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Preface for Millard Beatty

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Preface

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In Recognition of the 90th Birthday of Millard Beatty

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Professor Beatty has contributed a wide variety of research papers and book articles on topics in finite elasticity, continuum mechanics and classical mechanics, including some fundamental experimental work. His works are clear and informative and expose a didactic quality. In the following, we briefly touch upon some of the highlights of his research involvement throughout the years.

1 Synopsis of the Works of Millard Beatty

Beatty's earliest work concerns elastic stability theory in which he studied the uniqueness of solutions to problems of small static and dynamic motions superimposed on a possibly finitely deformed equilibrium configuration of an arbitrary hyperelastic body under dead load forces and subject to a zero moment condition for boundary conditions of place and tractions. He showed that the conditions of dead load stability of the underlying equilibrium state yielded a uniqueness theorem for the superimposed problem, and guaranteed that dynamically superimposed stress waves are admissible. He then showed that similar results follow within couple stress theory, based upon the introduction of a generalized Betti reciprocal theorem for small, superimposed deformations. Within this context, he then developed an extended form of Castigliano's classical theorems which relate the body surface point displacement and rotation to the corresponding surface force and couple acting thereon. This led him to a generalization of the theory to study certain hyperelastic Cosserat continua with constrained material directors and to explore hyperelastic materials of grade 2. Applications of these and various other aspects of finite elasticity and elastic stability theory were reported in several following articles, including the estimation of safe loads in special cases, specific results for incompressible materials, and problems of instability of a normally

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loaded half-space. Specific examples included the study of a fiber-reinforced thick slab and a thick-walled cylindrical tube under axial loading.

Several experimental studies in collaboration with Beatty's students explored the compression buckling of rectangular and circular rubber rods and cylinders, small, superimposed transverse vibrations of a finitely stretched rubber cord, axial buckling of thick-walled, helically wound composite paper tubes, stress-softening of rubber materials, and the inflation of balloons. James Bell's influence in the art of experimental mechanics is evident throughout these works as is exhibited in Beatty's 1977 review article "Elastic Stability of Rubber Bodies in Compression", in the ASME volume "Finite Elasticity". In addition to a thorough critique of the empirical shape factor theory prevalent in engineering design treatments, Beatty presented new data that confirmed that the critical axial load for rubber columns that fail by bending with large deflections is the classical Southwell critical load.

Beatty continued with his formulation of the theory of stress-softening in both compressible and incompressible isotropic materials, and he illustrated this theory with examples. He developed a general constitutive equation to account for the stress-softening Mullins effect in uniaxial experiments and studied its effects on the transverse vibration of a rubber string and membrane, and on the inflation of balloons. The familiar effect of preconditioning by stretching on the primary inflation of a balloon was characterized in his study of the Mullins effect in equibiaxial deformation. Applications in pure shear, simple shear, triaxial stretch of rubberlike and biological materials, and the torsion, extension and inflation of cylindrical tubes were described. He then introduced a phenomenological model containing an exponential front factor softening parameter in the constitutive equation for isotropic, incompressible rubberlike materials and showed that the results for the Arruda-Boyce and James-Guth parent material models compared favorably with simple extension data by Johnson and by Mullins and Tobin. The appreciable error that may be introduced into test data due to the reinforcement effect of a strain gage applied to a soft material was studied numerically and a method to reduce it was described.

Beatty then turned his attention to the interesting subject of universal relations for isotropic materials, including those with certain inextensible fibers, and universal quasi-static motions for certain viscoelastic materials of differential type. The physical properties of rubberlike, cellulosic, polymeric, and biological materials were studied, which led him to introduce the "Poisson function" to better characterize the finite simple extension of isotropic, elastic solids in terms of the data that was reported for several rubber-like materials. Extending the scope of this work, Beatty went on to develop necessary and sufficient conditions on the form of the strain energy function for a general compressible, isotropic hyperelastic material in order that antiplane, axisymmetric and helical shear deformations may be produced by application of surface tractions alone, a problem related to one that his thesis adviser, Jerry Ericksen, introduced years earlier; results are illustrated in several examples. He discovered that while these controllable axisymmetric, antiplane shear deformations are possible in a particular Hadamard material, they cannot be sustained in a certain Blatz-Ko compressible, foamed polyurethane rubber material. This is the same polyurethane material for which Beatty later showed that unstable equilibrium states exist in the small amplitude superimposed horizontal motion of a load supported symmetrically by rubber springs.

