in the case of many enthememattic arguments, it is not possible to say how the missing premises should be supplied. Such cases show that the *applicability* of logic is sometimes limited by inability to state all premises of certain real-life arguments. Similarly, in the case of inductive logic, inability to state the premises of inductive arguments would show not that inductive logic is impossible, but only that its applicability is limited. To have a logic, it is sufficient to be able to calculate certain relationships between sentences. A logic does not tell us how to read others' minds, or to generate a list of everything we know.

A more serious obstacle to the construction of an inductive logic emerges from Goodman's (1955:17–27 and 66–81) work on the problem of lawlikeness. As was seen above, an inductive logic involves syntactic criteria of confirmation. One type of confirmation with which an inductive logician would be particularly anxious to deal is the confirmation of scientific laws by their instances. At first it might seem that in this case a syntactic criterion is near to hand. It could be said that a statement of the form "All Fs are Gs" is confirmed by a statement of the form "A is F and a is G." But this is where

Goodman's problem comes in. As long as we think in terms of statements like "All crows are black," our criterion of confirmation seems all right. Unfortunately not all statements of the form "All Fs are Gs" are confirmed by their instances. "All the men attending this football game are third sons," for example, is not confirmed by "Jones is attending this football game and is a third son." Generalizations confirmed by their instances are termed "lawlike." The problem with our proposed syntactic criterion of confirmation is that it would incorrectly accord confirmation by their instances to non-lawlike generalizations. In order to treat the question of the confirmation of scientific laws by their instances in our inductive logic, it would be necessary to distinguish, on syntactical grounds, between lawlike and non-lawlike generalizations. Unfortunately, this does not seem to be possible. "All crows are black," and "All the men attending this game are third sons" are both of the form "All Fs are Gs." They are, in short, syntactically equivalent. The problem of the confirmation of scientific laws by their instances thus appears to be beyond the scope of any inductive logic. Hence, insofar as inductive logic is conceived as aspiring to deal with this problem, it must be concluded that such a logic is probably impossible.

Does it then follow that induction, and therefore science itself, is illogical or irrational? No. Construction of any logic is begun with a stock of inferences regarded as rationally justified, and another stock of inferences regarded as not rationally justified. Some syntactic relation between sentences which is coextensive with the relation of rational justification for the sentences in question is then sought. To succeed in this effort is to invent a logic. It would be a mistake, however, to view the construction of the logic as demonstrating the rationality of the inferences in question. On the contrary: the rationality of the inferences is presupposed in the construction of the logic. The question whether a logic can be constructed for a given class of inferences is not the question whether those inferences are rational; it is the question whether their rationality has a syntactic correlate. The conclusion that inductive inferences cannot be treated formally should shake our faith in the rationality of science no more than Gödel's incompleteness theorem has shaken our faith in the rationality of mathematics.

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