Further investigations of Beatty included: his work "On the Foundation Principles of General Classical Mechanics" in which the fundamental principles are deduced from Noll's principle of frame indifference of mechanical power; his development of the Arruda-Boyce constitutive equation for elastomers from a full network molecular based model; his derivation of the constitutive equation for the phenomenological Gent model of rubber elasticity as a Padé approximant within a general constitutive framework for limited elastic molecular based materials; and his demonstration of universal relations that follow from the constitutive equation for isotropic, nonlinear elastic materials. Along the way, Beatty also developed a number of special mathematical results which dealt with: determinants having complex conjugate elements; general integrals arising in plane finite elasticity; and an integral identity applicable to continuum mechanics. His 1987 review article on "Topics in Finite Elasticity: Hyperelasticity of Rubbers, Elastomers and Biological Tissues, with Examples," updated in 1996 as a book chapter titled "Introduction to Nonlinear Elasticity," continues to be widely referenced as a basic source for readers in nonlinear elasticity. In addition, Beatty's "Seven Lectures on Finite Elasticity" in the CISM volume "Topics in Finite Elasticity" presented at Udine in 2000 is an informative supplement to this article with its inclusion of additional applications to cellular solids, controlled deformations of compressible hyperelastic materials, vibrations of a load on viscohyperelastic shear mounts, hyperelastic Bell materials, and stress-softening in rubberlike materials.

Beatty showed broad interest in, and contributed to a basic understanding of the nonlinear dynamical behavior of finite amplitude free and driven oscillations of a load supported by various rubberlike supports, including members exhibiting stress-softening and limited extensibility. In related work, he reported on problems of small superimposed oscillation together with extensive experiments on the small transverse vibration of stretched rubber cords. His analysis regarding the transverse impact of a hyperelastic stretched string showed remarkably good agreement with the high velocity impact tests of Haddow and Wagner. In 1983, Beatty published the interesting paper "Finite Amplitude Oscillations of a Simple Rubber Support System" the exact solution to the longitudinal free vibration problem for a neo-Hookean oscillator and therein the general oscillatory motion and the period in terms of a complementary Heuman lambda function. The result delivered simple upper and lower bounds on the period expressed in terms of this special function. This seminal work led to exact and approximate solutions to other physically based dynamical nonlinear elasticity problems, including the finite amplitude oscillations of a load supported by simple shear mounts, the coupled oscillations of a load under shearing and combined torsion and extension, the radial oscillations of limited elastic thick-walled tubes, the finite amplitude oscillations of a load supported by a highly elastic tubular shear spring, and the small radial oscillations superimposed on the finite inflation of spherical and cylindrical shells. In following related studies Beatty included viscohyperelastic, stress-softening, limited elastic, and driven vibrational effects.

In a series of fundamental papers, the first of which, Part 1, appeared in 1992 in this journal, Beatty, together with Michael Hayes, developed the constitutive theory of an isotropic Bell constrained material. They were motivated by the empirical internal kinematic material constraint that was grounded in the great body of large deformation experiments by James Bell on a variety of annealed polycrystalline metals within the context of finite strain plasticity. Part 1 studied the homogeneous deformation of isotropic, hyperelastic Bell constrained materials. In this constrained theory, the kinematics alone showed that isochoric deformations are not possible - the material volume must decrease in every deformation. Still, they found that every such material behaves in small deformations like an incompressible material whose Poisson function in every simple extension, however large, has the constant value 1/2. The theory delivered some remarkable results that seem to validate some empirical results of Bell for plasticity. Overall, where comparisons were possible, the results of Beatty and Hayes concurred with Bell's empirical conclusions. The book chapter "Hyperelastic Bell Materials: retrospection, experiment, theory" published in 2001 provides an overview of Bell's experimental results and the aforementioned theoretical work.

Part 2 of the joint work on Bell constrained materials studied nonhomogeneous deformations. This included the bending and stretching of a rectangular block; the bending, extension and azimuthal shearing of an annular wedge; and the radial deformation of a spherical shell and membrane. A surprising result confirmed that eversion of a Bell constrained thick spherical shell is possible only when the wall thickness is less than its inside radius. Development of the theory of small superimposed deformations and waves and the propagation of small amplitude torsional waves followed in Part 3. For an infinitesimal twist superimposed on a pure homogeneous uniaxial extension or compression of a uniform prism, they found that the torsional couple may vanish in compression but not in extension if the prism is neither a solid circular cylinder nor a circular cylindrical tube. While this three-part work concluded their joint research on Bell materials, later, in 2005, Beatty and Hayes got together again and arranged for the production, and served as co-editors of, the special volume: "Mechanics and Mathematics of Crystals – Selected Papers of J. L. Ericksen".

Beatty continued with his investigation of the instability of a Bell constrained thick- and thin-walled cylindrical tube subject to averaged compressive dead loads over its ends, a thick plate under compression and a half-space under biaxial loading. His estimate for the critical buckling load of thin plates derived from the thick plate analysis for hyperelastic Bell constrained materials were found to coincide with the corresponding classical relation. Based upon the Hadamard criterion ensuring that propagating incremental wave speeds are real, Beatty studied the stability of a Bell constrained material subject to a pure homogeneous deformation. Among other things, he showed that the natural state is the only materially stable equilibrium state of a Bell constrained material under pressure loading.

Finally, Professor Beatty's passion for teaching is exhibited in a number of his didactic related articles on classical mechanics and is emphasized further in two volumes on the principles of engineering mechanics. The book "Dynamics – The Analysis of Motion" presents many illustrative examples that emphasize the predictive value of the principles of mechanics. Notable examples include the discovery of the planet Neptune, the U.S. Navy torpedo failures in World War II, the Foucault's pendulum experiments, the erratic ballistics in the 1914 battle of the Falkland Islands, and the general law of mutual internal action as derived by Noll using the principle of material frame indifference from continuum mechanics. This book contains a foundation for studies in advanced dynamics including Euler's formulation of the general principles of mechanics for all material bodies and Lagrange's invariant formulation of the equations of classical mechanics. Each chapter concludes with a list of annotated references for expanded study.

On every appropriate happening, Millard Beatty has expressed profound gratitude to his thesis advisor and principal teacher Jerry Ericksen in nonlinear elasticity, stability theory, and tensor analysis, to Clifford Truesdell his thesis reader and teacher in the non-linear field theories of mechanics, and to James Bell his senior research advisor and motivational teacher in vibrations, nonlinear mechanics, solid mechanics and experimental solid mechanics. Other influential scholars whose classes or lectures he attended when he was a Graduate Student at the Johns Hopkins University are Bernard Coleman, Oscar Dillon, Walter Noll, Owen Phillips, Ronald Rivlin, James Serrin, and Richard Toupin. In all, a most remarkable experience.

Beatty has noted on occasion that much of his research and didactic activity would not have been possible without the collaboration of some remarkable colleagues, talented postdoctoral fellows, and exceptional former students both undergraduate and graduate, especially their contribution to his various experimental programs. His list of publications exemplifies these mutual and cooperative accomplishments. But beyond that, Millard served as a mentor to many faculty and young scientists, and was a much beloved and respected chairman who always stood ready to protect the academic excellence in the Engineering Mechanics department at the University of Nebraska, Lincoln.

2 Principal Journal and Conference Publications of Millard Beatty

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